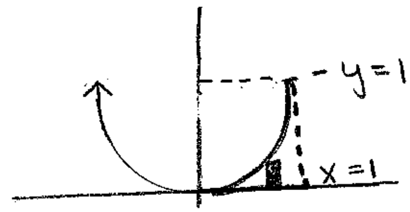
Math 2471 Calculus III – Test 2 Name KEY

This test consists of 5 questions with the point value for each problems is in the parenthesis. Please show all of the necessary steps.

1. (10) Reverse the order of integration and integrate showing your steps.

$$\int_0^1 \int_{\sqrt{y}}^1 \frac{3\,dx\,dy}{1+x^3}$$



$$\int_0^1 \int_0^{x^2} \frac{3}{1+x^3}\,dy\,dx$$

$$= \int_0^1 \frac{3y}{1+x^3}\Big|_0^{x^2} dx = \int_0^1 \frac{3x^2}{1+x^3}\,dx$$ let $u = 1+x^3$, $du = 3x^2dx$

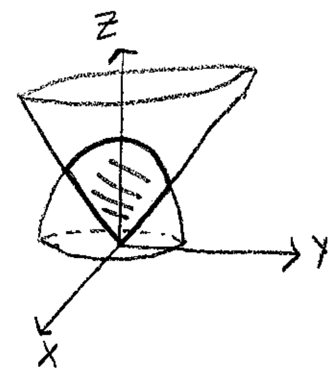
$$= \int u^{-1}du = \ln(1+x^3)\Big|_0^1 = \ln(2)-\ln(1)$$

$$= \ln(2)$$

2. (10) Find the volume bound by the paraboloid $z = 2 - x^2 - y^2$ and the cone $z = \sqrt{x^2+y^2}$

$z = 2 - r^2$ & $z = r$

Intersection
$2 - r^2 = r$
$r^2 + r - 2$
$(r+2)(r-1)$
$-2, 1$
solution



$$\int_0^{2\pi} \int_0^1 (2 - r^2 - r)\,r\,dr\,d\theta$$

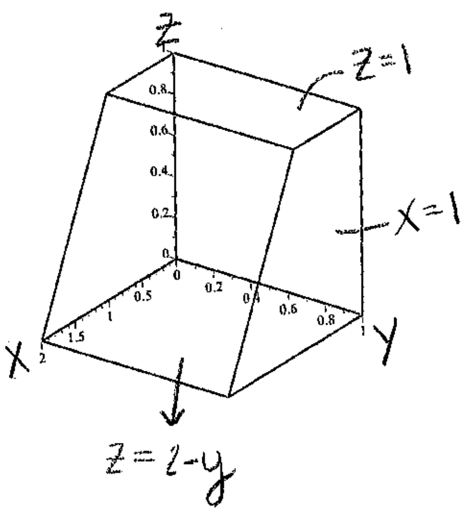
$$= \int_0^{2\pi} \int_0^1 (2r - r^3 - r^2)\,dr\,d\theta = \int_0^{2\pi} \left(r^2 - \frac{1}{4}r^4 - \frac{1}{3}r^3\right)\Big|_0^1 d\theta$$

$$= \int_0^{2\pi} 1 - 1/4 - 1/3\,d\theta = \int_0^{2\pi} \frac{5}{12}\,d\theta = \frac{5}{12}\theta\Big|_0^{2\pi} = \frac{5\pi}{6}$$

should be x

3. (8) Find the limits of integration of the triple integral where the volume is bound by $x=0, x=1, y=0, z=0, z=1,$ and $z=2-y$. The picture of the volume is below.

$$\iiint_V f(x,y,z)\,dV$$

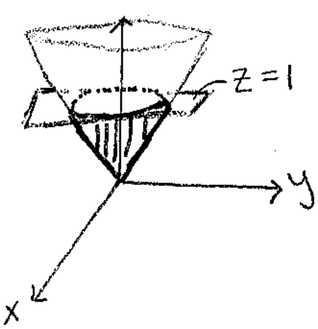


Front view along y direction

$$\int_0^1\int_0^1\int_0^{2-z} f(x,y,z)\,dx\,dz\,dy$$

4. (10) Find the limits of integration of the triple integral in both cylindrical and spherical coordinates where the volume is bound by the cone $z=\sqrt{x^2+y^2}$ and the plane $z=1$. DO NOT EVALUATE.

$$\iiint_V xyz\,dV$$



Cylindrical: $\int_0^{2\pi}\int_0^1\int_r^1 r^2\cos\theta\sin\theta\, z\,dz\,r\,dr\,d\theta$

Spherical: $\int_0^{2\pi}\int_0^{\pi/4}\int_0^{1/\cos\phi} \rho\cos\theta\sin\phi\cdot\rho\sin\theta\sin\phi\cdot\rho\cos\phi\cdot\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

if $z=0$ $\rho\cos\theta=0$

$z=1$ $\rho\cos\theta=1$

5. (12) Classify the critical points for $z = x^2y - x^2 + y^2 - 18y$.

$f_x = 2xy - 2x = 2x(y-1)$

$f_y = x^2 + 2y - 18$

means $x=0$ or $y=1$

if $x=0$: $2y-18=0$
$2y=18$
$y=9$

if $y=1$: $x^2-16=0$
$(x+4)(x-4)=0$
$x=-4,4$

The critical points are: $(0,9)(-4,1)(4,1)$

$f_{xx} = 2y-2$

$f_{xy} = 2x$

$f_{yy} = 2$

To find classification:
$f_{xx}f_{yy} - f_{xy}^2$

@Point $(0,9)$
$[2(9)-2](2)-(0)^2$
$16(2)$
$32>0$

$\Delta f > 0$ & $f_{xx} > 0$
thus $(0,9)$ is a min

@Point $(4,1)$
$[2(1)-2](2)-8^2$
$0(2)-64$
$-64<0$
$\Delta f<0$ thus $(4,1)$ is a Saddle

@Point $(-4,1)$
$[2(1)-2](2)-(-8)^2$
$0(2)-64$
$-64<0$
$\Delta f<0$ thus $(-4,1)$ is a saddle