Chapter 10 Helper Sheet

10-1

The outcomes for a specified event are called *favorable outcomes*. When all outcomes are equally likely, the **theoretical probability** of the event can be found using the following.

$$\text{Theoretical probability} = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

The probability of event A is written as $P(A)$.

Probability of the Complement of an Event

The probability of the complement of event A is

$P(\overline{A}) = 1 - P(A)$.

$$\text{Experimental probability} = \frac{\text{Number of successes}}{\text{Number of trials}}$$

Fundamental Counting Principle: If one event can occur in m ways and another event can occur in n ways, then the number of ways that both events can occur is $m \cdot n$. The Fundamental Counting Principle can be extended to three or more events.

10-2

Two events are **independent events** when the occurrence of one event does not affect the occurrence of the other event.

Probability of Independent Events

Words Two events A and B are independent events if and only if the probability that both events occur is the product of the probabilities of the events.

Symbols $P(A \text{ and } B) = P(A) \cdot P(B)$

Probability of Dependent Events

Words If two events A and B are dependent events, then the probability that both events occur is the product of the probability of the first event and the conditional probability of the second event given the first event.

Symbols $P(A \text{ and } B) = P(A) \cdot P(B|A)$

Combinations

Formula The number of combinations of n objects taken r at a time, where $r \le n$, is given by

$${}_nC_r = \frac{n!}{(n-r)! \cdot r!}.$$

10-4

Probability of Compound Events

If A and B are any two events, then the probability of A or B is

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

If A and B are disjoint events, then the probability of A or B is

$P(A \text{ or } B) = P(A) + P(B)$.

Permutations

Formulas	Examples
The number of permutations of n objects is given by ${}_nP_n = n!$.	The number of permutations of 4 objects is ${}_4P_4 = 4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$.
The number of permutations of n objects taken r at a time, where $r \le n$, is given by ${}_nP_r = \frac{n!}{(n-r)!}$.	The number of permutations of 4 objects taken 2 at a time is ${}_4P_2 = \frac{4!}{(4-2)!} = \frac{4 \cdot 3 \cdot \cancel{2!}}{\cancel{2!}} = 12$.

10-5

A **permutation** is an arrangement of objects in which order is important. For instance, the 6 possible permutations of the letters A, B, and C are shown.

ABC ACB BAC BCA CAB CBA

A **combination** is a selection of objects in which order is *not* important.

Binomial Expansions

In Section 4.2, you used Pascal's Triangle to find binomial expansions. The table shows that the coefficients in the expansion of $(a + b)^n$ correspond to combinations.

	n	Pascal's Triangle as Numbers	Pascal's Triangle as Combinations	Binomial Expansion
0th row	0	1	${}_0C_0$	$(a+b)^0 = 1$
1st row	1	1 1	${}_1C_0 \quad {}_1C_1$	$(a+b)^1 = 1a + 1b$
2nd row	2	1 2 1	${}_2C_0 \quad {}_2C_1 \quad {}_2C_2$	$(a+b)^2 = 1a^2 + 2ab + 1b^2$
3rd row	3	1 3 3 1	${}_3C_0 \quad {}_3C_1 \quad {}_3C_2 \quad {}_3C_3$	$(a+b)^3 = 1a^3 + 3a^2b + 3ab^2 + 1b^3$

The results in the table are generalized in the **Binomial Theorem**.

Core Concept

The Binomial Theorem

For any positive integer n, the binomial expansion of $(a + b)^n$ is

$$(a + b)^n = {}_nC_0\, a^n b^0 + {}_nC_1\, a^{n-1} b^1 + {}_nC_2\, a^{n-2} b^2 + \cdots + {}_nC_n\, a^0 b^n.$$

Notice that each term in the expansion of $(a + b)^n$ has the form ${}_nC_r\, a^{n-r} b^r$, where r is an integer from 0 to n.

10-3

A **two-way table** is a frequency table that displays data collected from one source that belong to two different categories.

Each entry in the table is called a **joint frequency**. The sums of the rows and columns are called **marginal frequencies**, which you will find in Example 1.

		Attendance	
		Attending	Not Attending
Class	Freshman	25	44
	Sophomore	80	32

joint frequency

A **random variable** is a variable whose value is determined by the outcomes of a probability experiment. For example, when you roll a six-sided die, you can define a random variable x that represents the number showing on the die. So, the possible values of x are 1, 2, 3, 4, 5, and 6. For every random variable, a *probability distribution* can be defined.

Core Concept

Probability Distributions

A **probability distribution** is a function that gives the probability of each possible value of a random variable. The sum of all the probabilities in a probability distribution must equal 1.

Probability Distribution for Rolling a Six-Sided Die						
x	1	2	3	4	5	6
$P(x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

One type of probability distribution is a **binomial distribution**. A binomial distribution shows the probabilities of the outcomes of a *binomial experiment*.

Core Concept

Binomial Experiments

A **binomial experiment** meets the following conditions.

- There are n independent trials.
- Each trial has only two possible outcomes: success and failure.
- The probability of success is the same for each trial. This probability is denoted by p. The probability of failure is $1 - p$.

For a binomial experiment, the probability of exactly k successes in n trials is

$$P(k \text{ successes}) = {}_nC_k\, p^k (1 - p)^{n-k}.$$

Remember that:

$${}_nC_r = \frac{n!}{(n-r)! \cdot r!}.$$