CDI Parametric Study

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Analysis of the Small-Deformation Connecting Rod: Slider Crank Mechanism

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OVERVIEW

Many mechanical applications require the use of both rigid and deformable bodies, making the integration of flexible body dynamics into multibody system (MBS) codes significant. The *Floating Frame of Reference* (FFR) formulation is an approximation method for modeling small-deformation flexible bodies that may undergo large translations and rotations in MBS analyses. In order to define a unique displacement field using the FFR formulation, the rigid body modes associated with the element shape functions must be eliminated. The process of eliminating the rigid body modes is achieved by imposing a set of *reference conditions*, which are not a unique set, yet should be chosen to be consistent with the kinematic constraints imposed on the boundaries of the deformable body. The fact that the set of boundary conditions is not unique, coupled with the responsibility of the system designer to choose this set, can lead to error. A benchmark example for the use of the FFR approach and the use of such reference conditions in MBS analysis is the small-deformation flexible connecting rod of a slider-crank mechanism; this benchmark example is presented in this document.

It is shown that the FFR deformation results converge to a semi-analytical solution as the connecting rod becomes divided into more elements, thereby more evenly distributing the inertia. It is also shown that, in some cases, *free-free* reference conditions may also be used if the rigid body modes are manually identified and removed.

DEFORMABLE CONNECTING ROD

A deformable body has infinite degrees of freedom, which is not appropriate for the computer methods used in MBS analyses. Therefore, using the finite element approach to produce an approximate solution using a finite number of coordinates, the deformable connecting rod is defined using a discrete number of elements. The procedure for designing the connecting rod is shown in Fig. 1.

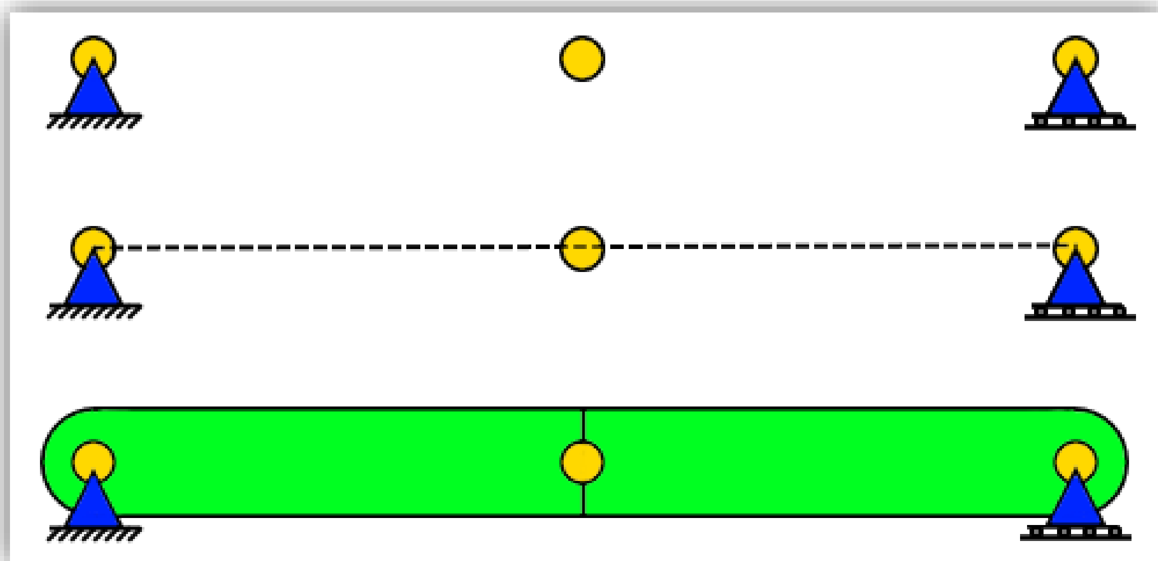

Figure 1. *The design process of a discretized deformable body. First, the nodal locations as well as the reference conditions are specified (TOP). The next step is to specify the element connectivity, or which nodes make up each element (MIDDLE). Finally, the element geometric and inertial properties are specified in order to fully define the deformable body (BOTTOM).*

The connecting rod used in this investigation consists of two Euler-Bernoulli (EB) beam elements (Shabana, 2013). The EB beam element has two nodes, or points at which elements are connected, that are located on either end of the element. The first step in defining the connecting rod is to specify the location of the required nodes. The connecting rod used in this investigation has length $L^3 = 3.048 \times 10^{-1}\,\text{m}$, and the nodal locations are given in table 1 for a two-element discretization.

Node Number	Symbolic	Value (m)
1	$(-L/2, 0)$	-1.524×10^{-1}
2	$(0, 0)$	0
3	$(L/2, 0)$	1.524×10^{-1}

Table 1. *The nodal locations of the connecting rod used in this investigation, discretized into two elements, and defined with respect to a connecting rod coordinate system that is located at the midpoint of the two end-nodes.*

The *reference conditions* of the rod can be specified at this time, which will be discussed in the following section. The next step is to define the element connectivity, or which nodes make up each element. For the example shown in Fig. 1, the element connectivity is given in table 2.

Element Number	First Node Number	Second Node Number
1	1	2
2	2	3

Table 2. *The element connectivity specification for the connecting rod used in this investigation, discretized into two elements.*

Finally, the geometric and inertial properties of the elements are specified to fully define the connecting rod. For the two-element discretization of the connecting rod used in this investigation, the element properties are given in table 3.

Element Number	Mass (kg)	Cross-Sectional Area (m^2)	Second Moment of Area (m^4)	Length (m)	Modulus of Elasticity (N/m^2)
1	3.781×10^{-2}	3.167×10^{-5}	7.981×10^{-11}	1.524×10^{-1}	2.068×10^{11}
2	3.781×10^{-2}	3.167×10^{-5}	7.981×10^{-11}	1.524×10^{-1}	2.068×10^{11}

Table 3. *Element geometric and inertial properties for the connecting rod used in this investigation, discretized into two elements*

REFERENCE CONDITIONS & THE ELIMINATION OF RIGID BODY MODES

The shape functions of the EB element, or the interpolation functions that describe the element kinematics between the two nodes, include rigid body modes of "deformation". The rigid body modes define the case of zero strain during arbitrary motion, and are shown in Fig. 2.

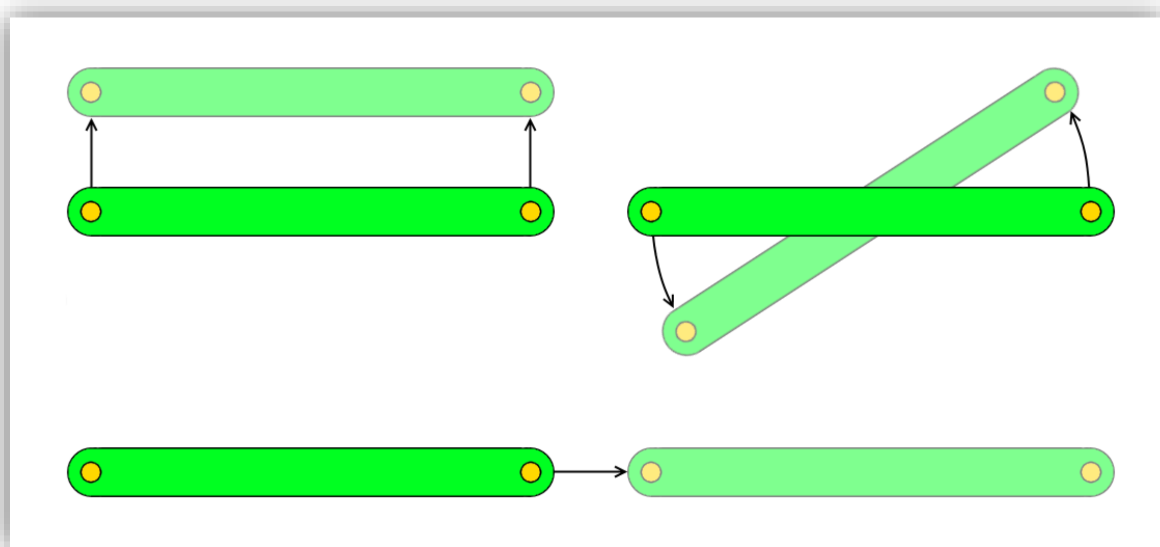

Figure 2. *Rigid Body Modes of the Connecting Rod.*

However, these rigid body modes must be eliminated in order to define a unique displacement field with respect to the body reference. This can be done by imposing a set of *reference conditions*, which define the nature of the deformable body axes, in the form of linear kinematic constraints. For the case of a connecting rod with revolute joints at each end, simply-supported reference conditions will provide the correct deformation shape, as shown in Fig. 3.

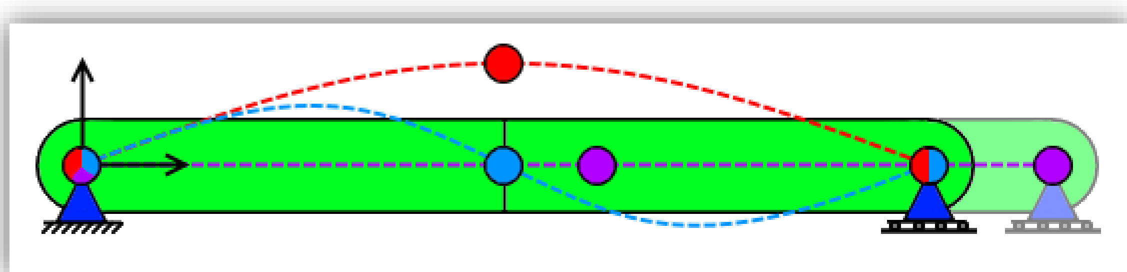

Figure 3. *Examples of the connecting rod modes of deformation, using simply-supported reference conditions. Two bending modes (RED/BLUE) and one axial mode (PURPLE).*

In this case, the connecting rod coordinate system may be located at the left node. However, the lateral axis of this coordinate system will always be in the direction of the other end-node, rather than along the beam centerline, and therefore is not considered “fixed” to the connecting rod: it is this nature of the body reference that gives rise to the name *Floating Frame of Reference*. The deformation shapes provided by the simply-supported reference conditions must match the kinematic constraints imposed by the revolute joints and the boundaries of the connecting rod in order to give the correct deformation results.

SIMULATION AND RESULTS

The following simulation of a slider-crank mechanism with a small-deformation flexible connecting rod was carried out using SIGMA/SAMS, a multi-purpose multibody dynamics simulation and analysis tool. The model considered is shown in Fig. 4.

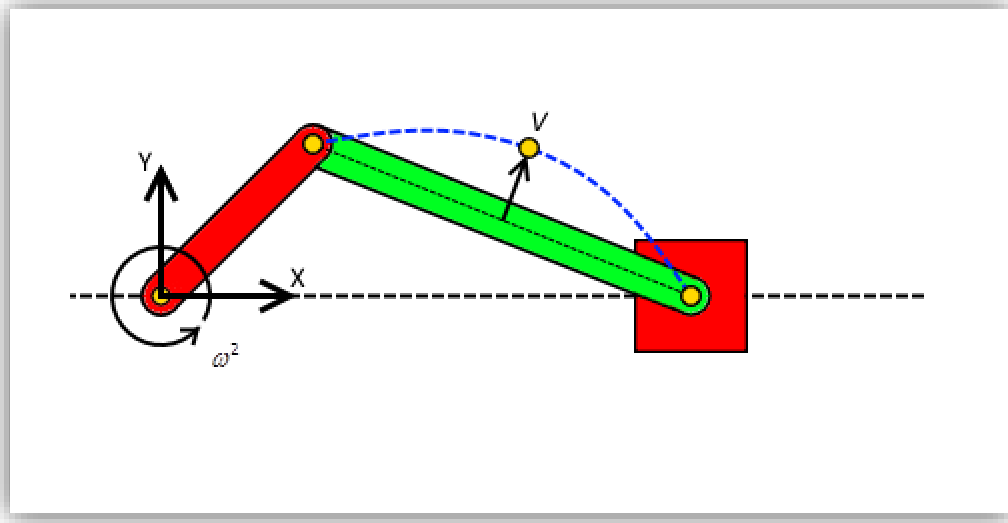

Figure 4. *Slider-crank mechanism*

The crankshaft has length $L^2 = 1.524 \times 10^{-1}$ m and a constant angular velocity $\omega^2 = 124.8$ rad/s. The slider-block is considered massless and is constrained to translate along the X-axis only. The deformation V is measured perpendicular from the axis that connects the two end-nodes of the connecting rod, and is normalized by the length of the connecting rod. The dimensionless deflection V is given in Fig. 5. The connecting rod is then discretized into four, and then six, elements and the simulations is ran again. The results from the finer-meshed simulations are also presented in Fig. 5. To validate the solution, a semi-analytical solution provided by (Chu and Pan, 1975) is also plotted in Fig. 5. The simulations converge to the analytical solution as the element mesh becomes finer: this is because a lumped-mass scheme is used in the inertia calculations. As the mass of the discretized connecting rod becomes more distributed, the inertia becomes more representative of a continuous body.

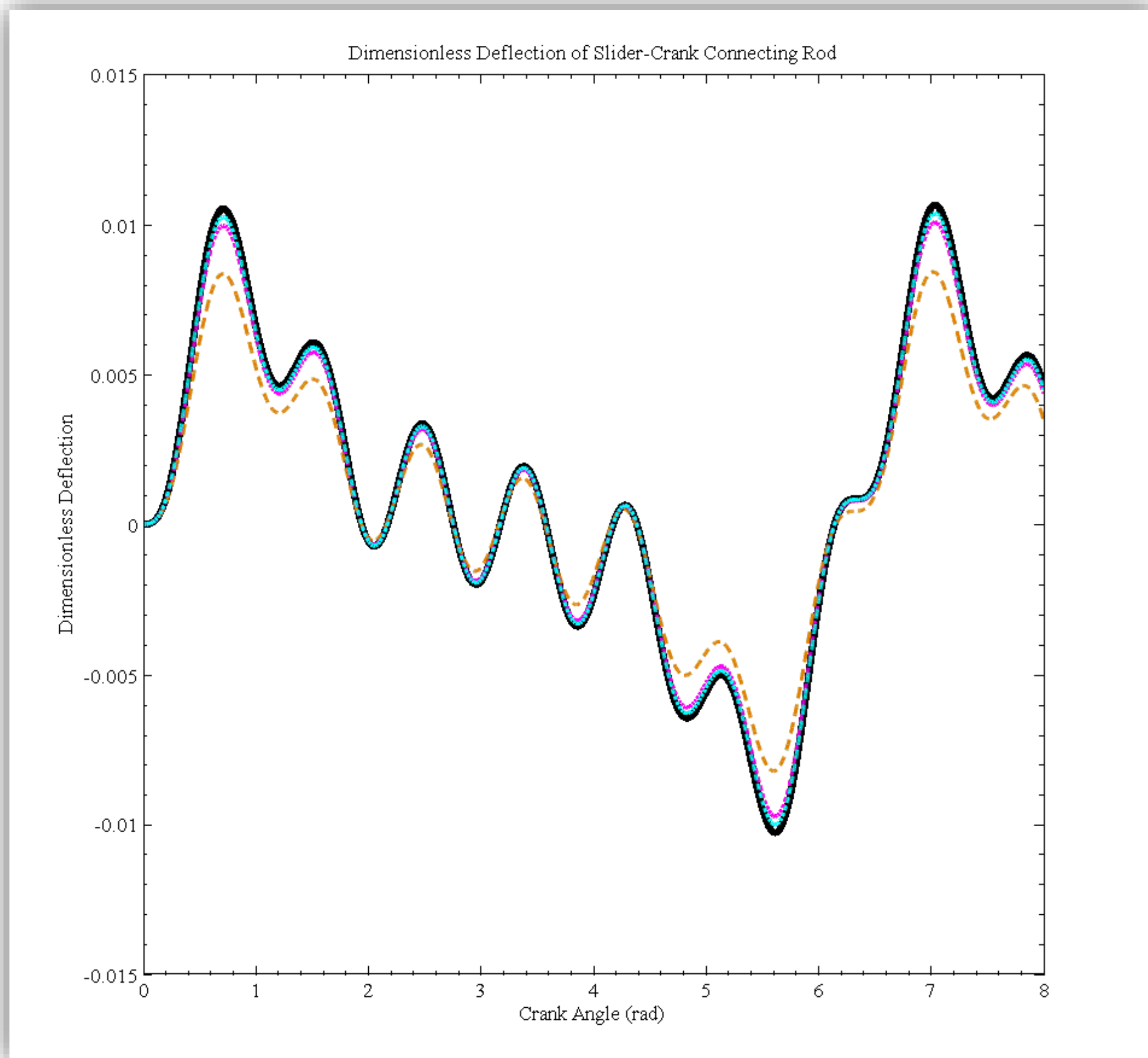

Figure 5. *Dimensionless deflection of the middle node of the connecting rod for two-element (ORANGE), four-element (MAGENTA), and six-element (CYAN) discretization. A semi-analytical provided by (Chu and Pan, 1975) is shown for validation (BLACK).*

Free-Free Reference Conditions

It is possible in some cases to, rather than specify reference conditions that remove the rigid body modes, manually identify and remove the rigid body modes that result from the singularity of the stiffness matrix. In this case, the body reference will be located at the instantaneous center of mass and therefore care should be taken in measuring the deformation with respect to the body reference. For the two-element discretization of the connecting rod, the dimensionless deflection of the connecting rod using free-free reference conditions is shown in Fig. 6.

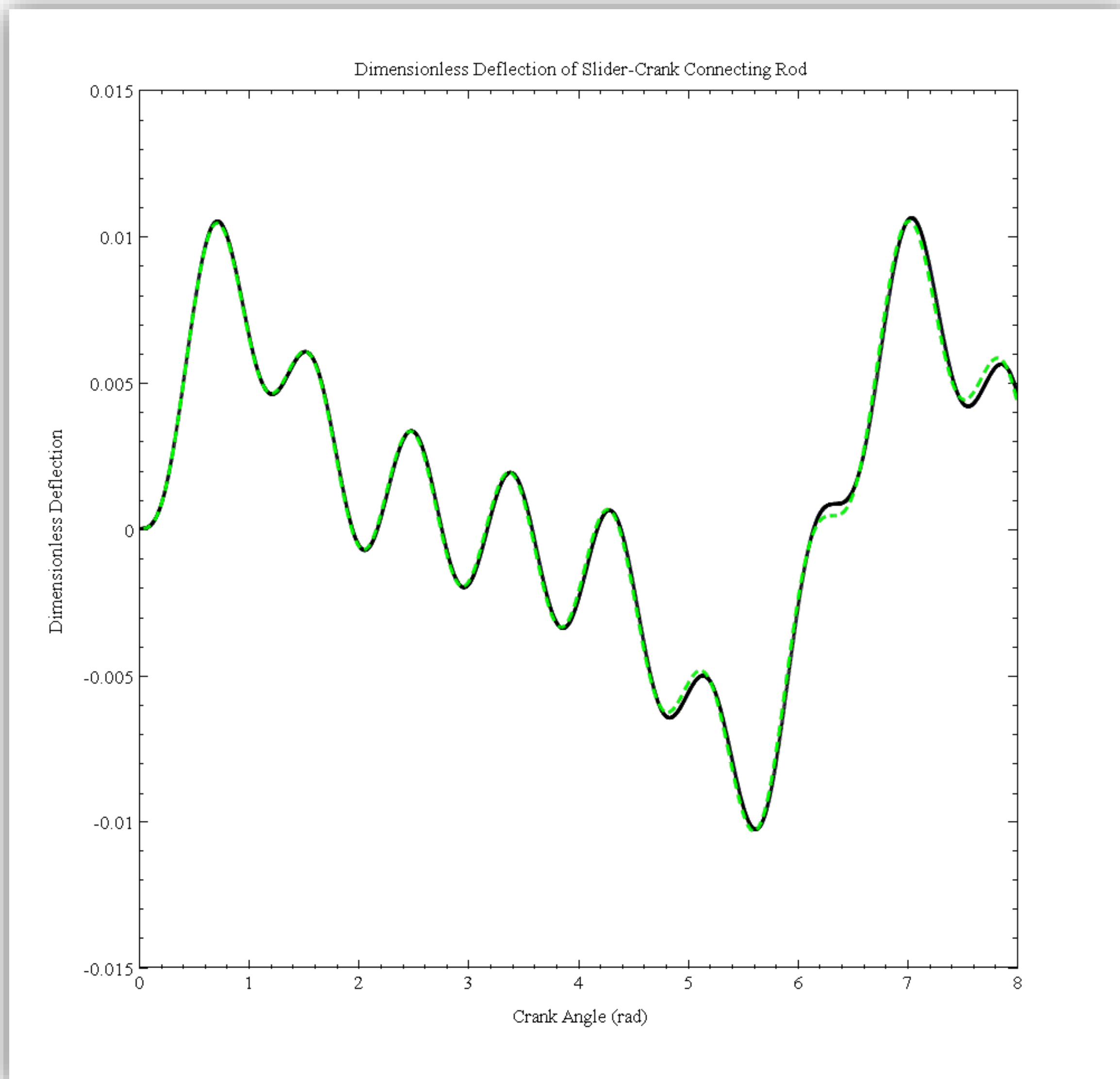

Figure 6. *Dimensionless deflection of the middle-node of the two-element discretization of the connecting rod (GREEN). The semi-analytical solution provided by (Chu and Pan, 1975) is given for validation (BLACK).*

DISCUSSION

Each mode shape included in the modal transformation matrix has an associated frequency. Table 4 lists the frequencies associated with the 6 modes of deformation included in the analysis of the two-element connecting rod for both the simply-supported as well as free-free reference conditions.

Mode Number	Simply Supported Ref. Conditions	Free-Free Ref. Conditions
1	1.385×10^{2} Hz	3.133×10^{2} Hz
2	6.123×10^{2} Hz	9.807×10^{2} Hz
3	1.539×10^{3} Hz	2.452×10^{3} Hz
4	2.806×10^{3} Hz	3.918×10^{3} Hz
5	4.323×10^{3} Hz	9.294×10^{3} Hz
6	1.510×10^{4} Hz	1.859×10^{4} Hz

Table 4. *Frequencies associated with the connecting rod modes of deformation, for the two-element discretization.*

The first three modes of deformation are shown in Fig. 7 in order to demonstrate that the modes of deformation, although calculated using different sets of reference conditions, can represent the same deformation of the connecting rod.

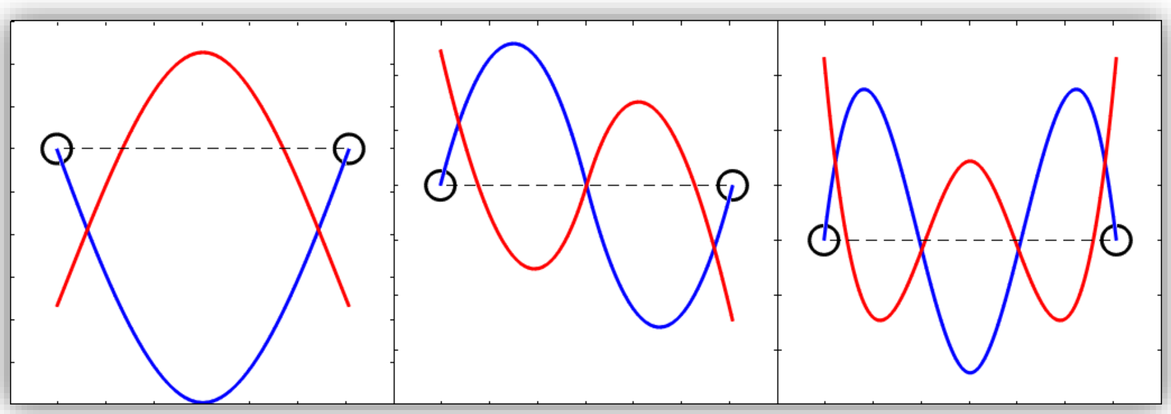

Figure 7. *First three modes of deformation for the connecting rod, discretized into two elements, for simply-supported (BLUE) and free-free (RED) reference conditions.*

Taking both table 4 and fig. 7 into account, it is clear that sets of modes that represent the same deformation shapes can have different frequencies. Caution must be taken then when setting a "cutoff frequency" in MBS codes without first understanding the reference conditions of the elastic bodies, the modal deformation shapes, as well as the frequencies associated with the selected modes.

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FINAL REMARKS

The *Floating Frame of Reference (FFR)* formulation was used in this investigation in order to approximate the deflection of a small-deformation flexible slider-crank connecting rod discretized into a number of elements. The process of defining a set of *reference conditions* in order to remove the rigid body modes inherited from the finite element approach was also discussed. Numerical results were presented for simply supported reference conditions and show that the approximate solution converges to a semi-analytical solution as the beam is meshed. The concept of *free-free* reference conditions was also discussed and results were presented. Finally, the modal frequencies were compared for the two sets of reference conditions used in this study, and modal deformation shapes were presented. It was shown that deformation modes, although representing the same deformation shape, can have different frequencies, and therefore caution must be taken when selecting a cutoff frequency in MBS codes.

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James O’Shea is a Research and Development Engineer for Computational Dynamics Inc. and has interests in the Floating Frame of Reference Formulation for the modeling of small-deformation elastic systems, railroad vehicle system dynamics and stability, as well as interface development. James earned a Bachelor’s Degree in Mechanical Engineering from the University of Illinois (UIC) at Chicago and is currently working on his Ph.D. at UIC as well.

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Shabana, A. A. (2013). *Dynamics of multibody systems*. Cambridge university press.

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