

1 March 2013

## A Higgs Family: Part I

by

### I. Motivation

The discovery of a good candidate for a “vanilla” Higgs boson reopens more sharply the issue of how many other spinless bosons inhabit the Higgs sector. In this note, we conjecture that there are a lot more, which are related to the symmetry associated with transformation of the three generations of quarks and leptons amongst each other.

The basis for this study goes all the way back to the Gell-Mann current-algebra ideas of the 1960's, when he introduced the concept of flavor. Back then the chiral  $U(3) \times U(3)$  flavor group was generated by the algebra of charges associated with the vector and axial currents defined in terms of the three known quarks  $u, d, s$ . Since then, the flavor group has grown to a chiral  $U(6) \times U(6)$ , generated by the six quarks  $u, c, t, d, s, b$ . Within this group can be found the electroweak group  $SU(2) \times U(1)$ . In addition, within the flavor group one can also find a chiral  $U(3) \times U(3)$  family group, generated by the charges associated with  $u, c, t$ . Electroweak symmetry guarantees that the  $d, s, b$  quarks also transform as a chiral family triplet.

However, nothing guarantees that the leptons fall into the same pattern. But they do. This suggests that the family symmetry possesses a kind of universality, perhaps to be eventually comprehended at or beyond the GUT level of consideration. But in any case, this universality feature does raise the question of whether the Higgs sector also contains family multiplets which are non-singlet. Just because the gluon, photon,  $W$ , and  $Z$  gauge degrees of freedom are family singlet, this does not imply the same for the Higgs degrees of freedom. And, given that “flavor mixing” (more accurately, “family mixing”) has so much to do with the mass matrices of quarks and leptons, which in turn have everything to do with the Higgs sector, it would seem very reasonable to entertain this possibility.

Exploring this option is the purpose of this note. If the Higgs sector is larger than the “vanilla”, family-singlet, complex electroweak doublet, what else might be out there? The basic hypothesis here will be that the Higgs sector transforms as a nonet under the chiral family group, as well as transforming as the usual complex doublet under the electroweak symmetry group. This implies that there are  $9 \times 4 = 36$  self-conjugate degrees of freedom in the Higgs sector. Since we observe 4 so far (the three Goldstone modes eaten by the  $W$  and  $Z$ , plus the state at 125 GeV), this means that there are 32 more to be found. In the usual way of classifying particles, this breaks down into 8 more positively charged particles (plus of course their electrically negative counterparts), 4 more self-conjugate neutral particles analogous to the Higgs, and 6 more neutral particles with nontrivial flavor quantum numbers (plus of course their 6 antiparticles).

The strategy which we here adopt will be to start with a description which accounts for only the third generation masses. Only later, will the second generation masses and mixings be addressed, still keeping the first generation massless. And only after some success at that level would the full problem, including CP violation effects, be addressed. In this note, it will be quite enough to simply consider the first of these three stages.

In what follows, we take the gaugeless limit. In that limit, the W and Z bosons become massless. Their transverse degrees of freedom decouple from the Higgs sector. The longitudinal degrees of freedom, which we denote as  $\vec{w}$ , are massless Goldstone modes and do not decouple.

In Section II we set up the formalism and consider the structure of the Higgs and Yukawa Lagrangians. In Section III we examine a highly symmetric limit of the spontaneous symmetry breaking pattern, one that turns out to be surprisingly realistic. Section IV generalizes the description, but still retaining family symmetry. In Section V, we have a first look at the phenomenological consequences. It turns out that the two-photon decay of the Higgs boson is sensitive to the existence of the other members of this Higgs family, and Section VI is devoted to this issue. In Section VII we have another look at the LHC discovery potential regarding the new states, and also look at what will be needed in order to take the next step---namely, to create a realistic phenomenology of second-generation masses and mixings.

## II. The Formalism

Left-handed quarks transform as a sextet under the  $SU(6) \times SU(6)$  chiral flavor group, as doublets under the electroweak group  $SU(2)$ , and as triplets under the chiral  $SU(3) \times SU(3)$  family group. These latter groups are subgroups of the flavor group. We order the quark sextet as (u, c, t, d, s, b) and write

$$[Q\bar{Q}] = \begin{pmatrix} u\bar{u} & u\bar{c} & u\bar{t} & u\bar{d} & u\bar{s} & u\bar{b} \\ c\bar{u} & c\bar{c} & c\bar{t} & c\bar{d} & c\bar{s} & c\bar{b} \\ t\bar{u} & t\bar{c} & t\bar{t} & t\bar{d} & t\bar{s} & t\bar{b} \\ \hline d\bar{u} & d\bar{c} & d\bar{t} & d\bar{d} & d\bar{s} & d\bar{b} \\ s\bar{u} & s\bar{c} & s\bar{t} & s\bar{d} & s\bar{s} & s\bar{b} \\ b\bar{u} & b\bar{c} & b\bar{t} & b\bar{d} & b\bar{s} & b\bar{b} \end{pmatrix}$$

With subscripts attached in an obvious way, this 6 x 6 array transforms under the chiral flavor group as follows:

$$[Q_L\bar{Q}_R] \rightarrow U_L [Q_L\bar{Q}_R] \bar{U}_R^{-1}$$

$$[Q_R Q_L] \equiv [Q_L\bar{Q}_R]^\dagger \rightarrow U_R [Q_R Q_L] \bar{U}_L^{-1}$$

The 6 x 6 unitary matrices  $U_L$  and  $U_R$  of course include the family and electroweak transformations. The latter are 3 x 3 block diagonal, while the former act only within the 3 x 3 family blocks.

The Higgs nonet  $\Phi$  is also described by a 6 x 6 matrix. We decompose it in 3 x 3 block form in a way that imitates the linear sigma-model description of the standard-model vanilla Higgs sector:

$$\Phi \equiv \frac{1}{\sqrt{2}} (H + i\vec{\tau} \cdot \vec{\omega}) = \frac{1}{\sqrt{2}} \begin{pmatrix} H + i\omega_3 & i\omega_1 + \omega_2 \\ i\omega_1 - \omega_2 & H - i\omega_3 \end{pmatrix} \quad \Phi \rightarrow U_L \Phi U_R^{-1}$$

The adjoint of this Higgs field is, just like in the simple vanilla model, not identical. But it is not independent either, provided the fields  $H$ ,  $\omega_1$ ,  $\omega_2$ , and  $\omega_3$  are described by 3 x 3 hermitian matrices:

$$\bar{\Phi} \equiv \frac{1}{\sqrt{2}} (H - i\vec{\tau} \cdot \vec{\omega}) = \frac{1}{\sqrt{2}} \begin{pmatrix} H + i\omega_3 & i\omega_1 + \omega_2 \\ i\omega_1 - \omega_2 & H - i\omega_3 \end{pmatrix} \quad \bar{\Phi} \rightarrow U_R \bar{\Phi} U_L^{-1}$$

The Yukawa coupling term in the Lagrangian can now be constructed:

$$\mathcal{L}_{\text{Yukawa}} = \frac{1}{v} \left\{ \text{Tr} \Phi M [Q_R \bar{Q}_L] + h.c. \right\}$$

Here the parameter  $v$  is introduced, so that the matrix  $M$  can eventually be interpretable as a (generalized) quark mass matrix. This matrix  $M$  must live in the right-handed sector only, and it cannot carry charge. Therefore it must be  $SU(2)_R$  diagonal:

$$M = \begin{pmatrix} M_{\text{up}} & 0 \\ 0 & M_{\text{down}} \end{pmatrix} = \left( \frac{1 + \tau_3}{2} \right) M_{\text{up}} + \left( \frac{1 - \tau_3}{2} \right) M_{\text{down}}$$

### III. Spontaneous Symmetry Breaking

We now turn to the structure of the Higgs potential itself. The most general form of that potential which is consistent with the family and electroweak symmetries contains many terms. We will not try for a general analysis ab initio, but restrict initial attention to an extremely simple ansatz. It is the product of a considerable amount of trial and error (especially error), and encapsulates the most important features of the slightly more complicated version presented in Section IV. Remarkably, it also seems to be quite realistic.

We write for the Higgs potential the following simple form:

$$\begin{aligned}
 V &= \frac{m^2}{2} \text{Tr} \bar{\Phi} \Phi + \frac{\lambda_0}{4} (\text{Tr} \bar{\Phi} \Phi)^2 - \frac{m_3^2}{2} \text{Tr} \bar{\Phi}_3^3 \Phi_3^3 \\
 &= \frac{m^2}{2} \text{Tr} (H - i\vec{t} \cdot \vec{\omega}) (H + i\vec{t} \cdot \vec{\omega}) + \frac{\lambda_0}{16} \left[ \text{Tr} (H - i\vec{t} \cdot \vec{\omega}) (H + i\vec{t} \cdot \vec{\omega}) \right]^2 - \frac{m_3^2}{2} \left[ (H_3^3)^2 + |\vec{\omega}_3^3|^2 \right] \\
 &= \frac{m^2}{2} \text{tr} (H^2 + |\vec{\omega}|^2) + \frac{\lambda_0}{4} \left[ \text{tr} (H^2 + |\vec{\omega}|^2) \right]^2 - \frac{m_3^2}{2} \left[ (H_3^3)^2 + |\vec{\omega}_3^3|^2 \right]
 \end{aligned}$$

Here the electroweak trace has been taken. The instruction  $\text{tr}$  acts on the  $3 \times 3$  hermitian matrices  $h$ ,  $w_1$ ,  $w_2$ ,  $w_3$ .

Note that we have included a rogue mass term which breaks family symmetry. But the remaining terms in the potential not only retain the full chiral flavor symmetry, but express an  $O(36)$  global symmetry. And, with  $m^2$  chosen to be positive, there is not yet any spontaneous symmetry breaking present at this stage. The rogue mass term explicitly breaks that symmetry down to  $O(32) \times O(4)$ . When  $m_3^2 > m^2$  (which we assume to be the case), spontaneous symmetry breaking does occur within the  $O(4)$  sector, leading to the usual "vanilla" Higgs phenomenology therein.

We now go through the details. We quite naturally assume the vev of the field  $\Phi$  has the form

$$\begin{aligned}
 \langle \Phi \rangle &= \frac{v}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} & H_3^3 &\equiv v + h_3^3 \\
 & & H_i^{\dot{a}} &= h_i^{\dot{a}} \text{ otherwise}
 \end{aligned}$$

Upon expanding things out, the form of the potential becomes

$$\begin{aligned}
 V &= \left[ (m^2 - m_3^2) \frac{v^2}{2} + \frac{\lambda_0 v^4}{4} \right] + (h_3^3 v) \left[ (m^2 - m_3^2) + \lambda_0 v^2 \right] \\
 &\quad + \lambda_0 v^2 (h_3^3)^2 + \frac{1}{2} \text{tr} (h^2 + |\vec{\omega}|^2) [m^2 + \lambda_0 v^2] - \frac{1}{2} m_3^2 \left[ (h_3^3)^2 + |\vec{\omega}_3^3|^2 \right] + \dots
 \end{aligned}$$

We have only kept the terms necessary to determine the vev and to determine the masses of the dynamical degrees of freedom.

The minimum condition is easy to work out.

$$m_3^2 - m^2 = \lambda_0 v^2$$

The value of the Higgs potential  $V$  at the minimum is

$$V_{\min} = \frac{1}{4}(m^2 - m_3^2)v^2 = -\frac{\lambda_0 v^4}{4}$$

With these results we turn to the classification of the masses of the bosons. The quadratic piece of this potential can be read off the above expression for the potential. We identify three kinds of bosons within the multiplet which are, in the long run, best considered separately. For each category, a short calculation leads to the following results for the quadratic piece of the potential:

1: Subgroup Fields (those containing no index 3):

$$V = \frac{1}{2}[(h_1^2)^2 + (h_2^2)^2 + 2|h_1^2|^2 + |\vec{w}_1^2|^2 + |\vec{w}_2^2|^2 + 2|\vec{w}_1^2|^2](\lambda_0 v^2 + m^2) + \dots$$

$$M_s^2 = m_3^2$$

2: Coset Fields (Those located off the diagonal and containing only one index 3):

$$V = \frac{1}{2}[(h_1^3)^2 + (h_2^3)^2 + |\vec{w}_1^3|^2 + |\vec{w}_2^3|^2](\lambda_0 v^2 + m^2) + \dots$$

$$M_c^2 = m_3^2$$

3: Third Generation Fields (those containing two indices 3):

The three  $\vec{w}$ -fields are massless Goldstone bosons:

$$V = \frac{1}{2}|\vec{w}_3^3|^2(\lambda_0 v^2 + m^2 - m_3^2) + \dots = 0 + \dots$$

$$m_w^2 = 0$$

They will be "eaten" by the W and Z bosons, when the gauge interactions are restored to the formalism.

The remaining state is the clear candidate for the newly-discovered Higgs boson, and has a unique mass:

$$V = \frac{1}{2}(h_3^3)^2 [2\lambda_0 v^2 + \lambda_0 v^2 + m^2 - m_3^2] + \dots = \lambda_0 v^2 (h_3^3)^2 + \dots$$

$$m_h^2 = 2(m_3^2 - m^2) = 2\lambda_0 v^2$$

With this identification, it follows that the vev  $v$  which we have defined is in fact the same as in standard electroweak phenomenology:  $v = 246 \text{ GeV}$ .

The results obtained so far clearly must be viewed as a first approximation. In real life, there are plenty of reasons to believe that the Lagrangian for the effective field theory for a complicated Higgs sector like this example will possess just as much clutter as the corresponding effective field theory for quarks and leptons. However, by giving ab initio all the new degrees of freedom a large common mass (which we assume is at least as large as the top-quark mass), we have a reasonable starting point for considering the phenomenology of these new states. We will assume that the complications which are left out can be considerable, but not large enough to grossly deform the basic pattern established above.

#### IV. Breaking Flavor Symmetry

In this section we generalize the description to include some contributions to the quartic potential which break flavor symmetry but which maintain family symmetry. These contributions will break the huge degeneracy present in the simple model of the previous section. The modified potential is as follows:

$$V = \frac{m^2}{2} \text{Tr} \bar{\Phi} \Phi + \frac{\lambda_0}{4} (\text{Tr} \bar{\Phi} \Phi)^2 - \frac{m_3^2}{2} \text{Tr} \bar{\Phi}_3^3 \Phi_3^3 \\ - \frac{\lambda_1}{4} \text{Tr} \bar{\Phi} \Phi \bar{\Phi} \Phi - \frac{\lambda_2}{4} \text{Tr} \tau_\alpha \bar{\Phi} \Phi \tau_\alpha \bar{\Phi} \Phi$$

From this point, easy calculations, similar to those described in Section III, lead to the following modifications of the conclusions reached there.

- 1.) The computation of the vev  $v$  is modified by the replacement:

$$(m_3^2 - m^2) = \left( \lambda_0 - \frac{\lambda_1}{2} - \frac{3\lambda_2}{2} \right) v^2$$

- 2.) The formula for the masses of the 16 subgroup fields, which we denote generically as  $s$  for self-conjugate fields and  $S$  otherwise, becomes:

$$V = \frac{1}{2} \left[ (h_1^2)^2 + (h_2^2)^2 + 2|h_1^2|^2 + |\vec{\omega}_1|^2 + |\vec{\omega}_2|^2 + 2|\vec{\omega}_1|^2 \right] (\lambda_0 v^2 + m^2)$$

$$M_s^2 = \lambda_0 v^2 + m^2 = m_3^2 + \frac{1}{2} (\lambda_1 + 3\lambda_2) v^2$$

- 3.) The 4 neutral coset fields, which we denote generically as  $G$ , show themselves as would-be pseudo-Goldstone modes (they become Goldstone when  $m^2 < 0$  and  $m_3$  vanishes):

$$V = \left[ |h_1^3|^2 + |h_2^3|^2 \right] \left[ \lambda_0 v^2 + m^2 - \frac{1}{2} (\lambda_1 + 3\lambda_2) v^2 \right] + \dots = m_3^2 \left[ |h_1^3|^2 + |h_2^3|^2 \right]$$

$$M_G^2 = m_3^2$$

- 6 -

4.) The 8 charged coset fields, which we denote generically as  $C$ , are shifted in mass:

$$\mathcal{V} = [|\vec{\omega}_1^3|^2 + |\vec{\omega}_2^3|^2] [\lambda_0 v^2 + m^2 - \frac{1}{2}(\lambda_1 + \lambda_2)v^2] + \dots = (m_3^2 + \lambda_2 v^2) [|\omega_1^3|^2 + |\omega_2^3|^2] + \dots$$

$$M_C^2 = m_3^2 + \lambda_2 v^2$$

5.) The third-generation fields  $\vec{\omega}_3^3$  remain as Goldstone bosons, hence remain prime candidates for becoming the longitudinal components of  $W$  and  $Z$ :

$$\mathcal{V} = \frac{1}{2} |\vec{\omega}_3^3|^2 [\lambda_0 v^2 + m^2 - m_3^2 - \frac{1}{2}(\lambda_1 + 3\lambda_2)v^2] + \dots = 0 + \dots$$

$$m_\omega^2 = 0$$

6.) The mass of the Higgs-like field is in a sense modified, but in another sense is unchanged:

$$\begin{aligned} \mathcal{V} &= \frac{1}{2} (h_3^3)^2 [3\lambda_0 v^2 + m^2 - m_3^2 - \frac{3}{2}(\lambda_1 + 3\lambda_2)v^2] + \dots \\ &= \frac{1}{2} (h_3^3)^2 [2\lambda_0 - \lambda_1 - 3\lambda_2] v^2 + \dots = (h_3^3)^2 (m_3^2 - m^2) \end{aligned}$$

$$m_h^2 = 2(m_3^2 - m^2)$$

## V. Phenomenology

At this point, the model appears to be credible, although clearly incomplete. Only the third-generation phenomenology is addressed, and additional couplings--and almost certainly even more Higgs degrees of freedom---will have to be introduced. Nevertheless quite a bit can be already said at the phenomenological level. There is a new spectroscopy defined, and this section is devoted to a first, rough, exploration of this crucial issue. However, there is much uncertainty. We begin by assuming that the 32 new states all have approximately the same mass, chosen to be between 200 GeV and 400 GeV, with the splittings small compared to 100 GeV. Note that this assumption requires a certain amount of fine tuning of the input mass parameters  $m$  and  $m_3$ . This feature encourages us to keep the mass scale of the 32 new states low. In any case, this phenomenological scenario will be revisited in Section VII.

The phenomenology of these new heavy states ---  $s$ ,  $S$ ,  $G$ , and  $C$  --- appears to be relatively robust. This occurs because there is a dominant production mechanism, along with dominant decay mechanisms, for these states. The production mechanism exploits their electroweak doublet nature, in particular their couplings to the (transverse!)  $W$ 's and  $Z$ 's:

$$q + \bar{q} \rightarrow Z \rightarrow \begin{cases} S + \bar{S} \\ C + \bar{C} \end{cases}$$

This rate is very easily normalized to Drell-Yan production of, say, muon pairs, in a way analogous to the famous parameter  $R$  which permeates the phenomenology of electron-positron collider physics. In particular,

$$R_{LHC} \sim \sum_{x_i = S, C} \frac{\sigma(q\bar{q} \rightarrow x_i \bar{x}_i)}{\sigma(q\bar{q} \rightarrow \mu^+ \mu^-)}$$

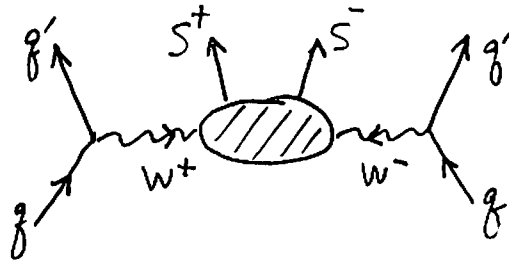
Likewise, there exist prompt decay processes. The  $C$  and  $G$  states decay promptly to a  $t$  quark and an antiquark in the first or second generation. The  $s$  and  $S$  states couple virtually to  $\bar{G}G$ ,  $\bar{G}C$ , or  $\bar{C}C$  intermediate states, which then decay either into  $t \bar{q} \bar{b} q'$  or into  $b \bar{q} \bar{t} q'$  final states, where again the quarks  $\bar{q}$  and  $q'$  are in the first or second generation. The lifetimes in all cases appear too short to be detectable. None of these decay modes open up easy search strategies at the LHC.

The hallmark of this scenario is the appearance of single events leading to final states containing a  $t \bar{t}$  pair plus two or four additional jets created by first or second generation  $q$ 's and  $\bar{q}$ 's. Above  $S \bar{S}$  threshold there will also be ( $t t + 4$  jet) final states, which may well be quite background-free.

What about existing data? I have only done a cursory search of the ATLAS public papers, in order to get a rough feel for the situation. First of all, for our chosen mass range, the total yield of  $C^+ C^-$  can be estimated to be of order  $1 / \text{fb}^{-1}$ . This is determined via normalization to the observed Drell-Yan dimuon spectrum. This is a very small number, and already rules out the possibility of a positive signal in present data, which is of order  $20 \text{fb}^{-1}$ .

There is another production mechanism that might be worth exploration, namely  $W^+ W^-$  fusion. An example is illustrated in the following figure:





Other than the tagging jets associated with the final state quarks  $q'$ , which lie in the leading particle regions, and the "fireball" containing the decay products, the remaining underlying event should be in general very quiet, so quiet that it may contain rapidity gaps. Therefore the presence of multitops in the final state might be much more background free. However, the production cross section is clearly much too small for this year's LHC data.

The bottom line is that it is very unlikely that a sensitive search for C's and/or S's can be made with the existing LHC data sample. Future prospects will be addressed in Section VII.

However, this is not the end of the story. It is possible that phenomena at the one-loop level may bear information. Two candidate phenomena come to mind. The first is the muon anomalous moment. A rough estimate based on the Feynman diagram shown below leads to the expression

$$\frac{(g-2)}{2} \sim \frac{m_\tau m_\mu}{\sqrt{2} m_G^2} \sim 10^{-11}$$

Even without a serious calculation, this looks to be too small to account for the extant discrepancy, of order  $3 \times 10^{-9}$ .

A more promising phenomenon is corrections to the  $\gamma\gamma$ ,  $W^+W^-$ , and  $Z Z^*$  final states in Higgs decay, due to virtual loops involving the new members of the Higgs family. This turns out to be of crucial importance, and is the subject of the next section.

## VI. Higgs Decay to Two Photons

The theory of this decay has recently been reviewed by many authors, thanks to a claim by Gastmans, Wu, and Wu that the standard calculation was wrong. Present consensus is that Gastmans et. al. are in fact wrong. An especially clear exposition is given by Marciano et. al. in arXiv 1109.5304, and we follow their discussion. (A problem set in Peskin and Schroeder, p. 777, is also a useful source.)

The standard calculation is dominated by the W-boson loop, but includes a piece contributed by a top-quark loop. This latter piece is a relatively straightforward computation, very similar to what we will encounter from charged S and C loops. We therefore can normalize our computation to the corresponding top-quark computation.

The formula for the width given by Marciano et. al. has the form

$$\Gamma(H \rightarrow \gamma\gamma) \sim \frac{\alpha}{\pi^3} \cdot \frac{m_h^3}{v^2} |F|^2$$

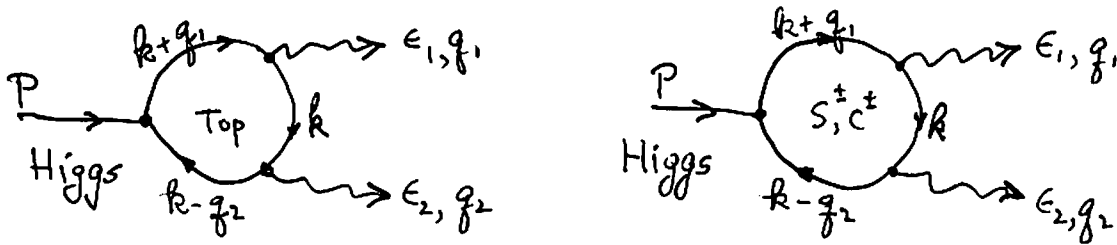
It is valid when the Higgs mass is small compared to twice the mass running around in the loop. We will in what follows make that approximation. In this limit the quantity F has the form

$$F = 7 - \frac{4}{3} \cdot (3) \cdot \left(\frac{2}{3}\right)^2 = 7 - \frac{16}{9}$$

The factor 7 is contributed by the W-loop, while the remainder is due to the top quark loop. We have explicitly factored out the factor 3 for color and the factor (2/3) for top-quark charge, because the scalar loops will not share these contributions.

It is notable that the formula for the Higgs width into gammas depends only upon the electroweak vev, the Higgs mass, and the fine structure constant, and not explicitly upon the mass running around the loop, which can therefore be quite large. Therefore we can anticipate nontrivial contributions from the S and C charged bosons, no matter what their mass is.

We now calculate the new contribution in detail by comparing each step with the corresponding one for the top quark loop. The relevant Feynman diagrams are shown below: (The exchange diagrams will just double the amplitudes, so we ignore them.)



In what follows, we define the coupling of the Higgs to the relevant charged scalar boson as follows:

$$[\text{Higgs coupling}]_i = \left(\frac{2M_i^2}{v}\right) \cdot f_i$$

The relevant pieces of the amplitude are then given by the following expressions:

$$M_{top} \propto (-i) \left(\frac{m}{v}\right) \cdot (3) \cdot \left(\frac{2}{3}\right)^2 \frac{[\text{Trace}]}{[\text{Denominators}]} \quad M_{\text{scalar}} \propto \left(\frac{2M^2}{v}\right) \cdot f \cdot \frac{\epsilon_1 \cdot (2k+q_1) \epsilon_2 \cdot (2k-q_2)}{[\text{Denominators}]}$$

$$[\text{Trace}] = \text{Tr} (k + q_1 + M) \not{\epsilon}_1 (k + M) \not{\epsilon}_2 (k - q_2 + M)$$

$$[\text{Denominators}] = [(k+q_1)^2 - M^2] [k^2 - M^2] [(k-q_2)^2 - M^2]$$

We combine denominators using Feynman parameters, and shift the loop-momentum variable appropriately:

$$k' = k + q_1 \alpha_1 - q_2 \alpha_2$$

The Feynman denominator has the form

$$\begin{aligned} [\text{Denominators}] &\Rightarrow [k^2 - M^2 + 2k \cdot (q_1 \alpha_1 - q_2 \alpha_2)]^3 = [k'^2 - M^2 + 2q_1 q_2 \alpha_1 \alpha_2]^3 \\ &= [k'^2 - M^2 + m_k^2 \alpha_1 \alpha_2]^3 \approx [k'^2 - M^2]^3 \end{aligned}$$

We see that it becomes quite simple in the light-Higgs limit.

There are delicate issues of regularization in the general computation, but these are finessed by only considering the coefficient of  $(\epsilon_1 \cdot q_2)(\epsilon_2 \cdot q_1)$ . For the two cases of interest here, the relevant amplitudes are, after integration over  $k'$ , as follows. For the top-quark amplitude,

$$M_{top} \propto -\left(\frac{M}{v}\right) \cdot 3 \cdot \left(\frac{2}{3}\right)^2 \cdot \frac{1}{M^2} [\text{Trace}]$$

$$\begin{aligned} [\text{Trace}] &= \text{Tr} [\not{q}_1(1-\alpha_1) + \not{q}_2 \alpha_2 + M] \not{\epsilon}_1 [-\not{q}_1 \alpha_1 + \not{q}_2 \alpha_2 + M] \not{\epsilon}_2 [-\not{q}_1 \alpha_1 - \not{q}_2(1-\alpha_2) + M] + \dots \\ &= 4M (\epsilon_1 \cdot q_2)(\epsilon_2 \cdot q_1) \cdot \frac{4}{v} [1 - 4\alpha_1 \alpha_2] + \dots \end{aligned}$$

Therefore,

$$M_{top} \propto -3 \cdot \left(\frac{2}{3}\right)^2 \cdot (\epsilon_1 \cdot q_2)(\epsilon_2 \cdot q_1) \cdot [1 - 4\alpha_1 \alpha_2] + \dots$$

For the corresponding scalar amplitude,

$$\begin{aligned} M_{\text{scalar}} &\propto \left(\frac{2M^2}{v}\right) \cdot f \cdot \frac{1}{M^2} \cdot 4 (\epsilon_1 \cdot q_2 \alpha_2)(\epsilon_2 \cdot q_1 \alpha_1) + \dots \\ &= -(\epsilon_1 \cdot q_2)(\epsilon_2 \cdot q_1) \cdot \frac{8f}{v} \cdot \alpha_1 \alpha_2 + \dots \end{aligned}$$

All that is left to do is to compare the values of the integrals over the Feynman parameters:

$$\int_0^1 d\alpha_1 \int_0^{(1-\alpha_1)} d\alpha_2 [1 - 4\alpha_1\alpha_2] = \frac{1}{3} \quad \int_0^1 d\alpha_1 \int_0^{(1-\alpha_1)} d\alpha_2 \alpha_1\alpha_2 = \frac{1}{24}$$

Putting the pieces together, we find the correction to the book value for the decay width:

$$\frac{\Gamma^2(\text{family})}{\Gamma^2(\text{vanilla})} = \frac{\left| 7 - \frac{4}{3} \left[ 3 \cdot \left(\frac{2}{3}\right)^2 + \sum_{i=1}^8 f_i \right] \right|^2}{\left| 7 - \frac{16}{9} \right|^2} = \left| 1 - \frac{3}{47} \sum_{i=1}^8 f_i \right|^2$$

We now turn to the computation of the coupling factors  $f_i$ .

We begin with the  $S^\pm$  contribution; its Yukawa coupling to the Higgs is

$$\begin{aligned} [\text{Higgs coupling}]_S &= 2\lambda_0 v \\ &= \frac{2}{v} (M_S^2 - m^2) = \frac{2M_S^2}{v} \left( 1 - \frac{m^2}{M_S^2} \right) \end{aligned}$$

Therefore,

$$f_S = \left( 1 - \frac{m^2}{M_S^2} \right) = 1 - \frac{M_3^2}{M_S^2} + \frac{(m_3^2 - m^2)}{M_S^2} = 1 - \frac{M_G^2}{M_S^2} + \frac{m_a^2}{2M_S^2}$$

The result for the  $C^\pm$  contribution is very similar:

$$\begin{aligned} [\text{Higgs coupling}]_C &= (2\lambda_0 - \lambda_1 - \lambda_2) v^2 \\ &= \frac{2}{v} (M_C^2 - m^2) = \frac{2M_C^2}{v} \left( 1 - \frac{m^2}{M_C^2} \right) \end{aligned}$$

Therefore, as before,

$$f_C = 1 - \frac{M_G^2}{M_C^2} + \frac{m_a^2}{2M_C^2}$$

The enhancement factor for the  $h \rightarrow \gamma\gamma$  decay then becomes

$$\frac{\Gamma(\text{family})}{\Gamma(\text{vanilla})} = \left| 1 - \frac{12}{27} \left[ 2 - \frac{M_G^2}{M_S^2} - \frac{M_G^2}{M_C^2} + \frac{m_h^2}{2M_S^2} + \frac{m_h^2}{2M_C^2} \right] \right|^2$$

The present data on this process hints of a significant excess. If we assume

$$1 \leq \frac{\Gamma(\text{family})}{\Gamma(\text{vanilla})} \leq 2,$$

this leads to the condition

$$0 \geq \left[ 2 - \frac{M_G^2}{M_S^2} - \frac{M_G^2}{M_C^2} + \frac{m_h^2}{2M_S^2} + \frac{m_h^2}{2M_C^2} \right] \geq (-1.6)$$

The Higgs contributions are quite small. For  $M_S \sim M_C \sim 300$  GeV,

$$\frac{m_h^2}{2M^2} \sim 0.1$$

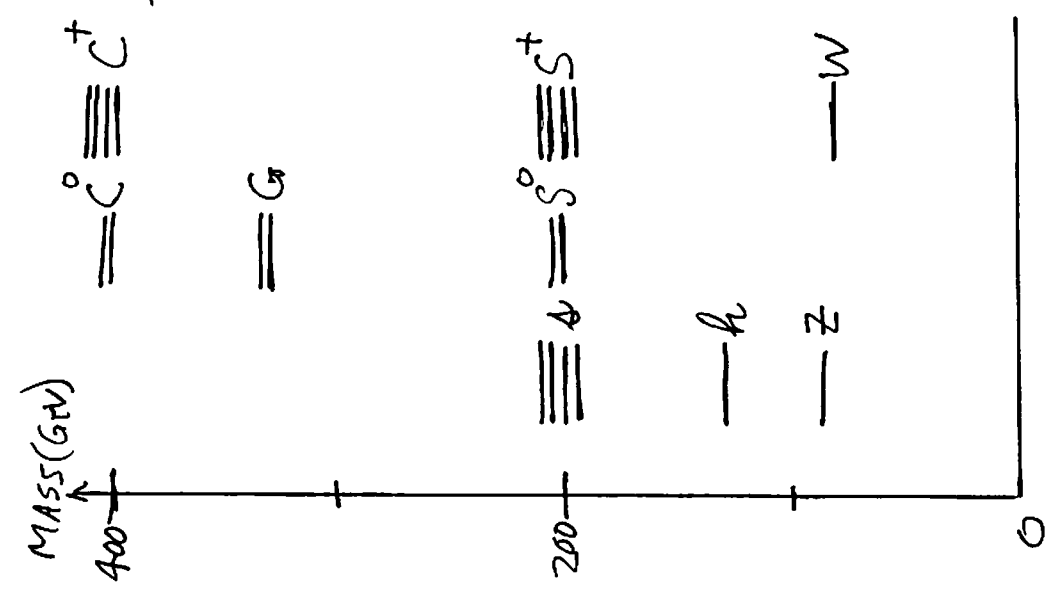
Using this estimate, our bound simplifies:

$$3.4 \gtrsim \left[ \frac{M_G^2}{M_S^2} + \frac{M_G^2}{M_C^2} \right] \gtrsim 2.2$$

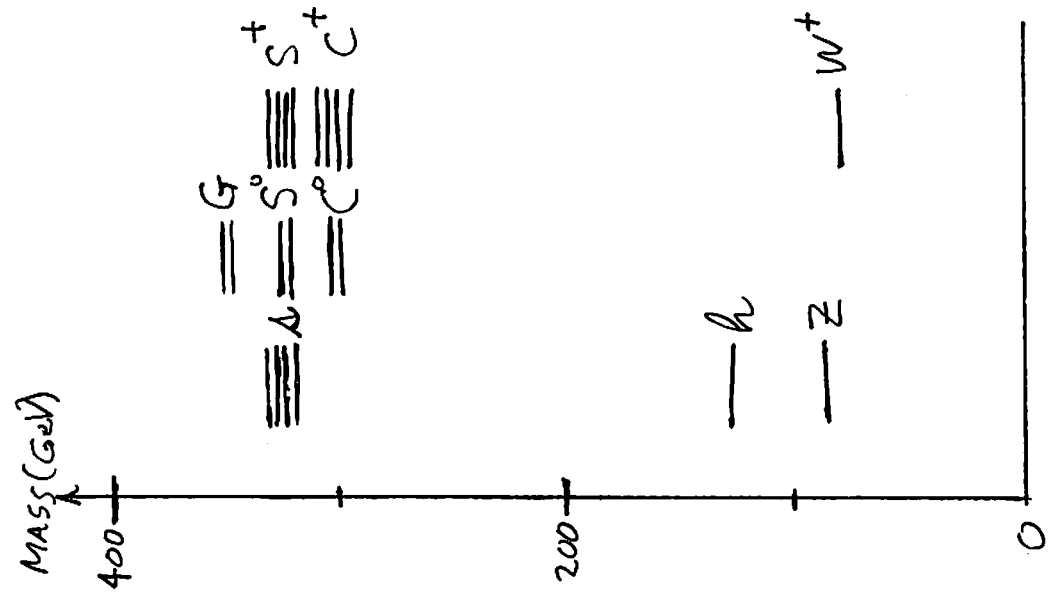
There are three classes of options:  $M_S > M_C$ ,  $M_S \cong M_C$ ,  $M_S < M_C$ . We illustrate typical cases on the next page. In all three cases the decay phenomenology is essentially the same.

The bottom-line question is obviously what data, present or future, provide the most stringent tests of these ideas? I see the leading candidates as (1) a nonstandard Higgs  $\rightarrow \gamma\gamma$  decay width, and (2) evidence for pair production of the charged subgroup states  $S^+ S^-$ . A few percent of those final states will contain (a) a same-sign charged-lepton pair, (b) large missing transverse energy, (c) 4 quark-antiquark jet pairs, with in principle (d) 4 b-tags. The total system would look like a more-or-less isotropic "fireball" containing 12 components, each of which shares the total energy fairly equally.

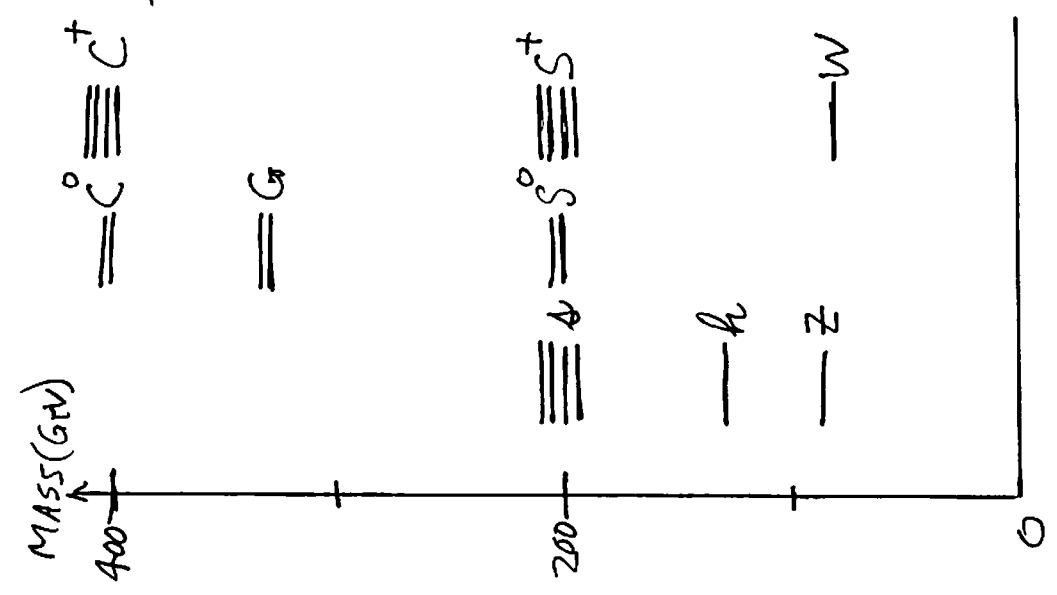
# A HIGGS FAMILY



OPTION A



OPTION B



OPTION C

DECAYS:  $C, G \rightarrow t\bar{t}$

$S \rightarrow \begin{cases} t\bar{t} b\bar{b}' \\ b\bar{b}' t\bar{t}' \end{cases}$

$\{g, g'\} = u, d, c, s$

## VII. Concluding Remarks

It is clear that the contents of this note comprise only a cartoon for a more serious description of the Higgs sector. In the introduction we divided the task into three stages. Only the first stage---a description appropriate only within the approximation of neglecting all first and second generation masses and mixings---has been considered. This note remains incomplete even at this first stage. The couplings of the new Higgs family members to the gauge bosons were neglected and should be restored. This will for sure alter the mass spectrum. In addition, more symmetry-breaking interaction terms should be included in the Higgs potential. This will no doubt be a necessity in order to allow the renormalization of gauge-boson loop contributions.

Even so, we regard the main idea of this note to be quite robust. The essence of what has been done would apply to alternate versions of the family extension involving  $N$  extra complex electroweak doublets instead of our choice of 8. To see this, change notation a little and write for the Higgs potential

$$V = -\frac{m^2}{2} \text{Tr} \bar{\Phi} \Phi + \frac{\lambda}{4} (\text{Tr} \bar{\Phi} \Phi)^2 + \sum_{i=1}^N \left[ \frac{m_i^2}{2} \text{Tr} \bar{\Psi}_i \Psi_i + \lambda_i \text{Tr} \bar{\Psi}_i \Psi_i \bar{\Phi} \Phi \right] + \dots$$

Here  $\Psi_i$  label the new doublets and  $\Phi$  the vanilla Higgs doublet, i.e. what was previously defined as  $\Phi_3$ . As long as the mass terms of the new doublets allow no spontaneous symmetry breaking, the bare masses  $m_i$  of the new states can be set to large values. In addition, the phenomenology is insensitive to details of the (unwritten) quartic couplings of the new doublets to each other. The main effect of the quartic terms which couple the new states to the vanilla sector will be to shift the bare masses  $m_i$  of the new, heavy states to new values  $M_i$ . And it is easy to see that the effect of the new states on the  $h \rightarrow \gamma\gamma$  decay amplitude of the Higgs boson will be in proportion to these mass shifts:

$$f_i = \left( \frac{M_i^2 - m_i^2}{M_i^2} \right)$$

Given the robust character of the basic idea, one is free to entertain other versions of family structure. For example, one might posit that the family symmetry appropriate to an effective field theory at the 1 TeV mass scale could be as low as  $O(3)$ , with the new fields transforming as a second rank tensor. This could lead to 20 new states instead of our 32.

However, I will stick to the present choice until deterred by some kind of obstacle. The main issue will be to find a good way to generalize the scheme to second-generation phenomenology. The main challenges at this level are considerable, but at least can be enumerated. A mechanism for giving masses to  $c$ ,  $s$ , and  $\mu$  must be identified. And the most central issue will be to understand the origin of family mixing, and account for the value of  $V_{cb}$ . However, the issues of CP violation and of the origin of the tiny first-generation masses can still be set aside.

There are also interesting opportunities for linking these ideas to some of the bigger issues of particle theory which extend beyond the standard-model phenomenology. One is whether family structure in the Higgs sector can peacefully coexist with the ideas of grand unification. The answer seems to be a tentative yes. The family of electroweak doublets generalize to a family of (complex) Higgs 5 representations at the SU(5) level. This implies that the Higgs family will also contain  $N$  new spin-zero, electroweak-singlet, color-triplet, charge  $1/3$  scalar bosons, in addition to the one triplet involved in the notorious doublet-triplet splitting problem of the standard version. In a complete version of the theory, these bosons could induce proton decay. Therefore we anticipate that their masses are well above the LHC energy scale, and not directly involved in present-day phenomenology. However, their coupling pattern to quarks and leptons appears to be interesting, and their possible relevance should be kept in mind as we proceed further.