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Exploration of Adaptive Gait Patterns with a Reconfigurable Linkage Mechanism

Shunsuke Nansai, Nicolas Rojas, Mohan Rajesh Elara, and Ricardo Sosa

Abstract— **Legged robots are able to move across irregular terrains and some can be energy efficient, but are often constrained by a limited range of gaits which can limit their locomotion capabilities considerably. This paper reports a reconfigurable design approach to robotic legged locomotion that produces a wide variety of gait cycles, opening new possibilities for innovative applications. In this paper, we present a distance-based formulation and its application to solve the position analysis problem of a standard Theo Jansen mechanism. By changing the configuration of a linkage, our objective in this study is to identify novel gait patterns of interest for a walking platform. The exemplary gait variations presented in this work demonstrate the feasibility of our approach, and considerably extend the capabilities of the original design to not only produce novel cum useful gait patterns but also to realize behaviors beyond locomotion.**

I. Introduction

Legged platforms are one of the most versatile design strategies for efficient robot locomotion. The choice of leg-like mechanisms often responds to design requirements such as the ability to move through irregular terrains or to increase stability, maneuverability or energy efficiency. One of the main challenges of using legged robots in practical applications is how to control and adapt their gait —*i.e.*, finding a suitable and adaptable foot displacement trajectory [1]. Legged animals coordinate a wide range of components and systems to walk adaptively and efficiently under a variety of speeds, terrains and task goals including chasing, courtship and stealth. However, in nature, individuals are limited by their species morphology and are only able to change their gait from a limited number of alternatives. This paper reports a reconfigurable approach to robotic legged locomotion that produces a wide variety of gait curves, opening new possibilities for innovative applications. The main departure from the state of the art in this area is that large solution spaces are generated using a mechanism of mobility one, producing gait variance via parametric changes of the leg assembly.

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Our ultimate aim is to build robots that can entirely redesign their morphologies according to changes in the environment and to reflect their learning of new abilities. This paper represents an initial step towards robots that self-design themselves. We envision robots equipped with reconfigurable legs to generate a large number of locomotion capabilities during their deployment. Alternatively, for tasks that require single-purpose mechanisms, our approach can be used as a design method to explore and select optimal yet feasible configurations that can be used to build specialized and efficient walking robots.

The literature shows a variety of design strategies to generate gait patterns, including: adaptive locomotion control [2], structural combination of rigid and tensile structural elements [3], morphological computation [4], oscillator controller with pneumatic actuators [1] and biomimetic adaptations based on ground contact timing [5] or using sensorimotor coordination [6]. A reconfigurable design approach is presented in this paper where a robot varies its hardware morphology by parametric changes of its components and sub-assemblies. For example, re-shaping a robot's legs by modifying length ratios in 8 links in this case creates a large space of gait patterns that the robot can explore and test directly in its environment in order to obtain real feedback and learn transformation rules and heuristics at two complimentary levels: the single leg and the n-legged assembly that enables its walking capabilities at a given time.

Nested reconfiguration is an emerging research topic that refers to the capacity of modular robots to perform intra- and inter-reconfigurations. The reconfigurable single Jansen mechanism presented in this paper is an initial design and implementation of nested-reconfigurable legged robots. This design supports homogeneous and heterogeneous walking platforms, from two-legged robots with symmetrical legs to front/back leg specialization in four or more legged robots. It also supports adaptive changes in response to the failure of one leg, or to use a leg as a tool other than to perform locomotion.

The rest of the paper is organized as follows. Section 2 introduces the distance-based formulation for kinematic chains and applies this simplified bilateration method to address the position analysis problem of the robotic leg used in this study. Section 3 identifies a sample set of gaits produced by a reconfigurable design of this mechanism and section 4 presents a procedure for transforming a leg between two given configurations that produce different gaits with clear potential for future applications. Section 5 concludes this study and discusses future work.

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II. Reconfigurable Single Jansen Mechanism

The Theo Jansen Mechanism ("Jansen leg" is the term used in this paper) is a planar linkage enabling mobility. It is a highly efficient walking mechanism that can operate with only one actuator. The main challenges in designing a reconfigurable version of this type of mechanism include the development of efficient approaches to solve the position analysis problem and trace the corresponding coupler curves, the definition of metrics for the novelty and utility of the resulting foot trajectories, the development of heuristics to guide the reconfiguration process, and the non-trivial process of implementing theoretical designs generated analytically into physical mechanisms.

The foot trajectory of the standard Jansen leg is remarkably similar to certain animals [7]. The aims of our study include finding novel gait patterns for the Jansen leg that satisfy one of the following goals: a) mimicry of different animal species; b) significant improvements of the locomotion efficiency in non-even surfaces, a range of materials, and external perturbations such as strong winds; and c) transform locomotion mechanisms into tools for different behaviors such as manipulation, drilling, etc.

A. Kinematics of a Jansen Leg: Bilateration

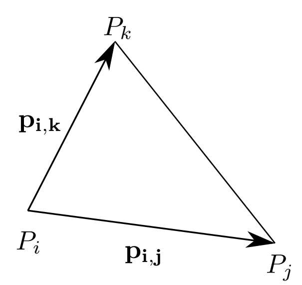


Fig. 1. The bilateration problem.

The bilateration problem consists in finding the feasible locations of a point, say P_k, given its distances to two other points, say P_i and P_j, whose locations are known. Then, according to Fig. 1, the solution to this problem, in matrix form, can be expressed as:

$$\mathbf{p}_{i,k} = \mathbf{Z}_{i,j,k}\, \mathbf{p}_{i,j} \quad (1)$$

where $\mathbf{p}_{i,j} = \overrightarrow{P_i P_j}$ and

$$\mathbf{Z}_{i,j,k} = \frac{1}{2\, s_{i,j}} \begin{bmatrix} s_{i,j} + s_{i,k} - s_{j,k} & -4\, A_{i,j,k} \\ 4\, A_{i,j,k} & s_{i,j} + s_{i,k} - s_{j,k} \end{bmatrix}$$

is called a *bilateration matrix*, with $s_{i,j} = d_{i,j}^2 = \|\mathbf{p}_{i,j}\|^2$, the squared distance between P_i and P_j, and

$$A_{i,j,k} = \pm\frac{1}{4}\sqrt{(s_{i,j}+s_{i,k}+s_{j,k})^2 - 2\,(s_{i,j}{}^2+s_{i,k}{}^2+s_{j,k}{}^2)}, \quad (2)$$

the oriented area of $\triangle P_i P_j P_k$ which is defined as positive if P_k is to the left of vector $\mathbf{p}_{i,j}$, and negative otherwise. It can be observed that the product of two bilateration matrices is commutative. Then, it is easy to prove that the set of bilateration matrices, *i.e.*, matrices of the form $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$, constitute a commutative group under the product and addition operations. Moreover, if $\mathbf{v} = \mathbf{Z}\mathbf{w}$, where $\mathbf{Z}$ is a bilateration matrix, then $\|\mathbf{v}\|^2 = \det(\mathbf{Z})\, \|\mathbf{w}\|^2$. The interested reader is addressed to [8] for a derivation of (1) and its properties.

It has been shown that by using bilateration matrices, the position analysis problem of linkages is greatly simplified –see, for instance, [9], [10]. This problem consists of finding the feasible assembly modes that a kinematic chain can adopt. An assembly mode is a possible relative transformation between the links of a kinematic chain or linkage. When an assignment of positions and orientations is made for all links with respect to a given reference frame, an assembly mode is called a configuration. Next, we present how to apply the bilateration method for solving the position analysis problem of a Jansen leg.

B. Kinematics of a Jansen Leg: System of equations

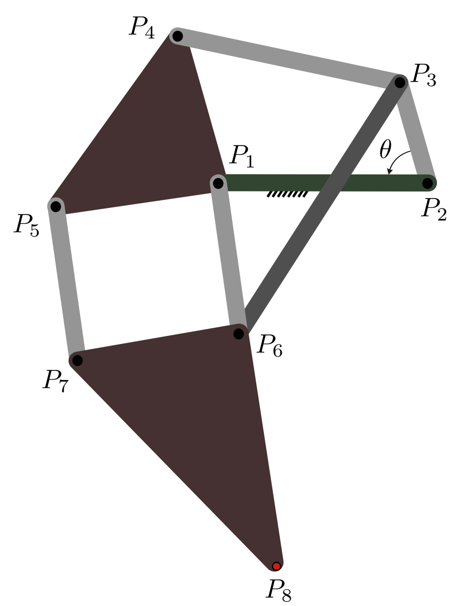


Fig. 2. The Jansen leg, a one-mobility planar linkage of eight links and three independent loops.

Figure 2 shows a Jansen leg. This planar linkage of mobility one consists of six binary links, one ternary link, and a coupler link with three independent loops. The centers of the revolute joints of the binary links define the line segments $\overline{P_1P_2}$, $\overline{P_1P_6}$, $\overline{P_2P_3}$, $\overline{P_3P_4}$, $\overline{P_3P_6}$, and $\overline{P_5P_7}$, those for the ternary link define the triangle $\triangle P_1P_4P_5$, and those for the coupler link with point P_8, the foot of a Jansen leg, define the triangle $\triangle P_6P_7P_8$. The position analysis problem for a Jansen leg corresponds to, given the dimensions of every link, the position of the revolute joint centers P_1 and P_2, and an angle θ for the input link, calculating all the feasible Cartesian locations of point P_8. To this end, instead of using joint angles through independent loop-closure equations, we will use squared distances and bilateration matrices to compute the corresponding values of P_8.

First, let us compute $\mathbf{p}_{1,3}$ from θ and the location of the revolute joint centers P_1 and P_2. That is,

$$\mathbf{p}_{1,3} = \mathbf{Z}_{1,2,3}\,\mathbf{p}_{1,2} \tag{3}$$

with $s_{1,3} = s_{1,2} + s_{2,3} - 2\,d_{1,2}\,d_{2,3}\,\cos\theta$. Now, following a simple geometric constructive process from $\mathbf{p}_{1,3}$, we get

$$\mathbf{p}_{1,4} = \mathbf{Z}_{1,3,4}\,\mathbf{p}_{1,3}, \tag{4}$$

$$\mathbf{p}_{1,5} = \mathbf{Z}_{1,4,5}\,\mathbf{p}_{1,4} = \mathbf{Z}_{1,4,5}\mathbf{Z}_{1,3,4}\,\mathbf{p}_{1,3}, \text{ and} \tag{5}$$

$$\mathbf{p}_{1,6} = \mathbf{Z}_{1,3,6}\,\mathbf{p}_{1,3}. \tag{6}$$

Thus,

$$\mathbf{p}_{5,6} = -\mathbf{p}_{1,5} + \mathbf{p}_{1,6} = (-\mathbf{Z}_{1,4,5}\mathbf{Z}_{1,3,4} + \mathbf{Z}_{1,3,6})\,\mathbf{p}_{1,3}. \tag{7}$$

Then,

$$s_{5,6} = \det(-\mathbf{Z}_{1,4,5}\mathbf{Z}_{1,3,4} + \mathbf{Z}_{1,3,6})\,s_{1,3}. \tag{8}$$

Finally, from $\mathbf{p}_{5,6}$, we get

$$\mathbf{p}_{6,7} = -\mathbf{Z}_{6,5,7}\,\mathbf{p}_{5,6}, \tag{9}$$

$$\mathbf{p}_{6,8} = \mathbf{Z}_{6,7,8}\,\mathbf{p}_{6,7} = -\mathbf{Z}_{6,7,8}\,\mathbf{Z}_{6,5,7}(-\mathbf{Z}_{1,4,5}\mathbf{Z}_{1,3,4} + \mathbf{Z}_{1,3,6})\,\mathbf{p}_{1,3}. \tag{10}$$

Equation (10) defines the location of point P_8, the foot of a Jansen leg. This equation depends on the set of link dimensions (S), the angle of the input link (θ), the orientation sign of the oriented areas $A_{1,2,3}$, $A_{1,3,4}$, $A_{1,3,6}$, and $A_{6,5,7}$, and the location of P_1 and P_2, the centers of the grounded revolute joints. For a given set of values for all these variables, a specific configuration of a Jansen leg is determined, that is, the point P_8 is uniquely defined. We represent a configuration of a Jansen leg as $(S,\,\theta,\,\eta,\,P_1,\,P_2)$ where $\eta = 0,\ldots,15$ specifies the combination of signs for the areas $A_{1,2,3}$, $A_{1,3,4}$, $A_{1,3,6}$, and $A_{6,5,7}$. Thus, for example, $\eta = 10 = (1010)_2 \equiv +-+-$ implies that $A_{1,2,3} > 0$, $A_{1,3,4} < 0$, $A_{1,3,6} > 0$, and $A_{6,5,7} < 0$.

III. Identification of Foot Trajectories for Intra-reconfigurability

Figure 3(top) presents all possible locations of point P_8 in all assembly modes, computed from equation (10), for sampled values of θ at increments of $\frac{1}{100}$ for the case in which $s_{1,2} = 1073.55$, $s_{1,4} = 553.61$, $s_{1,5} = 631.72$, $s_{1,6} = 552.63$, $s_{2,3} = 117.38$, $s_{3,4} = 1216.96$, $s_{3,6} = 1468.58$, $s_{4,5} = 1045.75$, $s_{5,7} = 572.84$, $s_{6,7} = 642.22$, $s_{6,8} = 1292.26$, and $s_{7,8} = 1900.87$, with $P_1 = (0,0)^T$ and $P_2 = (32.436, 4.632)^T$. In this procedure, for each value of θ, sixteen possible locations for the point P_8 are calculated, one per each combination of signs for the oriented areas $A_{1,2,3}$, $A_{1,3,4}$, $A_{1,3,6}$, and $A_{6,5,7}$ [11]. From Fig. 3(top), a result that contains no information on the connectivity of each sample to its neighbors, we observe that point P_8 shapes to different trajectories. In fact, since a Jansen leg corresponds to a pin-jointed planar linkage of mobility one, any arbitrary point on it generates a plane curve, called coupler curve, when the mechanism moves.

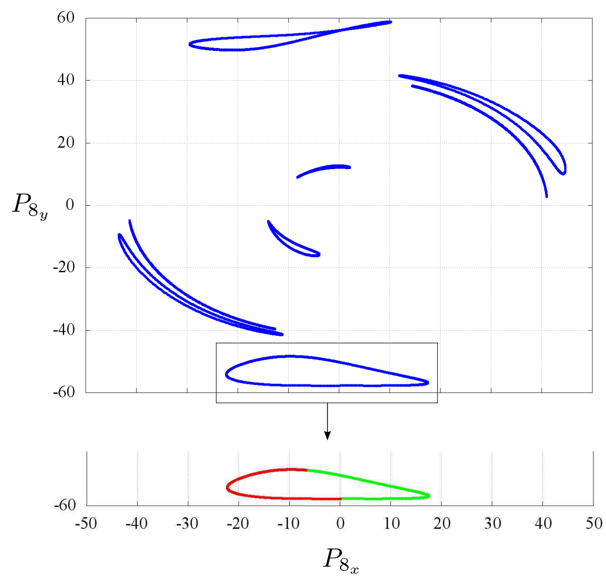


Fig. 3. **Top:** Possible locations of point P_8, the foot, in all assembly modes of a standard Jansen leg. The lowest sampled curve corresponds to the trajectory used in walking platforms. **Bottom:** Traced foot trajectory of a standard Jansen leg. The green path corresponds to the assembly mode family given by $A_{1,2,3} > 0$, $A_{1,3,4} > 0$, $A_{1,3,6} < 0$, and $A_{6,5,7} < 0$. In the red path, $A_{1,2,3} < 0$, $A_{1,3,4} > 0$, $A_{1,3,6} < 0$, and $A_{6,5,7} < 0$.

A novel approach to trace coupler curves, that takes advantage from the geometric information of bilateration-based equations, has been recently discussed in [10]. By tracing we mean that the connectivity between samples is known. Following such method, for the particular case of a Jansen leg, the curve generated from a known initial feasible configuration $(S,\,\theta,\,\eta,\,P_1,\,P_2)$ by point P_8 —the foot trajectory— is traced following these steps:

1) Compute P_8 using equation (10).
2) Evaluate the oriented areas $A_{1,2,3}$, $A_{1,3,4}$, $A_{1,3,6}$, and $A_{5,6,7}$ with the current value of θ. If any of them is equal to zero, the current configuration –*i.e.*, the current tuple $(S,\,\theta,\,\eta,\,P_1,\,P_2)$– belongs to more than one family of assembly modes (combinations of signs of the oriented areas $A_{1,2,3}$, $A_{1,3,4}$, $A_{1,3,6}$, and $A_{5,6,7}$) and the leg movement may evolve along different paths. Identify all these families, that is, determine all feasible values that η can assume.
3) Increase θ at a specified rate. When θ reaches the limit imposed by the triangular inequalities associated to $\triangle P_1P_2P_3$, $\triangle P_1P_3P_4$, and $\triangle P_1P_3P_6$, start to decrease the variable.
4) Repeat steps 1 to 3 for each tuple $(S,\,\theta,\,\eta,\,P_1,\,P_2)$ until the whole range of θ has been evaluated.

In a standard Jansen leg, the foot trajectory used in walking platforms –the lowest sampled curve in Fig. 3(top)– can be easily traced following the above procedure from any θ between 0 and π with $A_{1,2,3} > 0$, $A_{1,3,4} > 0$, $A_{1,3,6} < 0$, and $A_{5,6,7} < 0$. The corresponding result is depicted in Fig. 3(bottom). It is interesting to note that the standard foot trajectory of a Jansen leg resembles to the plantigrade locomotion of some terrestrial animals. In fact, this trajectory is quite similar to those of the ankles of rats during single step cycles [7].

It is well-known that a change in the link dimensions of a linkage generates new and different coupler curves. With this basic principle in mind, our objective in this study is to identify whether by performing small variations in the lengths of the links of a standard Jansen leg, novel foot trajectories of interest for a walking platform can be obtained. The final goal of this idea is to integrate intra-reconfigurable legs into walking platforms with inter-reconfiguration characteristics that improve the mechanism's adaptability and scope. To this end, a simple exploratory design study is carried out where we vary the standard dimensions of the links in $\pm 20\%$, first link by link, later in couples, and finally in trios, and register the resulting coupler curves –computed using the discussed procedure– to detect useful gait patterns for future innovative applications in robotics. Following the proposed scheme, we present here five gait patterns of interest for nested reconfiguration applications generated through minimal changes to linkage configurations. Next, each of them is briefly described.

A. Digitigrade locomotion (Cat walking)

The standard foot trajectory of a Jansen leg corresponds to a kind of plantigrade locomotion. A plantigrade is an animal that stands or walks with its podials, such as humans regularly do. In contrast, digitigrades walk on their digits or toes. Example of these kind of animals include dogs, cats, many other mammals, and most birds. Since in each step of digitigrades less foot is touching the surface, these animals present less friction and waste of energy than plantigrades. In consequence, digitigrades tend to be very fast runners [12]. This fact makes digitigrade locomotion of great interest for the development of walking platforms.

Table I depicts the foot trajectory of a Jansen leg when the length of the binary link connecting the revolute joint centers P_5 and P_7 is increased by 20% respect to its standard value -First row, column "single link". The shape of this curve is quite similar to the gait cycle of a cat [13, Fig. 2]. This result is relevant because it shows that by modifying the link dimensions of a standard Jansen leg, that is, by reconfiguring a mechanism of mobility one, we can switch from a plantigrade locomotion to a digitigrade locomotion. Columns "couple of links" and "trio of links" in the first row of Table I present other modifications in the link dimensions that further yield digitigrade behaviors. The foot trajectories of the combinations with the $\star$ symbols in the columns are depicted. In each of these figures, the gait of the standard Jansen leg is presented for reference as a black curve. These conventions apply in all next cases.

B. Obstacle avoidance

The height of the foot trajectory of a standard Jansen leg is 9.38 units (Fig. 3), that is a 16.26% of the total height of the leg, a value computed from the grounded revolute joint center $P_1 = (0,0)^T$ to the lowest point of the foot trajectory. Therefore, a walking platform based on standard Jansen legs cannot in principle overpass obstacles higher than this limit (> 9.38 units). This is an important drawback because although a Jansen-based system is extremely efficient, its operability in rough terrain is reduced.

Second row, column "single link", of Table I shows the foot trajectory of a Jansen leg when the side length associated to points P_7 and P_8 of the coupler link $\triangle P_6 P_7 P_8$ is decreased by 20% respect to its standard value. In this case, the height of the foot trajectory is 21.91 units, that is a 36.74% of the total height of the leg. This new height is more than twice the foot trajectory of a standard Jansen leg. This intra-reconfigurability characteristic could be of interest for applications in uneven terrains —think, for instance, in a team of walking platforms for space exploration missions, that is, exploration of asteroids, comets, planets, and so on. Similar obstacle avoidance patterns can be obtained by simultaneously changing different link lengths of a Jansen leg. These results are presented in the second row, columns "couple of links" and "trio of links", of Table I.

C. Jam avoidance (Walking on mud)

In soft terrains, walkers can easily get stuck because of the soil conditions. To overcome such situations, a change in the walker's gait cycle has to be introduced, a versatility that a walking platform based on standard Jansen legs does not offer in its current form. For example, a transition from a dry soil to a semi-wet mud terrain, the typical foot trajectory of a Jansen leg seems inadequate because of the rigidity variation of the soil in the two scenarios. A gait with the potential to solve this issue is depicted in the third row, column "single link", of Table I. Such curve is obtained by increasing 20% the distance between the grounded revolute joint centers P_1 and P_2. Beyond its height, this type of trajectory is of interest because facing the soil with an arc shape, while maintaining a step length close to the original one, allows to extract material and look for a suitable support point at the same time. As in the other intra-reconfigurability characteristics previously discussed, similar jam avoidance patterns can be obtained by simultaneously changing different link lengths of a Jansen leg. The corresponding combinations of link dimensions are presented in the third row, columns "couple of links" and "trio of links", of Table I.

D. Step climbing

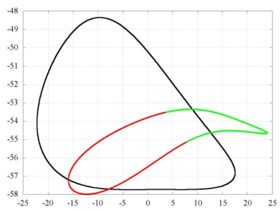
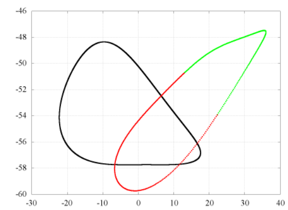
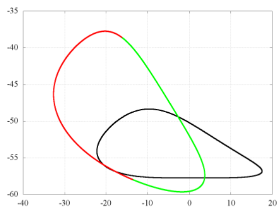
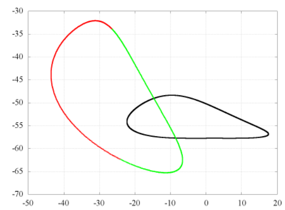
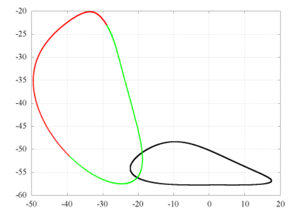
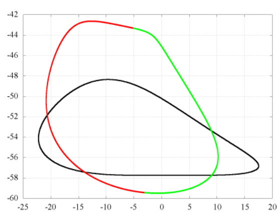
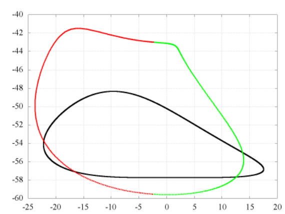
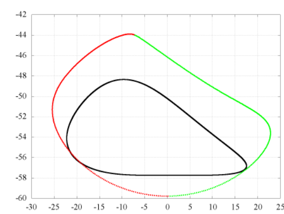
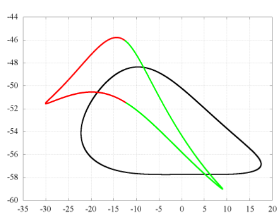
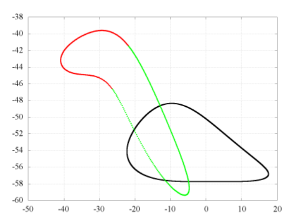
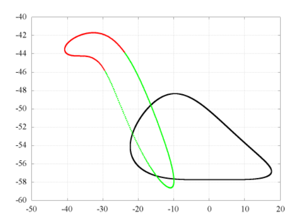
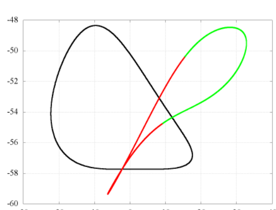
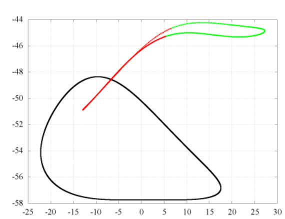
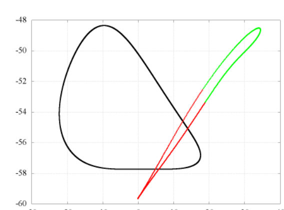
Fourth row of Table I presents the modifications in the link dimensions of a standard Jansen leg that yield foot trajectories for climbing steps. This curve results from increasing 16% the distance between the grounded revolute joint centers P_3 and P_6. These gamma-like patterns are more appropriate for climbing steps than the normal foot trajectory due to the significantly shorter contact line with the floor —think, for example, the length of the steps.

E. Drilling motion

In kinematics of mechanisms, reciprocating motion is in general defined as a recurrent up-and-down or back-and-forth movement. It is normally associated to a repetitive straight

TABLE I

IDENTIFIED FOOT TRAJECTORY PATTERNS OF INTEREST FOR NESTED RECONFIGURATION APPLICATIONS

Pattern	Single link	Couple of links	Trio of links
Digitigrade locomotion *Cat walking*	$d_{5,7}: +20\% \star$	$d_{1,2}: +10\%$, $d_{7,8}: +20\%$ $d_{3,4}: -12\%$, $d_{7,8}: +20\% \star$ $d_{5,7}: +10\%$, $\triangle P_1P_4P_5: -20\%$ $d_{3,4}: +12\%$, $\triangle P_6P_7P_8: +20\%$	$d_{1,2}: -8\%$, $d_{5,7}: +20\%$, $d_{3,6}: -20\% \star$ $d_{3,4}: -12\%$, $d_{7,8}: +20\%$, $d_{5,7}: +20\%$ $d_{3,4}: -12\%$, $d_{7,8}: +20\%$, $\triangle P_1P_4P_5: -10\%$
			
Obstacle avoidance	$d_{7,8}: -20\% \star$	$d_{6,8}: +20\%$, $d_{7,8}: -20\% \star$ $d_{3,4}: -20\%$, $d_{3,6}: -20\%$	$d_{6,8}: +20\%$, $d_{7,8}: -20\%$, $d_{1,2}: +20\%$ $d_{6,8}: +20\%$, $d_{7,8}: -20\%$, $d_{3,4}: -20\% \star$
			
Jam avoidance *Walking on mud*	$d_{1,2}: +20\% \star$	$d_{1,2}: +20\%$, $d_{2,3}: +20\% \star$	$d_{1,2}: -20\%$, $d_{3,4}: -20\%$, $d_{3,6}: -20\% \star$
			
Step climbing	$d_{3,6}: +16\% \star$	$d_{2,3}: -10\%$, $d_{3,6}: +20\%$ $d_{3,6}: +16\%$, $d_{7,8}: -20\% \star$ $d_{3,4}: +20\%$, $d_{7,8}: +20\%$	$d_{1,2}: +20\%$, $d_{2,3}: +20\%$, $d_{3,6}: +20\%$ $d_{2,3}: -14\%$, $d_{3,6}: +20\%$, $d_{7,8}: -20\% \star$ $d_{3,4}: +20\%$, $d_{5,7}: +20\%$, $d_{7,8}: +20\%$
			
Drilling motion	$d_{7,8}: +20\% \star$	$d_{1,2}: -8\%$, $d_{5,7}: +20\%$ $d_{2,3}: +16\%$, $d_{6,8}: -20\% \star$ $d_{6,8}: -20\%$, $d_{7,8}: +2\%$	$d_{1,2}: +10\%$, $d_{3,4}: +20\%$, $d_{7,8}: +20\%$ $d_{2,3}: -20\%$, $d_{6,8}: -20\%$, $d_{7,8}: +4\%$ $d_{1,2}: +10\%$, $d_{7,8}: +20\%$, $\triangle P_1P_4P_5: -20\% \star$
			

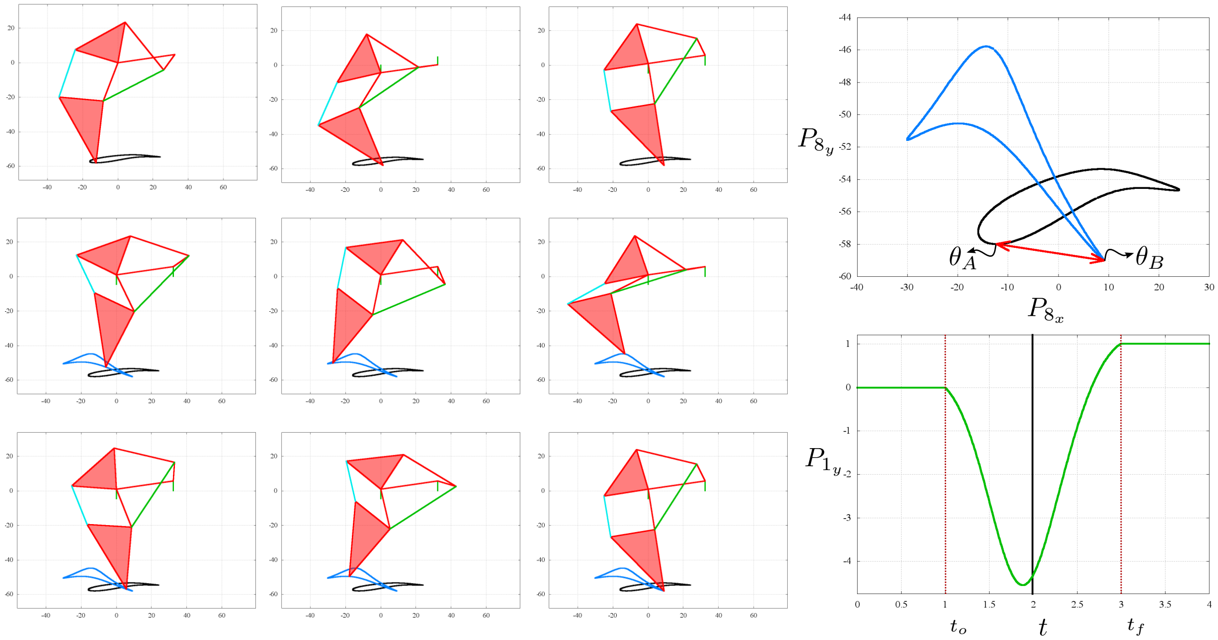


Fig. 4. Transformation from digitigrade locomotion ($d_{5,7}$: $+20\%$) to step climbing ($d_{3,6}$: $+16\%$). **Left:** Some steps of a simulation of the leg transformation, the binary links of variable dimension and the vertical movement of the grounded revolute joint centers are highlighted in cyan and light green. The corresponding foot trajectories are also depicted –digitigrade locomotion in black and step climbing in blue. **Right-top:** The lowest values of P_{8_y} for each of the foot trajectories in the transformation are connected by an arrow. These values are used to determine the initial and final input angles, θ_A and θ_B respectively, in the transformation process. **Right-below:** The evolution of P_{1_y} from $t = t_o$ to $t = t_f$.

line motion resulting from or giving rise to a full rotation. When considering complex planar linkages of mobility one —*i.e.*, mechanisms with coupler curves of order much higher than six, the degree of a four-bar linkage coupler curve, such standard concept should be extended because overlapping motions different to a straight line can be obtained in specific ranges of the input joint. An example of this kind of reciprocating motion in a double butterfly linkage, a mechanism whose coupler curves can reach order 48, is presented in [14]. Observe that this behavior also occurs in a standard Jansen leg as it can be verified in the sampled curves presented in Fig. 3(top).

Fifth row of Table I presents the modifications in the link dimensions of a standard Jansen leg that yield foot trajectories with reciprocating characteristics. These curves, product of increasing 20% the distance between joint centers P_7 and P_8, are of interest because their needle-like shapes can be used for drilling activities. With this resulting intra-reconfigurability characteristic we go one step further because it shows that by varying the link dimensions of a standard Jansen leg we can, in addition to modify the gait patterns of the walking platform, change the behaviour of the system. In this case, from a walker to a driller.

IV. Characterization of Leg Transformation

We have shown so far that by changing the link dimensions of a standard Jansen leg, a variety of gait patterns of interest for nested reconfiguration applications can be identified, i.e., digitigrade locomotion, obstacle avoidance, jam avoidance, step climbing, and drilling motion. An important design challenge is how to perform a proper transformation between gait patterns. By proper we mean, for example, that undesired floor contacts must be avoided during the transformation process. Note that the answer to this question has implications in the control and design of the proposed Jansen leg with variable link dimensions.

Following the above discussion, we have devised a simple procedure for transforming a reconfigurable Jansen leg from a pattern A to a pattern B, where both A and B are different and belong to the set of patterns {plantigrade locomotion, digitigrade locomotion, obstacle avoidance, jam avoidance, step climbing, and drilling motion} as described in section III. As a proof of concept, we consider the transformation from digitigrade locomotion ($d_{5,7}$: $+20\%$) to step climbing ($d_{3,6}$: $+16\%$). The proposed method is as follows:

1) For pattern A, from the current location of the grounded revolute joint centers P_1 and P_2, determine the lowest value of P_{8_y} and the corresponding input angle θ, say θ_A, using equation (10) with increases of θ at a specified rate. For our transformation example from digitigrade locomotion to step climbing, with $P_1 = (0, 0)^T$, $P_2 = (32.436, 4.632)^T$, and increments of $\frac{1}{100}$ for θ, we get $\theta_A = 5.48$ rad [Fig. 4(right-top)].
2) Repeat step 1 for pattern B. In our example, $\theta_B = 1.27$ rad [Fig. 4(right-top)].
3) Define the transformation time values. The transformation starts at time $t = t_o$ and finalizes at time $t = t_f$. Transformation time $\Delta t = t_f - t_o$. For the case study, $\Delta t = 2$ with $t_o = 1$ and $t_f = 3$ [Fig. 4(right-bottom)].
4) From patterns A and B, determine the link dimensions that have to be changed from a l_o value at time $t = t_o$

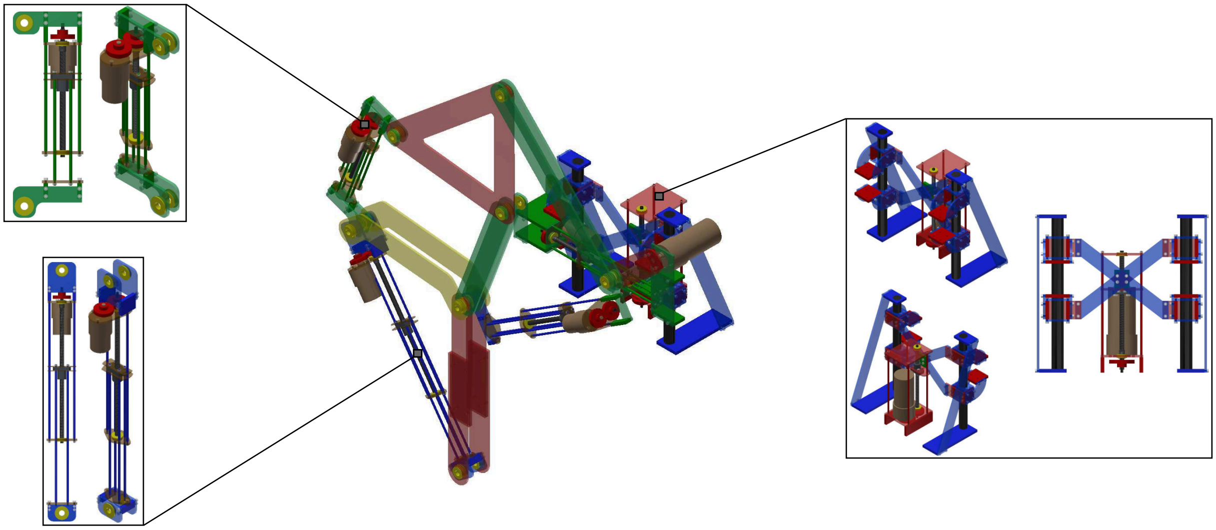


Fig. 5. A fully-functional reconfigurable Jansen leg with four actuators suitable for the transformation procedure presented in section IV ("single link" column) **(center)**. Zoomed areas present details of the grounded system **(right)** and some of the links of variable dimension **(left)**.

(pattern A) to a l_f value at time $t = t_f$ (pattern B). That is, in the case of our example:

Link dimension	$\mathbf{l_o}$	$\mathbf{l_f}$
$d_{5,7}$	28.7208 (+20%)	23.9340 (0%)
$d_{3,6}$	38.3220 (0%)	44.4535 (+16%)

5) Start the transformation, $t = t_o$. Set the input angle $\theta = \theta_A$ and determine the current location of the foot, say P_{8c}, from the values compute in step 1.
6) For each of the link dimensions determined in step 4, compute $l_c = \frac{l_f - l_o}{\Delta t}(t - t_o) + l_o$, where l_c is the current length of the corresponding variable link. Now, compute $\theta_c = \frac{\theta_B - \theta_A}{\Delta t}(t - t_o) + \theta_A$, where θ_c is the current value of the input angle.
7) From the fixed link dimensions, the corresponding l_c values of the variable links, the input angle θ_c, and the current locations of the grounded revolute joint centers P_1 and P_2, compute the new foot location, say P_{8n}, using equation (10).
8) Compute the offset $\Delta y = P_{8n_y} - P_{8c_y}$ and set $P_{8c} = P_{8n}$.
9) Update the vertical position of the grounded revolute joint centers P_1 and P_2 by subtracting Δy from both ordinates. For the case study, the evolution of P_{1_y} from $t = t_o$ to $t = t_f$ is depicted in Fig. 4(right-bottom).
10) Increase time t at a specified rate δt, that is, $t = t + \delta t$. In our example, $\delta t = \frac{1}{100}$.
11) Repeat steps 6 to 9 until $t = t_f$.

Fig. 4(left) shows some steps of a simulation of the leg transformation from digitigrade locomotion ($d_{5,7}$: +20%) to step climbing ($d_{3,6}$: +16%). The binary links of variable dimension and the vertical movement of the grounded revolute joint centers are highlighted in cyan and light green. As a complement, Fig. 5 presents the complete CAD design of a fully-functional reconfigurable Jansen leg with four actuators suitable for the suggested transformation procedure. In this design we consider transformations of patterns by changing the lengths of single links —*i.e.*, link combinations presented in column "single link" of Table I. Note that for such transformations only four length variables are needed, namely, $d_{1,2}$, $d_{3,6}$, $d_{5,7}$, and $d_{7,8}$. The accompanying video shows the simulation and CAD animation of four leg transformations, namely, i) from plantigrade locomotion to obstacle avoidance, ii) from obstacle avoidance to step climbing, iii) from step climbing to jam avoidance, and iv) from jam avoidance to drilling motion. The details of the proposed design as well as its corresponding implementation are out of the scope of the present report and are discussed elsewhere.

V. Conclusions

An original design approach has been presented in this paper to achieve adaptive gait patterns in legged robots for which Jansen linkage form the core. A planar mechanism of mobility one that is highly efficient and widely adopted in walking platforms has been modified based on the reconfiguration principle of variable allocation of joint positions. We have discussed novel approaches to address the position analysis problem and to characterize leg transformation in this reconfigurable design. Five gait patterns of interest have been identified, analyzed and discussed in relation to potential future applications. These exemplary gait variations considerably extend the capabilities of the original design not only to produce novel gait patterns but also to realize behaviors beyond locomotion. Finally, a fully-functional design is presented which enables all single-link transformations. Such reconfigurable linkage switches from a pin-jointed Grübler kinematic chain to a mechanism of mobility five with slider joints during the reconfiguration process.

Future work will include the online generation of gait patterns based on target evaluation functions. A four-legged robot is currently being assembled with these intra-reconfigurable legs to test different reconfiguration scenarios and control strategies for limb specialization and graceful degradation. Our long-term aim is to develop systematic methods to design nested-reconfigurable robots capable of transforming their intra- as well as inter- morphologies in response to needs associated with environment, task, or failures.

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