CAP 5993/CAP 4993
Game Theory

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Schedule

- HW4 due today.
- Project presentations on 4/18 and 4/20.
- Project writeup due 4/20.
- Final exam on 4/25.

Projects

- Presentations: 10 mins each + 2 mins for questions
  - 3 presentations on 4/18, I will be giving final lecture in first half
  - 6 presentations on 4/20
- Send titles and abstracts to me tonight
- Project (presentation + paper) worth 25% of grade. No specified subdivision between presentation and paper, but roughly 25% for presentation and 75% for paper.
- Prepare presentations in powerpoint, convert to pdf, and send me pdf file before the lecture of your presentation.
- Papers have a 10-page maximum limit
- Suggested format is AAMAS pdf: http://www.aamas2017.org/submission-instructions_aamas2017.php.

Auction summary

- Open-bid ascending auction (English auction)
  - Lowering hand when price reaches private valuation is equilibrium
- Open-bid descending auction (Dutch auction)
  - Strategically equivalent to sealed-bid first-price
- Sealed-bid first-price auction
  - Strategically equivalent to open-bid descending
- Sealed-bid second-price auction (Vickrey auction)
  - Truthfully bidding private valuation is equilibrium
  - Strategically similar to ebay auction (proxy bidding)

- Corollary: In the example, all four auction methods presented, the sealed-bid first-price auction, sealed-bid second price auction, open-bid ascending auction, and open-bid descending auction yield the seller the same expected revenue in equilibrium.

- This result is generalized in the Revenue Equivalence Theorem.

- A committee composed of 21 people needs to select one individual from among three candidates named A, B, and C. The committee members' preferences are given in the following table. Which candidate will/should be chosen?

No. committee members	First choice	Second choice	Third choice
1	A	B	C
7	A	C	B
7	B	C	A
6	C	B	A

- “Condorcet winner”: candidate who would defeat every other candidate by majority vote in a head-to-head competition.
- Is there a Condorcet winner in this game?

No. committee members	First choice	Second choice	Third choice
1	A	B	C
7	A	C	B
7	B	C	A
6	C	B	A

- Candidate C is a Condorcet winner. He would defeat A by a vote of 13 to 8 if the two of them were the sole candidates, and similarly would win by 13 votes to 8 votes against B.

No. committee members	First choice	Second choice	Third choice
1	A	B	C
7	A	C	B
7	B	C	A
6	C	B	A

- Who is the Condorcet winner now?

No. committee members	First choice	Second choice	Third choice
23	A	B	C
2	B	A	C
17	B	C	A
10	C	A	B
8	C	B	A

- Trick question, there isn't one!
- A defeats B by 33-27, B trounces C by 42-18, and C wins against A by 35-25.

No. committee members	First choice	Second choice	Third choice
23	A	B	C
2	B	A	C
17	B	C	A
10	C	A	B
8	C	B	A

- So there is no Condorcet winner: the preference ordering given by pairwise majority voting is not transitive. This has several implications. First, the order in which voting between candidates is conducted can affect the result – that is, if we first pit two candidates against each other and then have the winner between them compete against the third candidate, the order in which this is done may be crucial. If A and B compete head-to-head with the winner going up against C, then C will be selected. But if we first have A and C compete against each other with the winner squaring off against B, then B ends up being selected. And if B competes against C with the winner between them pitted against A, then A is the ultimate selection. The order of voting is absolutely crucial.

- Another consequence of the fact that the preference relation is not transitive is that in any voting method that generalizes majority vote between two candidates to a greater number of candidates, the results may depend on the presence or absence of a candidate who is not even the winner! For example, suppose that a certain voting method, which for two candidates chooses the winner by majority vote, leads to the selection of A. If B were to decline to participate, A and C would instead compete directly against each other – and then C would win by majority vote. In other words, B’s presence as a candidate can affect the results, even though B does not win when he competes. A similar phenomenon would exist if the voting method were to select B or C.

- The condition that the presence or absence of a candidate that is not selected by the procedure should not affect the results of a voting method is called the “independence of irrelevant alternatives.” The above example shows that it is not true that every voting system that in the presence of two candidates chooses one of them using majority vote satisfies independence of irrelevant alternatives.

- Another problem with pairwise majority voting is that even when a Condorcet winner exists, it is not always clear that he is the candidate who should be selected. We can check whether or not several well-known voting methods select the Condorcet winner when such a candidate exists. One popular method is to have each committee member vote for his or her most-preferred candidate, with the candidate receiving the most votes winning. If this method were to be adopted in the above example, then A would receive 8 votes, B would get 7 votes, and C only 6 votes, leading to the selection of candidate A and not the Condorcet winner C.
  - Used by many committees, including committees selecting candidates for public service positions in Britain.

- Another method chooses the winning candidate in a two-round process: in round one, every committee member votes for his or her most-preferred candidates. The two candidates who received the greatest number of votes in round one go on to compete against each other in round two, with the candidate garnering the most votes in round two ultimately selected as the winner. In the first example this method would lead to candidates A and B proceeding to a head-to-head competition in round two, where B would defeat A by 13 votes to 8; once again the Condorcet winner, candidate C, fails to be selected.
  - This is the method used to elect the President of France.

- Election results, therefore, are extremely sensitive to which voting method is adopted, and as we have seen, two very popular voting methods by-pass the Condorcet winner, when such a candidate exists, and may well end up selecting another candidate.

Manipulation

- In addition, the two methods discussed above can be subject to **manipulation**, in the sense that committee members have incentives to misrepresent their preferences in order to change the results. To see that, note that in the first example, under the voting system in which each committee member votes for only one candidate, and the candidate with the greatest number of votes is chosen, if the committee members who prefer C to B and B to A vote for B instead of C, then B, whom they prefer to A, will win instead of A. In the same example with a two-round voting method, if the committee members who rank A over C and C over B vote for C instead of A in the first round, then C and B will be the candidates competing in the second round, with C, whom they prefer to B, ultimately winning.

“Social choice” (voting)

- So far, the main question was choosing one alternative (e.g., candidate) from a set of alternatives (candidates). Suppose more generally that every individual in a given population has a preference ordering (or ranking) over a set of alternatives, and society in general seeks to derive, out of all the individual rankings, a single ranking representing society’s collective preferences among the alternatives: society’s first choice among the alternatives, society’s second choice, and so forth. In other words, the question before us is how to “aggregate” all the individual preference rankings into one preference ranking that can be interpreted as that of society’s.

Simple Majority Rule

- Suppose there are only two alternatives $A = \{a,b\}$. For each strict preference profile P^N we will denote the number of individuals who prefer a to b by $m(P^N)$. The simple majority rule is the social welfare function F defined by:
  - If $m(P^N) > n/2$ then society as a whole prefers a to b.
  - If $m(P^N) < n/2$ then society as a whole prefers b to a.
  - If $m(P^N) = n/2$ then society as a whole is indifferent between a and b.

Arrow's Impossibility Theorem

- Theorem [Arrow 1951]: If |A| >= 3, then every social welfare function satisfying the properties of unanimity and independence of irrelevant alternatives is dictatorial.
  - Unanimity: if all individuals in society prefer a to b, then society also prefers a to b.
  - IIA: Whether a is preferable to b depends only on the way individuals compare a to b. E.g., Ann ranked higher than Dan, then Tanya's grade changed (because she retook an exam), this should have no effect on relative ranking of Ann and Dan.
  - Dictatorship: a single voter has the power to always determine the group's preferences.

Gibbard-Satterthwaite Theorem

- Theorem [1973,1975]: Let G be a nonmanipulable social choice function satisfying the unanimity property. If |A| >= 3 then G is dictatorial.

- If we wish to apply a nondicatorial social choice function, there are necessarily situations in which one (or more) of the individuals has an incentive to report a preference relation that is different from his or her true preference relation.

Borda method

- Every voter ranks the candidates, from most preferred to least preferred. A candidate receives k points (called Borda points) from a voter if that voter ranks the candidate higher than exactly k other candidates. The Borda ranking of a candidate is given by the total number of Borda points he receives from all the voters. The winning candidate (called the Borda winner) is then the candidate who has amassed the most Borda points.

Range Voting

- **Range voting** or **score voting** is a voting method for single-seat elections, in which voters give each candidate a score, the scores are added (or averaged), and the candidate with the highest total is elected.

- Sports such as gymnastics rate competitors on a numeric scale, although the fact that judges' ratings are public makes it less likely for them to engage in blatant tactical voting.

- In most cases, ideal range voting strategy for well-informed voters is identical to ideal approval voting strategy, and a voter would want to give his least and most favorite candidates a minimum and a maximum score, respectively. If one candidate's backers engaged in this tactic and other candidates' backers cast sincere rankings for the full range of candidates, then the tactical voters would have a significant advantage over the rest of the electorate. When the population is large and there are two obvious and distinct front-runners, tactical voters seeking to maximize their influence on the result would give a maximum rating to their preferred candidate, and a minimum rating to the other front-runner; these voters would then give minimum and maximum scores to all other candidates so as to maximize expected utility.
  - **Approval voting** is a single-winner electoral system. Each voter may "approve" of (i.e., select) any number of candidates. The winner is the most-approved candidate.

- However, there are examples in which voting maximum and minimum scores for all candidates is *not* optimal. Exit poll experiments have shown that voters tend to vote more sincerely for candidates they perceive have no chance of winning. Thus range voting may yield higher support for third party and independent candidates, unless those candidates become viable, than other common voting methods, creating what has been called the “nursery effect.”

- Range voting advocates argue that range voting methods (including approval voting) give no reason to ever dishonestly rank a less-preferred candidate over a more-preferred one in 3-candidate elections. However, detractors respond that it provides motivation to rank a less-preferred and more-preferred candidate equally or near-equally (i.e., both 0-1 or both 98-99). This could lead to undemocratic results if different segments of the population used the strategy at significantly different rates. (Note that traditional first-past-the-post voting forces all candidates except one to be ranked equally, so that all voters are compressing their preferences equally.)

- Addressing these criticisms, the *Equal Vote Coalition*, a voting reform advocacy group, proposes a variant of range voting with an extra second round featuring the two top rated candidates in which the candidate with the majority of preference wins. It is claimed that the existence of a second round would discourage approval-style strategic ballots and exaggeration of ratings.