Math 1497 Calc 2

u- Substitution

$$\int_0^1 \frac{x\,dx}{x^2+1} \qquad u = x^2+1 \quad du = 2x\,dx$$

$$\text{so} \quad x\,dx = \frac{du}{2}$$

$x=0 \quad u = 0^2+1 = 1$

$x=1 \quad u = 1^2+1 = 2$

$$\int_1^2 \frac{1}{2}\frac{du}{u} = \frac{1}{2}\int_1^2 \frac{du}{u} = \frac{1}{2}\ln|u|\Big|_1^2 = \frac{1}{2}\ln|2|$$

$\because \ln|1| = 0$

2) Integration by Parts

Consider

$$u = xe^x \qquad du = (1\cdot e^x + xe^x)\,dx$$

or $d(xe^x) = 1\cdot e^x dx + x e^x dx$

Suppose I wanted to find

$$\int x e^x dx$$

would $d(xe^x) = e^x dx + x e^x dx$

be used ful?

Well $x e^x dx = d(xe^x) - e^x dx$

$$\int x e^x dx = \int d(xe^x) - \int e^x dx$$

$$= x e^x - e^x + C$$ so Yes!

Let consider this in general

$$d(uv) = u dv + v du$$

or $u dv = d(uv) - v du$

$$\int u\,dv = \int d(uv) - \int v\,du$$

so

$$\boxed{\int u\,dv = uv - \int v\,du}$$ ← integration by parts formula

Ex $\int x e^x\,dx$ already considered

Need to pick

$u = ?$

$dv = ?$

how choose

$u = x$ ↓ ↑ $dv = e^x dx$ so $u = x$ $\quad v = e^x$

$du = dx$ $\quad dv = e^x$

Form

$\Rightarrow uv - \int v\,du = x e^x - \int e^x dx = x e^x - e^x + C$ ✓

Ex 2 $\int x \sin x \, dx$

$u = x \qquad v = -\cos x$

$du = dx \qquad dv = \sin x \, dx$

$$= -x\cos x - \int -\cos x \, dx$$

$$= -x\cos x + \int \cos x \, dx$$

$$= -x\cos x + \sin x + c$$

Ex 3 $\int x \ln x \, dx$

$u = x$ ↑? Not obvious

$dv = \ln x \, dx$

try $u = \ln x \qquad v = x^2/2$

$du = \frac{1}{x} dx \qquad dv = x \, dx$

$$\Rightarrow \frac{x^2}{2}\ln x - \int \frac{x^2}{2}\cdot\frac{1}{x} dx = \frac{x^2}{2}\ln x - \frac{1}{2}\int x \, dx$$

$$= \frac{x^2}{2}\ln x - \frac{1}{2}\cdot\frac{x^2}{2} + c$$

Ex4 $\int \ln x \, dx$ only 1 term

$u = \ln x \qquad v = x$

$du = \frac{dx}{x} \qquad dv = 1 dx$

$= x \ln x - \int x \cdot \frac{1}{x} dx = x \ln x - \int 1 dx$

$= x \ln x - x + C$

Ex5 $\int x^2 e^{2x} dx \qquad u = x^2 \qquad v = e^{2x}/2$

$du = 2x dx \qquad dv = e^{2x} dx$

$= \frac{x^2 e^{2x}}{2} - \int \frac{e^{2x}}{2} 2x \, dx$

$= \frac{x^2 e^{2x}}{2} - \underbrace{\int x e^{2x} dx}_{\text{parts again!}}$

Ex6 $\int e^x \sin x \, dx$

$u = \sin x \qquad v = e^x$

$du = \cos x \, dx \qquad dv = e^x \, dx$

$$= \sin x \cdot e^x - \underbrace{\int e^x \cos x \, dx}_{\text{parts again}}$$

$u = \cos x \qquad v = e^x$

$du = -\sin x \, dx \qquad dv = e^x \, dx$

$$= e^x \sin x - \left[\cos x \cdot e^x - \int -\sin x \, e^x \, dx \right]$$

$$= e^x \sin x - e^x \cos x - \int e^x \sin x \, dx$$

$$2 \int e^x \sin x \, dx = e^x \sin x - e^x \cos x$$

$$\int e^x \sin x \, dx = \frac{1}{2} e^x \sin x - \frac{1}{2} e^x \cos x + C$$