Forecast:

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The Relaxation Times

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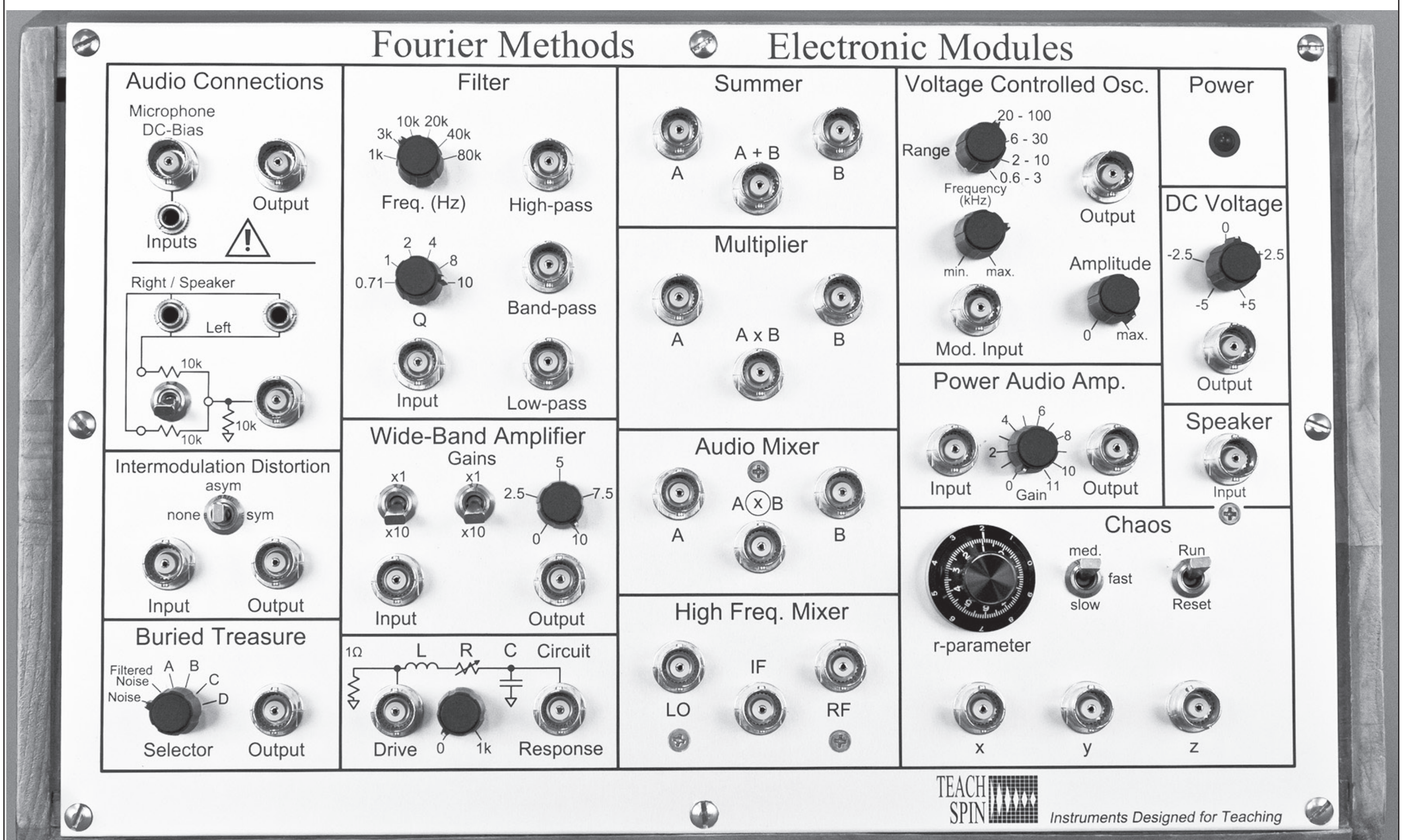
VOL. IV No. 6 *NEWSLETTER OF TEACHSPIN, INC.* *February 2014*

Fourier Methods: More Tools !



You might recall TeachSpin's "Fourier Methods" offering, featured in a previous newsletter; it is a set of experiments centered around the SR770 fast-Fourier-transform analyzer from Stanford Research Systems (www.teachspin.com, under Newsletters, HatTrick). We previously touched on the three 'physics packages' that come with the Fourier-Methods instrument: an acoustic resonator, a coupled-oscillator system, and a fluxgate magnetometer. But in this issue we'll focus on the capabilities of the **'Electronic Modules'** that are also a part of the Fourier Methods instrument package. The Electronic Modules, shown in the figure above, puts at the students' fingertips all sorts of handy modules that are needed for experiments and demonstrations of Fourier Methods.

What are all those modules for? *They are designed for processing electrical signals in ways that illustrate the power of Fourier thinking: the ability to see and understand a physical system in both the time, and the frequency, domains.* In the three examples that follow, we'll show you diagnostic signals in the time domain, as shown on an oscilloscope capture, and in the frequency domain, as revealed by the SR770.

Example 1: Modulation

Every textbook shows a picture like Fig. 1, an 'amplitude modulated sinusoid'. This signal was created using the following:

- 10kHz source from the SR770 (carrier wave)
- A separate signal generator for 400Hz modulation
- Modules: Summer, Multiplier, DC offset

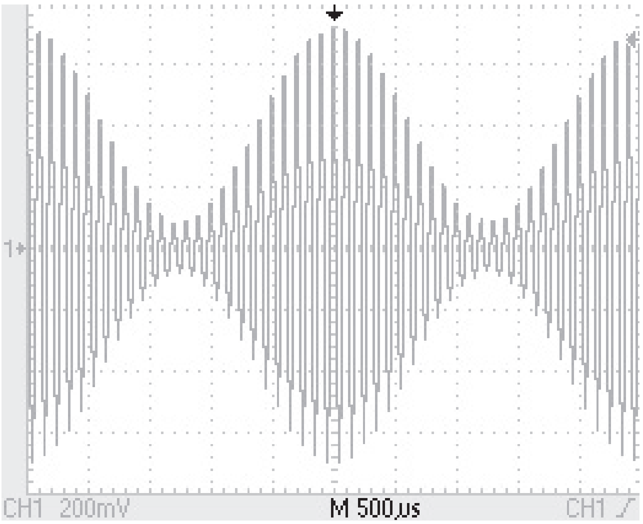


Fig. 1: An amplitude-modulated 10-kHz waveform

If the scope capture shown in Figure 1 gives the time-domain view, what is the corresponding frequency domain spectrum? The SR770 gives that in exquisite detail, shown in Figure 2. The SR770 has been formatted for high frequency resolution in the region around 10kHz and the vertical scale is logarithmic, extending over 4 decades (80 dB) of dynamic range. The frequency-domain description of this waveform is a 'carrier wave' at 10 kHz, plus two 'sidebands', here at 10.4 and 9.6 kHz. Users of the FM package can see, in detail, how this view changes in real time as they adjust frequency, or amplitude (or waveform!) of either generator.

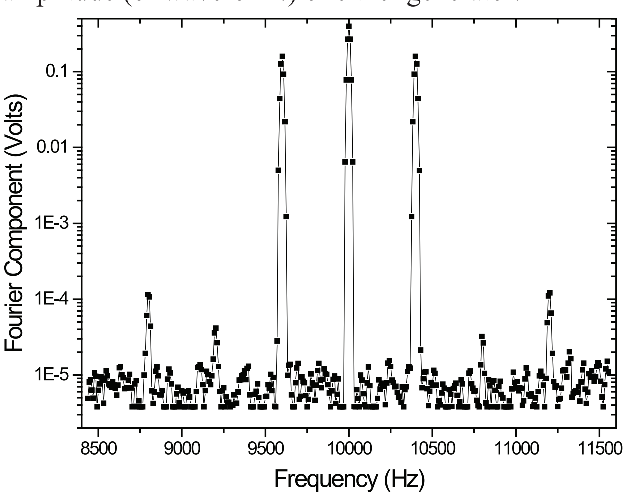


Fig. 2: The Fourier spectrum of the waveform of Fig. 1

But what about a simpler waveform, the mere real-time multiplication of the carrier wave sinusoid and the modulating sinusoid? That too has a (subtly different) time-domain representation (Fig. 3), and it too has a frequency-domain spectrum (Fig. 4). What's the novelty? You still get the sidebands, but the carrier is *missing*. This would be called a 'double-sideband, suppressed-carrier' waveform in communications parlance.

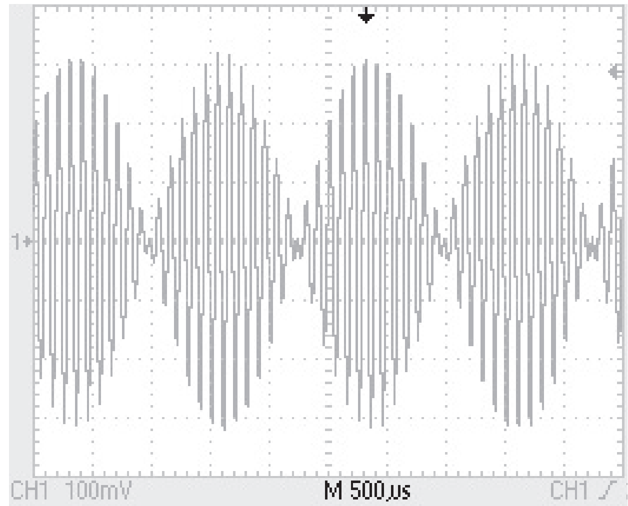


Fig. 3: The waveform of a simple *product* of two sinusoids

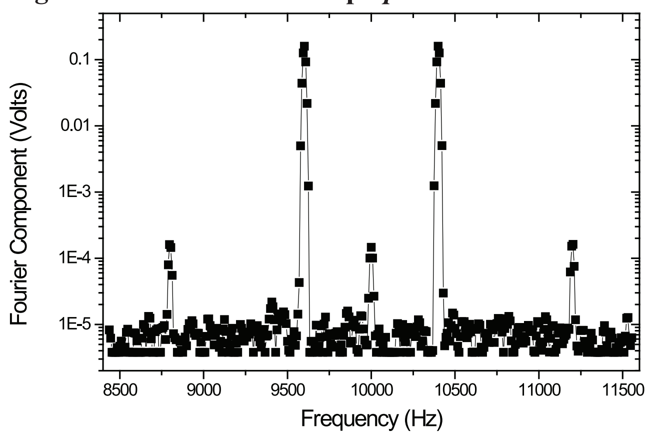


Fig. 4: The Fourier spectrum of the waveform of Fig. 3

The Fourier Methods curriculum discusses the concept of modulation in detail, and shows its connection to 'frequency mixing' in general. In fact there's a Module dedicated to Intermodulation Distortion, showing that **any** non-linear system, when driven by two frequencies, generates a plethora of new frequencies, the sum-frequency and the difference-frequency among them.

Example 2: Chaos

One of the biggest discoveries in classical physics in the last half-century is the phenomenon of chaos, the emergence of solutions of fully-deterministic differential equations which nevertheless show exponentially-decaying predictability into the future. A famous case of this phenomenon, the Lorenz Attractor, is featured among our Electronic Modules. It is an all-analog realization of the very system of differential equations in which Edward Lorenz discovered the phenomenon. From our Module, there emerge three voltages, directly proportional to the *x-, y-,* and *z*-functions of the Lorenz system, with the system's characteristic *r*-parameter variable by student control of a 10-turn knob.

A time-domain view of the voltages $V_x(t)$ and $V_y(t)$ appears in Fig. 5, showing the non-periodic, though fully deterministic, evolution of these signals. A parametric plot of V_z vs. V_x appears as the 'butterfly diagram' in Fig. 6. Both of these can be studied in detail in the time domain, but what is the *spectral* content of such waveforms? The SR770 makes it really easy to see. Non-periodic behavior results in a **continuous** spectrum, rather than a **line** spectrum, in the frequency domain. Fig. 7 shows a spectrum accumulated with a bit of time-averaging. That 'dip' in the spectrum near 250Hz we believe to be real, although we have not seen it recognized in the literature.

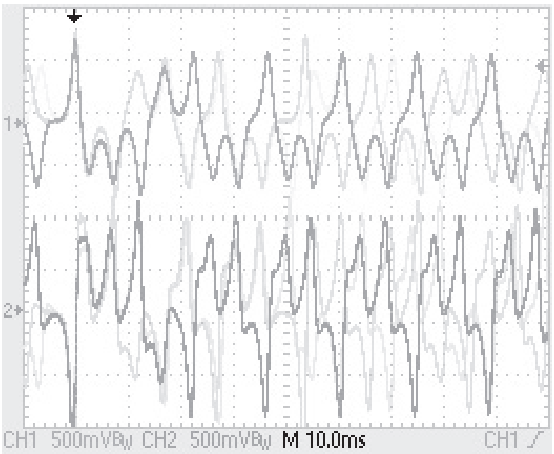


Fig. 5: $V_x(t)$ and $V_y(t)$ waveforms from a Lorenz attractor

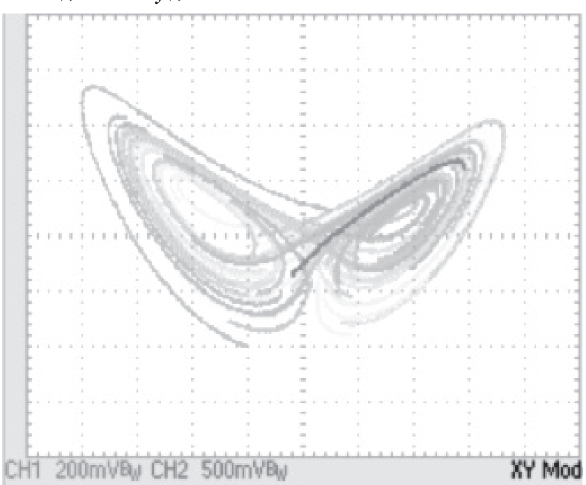


Fig. 6: V_z (up) and V_x (across) in a parametric plot of chaos

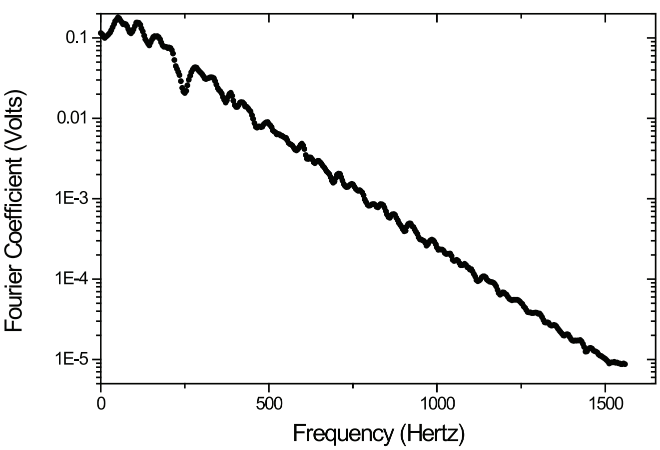


Fig. 7: The Fourier spectrum of Fig. 5's chaotic waveform

But there are settings of the *r*-parameter which put the Lorenz attractor into a 'periodic regime', and then the spectrum collapses to a line spectrum, composed of a fundamental plus its harmonics. See Fig. 8 for a view of the spectrum in this case, and note how high the peaks stand above the noise floor. If you know about the 'period-doubling bifurcation', you know that tiny adjustments of the r-parameter turn successive cycles-in-time from the AAA… pattern to an ABAB… pattern, which thus has double the period. It follows that a new fundamental frequency, at ***half*** the former fundamental frequency, must arise – and this is revealed, with exquisite sensitivity, by the SR770, since it occurs against a negligible background in the frequency domain. The same bifurcation can be **in**visible on a 'scope, so small are the changes involved.

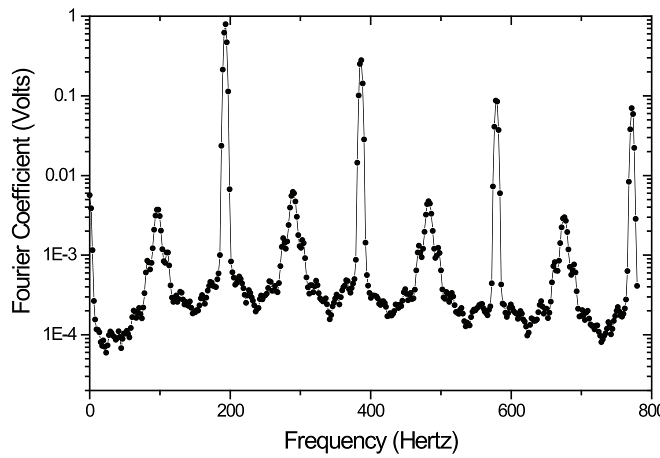


Fig. 8: Spectrum of attractor in a 'periodic window' -- note the onset of a sub-harmonic spectral line

The point is that the frequency domain can be the place in which subtle changes are most easily detected. This is one of the many advantages of Fourier-transform spectroscopy.

Example 3: Filtering

Consider using the Modules to form an example of a signal corrupted by interference (Figure 9). It's a superposition (made in the Summer) of a 3.3-kHz sinusoid of 70mV amplitude from a generator, plus some 'interference', derived by electrostatic pick-up from a paper-clip antenna, amplified 10^3-fold in the Wide-Band Amplifier Module. Maybe you can still see the signal on the 'scope trace, but you surely couldn't trigger on it.

The frequency-domain view of the same waveform on the SR770 (Fig. 10, using a log-frequency scale) reveals that the 'interference' is actually highly structured, consisting of lots of 45-kHz content and its harmonics. We blame this on the fluorescent light fixtures in our lab. That frequency-domain knowledge suggests the use of the Filter module to separate the signal from the interference. If we set the Filter to serve as a bandpass design, with a Q of 10 at its center frequency of 3.3 kHz, it offers a gain of 10 for the signal, but a 'gain' below $3.3/45 \approx 0.07$ for the interference.

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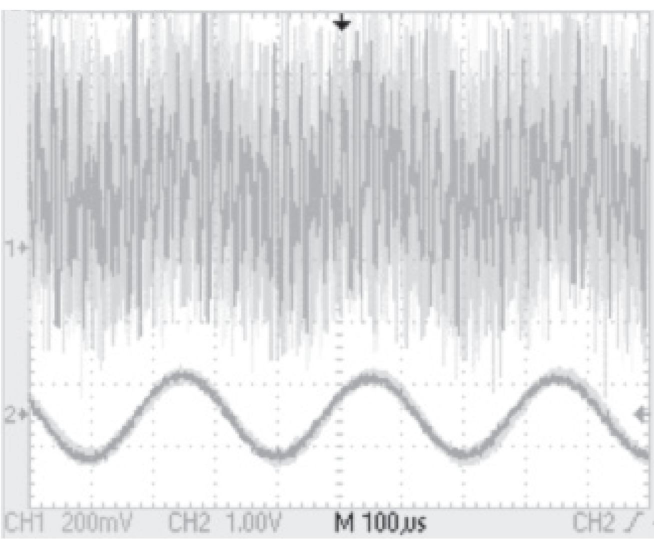

Fig. 9: 3.3-kHz sinusoidal signal, corrupted by interference

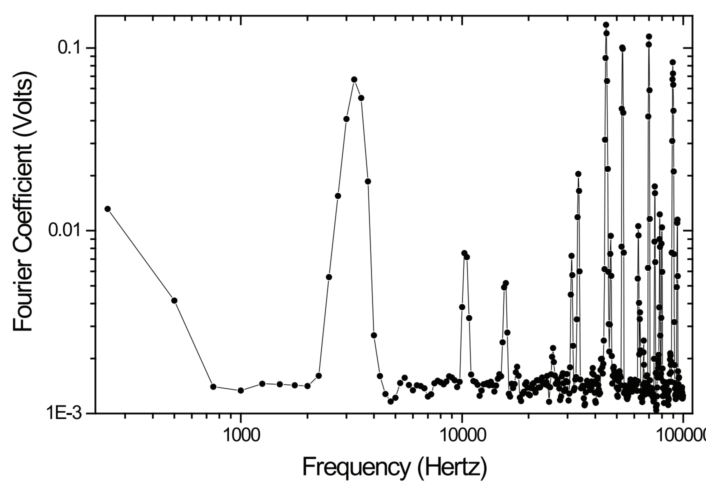

Fig. 10: The Fourier spectrum of a 3.3-kHz signal and the interference at 45-kHz (and harmonic)

The output of the filter has improved the signal-to-interference ratio by more than 10/0.07 or 140, and the lower trace in Fig. 9 shows the result – a signal beautifully 'cleansed' of its interference.

It's a frequency-domain view of the signal, with and without the filter, which enables students to understand why this works. And once they do, they can add genuine noise, ie. stochastic noise with a continuous spectrum, to the signal, using the 'Buried Treasure' module in place of the interference contamination illustrated above. Then they'll see to what extent the Filter module can still offer advantages in the output signal-to-noise ratio.