Math 3331 ODEs

Now we start to analytically solve 1st order ODEs. Probably the simplest are ODEs of the form

$$\frac{dy}{dx} = f(x)\,g(y)$$

As the left hand side is a product "$f(x) * g(y)$" these ODEs are referred to as "separable"!

If we treat $\frac{dy}{dx}$ as a fraction then we separate giving

$$\frac{dy}{g(y)} = f(x)\,dx$$

then integrate

$$\int \frac{dy}{g(y)} = \int f(x)\,dx + C$$

Note: we only need 1 $+C$ and will be set if IC/BC are given

Now we must remember those techniques of integration

Ex 1 $\quad \frac{dy}{dx} = 2x$

Sep. $\quad dy = 2x\,dx \quad$ so $\quad \int dy = \int 2x\,dx + c$

$\Rightarrow \quad y = x^2 + c \quad$ sol^n

Ex 2 $\quad \frac{dy}{dx} = y$

Sep $\quad \frac{dy}{y} = dx \quad$ so $\quad \int \frac{dy}{y} = \int dx + c$

$\ln|y| = x + c$ ← can we solve for y - yes!

If we ~~can~~ we usually do. So

$e^{\ln|y|} = e^{x+c} \Rightarrow \quad y = e^x \cdot e^c = \bar{c}e^x$

where $\quad \bar{c} = e^c \quad$ then drop the bar

sol^n $\quad y = ce^x$

Note: if $\int \frac{dy}{y} = \int dx \Rightarrow \ln|y| = x + \ln c$ ← $\ln c$ instead of c

$\Rightarrow \quad y = ce^x$ right away.

ex 3 $\frac{dy}{dx} = 2xy^2 + y^2 + 2x + 1$ ← separable ???

Looks kinda complicated .... but

$$\frac{dy}{dx} = (2x+1)y^2 + (2x+1)$$

$$= (2x+1)(y^2+1) \quad \leftarrow \text{now sep}$$

$$\frac{dy}{1+y^2} = (2x+1)dx$$

$$\int \quad \tan^{-1} y = x^2 + x + c$$

$$y = \tan(x^2 + x + c)$$

suppose we are told solve

$$\frac{dy}{dx} = (2x+1)(y^2+1), \quad y(0) = 0$$

what does $y(0)=0$ do for us. It sets the c

so bring the IC/BC in any place

$$\tan^{-1} y = x^2 + x + c$$

$$y(0) = 0 \Rightarrow \tan^{-1}(0) = 0^2 + 0 + c \Rightarrow c = 0$$

sol^n

$$\boxed{y = \tan(x^2 + x)}$$

Ex 4 $\quad \frac{dy}{dx} = y(1-y) \quad$ Sep ✓

$$\frac{dy}{y(1-y)} = dx$$

Partial Fractions

$$\int \left(\frac{1}{y} + \frac{1}{1-y} \right) dy = \int dx$$

$$\ln|y| - \ln|1-y| = x + \ln c$$

$$\frac{y}{1-y} = ce^x$$

So $\quad y = ce^x(1-y) \Rightarrow y = ce^x - ce^x y$

$$y(1+ce^x) = ce^x$$

So $\quad y = \dfrac{ce^x}{1+ce^x} \qquad (*)$

IC. $\quad y(0)=0, \quad y(0)=1$

so (i) $y(0)=0$

then $0 = \frac{ce^0}{1+ce^0} = \frac{c}{1+c}$

So $c=0$ so $y \equiv 0$ always

we can see this from the ODE

$y' = y(1-y)$ $y=0$, $y'=0$

LS $=0$ RS $=0$ ✓

(ii) $y(0)=1$

then $1 = \frac{ce^0}{1+ce^0} \Rightarrow 1 = \frac{c}{1+c} \Rightarrow 1+c=c$

$\Rightarrow 1=0$?

but $y \equiv 1$, $y'=0$

$y' = y(1-y)$

y' = LS $=0$, RS $=0$ so $y \equiv 1$ is a

sol^n but why is it not in (*)