Edexcel GCE

Core Mathematics C3

Silver Level S3

(Mark Scheme)

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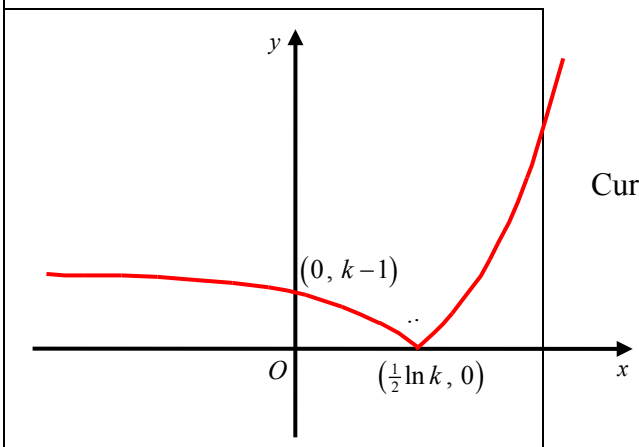
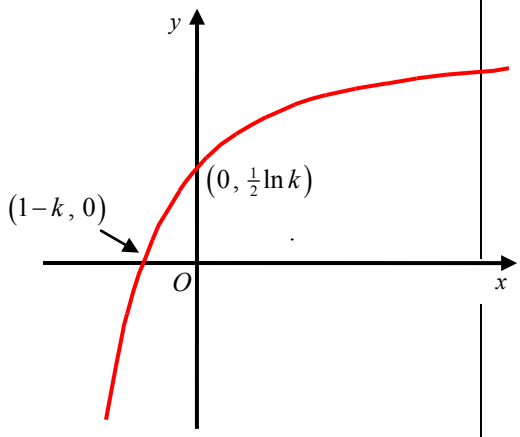
Question Number	Scheme	Marks
1. (a)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{(5x-1)}\right)=\frac{\mathrm{d}}{\mathrm{d}x}\left((5x-1)^{\frac{1}{2}}\right)$	
	$=5\times\frac{1}{2}(5x-1)^{-\frac{1}{2}}$	M1 A1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=2x\sqrt{(5x-1)}+\frac{5}{2}x^2(5x-1)^{-\frac{1}{2}}$	M1 A1ft
	At $x=2$, $\frac{\mathrm{d}y}{\mathrm{d}x}=4\sqrt{9}+\frac{10}{\sqrt{9}}=12+\frac{10}{3}$	M1
	$=\frac{46}{3}$ Accept awrt 15.3	A1 (6)
(b)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\sin 2x}{x^2}\right)=\frac{2x^2\cos 2x-2x\sin 2x}{x^4}$	M1, $\frac{\text{A1+A1}}{\text{A1}}$ (4) **[10]**
2.	$\mathrm{f}(x)=x^3+2x^2-3x-11$	
(a)	$\mathrm{f}(x)=0 \Rightarrow x^3+2x^2-3x-11=0$	M1
	$\Rightarrow x^2(x+2)-3x-11=0$	
	$\Rightarrow x^2(x+2)=3x+11$	
	$\Rightarrow x^2=\frac{3x+11}{x+2}$	
	$\Rightarrow x=\sqrt{\left(\frac{3x+11}{x+2}\right)}$	A1 **AG** (2)
(b)	Iterative formula: $x_{n+1}=\sqrt{\left(\frac{3x_n+11}{x_n+2}\right)}$, $x_1=0$	
	$x_2=\sqrt{\left(\frac{3(0)+11}{(0)+2}\right)}$	M1
	$x_2=2.34520788...$	A1
	$x_3=2.037324945...$	
	$x_4=2.058748112...$	A1 (3)
(c)	Let $\mathrm{f}(x)=x^3+2x^2-3x-11=0$	
	$\mathrm{f}(2.0565)=-0.013781637...$	M1
	$\mathrm{f}(2.0575)=0.0041401094...$	dM1
	Sign change (and $\mathrm{f}(x)$ is continuous) therefore a root α is such that $\alpha\in(2.0565, 2.0575)\Rightarrow\alpha=2.057$ (3 dp)	A1 (3) **[8]**

Question Number	Scheme	Marks
3. (a)	$\dfrac{4x-1}{2(x-1)} - \dfrac{3}{2(x-1)(2x-1)}$	
	$= \dfrac{(4x-1)(2x-1)-3}{2(x-1)(2x-1)}$ An attempt to form a single fraction	M1
	$= \dfrac{8x^2-6x-2}{\{2(x-1)(2x-1)\}}$ Simplifies to give a correct quadratic numerator over a correct quadratic denominator	A1 aef
	$= \dfrac{2(x-1)(4x+1)}{\{2(x-1)(2x-1)\}}$ An attempt to factorise a 3 term quadratic numerator	M1
	$= \dfrac{4x+1}{2x-1}$	A1 (4)
(b)	$f(x) = \dfrac{4x-1}{2(x-1)} - \dfrac{3}{2(x-1)(2x-1)} - 2, \quad x>1$	
	$f(x) = \dfrac{(4x+1)}{(2x-1)} - 2$	
	$= \dfrac{(4x+1)-2(2x-1)}{(2x-1)}$ An attempt to form a single fraction	M1
	$= \dfrac{4x+1-4x+2}{(2x-1)}$	
	$= \dfrac{3}{(2x-1)}$ Correct result	A1 * (2)
(c)	$f(x) = \dfrac{3}{(2x-1)} = 3(2x-1)^{-1}$	
	$f'(x) = 3(-1)(2x-1)^{-2}(2)$ $\pm k(2x-1)^{-2}$	M1 A1 aef
	$f'(2) = \dfrac{-6}{9} = -\dfrac{2}{3}$ Either $\dfrac{-6}{9}$ or $-\dfrac{2}{3}$	A1
		(3) **[9]**

Question Number	Scheme	Marks
4. (a)	$7\cos x + \sin x = R\cos(x-\alpha)$	
	$R = \sqrt{(7^2+1^2)} = \sqrt{50} = (5\sqrt{2})$	B1
	$\alpha = \arctan\left(\frac{1}{7}\right) = 8.13... = \text{awrt } 8.1^0$	M1A1
		(3)
(b)	$\sqrt{50}\cos(x-8.1) = 5 \Rightarrow \cos(x-8.1) = \frac{5}{\sqrt{50}}$	M1
	$x - 8.1 = 45 \Rightarrow x = 53.1^0$	M1A1
	AND $x - 8.1 = 315 \Rightarrow x = 323.1^0$	M1A1
		(5)
(c)	One solution if $\frac{k}{\sqrt{50}} = \pm 1, \Rightarrow k = \pm\sqrt{50}$ ft on R	M1 A1ft
		(2) **[10]**
5. (a)	$y = 4 - \ln(x+2)$	
	$\ln(x+2) = 4 - y$	
	$x + 2 = e^{4-y}$	
	$x = e^{4-y} - 2$	M1
	$\text{f}^{-1}(x) = e^{4-x} - 2$ oe	M1A1
		(3)
(b)	$x \leq 4$	B1
		(1)
(c)	$fg(x) = 4 - \ln(e^{x^2} - 2 + 2)$	M1
	$fg(x) = 4 - x^2$	dM1A1
		(3)
(d)	$fg(x) \leq 4$	B1ft
		(1)
		[10]

Question Number	Scheme	Marks
6. (a)	 Curve retains shape when $x > \frac{1}{2}\ln k$ Curve reflects through the x-axis when $x < \frac{1}{2}\ln k$ $(0, k-1)$ and $(\frac{1}{2}\ln k, 0)$ marked in the correct positions.	B1 B1 B1 (3)
(b)	 Correct shape of curve. The curve should be contained in quadrants 1, 2 and 3 (Ignore asymptote) $(1-k, 0)$ and $(0, \frac{1}{2}\ln k)$	B1 B1 (2)
(c)	Range of f: $\underline{f(x) > -k}$ or $\underline{y > -k}$ or $\underline{(-k, \infty)}$	B1 (1)
(d)	$y = e^{2x} - k \Rightarrow y + k = e^{2x}$ $\Rightarrow \ln(y+k) = 2x$ $\Rightarrow \frac{1}{2}\ln(y+k) = x$ Hence $f^{-1}(x) = \underline{\frac{1}{2}\ln(x+k)}$	M1 M1 A1 cao (3)
(e)	$f^{-1}(x)$: Domain: $\underline{x > -k}$ or $\underline{(-k, \infty)}$	B1ft (1) **[10]**

Question Number	Scheme	Marks
7. (a)	f(0.8) = 0.082, f(0.9)= –0.089	M1
	Change of sign ⇒ root (0.8,0.9)	A1 (2)
(b)	$f'(x) = 2x - 3 - \sin\left(\frac{1}{2}x\right)$	M1 A1
	Sets $f'(x) = 0 \Rightarrow x = \frac{3 + \sin\left(\frac{1}{2}x\right)}{2}$	M1A1* (4)
(c)	Sub $x_0 = 2$ into $x_{n+1} = \frac{3 + \sin\left(\frac{1}{2}x_n\right)}{2}$	M1
	x_1 = awrt 1.921, x_2 = awrt 1.91(0) and x_3 = awrt 1.908	A1, A1 (3)
(d)	[1.90775,1.90785]	M1
	f'(1.90775) = –0.00016... AND f'(1.90785) = 0.0000076...	M1
	Change of sign ⇒ x = 1.9078	A1 (3)
		[12]

Qu. No.	Scheme		Marks
8 (a)	$y = \dfrac{3 + \sin 2x}{2 + \cos 2x}$		
	Apply quotient rule: $\left\{\begin{array}{ll} u = 3 + \sin 2x & v = 2 + \cos 2x \\ \dfrac{du}{dx} = 2\cos 2x & \dfrac{dv}{dx} = -2\sin 2x \end{array}\right\}$		
	$\dfrac{dy}{dx} = \dfrac{2\cos 2x(2 + \cos 2x) - -2\sin 2x(3 + \sin 2x)}{(2 + \cos 2x)^2}$	Applying $\frac{vu' - uv'}{v^2}$ Any one term correct on the numerator Fully correct (unsimplified).	M1 A1 A1
	$= \dfrac{4\cos 2x + 2\cos^2 2x + 6\sin 2x + 2\sin^2 2x}{(2 + \cos 2x)^2}$		
	$= \dfrac{4\cos 2x + 6\sin 2x + 2(\cos^2 2x + \sin^2 2x)}{(2 + \cos 2x)^2}$	For correct proof with an understanding that $\cos^2 2x + \sin^2 2x = 1$.	
	$= \dfrac{4\cos 2x + 6\sin 2x + 2}{(2 + \cos 2x)^2}$ (as required)	No errors seen in working.	A1*
			(4)
(b)	When $x = \frac{\pi}{2}$, $y = \dfrac{3 + \sin \pi}{2 + \cos \pi} = \dfrac{3}{1} = 3$	$y = 3$	B1
	At $\left(\frac{\pi}{2}, 3\right)$, $\text{m}(\mathbf{T}) = \dfrac{6\sin \pi + 4\cos \pi + 2}{(2 + \cos \pi)^2} = \dfrac{-4+2}{1^2} = -2$	$\text{m}(\mathbf{T}) = -2$	B1
	Either **T**: $y - 3 = -2(x - \frac{\pi}{2})$ or $y = -2x + c$ and $3 = -2\left(\frac{\pi}{2}\right) + c \Rightarrow c = 3 + \pi$;	$y - y_1 = m(x - \frac{\pi}{2})$ with 'their TANGENT gradient' and their y_1; or uses $y = mx + c$ with 'their TANGENT gradient';	M1
	T: $y = -2x + (\pi + 3)$	$y = -2x + \pi + 3$	A1
			(4) **[8]**

Statistics for C3 Practice Paper Silver Level S3

Qu	Max score	Modal score	Mean %	Mean score for students achieving grade: ALL	A*	A	B	C	D	E	U
1	10		79	7.90		9.61	8.96	8.07	7.08	6.06	3.01
2	9		81	7.27	8.89	8.40	7.80	7.05	6.11	5.42	3.88
3	8		84	6.68		7.32	6.75	5.96	5.65	4.67	3.39
4	10		74	7.35	9.23	8.18	7.34	6.17	5.35	3.30	1.27
5	8		65	5.19	7.21	6.37	5.46	4.56	3.64	2.59	1.47
6	10		67	6.72		8.37	6.89	5.89	4.85	3.75	2.30
7	12		72	8.58	11.48	10.31	9.14	8.12	6.66	5.44	2.78
8	8		66	5.24	7.44	6.98	6.08	5.20	3.94	2.72	1.13
	75		**73**	**54.93**		**65.54**	**58.42**	**51.02**	**43.28**	**33.95**	**19.23**