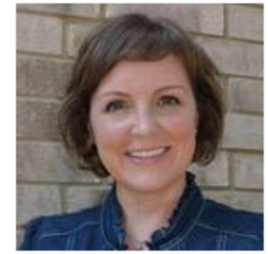
Explicit Instruction in Mathematics

Sarahpowellphd.com

Evidence-based mathematics resources for educators



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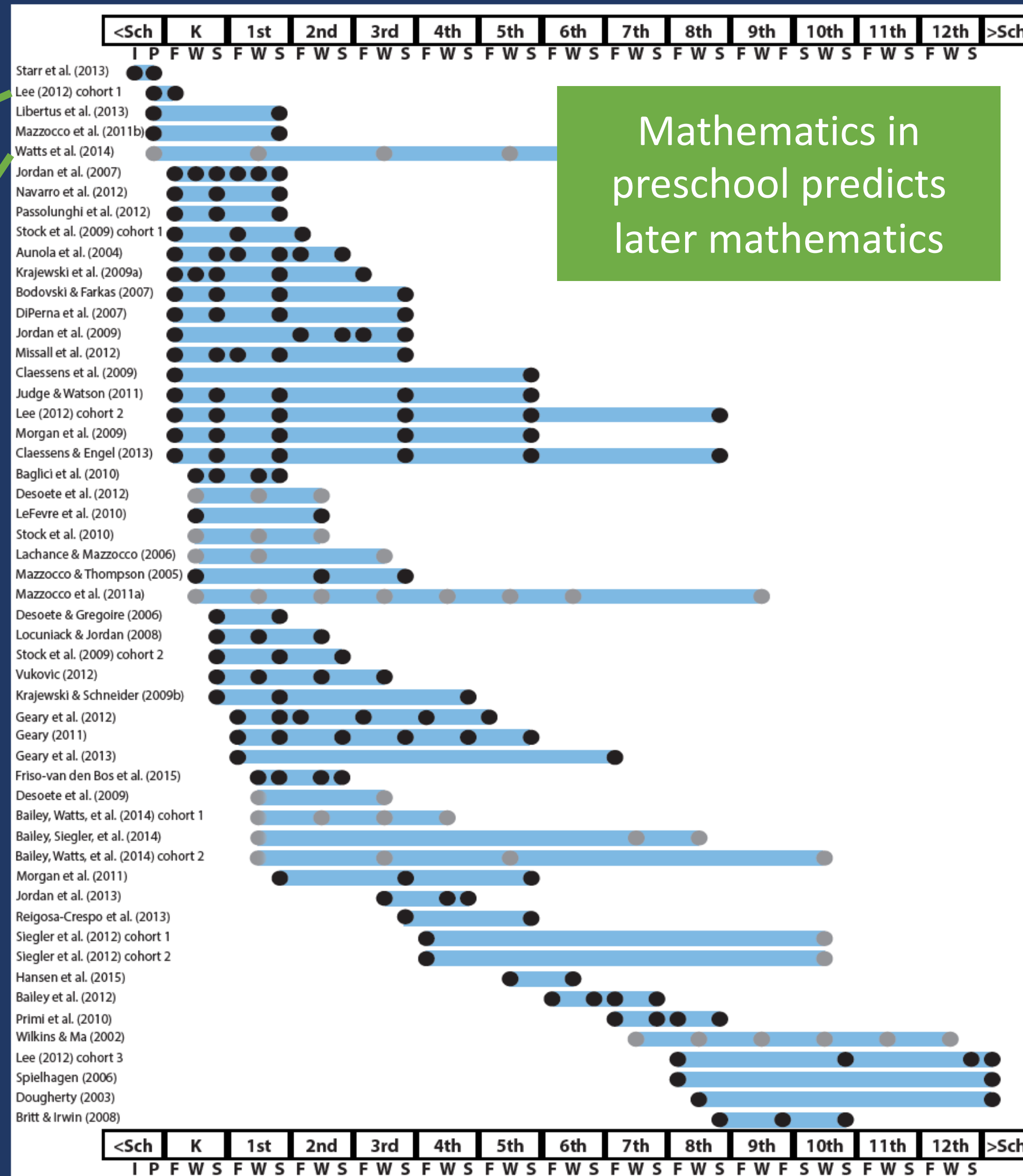
Broad math in preK predicted K broad math

Broad math in preK predicted grade 10 broad math

Mathematics in preschool predicts later mathematics

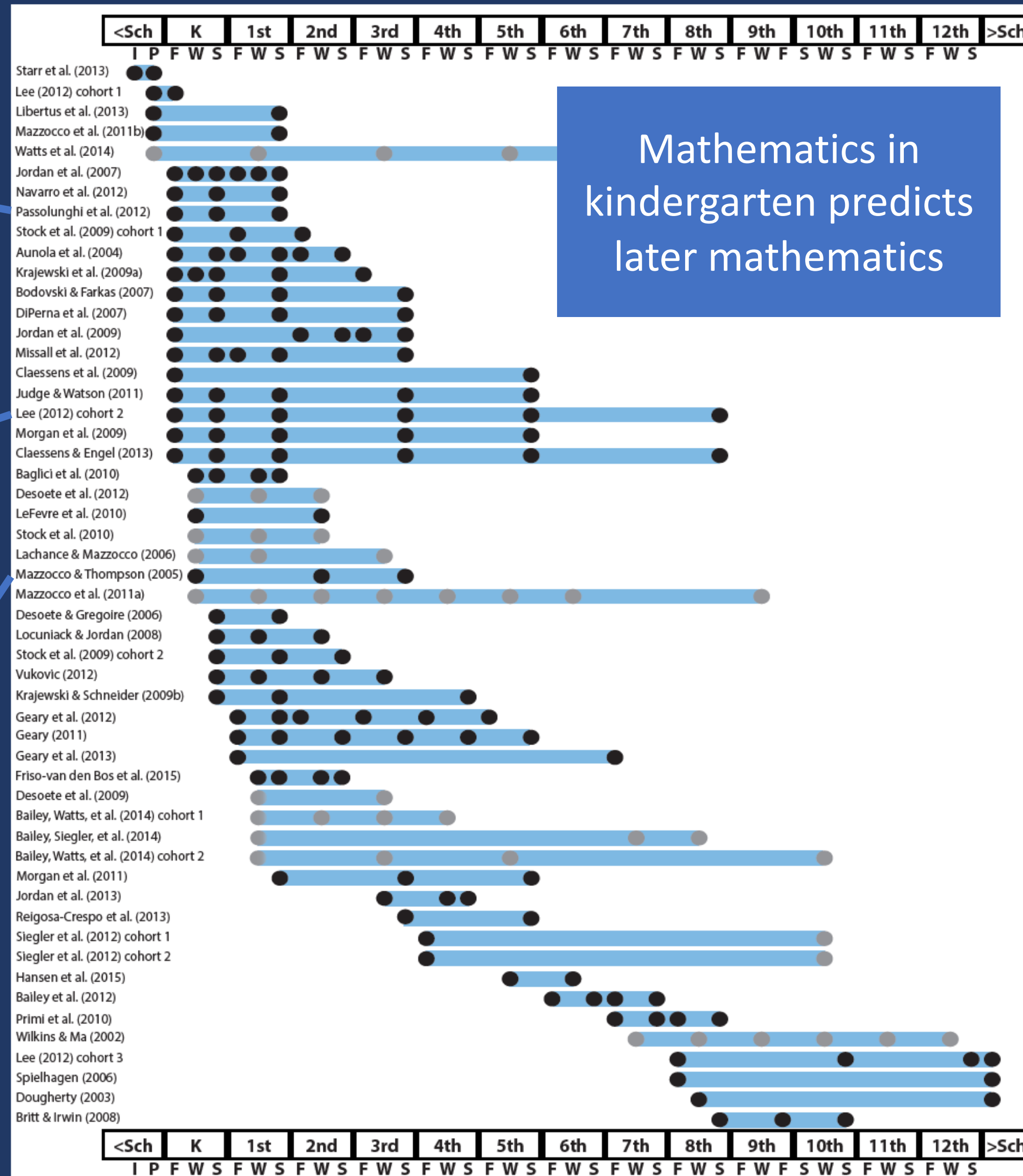


http://www.greatertexasfoundation.org/trajectories-of-mathematics-performance/

Counting in K predicted grade 1 broad math

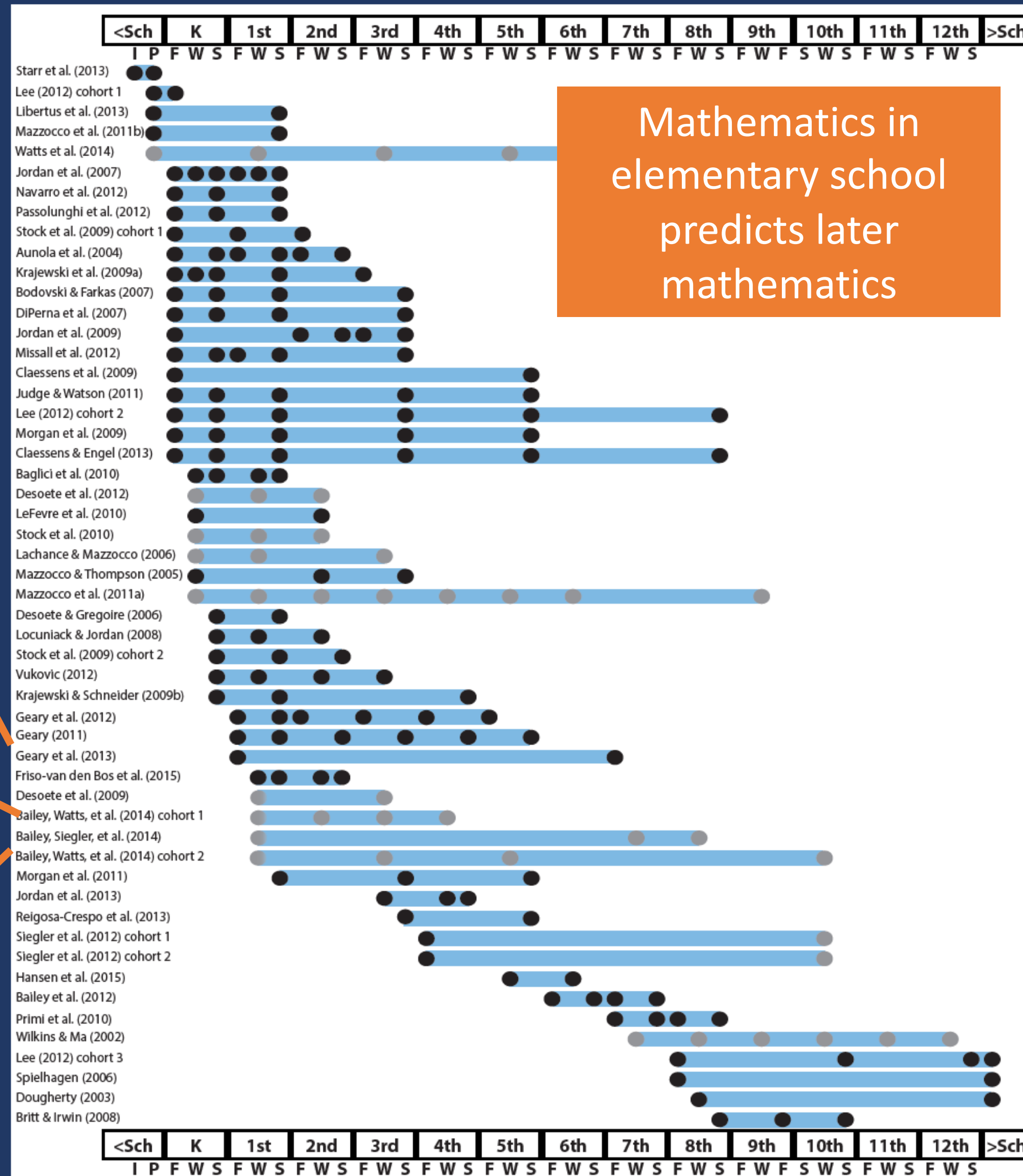
Broad math in K predicted grade 8 broad math

K math accurately predicted math performance below 10th percentile in grades 2 and 3 with 84% correct classification



http://www.greatertexasfoundation.org/trajectories-of-mathematics-performance/

Mathematics in elementary school predicts later mathematics



Addition influenced arithmetic with increasing importance from grades 1 to 5

Grade 1 arithmetic predicted arithmetic at grades 2, 3, and 4

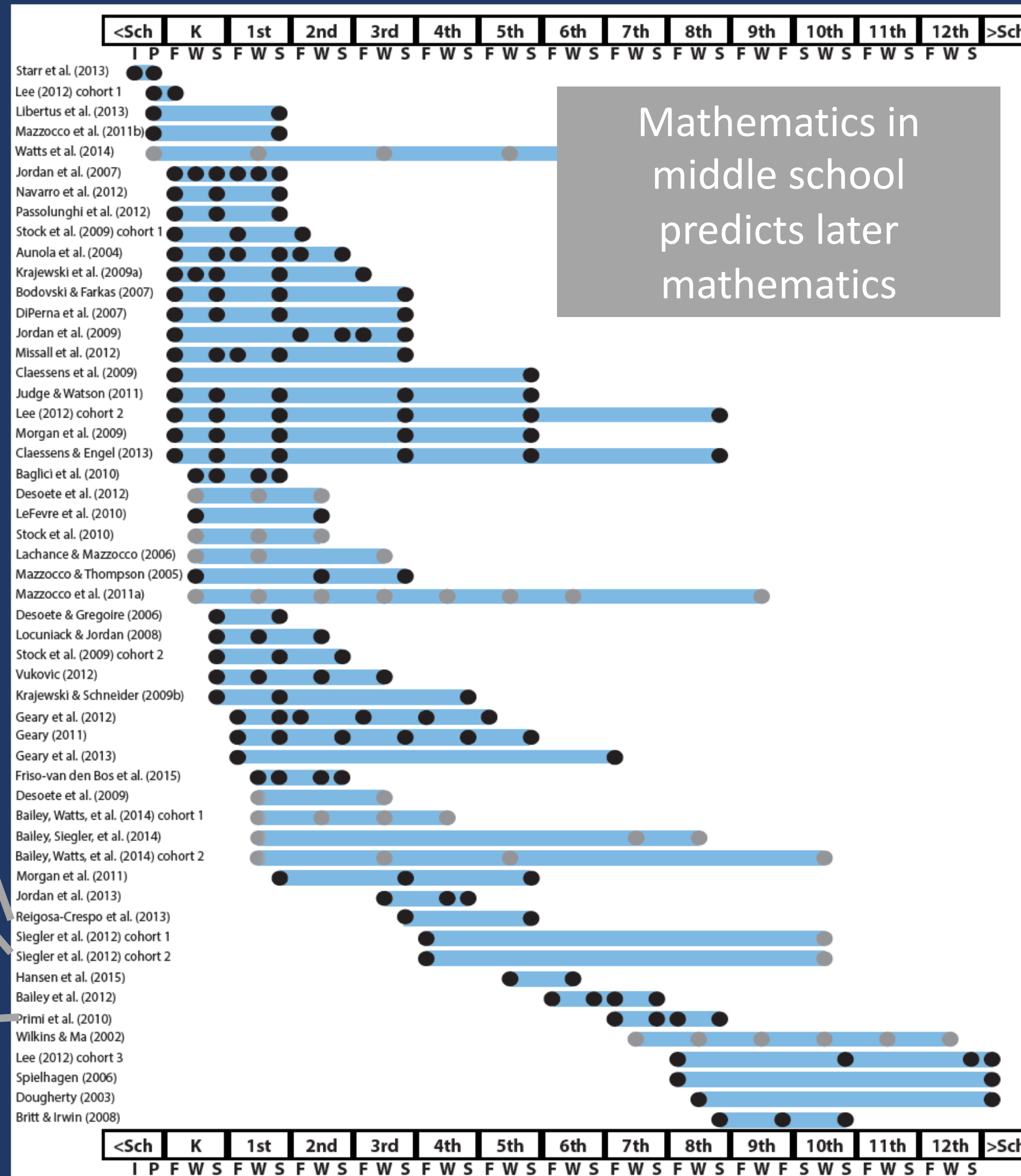
Grade 1 broad math predicted broad math at grades 3, 5, and 10

http://www.greatertexasfoundation.org/trajectories-of-mathematics-performance/

Counting and comparison in grades 2 or 4 predicted broad math 1 year later

Fractions at 10-12 years old predicted broad math 5 years later

Broad math in grade 7 predicted broad math in grade 8



Mathematics in middle school predicts later mathematics

http://www.greatertexasfoundation.org/trajectories-of-mathematics-performance/

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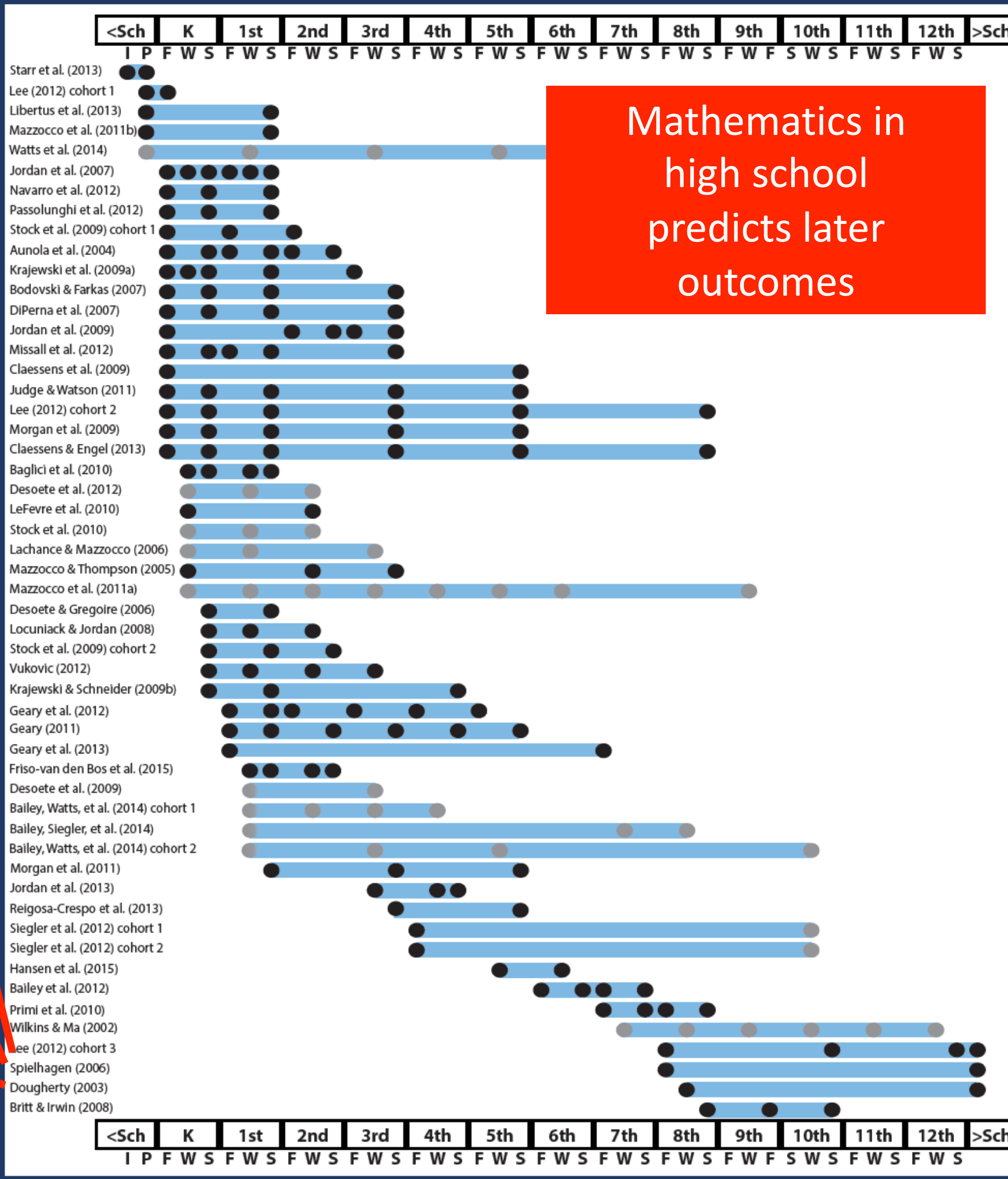
Evidence-based mathematics resources for educators

Broad math in grade 8 predicted completion of 4-year college degree

Students who took algebra in grades 8 took more advanced math courses and enrolled in 4-year colleges more often than students who took algebra in grade 9

Numeracy measured in adolescence impacted hourly earnings 7 to 15 years later



Mathematics in high school predicts later outcomes

http://www.greatertexasfoundation.org/trajectories-of-mathematics-performance/

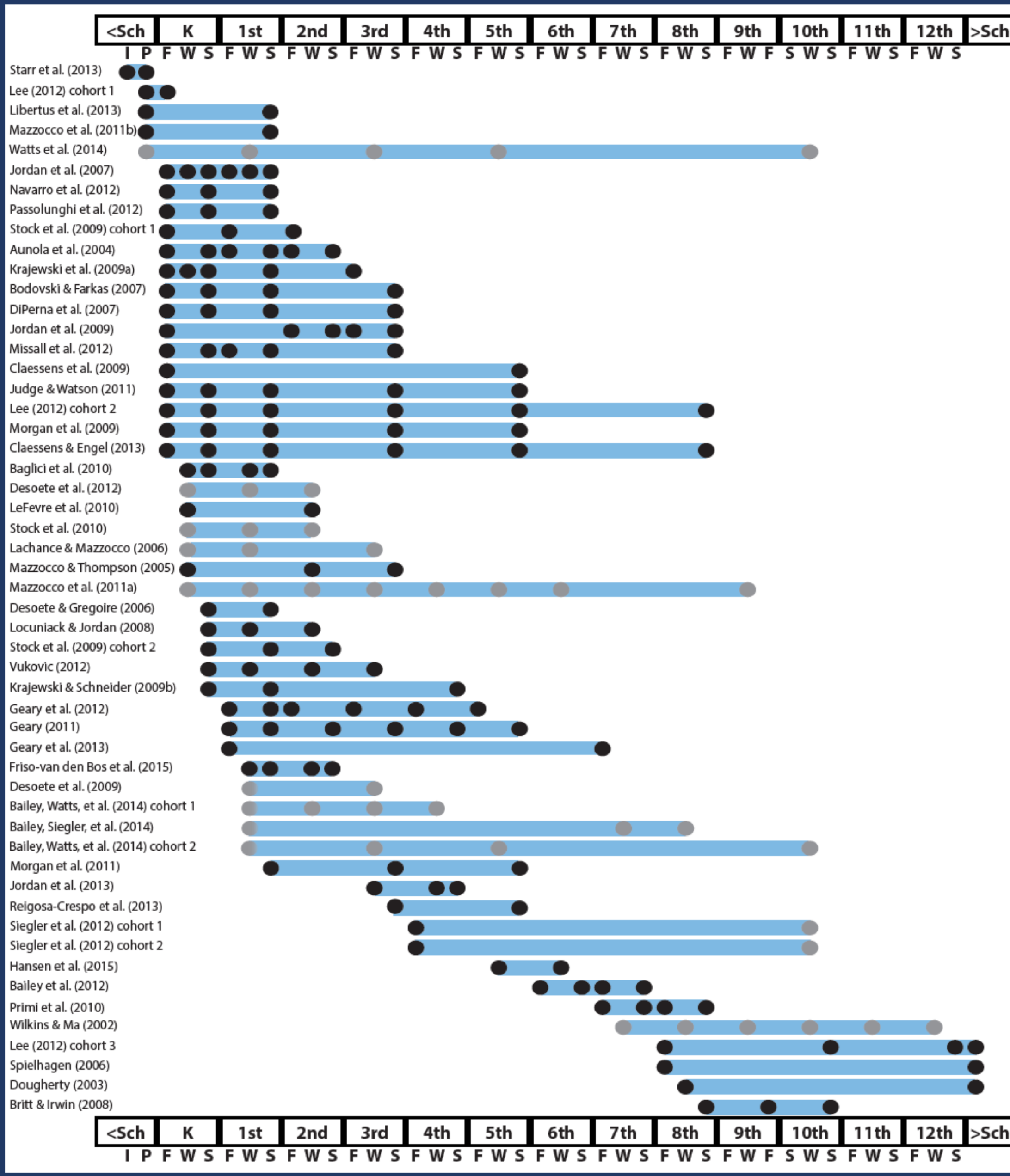
Mathematics in preschool predicts later mathematics

Mathematics in kindergarten predicts later mathematics

Mathematics in elementary school predicts later mathematics

Mathematics in middle school predicts later mathematics

Mathematics in high school predicts later outcomes



http://www.greatertexasfoundation.org/trajectories-of-mathematics-performance/

Learning Disabilities Research & Practice, *30*(4), 182–192

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Intensive Intervention in Mathematics

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Lynn S. Fuchs
Vanderbilt University

Students who demonstrate persistent mathematics difficulties and whose performance is severely below grade level require *intensive intervention*. Intensive intervention is an individualized approach to instruction that is more demanding and concentrated than Tier 2 intervention efforts. We present the elements of intensive intervention that teachers should consider when planning for, implementing, and monitoring intensive intervention in mathematics. Each of these elements is based on evidence from validated interventions. We also highlight strategies for intensifying instruction. We provide two examples of intensive intervention, one of which launches from a Tier 2 intervention platform and the other which is completely generated by a teacher. We conclude with considerations for intensive intervention in mathematics.

INTENSIVE INTERVENTION IN MATHEMATICS

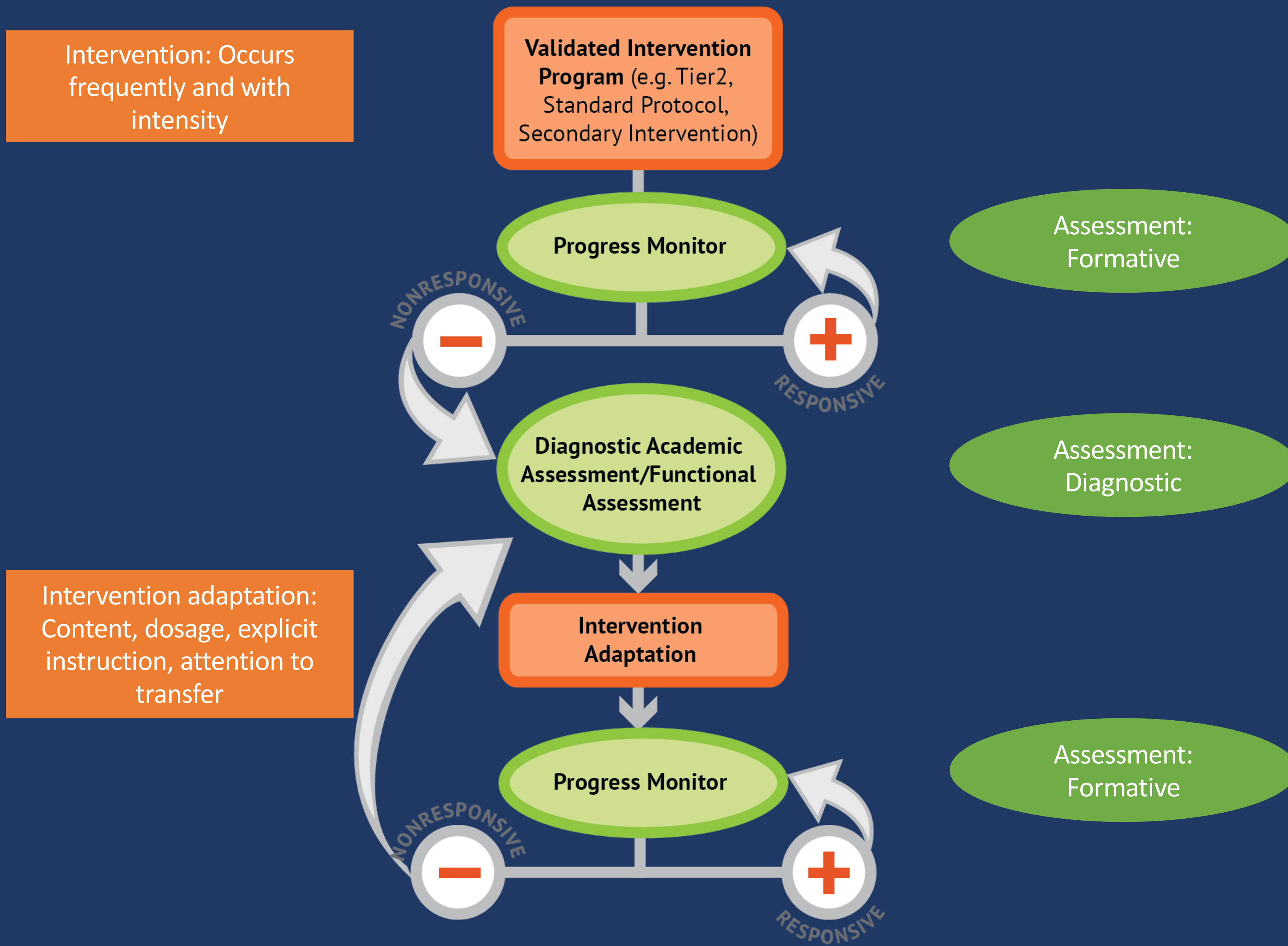
The mathematics performance of at-risk students can be improved with a secondary (i.e., Tier 2) intervention provided within a multitiered system of support (e.g., Bryant, Bryant, Gersten, Scammacca, & Chavez, 2008a; Fuchs, Fuchs, Craddock, et al., 2008). Some students, however, require more than a Tier 2 intervention. For students who demonstrate persistent mathematics difficulty (MD), that is, students whose performance is severely below grade level and who have not responded to Tier 2 intervention, *intensive intervention* is necessary. The purpose of this article is to introduce readers to the basics of intensive intervention for students with MD. Our model of intensive intervention relies on individualized instruction based on the needs of the student (Fuchs, Fuchs, & Vaughn, 2014; Vaughn & Wanzek, 2014; Wehby & Kern, 2014). This individualization often occurs at Tier 3 within the typical three-tier system of support (Stecker, Fuchs, & Fuchs, 2008).

Tier 3 intervention is more intensive, and the mathematics content and pedagogy are substantially different from that delivered at Tiers 1 and 2. This is necessary because a student only enters Tier 3 intensive intervention when instructional models employed at previous tiers have proven unsuccessful in meeting the student's needs. In an individualized approach to intensive intervention, frequent progress monitoring is essential. Teachers' use progress-monitoring data to make individualized, data-based decisions about the student's instructional program. Tier 3 is noticeably different from Tier 2 intervention, which involves a standard (nonindividualized) program, representing a single approach that is packaged in a manual.

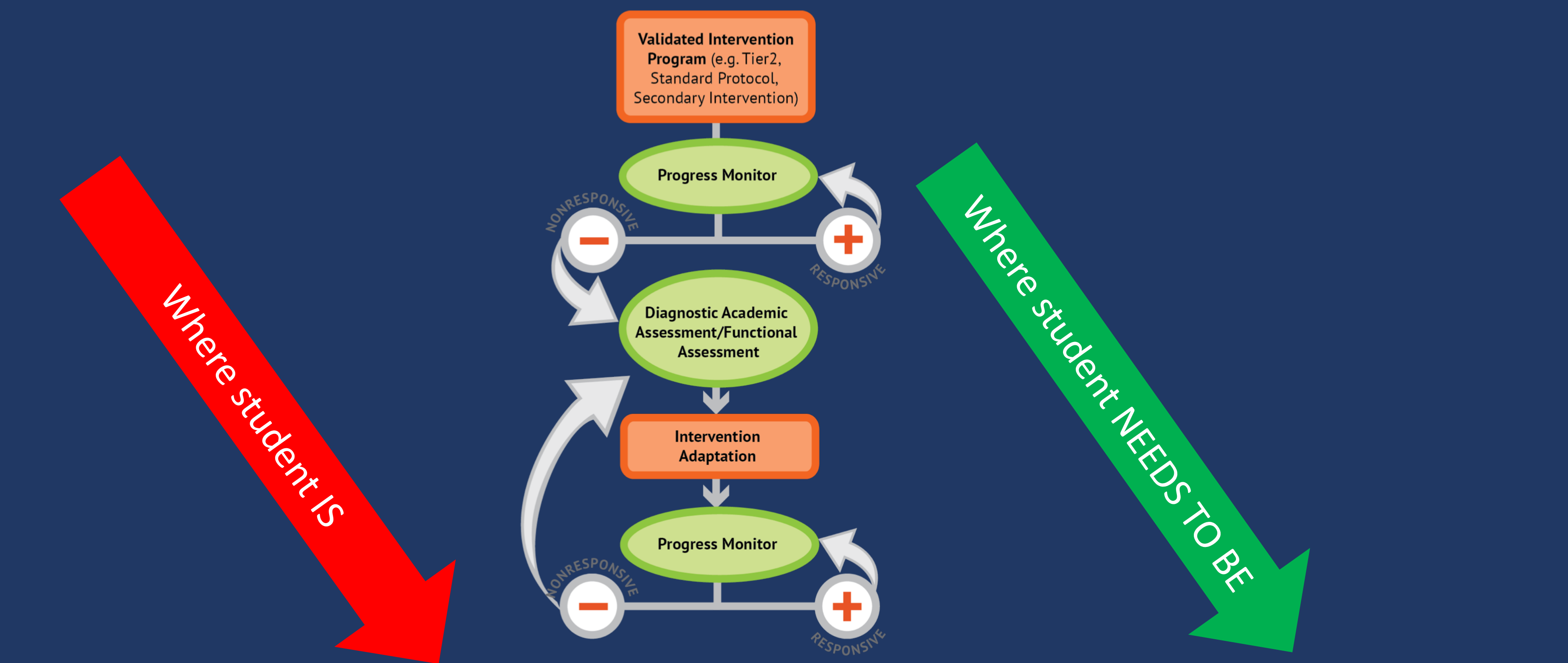
The intensive (individualized) intervention should be built upon existing structures, often starting with a validated Tier 2 *program* when available (McInerney, Zumeta, Gandhi, & Gersten, 2014). The Tier 2 program is used as a *platform*—a starting point from which the teacher modifies the validated, standard intervention, in response to ongoing progress-monitoring data, to formatively develop individualized, intensive intervention. It is important that the Tier 2 program be validated and address key mathematics deficits for the individual student. By *validated*, we mean that there is positive evidence, collected during at least one well-conducted randomized control trial, that the program improves the mathematics outcomes of students with MD in a Tier 2 intervention. To be clear, in this article, we refer to a Tier 2 *program* to denote its use at Tier 2, where the program is implemented with fidelity according to the procedures under which it was validated. For its use in intensive intervention, we use the term *platform* to denote a validated Tier 2 program that is modified in Tier 3 to meet a student's individual needs (i.e., the platform from which the individualization program is built).

When selecting a Tier 2 intervention program to use as a platform (i.e., starting point) for Tier 3 intensive intervention, it is important to consider whether the validated program provides evidence of efficacy for students with very severe or persistent MD (Fuchs, Fuchs, & Malone, 2015). The Academic Intervention Tools Chart from the National Center on Intensive Intervention (www.intensiveintervention.org) provides descriptions of efficacy studies of mathematics intervention programs, with summaries of results. Intervention developers are provided with the opportunity to report disaggregated results for students with very low mathematics performance from the larger sample of at-risk students who were included in the study. Selecting a program with demonstrated success for very low performers increases the likelihood of success with a student who has a history of poor response.

Requests for reprints should be sent to Sarah R. Powell, University of Texas at Austin. Electronic inquiries should be sent to srpowell@austin.utexas.edu.


Intervention: Occurs frequently and with intensity
Validated Intervention Program (e.g. Tier2, Standard Protocol, Secondary Intervention)
Progress Monitor
Assessment: Formative
NONRESPONSIVE
RESPONSIVE
Diagnostic Academic Assessment/Functional Assessment
Assessment: Diagnostic
Intervention adaptation: Content, dosage, explicit instruction, attention to transfer
Intervention Adaptation
Progress Monitor
Assessment: Formative
NONRESPONSIVE
RESPONSIVE

Let's think about designing the instructional platform…


Where student IS
Validated Intervention Program (e.g. Tier2, Standard Protocol, Secondary Intervention)
Progress Monitor
NONRESPONSIVE
RESPONSIVE
Diagnostic Academic Assessment/Functional Assessment
Intervention Adaptation
Progress Monitor
NONRESPONSIVE
RESPONSIVE
Where student NEEDS TO BE

Fluency

Fluently add and subtract within 5.

Fluently add and subtract within 5.

Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.

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Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or relationships.

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Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division…

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Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division…

Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Fluently add and subtract within 5.	Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.	Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or relationships.	Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division…	Fluently add and subtract multi-digit whole numbers using the standard algorithm.	Fluently multiply multi-digit whole numbers using the standard algorithm.

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Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Fluently multiply multi-digit whole numbers using the standard algorithm.

Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm.

Where student NEEDS TO BE

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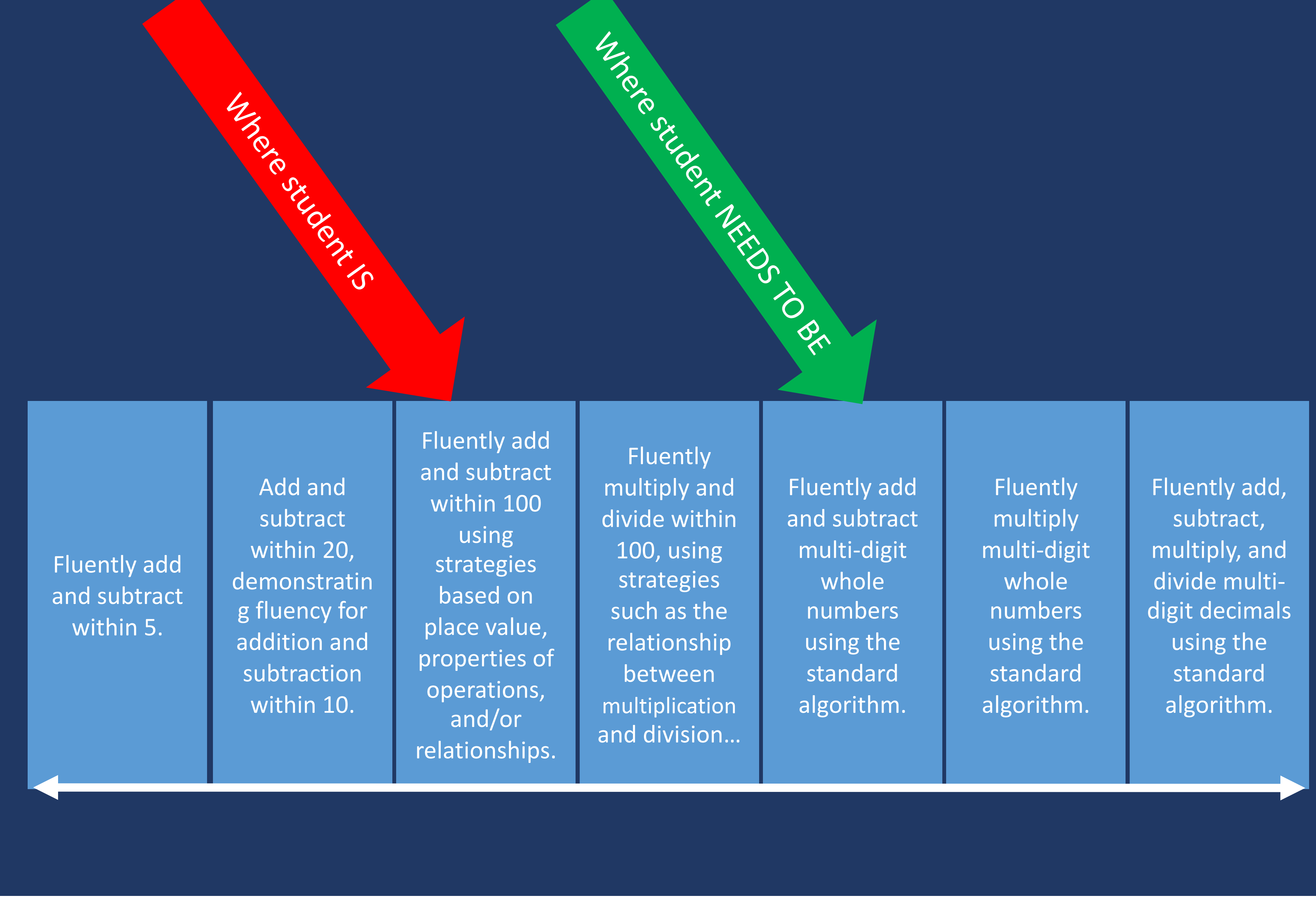
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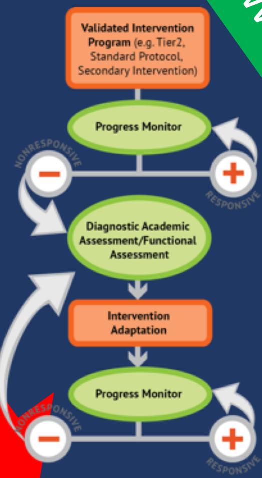
Where student IS

Where student NEEDS TO BE



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Place Value

Recognize that in a multi-digit number, a digit in one place represents ten times what it represents in the place to its right…

Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones.

Compose and decompose numbers from 11 to 19 into ten ones and some further ones…

Understand that the two digits of a two-digit number represent amounts of tens and ones.

Use place value understanding to round whole numbers to the nearest 10 or 100.

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Problem Solving

Solve real-world and mathematical problems leading to two linear equations in two variables.

Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions…

Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations…

Use multiplication and division within 100 to solve word problems…

Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20…

Solve addition and subtraction word problems, and add and subtract within 10…

Solve real-world and mathematical problems involving the four operations with rational numbers.

Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators …

Use addition and subtraction within 100 to solve one- and two-step word problems…

Solve addition and subtraction word problems, and add and subtract within 10…

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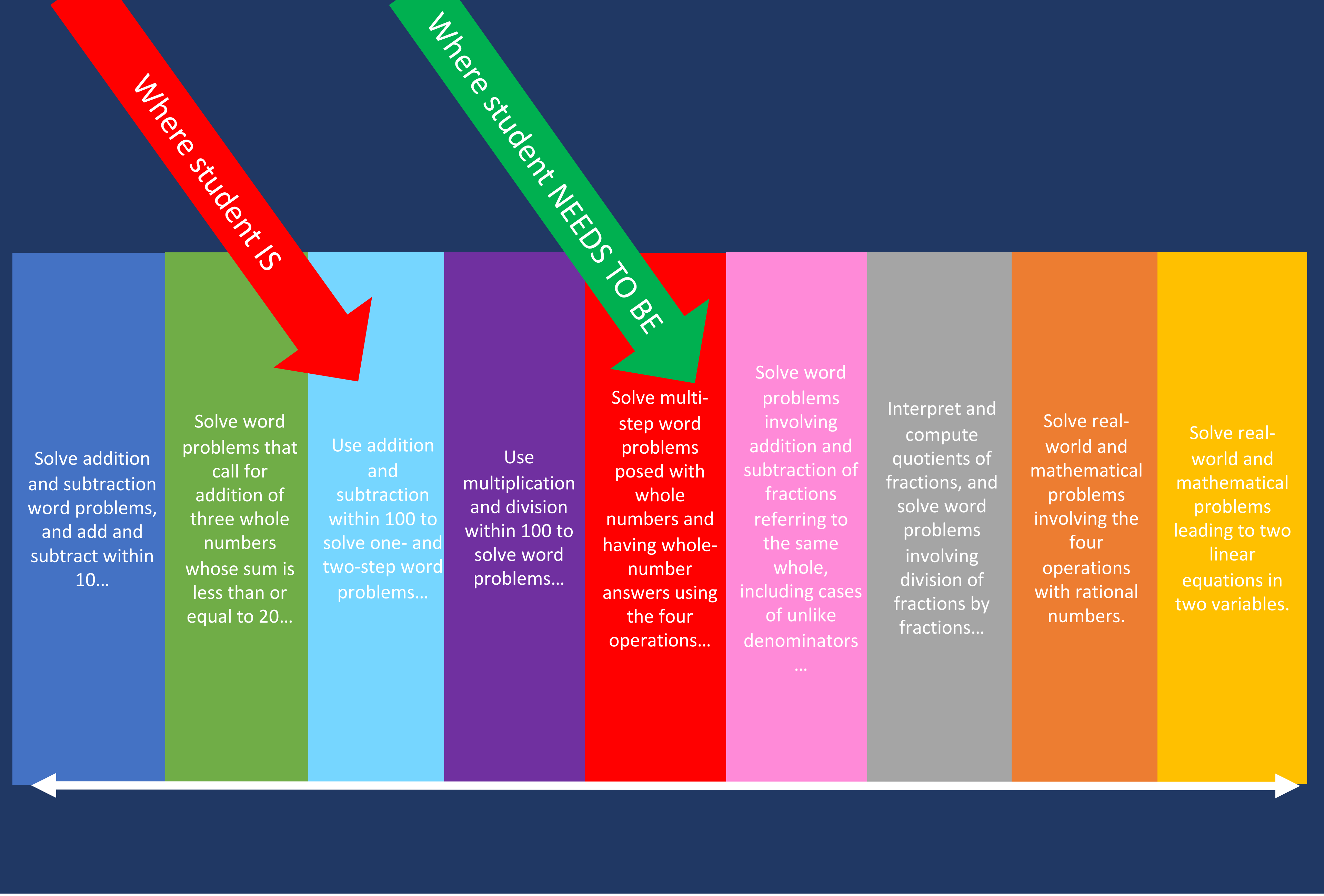
Where student IS
Where student NEEDS TO BE
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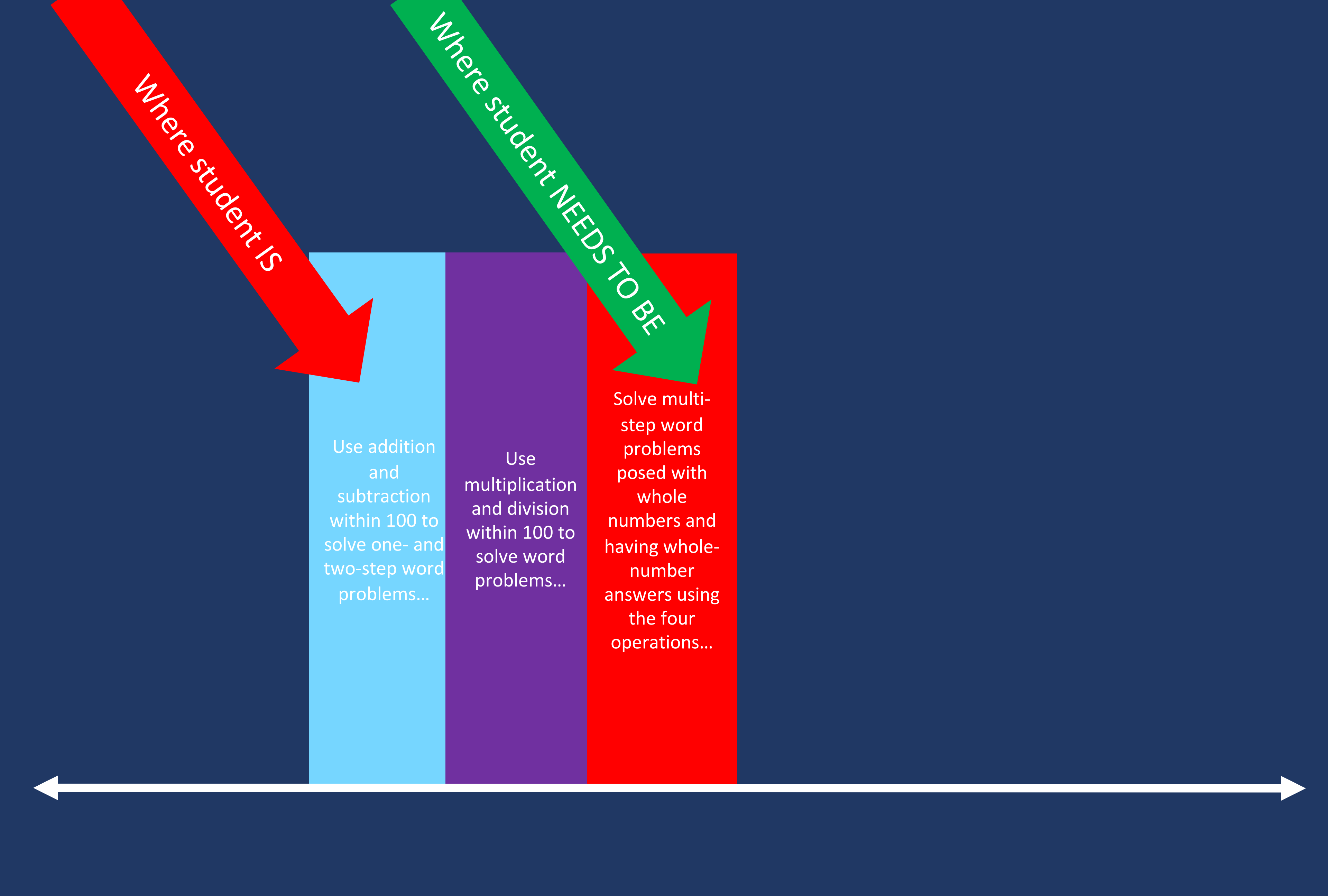
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Understand that the two digits of a two-digit number represent amounts of tens and ones.
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properties of operations, and/or
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Apply properties of operations as strategies to multiply and divide....
Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division...
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Fluently add and subtract multi-digit whole numbers using the standard algorithm.
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Let’s think about designing the instructional platform…

evidence-based practice

A practice that has shown **consistent and positive** results

An intervention (i.e., packaged program) that has shown **consistent and positive** results

evidence-based practice

evidence-based intervention

10 minutes

Grade 1
Unit 4 • Day 3

Instruction

Lesson 5: Crossing Decades

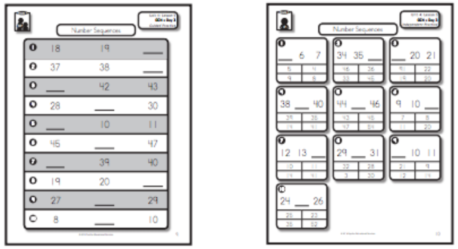
Skill	Objective(s)
Ordering and Comparing Numbers	• Students will identify 1- and 2- digit numbers. • Students will identify the missing number in 3-number sequences.

Vocabulary

- **Count up:** A strategy to find a number missing from the middle or end of a 3-number sequence. Students count forward from the least number.
- **Count down:** A strategy to find a number missing from the beginning of a 3-number sequence. Students count backward from the greatest number.

Materials

- Wipe board (T & S)
- Hundreds chart (T & S)
- Dry-erase marker (T & S)
- Teacher & Student Master, pp. 9-10



Bryant, Bryant, Roberts, Vaughn, Hughes, Porterfield, & Gersten (2011)

Week 1: The Big Parade

Day: 1, 2, 3, 4

Common Core Domains and Clusters Addressed:

Counting and Cardinality

- *Know Number Names and the Count Sequence*
- *Count to tell the Number of Objects*
- *Compare numbers*

Numbers and Operations in Base Ten

- *Extend the Counting Sequence*
- *Understand Place Value*

The new apprentice arrives in NumberShire and everyone is excited! The villagers decide to throw a parade, but they need help organizing it. Example lessons include comparing groups of floats in the parade and helping Minstrel Max bundle stacks of coins into a ten so that he can help fund the parade.

Week 2: Building and Baking

Day: 1, 2, 3, 4

Common Core Domains and Clusters Addressed:

Counting and Cardinality

- *Know Number Names and the Count Sequence*
- *Count to tell the Number of Objects*
- *Compare numbers*

Operations and Algebraic Thinking

- *Understand and Apply Properties of Operations and the Relationship between Addition and Subtraction*
- *Add and Subtract within 20*

Numbers and Operations in Base Ten

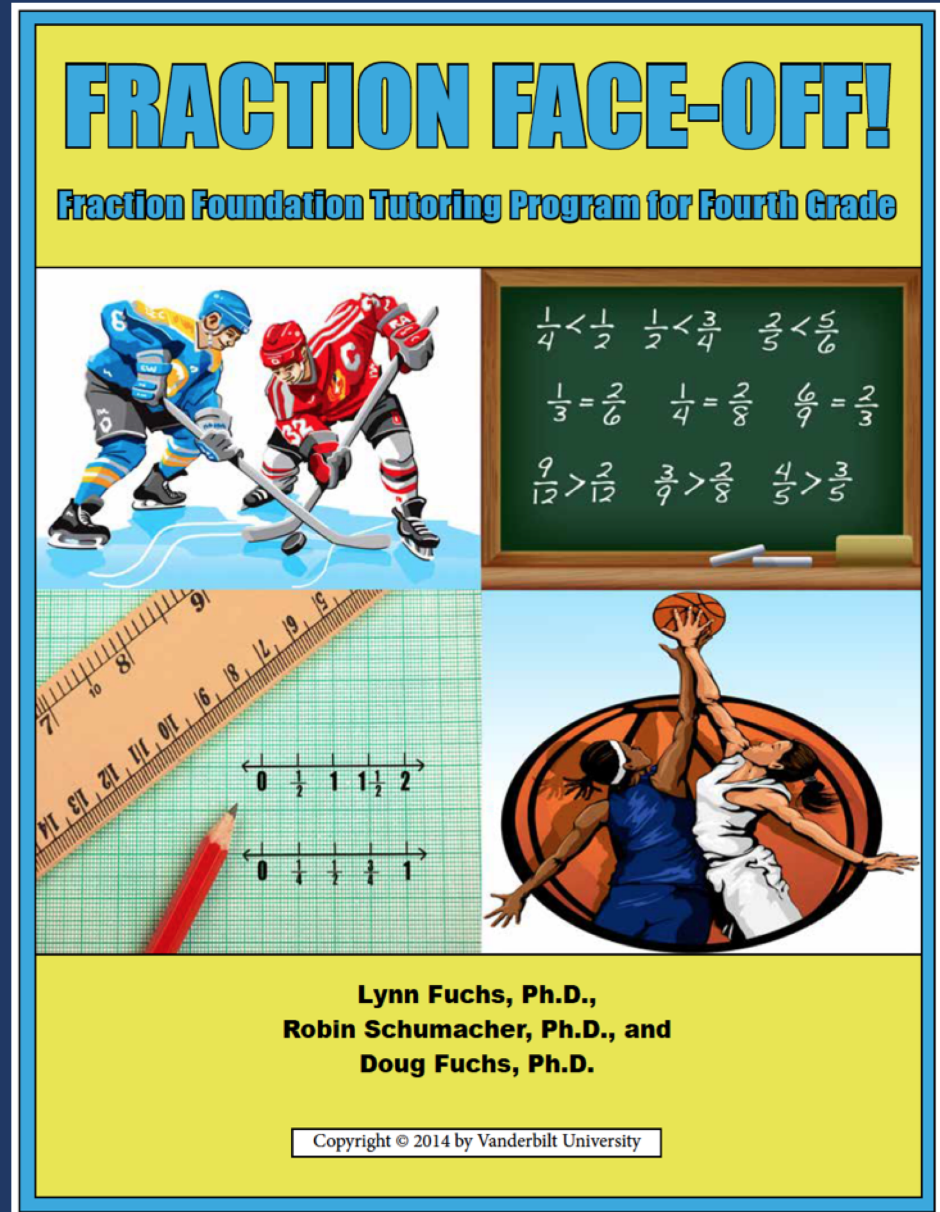
- *Extend the Counting Sequence*
- *Understand Place Value*

It's a busy week and there's a lot to do. First, Thatcher Tom stops by to spruce up the player's house. He needs help sorting individual blocks of wood and sticks of ten to make teen numbers. The bakery is really busy getting ready for a big event, so the apprentice stops by to help Night Owl the Baker. Challenges include writing numbers in frosting and adding one more cake to each order.

https://dibels.uoregon.edu/market/numbershire

evidence-based practice

evidence-based intervention



Fuchs, Schumacher, & Fuchs (2014)

evidence-based practice

evidence-based intervention

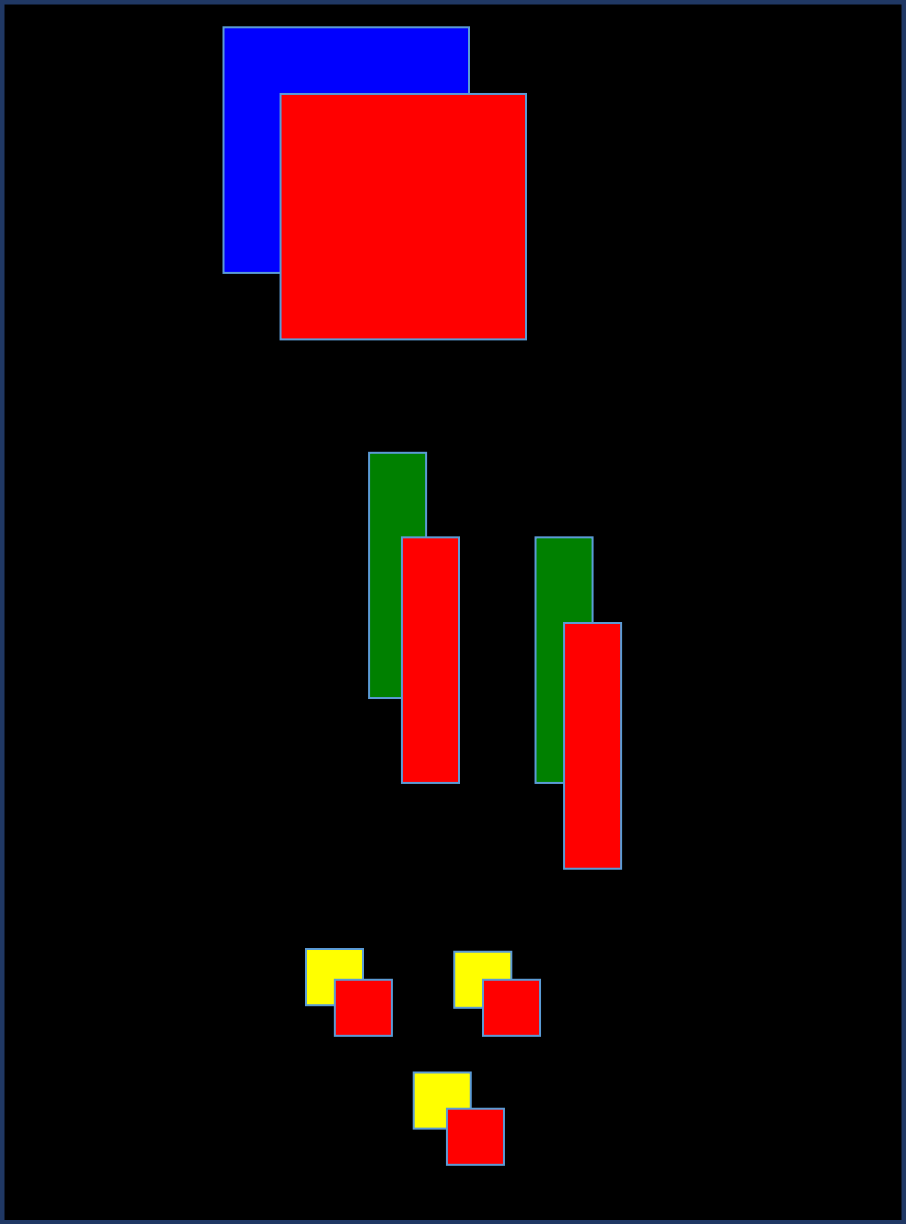
evidence-based strategy

A method or strategy that has shown **consistent and positive** results

Cover, Copy, Compare

9 × 6 54		8 × 6 48	
7 × 8 56		6 × 5 30	
9 × 9 81		7 × 9 63	
6 × 7 42		8 × 5 40	
8 × 8 64		7 × 7 49	

ebi.missouri.edu/wp-content/uploads/2013/09/EBI-Brief-Template-Cover-Copy-and-Compare.pdf



Strickland & Maccini (2012)

$$\begin{array}{r} 5 \\ +\;6 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ +\;2 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ +\;4 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ +\;6 \\ \hline \end{array}$$

Burns (2005)

evidence-based practice

evidence-based intervention

evidence-based strategy

promising practice

A method or strategy that has shown a **positive** result

sarahpowellphd.com
Evidence-based mathematics resources for educators

evidence-based practice

evidence-based intervention

evidence-based strategy

promising practice

~~no or negative evidence~~

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Evidence-based mathematics resources for educators

evidence-based practice

evidence-based intervention

evidence-based strategy

promising practice

Assessment data to show results

Improvement from before intervention

Improvement compared to no treatment students

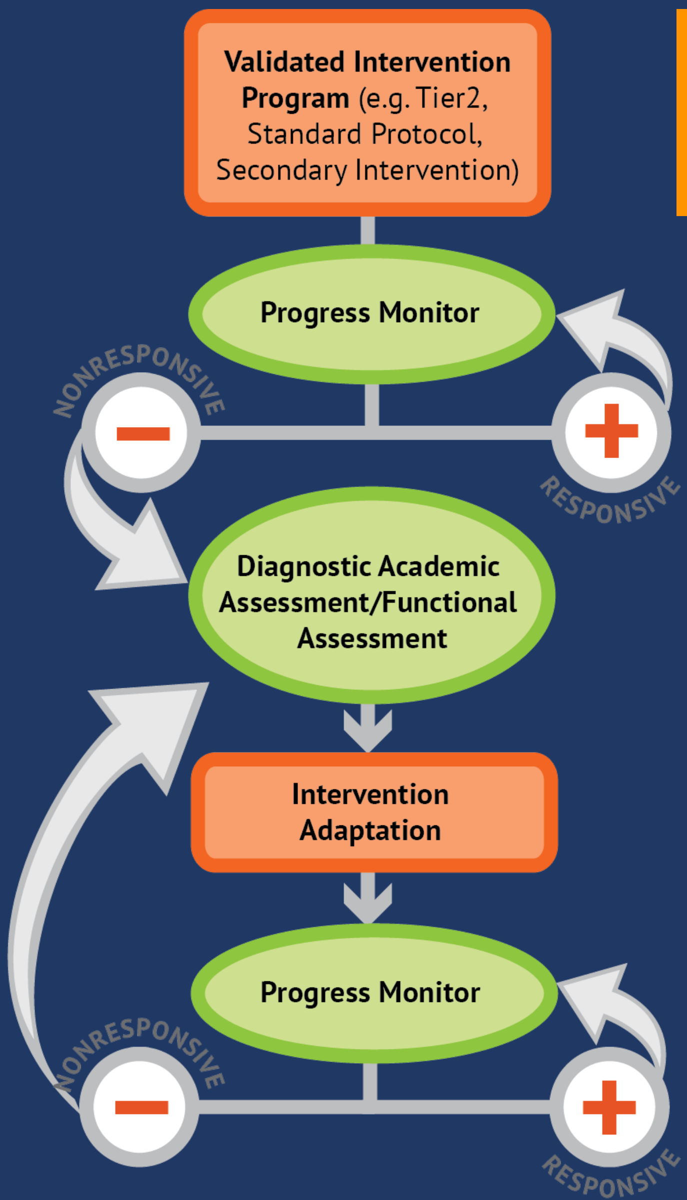
Replication

Multiple researchers

Multiple students

Multiple times

Setting and students similar to your own



Instructional Platform

INSTRUCTIONAL DELIVERY

- Explicit instruction
- Precise language
- Multiple representations

INSTRUCTIONAL STRATEGIES

- Fluency building
- Problem solving instruction

Modeling

Clear Explanation

Planned Examples

Practice

Guided Practice

Independent Practice

Supports

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

Goal and importance

Modeling

Clear Explanation

Planned Examples

"Today, we are learning about division. This is important because sometimes you have to share objects or things with your friends."

"Let's continue working with our three-dimensional shapes and volume. Understanding volume and calculating volume helps with measuring capacity."

Goal and importance

Model steps

Modeling

Clear Explanation

Planned Examples

"To solve 26 plus 79, I first decide about the operation. Do I add, subtract, multiply or divide?"

"The plus sign tells me to add. So, I'll add 26 plus 79. I'll use the partial sums strategy. First, I add 20 plus 70. What's 20 plus 70?"

"20 plus 70 is 90. I write 90 right here under the equal line. Where do I write 90?"

"Then I add 6 plus 9. What's 6 plus 9?"

"6 plus 9 is 15. So, I write 15 here."

"Finally, we add the partial sums: 90 and 15. 90 plus 15 is 105. So, 26 plus 79 equals 105."

Modeling

Clear Explanation

Planned Examples

Goal and importance

Model steps

With examples

"Today, we are learning about division. This is important because sometimes you have to share objects or things with your friends."

24 / 6 **28 ÷ 7** **35) 5**

Modeling

Clear Explanation

Planned Examples

Goal and importance

Model steps

With examples

With non-examples

"Today, we are learning about division. This is important because sometimes you have to share objects or things with your friends."

$32 \div 8$ $42 \div 7$ $25 - 5$

Modeling

Clear Explanation

Planned Examples

Goal and importance

Model steps

With examples

With non-examples

Design **modeling**

$18 \div 3$

$\frac{1}{2} + \frac{3}{4}$

$2x - 5 = 3$

Modeling	Practice
Clear Explanation	Guided Practice
Planned Examples	Independent Practice

Supports

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

Practice
Guided Practice Independent Practice

Practice

Guided Practice

Independent Practice

Teacher and student practice together

Practice

Guided Practice

Independent Practice

Student practices with teacher support

Practice

Guided Practice

Independent Practice

Design **practice**

$18 \div 3$ $\frac{1}{2} + \frac{3}{4}$ $2x - 5 = 3$

Modeling	Practice
Clear Explanation	Guided Practice
Planned Examples	Independent Practice

Supports

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

Supports

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

Low-level and high-level

"What is 7 times 9?"

"Which shape has 6 sides?"

"What do you do when you see a word problem?"

"Why do you have to regroup?"

"How would you solve this problem?"

"Why do you have to use zero pairs?"

Supports

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

Low-level and high-level

Classwide, individual, partner, write on paper, write on whiteboard, thumbs up, etc.

"Turn and discuss the formula for perimeter with your partner."

"Write the multiplication problem on your whiteboard."

"In your math journal, draw a picture to help you remember to term *parallelogram*."

Supports

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

Low-level and high-level

Classwide, individual, partner, write on paper, write on whiteboard, thumbs up, etc.

Affirmative and corrective

Supports

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

"Good work using your word-problem attack strategy."

"Let's look at that again. Tell me how you added in the hundreds column."

Low-level and high-level

Classwide, individual, partner, write on paper, write on whiteboard, thumbs up, etc.

Affirmative and corrective

Planned and organized

Supports

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

Supports

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

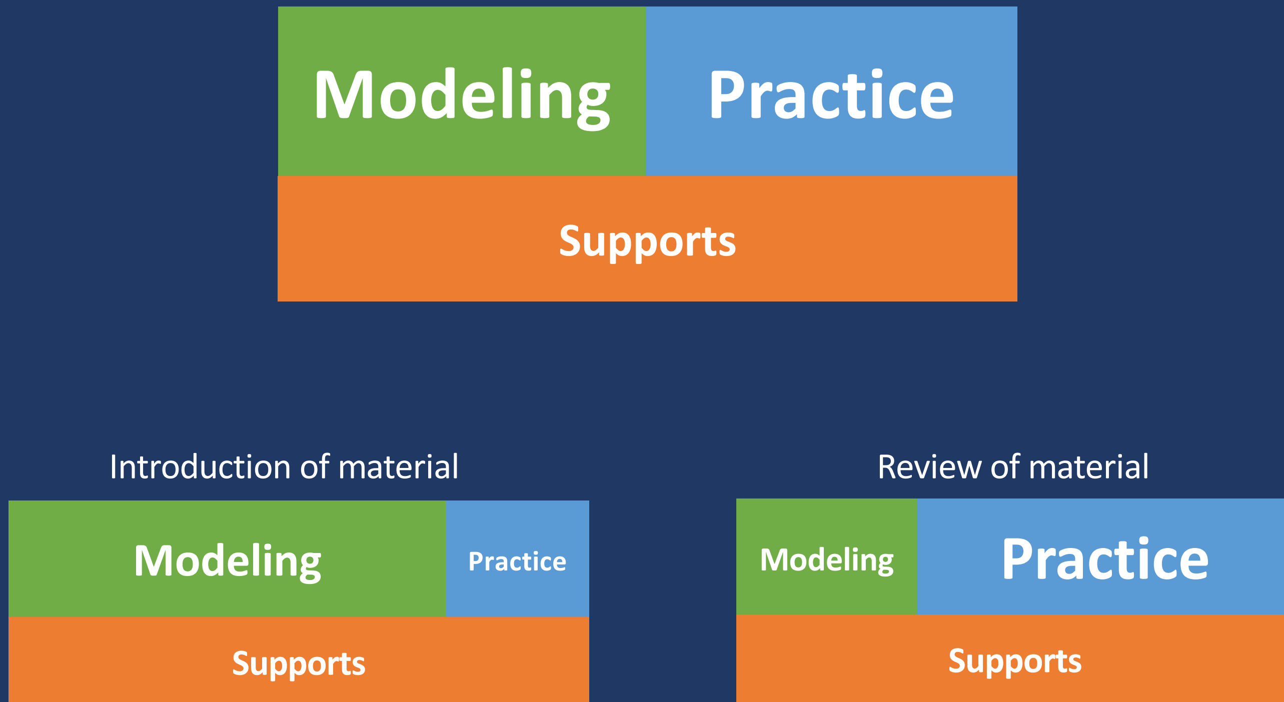
Design **supports**

$18 \div 3$ $\frac{1}{2} + \frac{3}{4}$ $2x - 5 = 3$

Modeling	Practice
Clear Explanation	Guided Practice
Planned Examples	Independent Practice

Supports

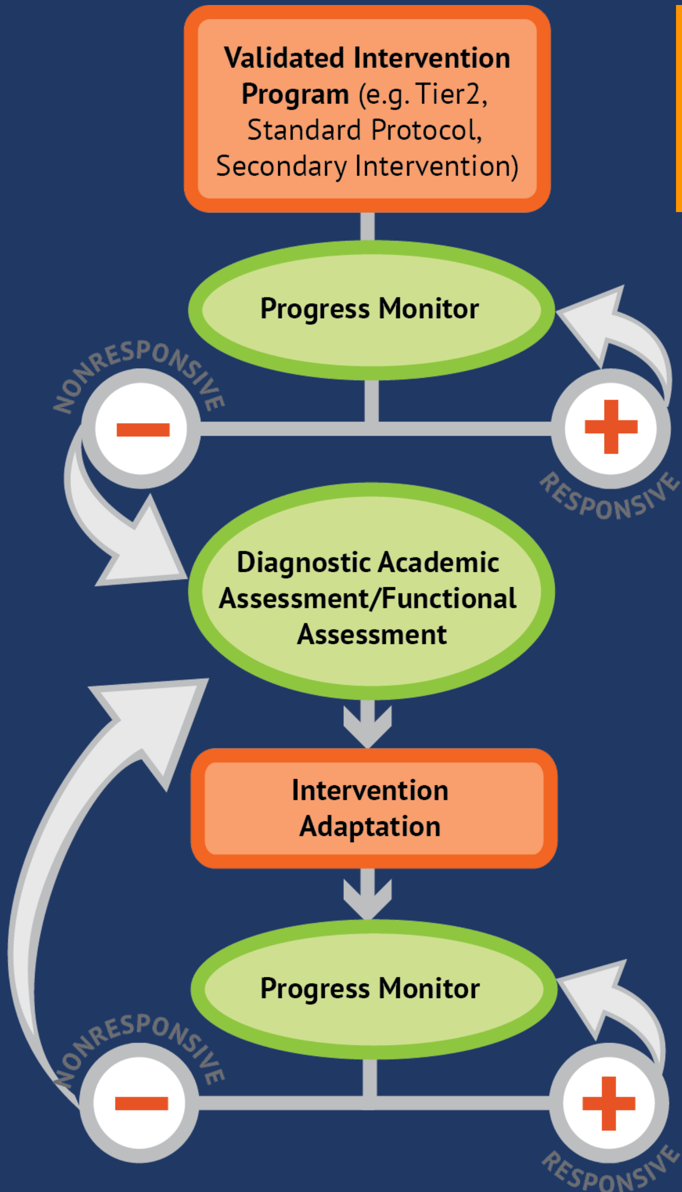
- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace


Modeling
Practice
Supports
Introduction of material
Modeling
Practice
Supports
Review of material
Modeling
Practice
Supports

How do you use explicit instruction?

- ❑ Model steps using concise language
- ❑ Provide guided practice opportunities
- ❑ Provide independent practice opportunities
- ❑ Use supporting practices during modeling and practice
  - ❑ Ask the right questions
  - ❑ Elicit frequent responses
  - ❑ Provide feedback
  - ❑ Be planned and organized

Instructional Platform



INSTRUCTIONAL DELIVERY

Explicit instruction

Precise language

INSTRUCTIONAL STRATEGIES

Use formal math language

Use terms precisely

1. Some math terms are shared with English but have different meanings

base

right

degree

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math terms are shared with English with similar meanings (but a more precise math meaning)

difference

even

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math terms are shared with English with similar meanings (but a more precise math meaning)

3. Some math terms are only used in math

numerator

trapezoid

parallelogram

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math terms are shared with English with similar meanings (but a more precise math meaning)

3. Some math terms are only used in math

4. Some math terms have more than one meaning

round

second

base

square

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math terms are shared with English with similar meanings (but a more precise math meaning)

3. Some math terms are only used in math

4. Some math terms have more than one meaning

5. Some math terms are similar to other content-area terms with different meanings

divide vs. Continental Divide

variable vs. variably cloudy

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math terms are shared with English with similar meanings (but a more precise math meaning)

3. Some math terms are only used in math

4. Some math terms have more than one meaning

5. Some math terms are similar to other content-area terms with different meanings

6. Some math terms are homographs

eight vs. ate

sum vs. some

rows vs. rose

base vs. bass

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math terms are shared with English with similar meanings (but a more precise math meaning)

3. Some math terms are only used in math

4. Some math terms have more than one meaning

5. Some math terms are similar to other content-area terms with different meanings

6. Some math terms are homographs

7. Some math terms are related but have distinct meanings

factor vs. multiple

hundreds vs. hundredths

numerators vs. denominator

Rubenstein & Thompson (2002)

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2. Some math terms are shared with English with similar meanings (but a more precise math meaning)

3. Some math terms are only used in math

4. Some math terms have more than one meaning

5. Some math terms are similar to other content-area terms with different meanings

6. Some math terms are homographs

7. Some math terms are related but have distinct meanings

8. An English math term may translate into another language with different meanings

mesa vs. tabla

Rubenstein & Thompson (2002)

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2. Some math terms are shared with English with similar meanings (but a more precise math meaning)

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6. Some math terms are homographs

7. Some math terms are related but have distinct meanings

8. An English math term may translate into another language with different meanings

9. English spelling and usage may have irregularities

four vs. forty

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings
2. Some math terms are shared with English with similar meanings (but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning
5. Some math terms are similar to other content-area terms with different meanings
6. Some math terms are homographs
7. Some math terms are related but have distinct meanings
8. An English math term may translate into another language with different meanings
9. English spelling and usage may have irregularities
10. Some math concepts are verbalized in more than one way

one-fourth vs. one quarter

skip count vs. multiples

Rubenstein & Thompson (2002)

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2. Some math terms are shared with English with similar meanings (but a more precise math meaning)

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7. Some math terms are related but have distinct meanings

8. An English math term may translate into another language with different meanings

9. English spelling and usage may have irregularities

10. Some math concepts are verbalized in more than one way

11. Informal terms may be used for formal math terms

rhombus vs. diamond

vertex vs. corner

Rubenstein & Thompson (2002)

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2. Some math terms are shared with English with similar meanings (but a more precise math meaning)

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10. Some math concepts are verbalized in more than one way

11. Informal terms may be used for formal math terms

Rubenstein & Thompson (2002)

Use formal math language

Use terms precisely

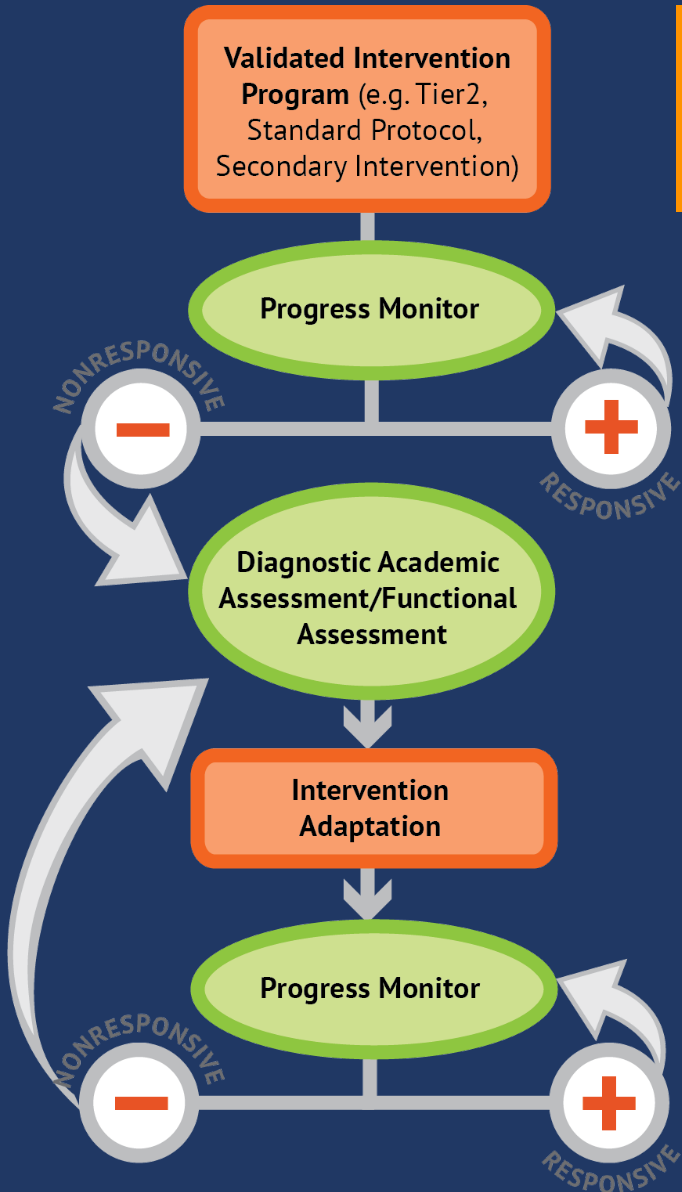
Design **language**

$18 \div 3$ **$\frac{1}{2} + \frac{3}{4}$** **$2x - 5 = 3$**

How do you attend to mathematical language?

- Understand formal mathematical language
- Plan for mathematical language to be precise
- Plan for mathematical definitions to be concise

Instructional Platform



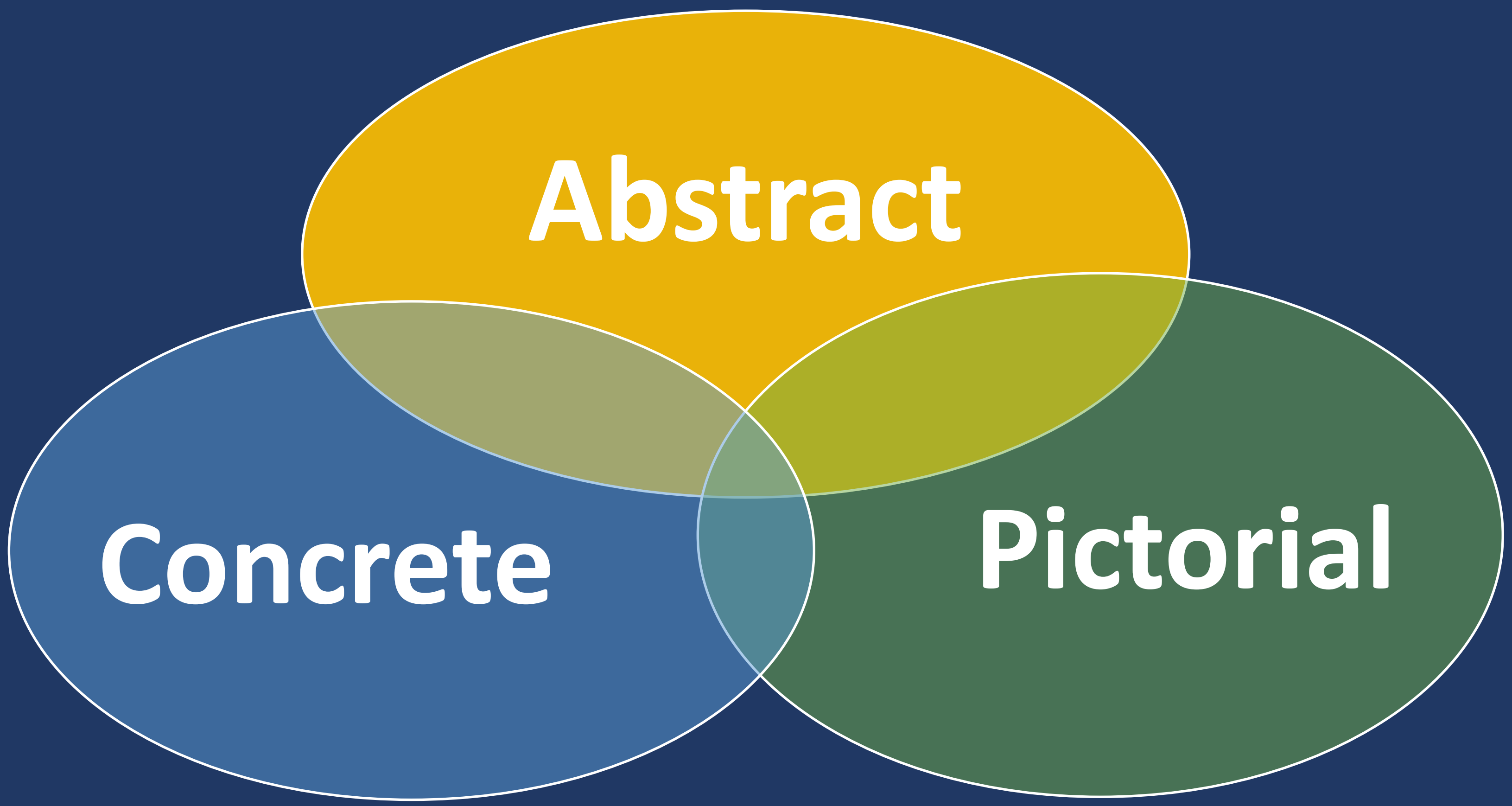
INSTRUCTIONAL DELIVERY

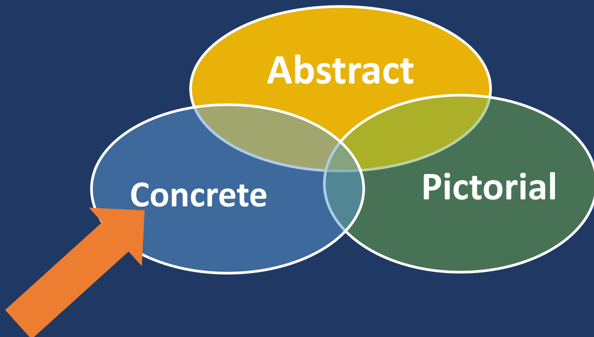
Explicit instruction

Precise language

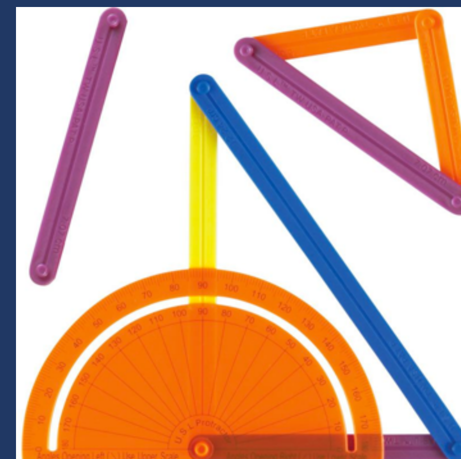
Multiple representations

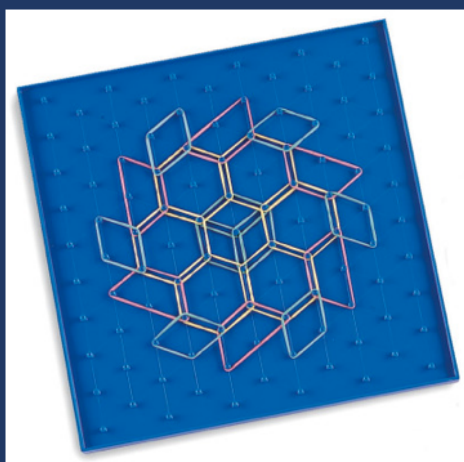
INSTRUCTIONAL STRATEGIES

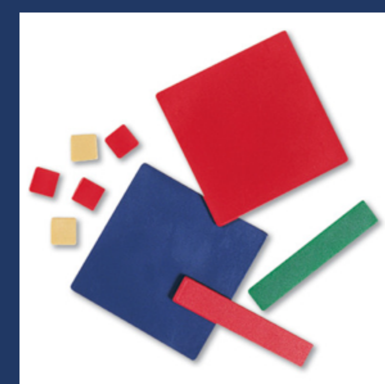

Abstract
Concrete
Pictorial

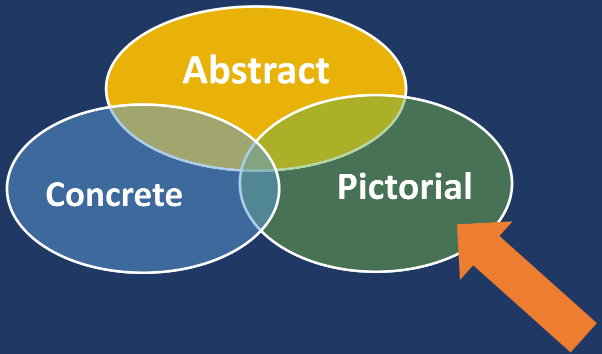


Three-dimensional objects

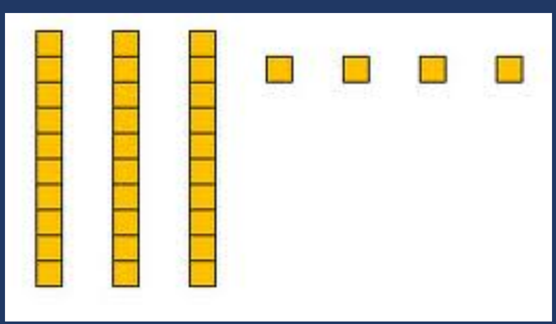


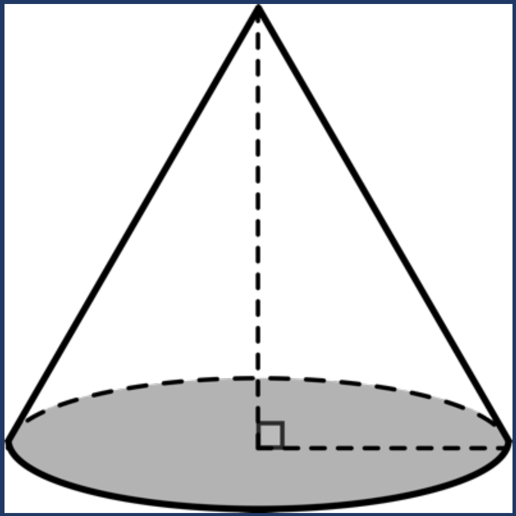


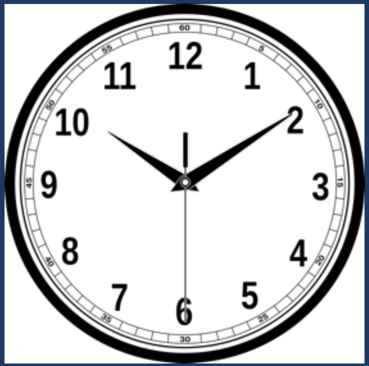


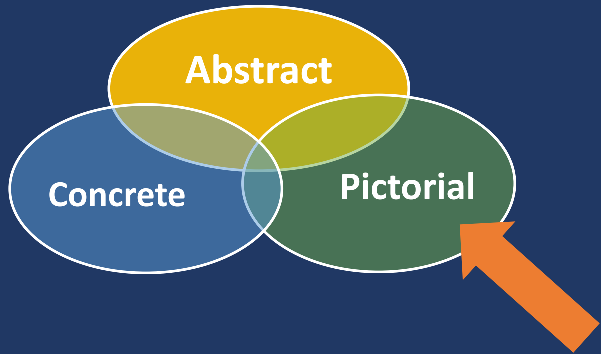


Two-dimensional images

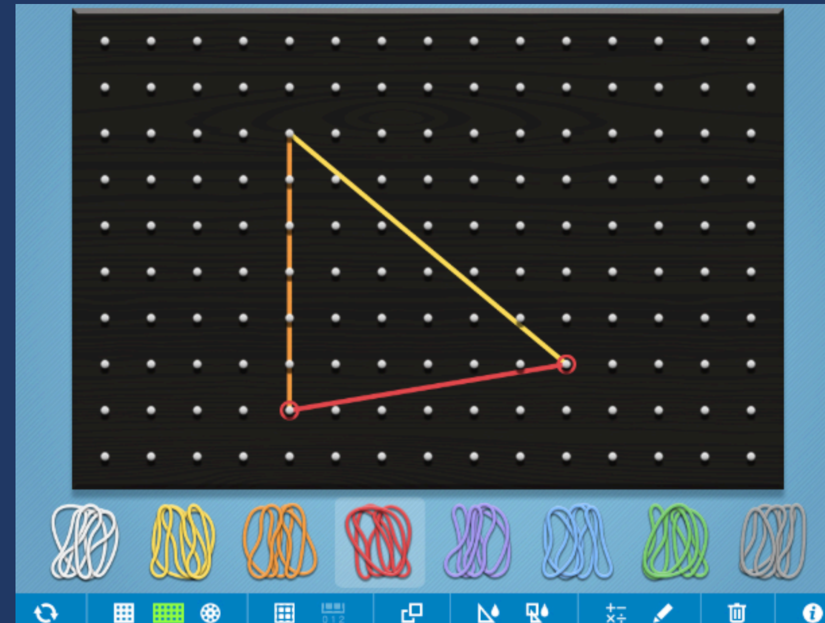


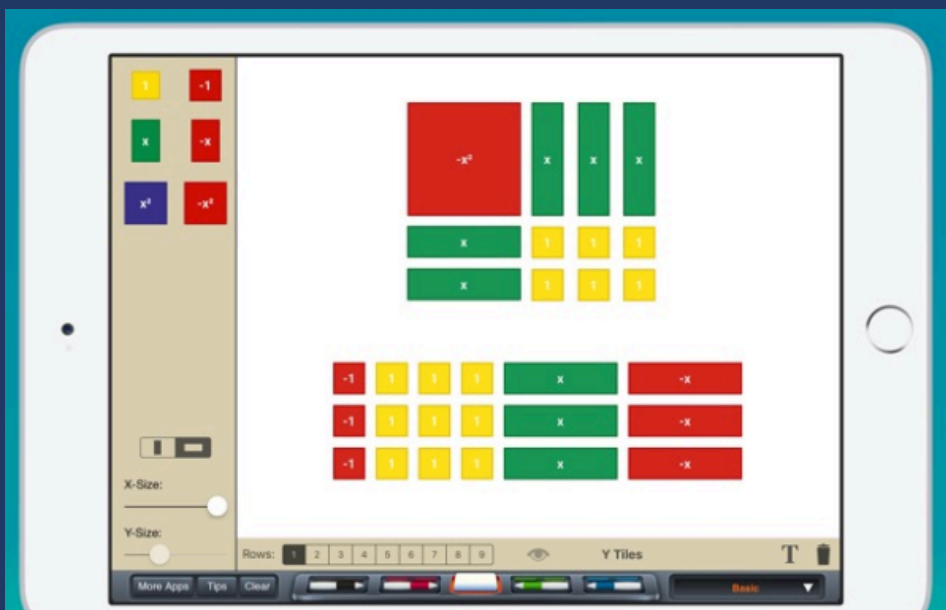


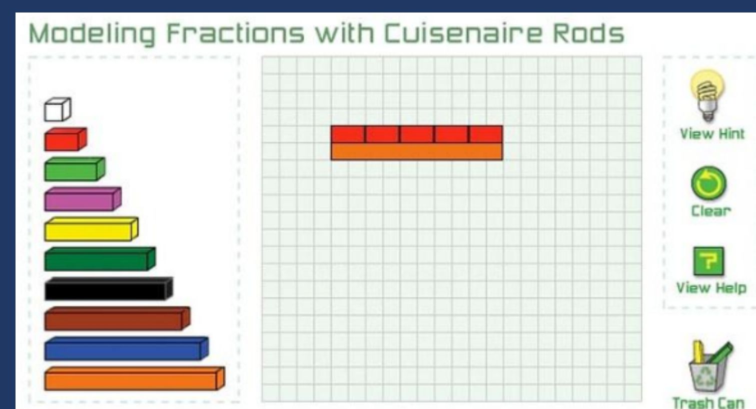


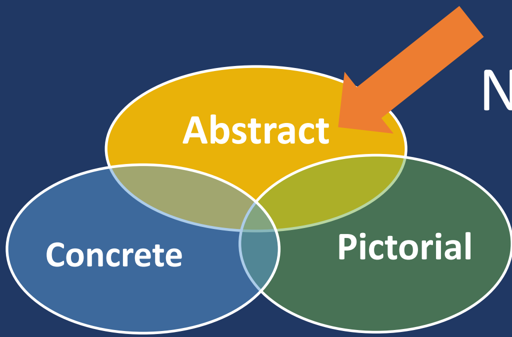


Two-dimensional images









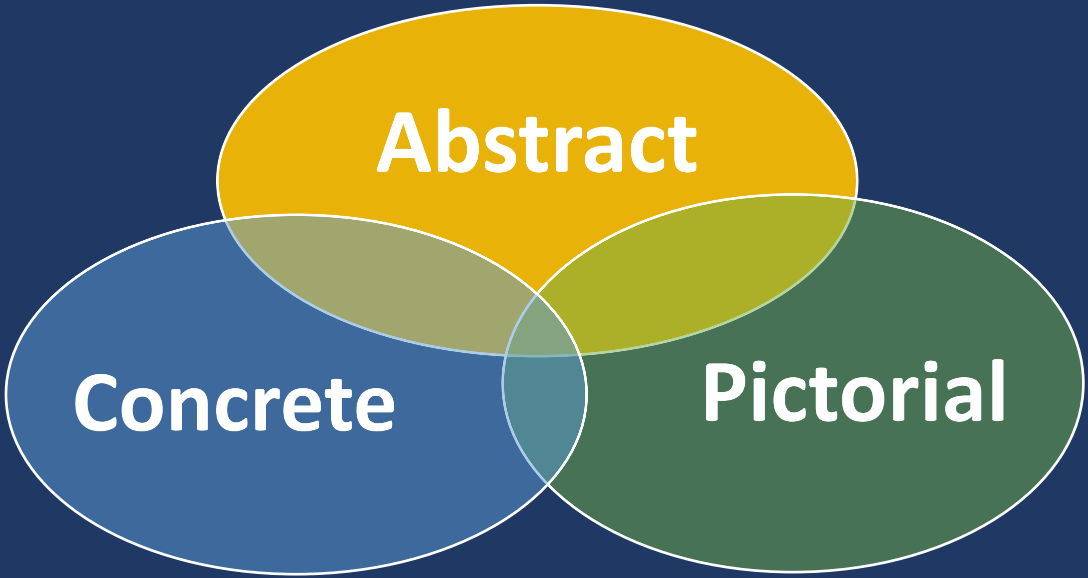
Numerals, symbols, and words

$34 = 3$ tens and 4 ones

$2 + 8 = 10$

$x - 6 = 8$

$$\begin{array}{r} 4{,}179 \\ +\quad 569 \\ \hline \end{array}$$

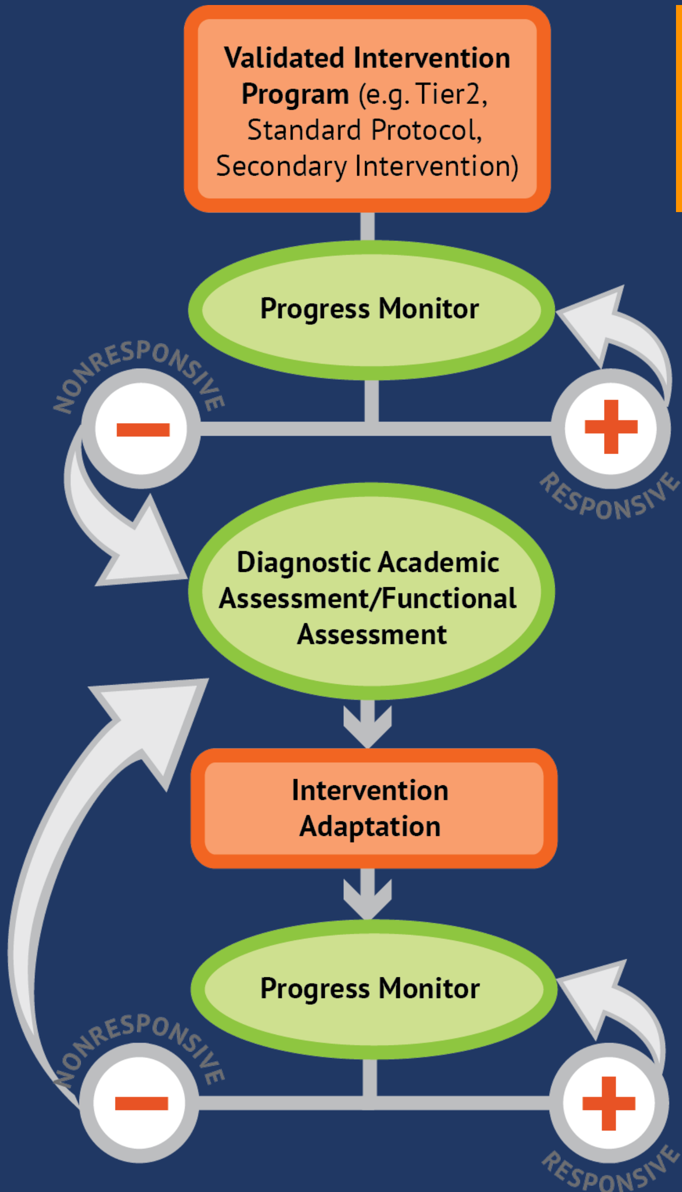


Design **multiple representations**

18 ÷ 3 **½ + ¾** **$2x - 5 = 3$**

How do you use multiple representations?

- Use three-dimensional concrete materials
- Use two-dimensional representations
- Ensure students understand mathematics with numbers and symbols (i.e., the abstract)



Instructional Platform

INSTRUCTIONAL DELIVERY

- Explicit instruction
- Precise language
- Multiple representations

INSTRUCTIONAL STRATEGIES

- Fluency building

Addition

Subtraction

Multiplication

Division

Addition

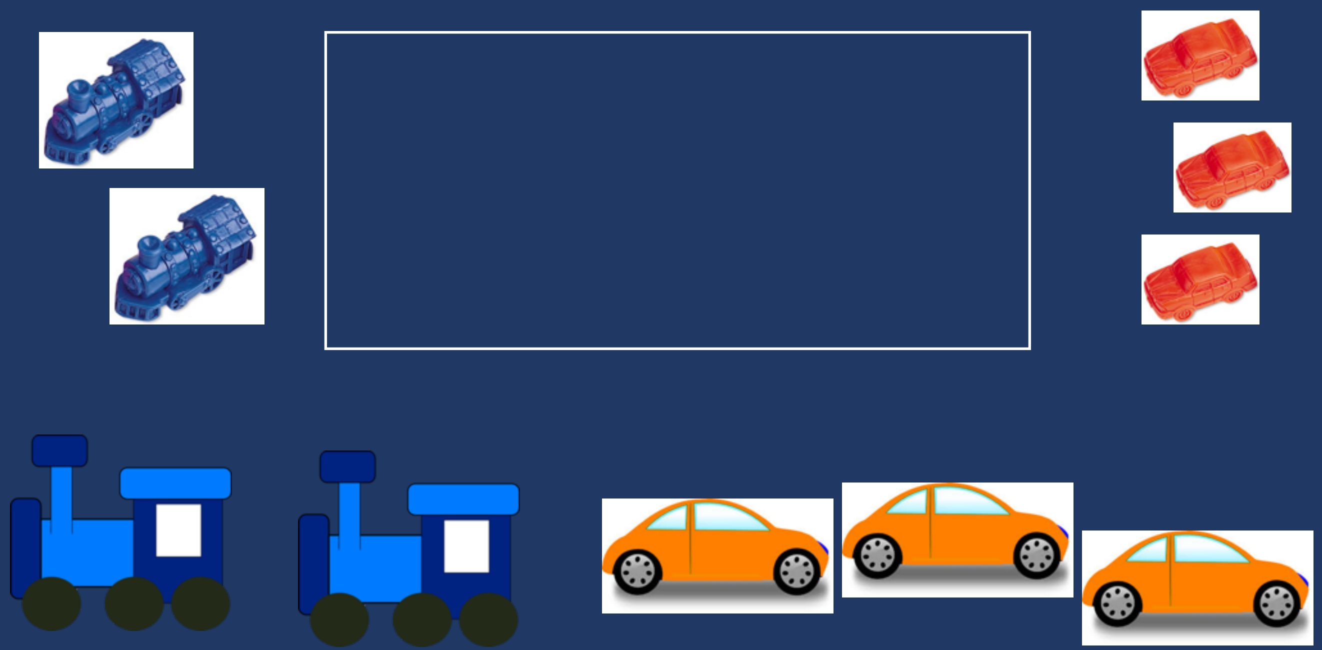
100 addition basic facts

- Single-digit addends sum to a single- or double-digit number

$$\begin{array}{rl} 5 & \text{(addend)} \\ +\quad 4 & \text{(addend)} \\ \hline 9 & \text{(sum)} \end{array}$$

Addition: Part-Part-Whole (Total)

Count one set, count another set, put sets together, count sum



$$2 + 3 = 5$$

Addition: Join (Change Increase)

Start with a set, add the other set, count sum

$$2 + 3 = 5$$

Total

Parts put together into a **total**

- Angie saw **4** cardinals and **5** blue jays. How many birds did Angie see?

  4 + **5** = **?**

Change

An amount that **increases** or decreases

- Pam had $4. Then she earned $3 for cleaning her room. How much money does Pam have now?

  4 + 3 = ?

Subtraction

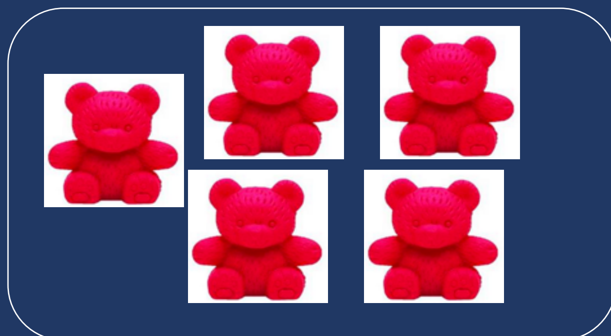
100 subtraction basic facts

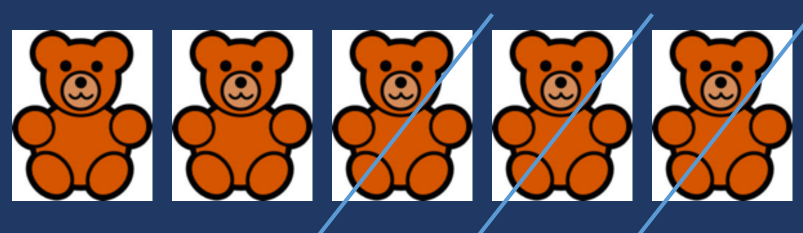
- Subtrahend and difference are single-digit numbers and minuend is single- or double-digit number

$$\begin{array}{rl} 16 & \textbf{(minuend)} \\ -\ \ 8 & \textbf{(subtrahend)} \\ \hline 8 & \textbf{(\underline{difference})} \end{array}$$

Subtraction: Separate (Change Decrease)

Start with a set, take away from that set, count difference

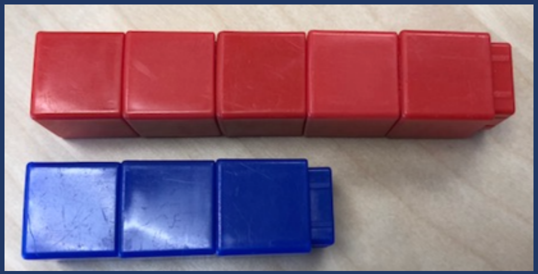


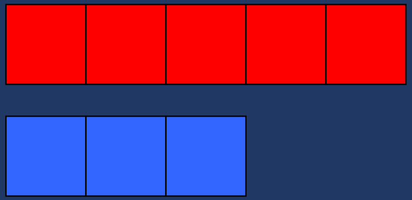


$$5 - 3 = 2$$

Subtraction: Compare (Difference)

Compare two sets, count difference





$$5 - 3 = 2$$

Change

An amount that increases or **decreases**

- Amanda had **9** cookies. Then she ate **2** of the cookies. How many cookies does Amanda have now?

  9 – **2** = **?**

Difference

Greater and less amounts compared for a difference

- Scott has **9** apples. Cathy has **4** apples. How many more apples does Scott have? (How many fewer does Cathy have?)
  - **9** – **4** = **?**

Multiplication

100 multiplication basic facts

- Multiplication of single-digit factors results in a single- or double-digit product

$$\begin{array}{r} 2 \\ \times \quad 3 \\ \hline 6 \end{array}$$

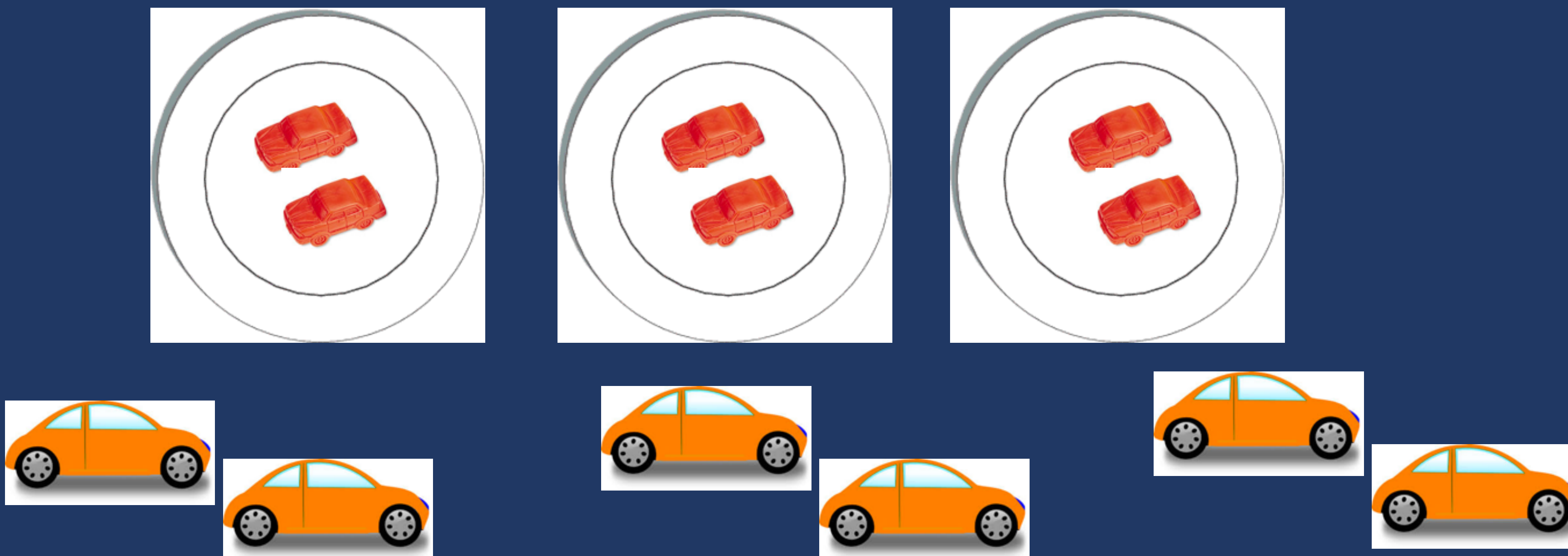
(factor)

(factor)

(product)

Multiplication: Equal Groups

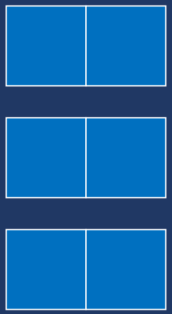
Show the groups, show the amount for each group, count product



$$3 \times 2 = 6$$

Multiplication: Array/Area

Make the array, count product



$$3 \times 2 = 6$$

Multiplication: Comparison

Show a set, then multiply the set

$$3 \times 2 = 6$$

Equal Groups

Groups multiplied by **number in each group** for a **product**

- Carlos has **2** bags of apples. There are **6** apples in each bag. How many apples does Carlos have altogether?

  - $2 \times 6 = ?$

Comparison

Set multiplied by a number of times for a product

- Beth picked 6 apples. Amy picked 2 times as many apples as Beth. How many apples did Amy pick?
  - $6 \times 2 = ?$

Division

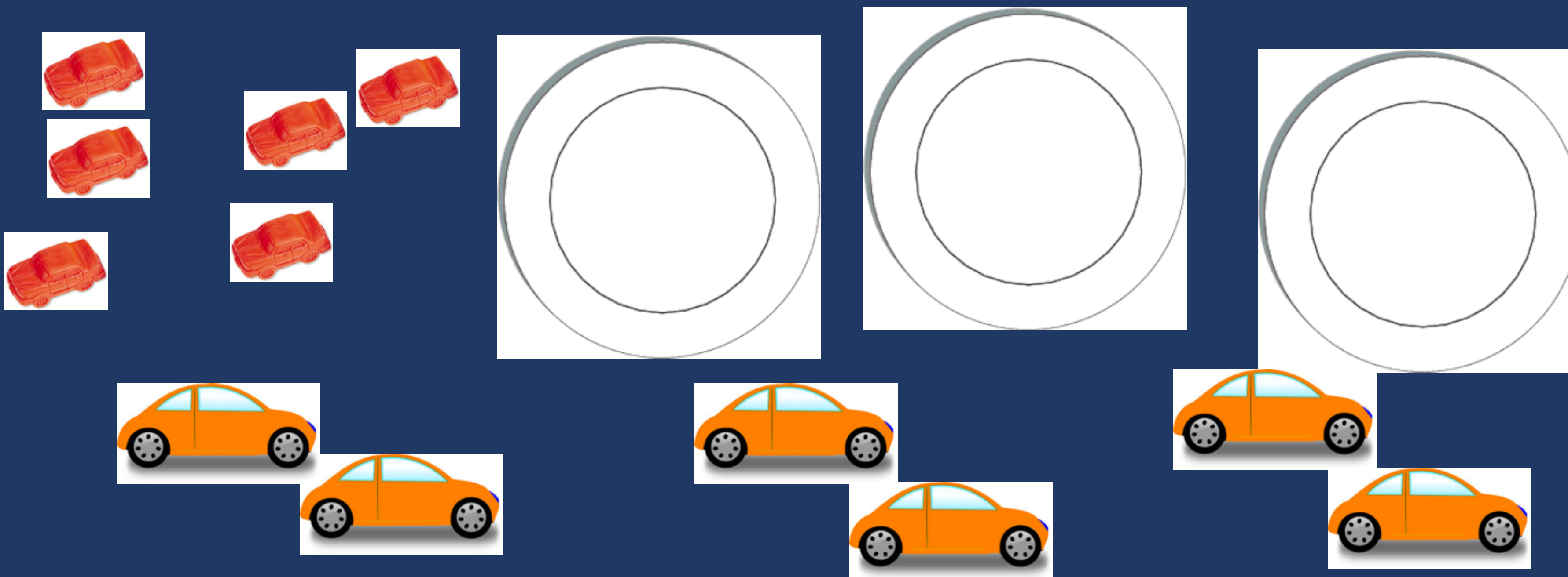
90 division basic facts

- Divisor and quotient are single-digit numbers and dividend is single- or double-digit number

8	÷	4	=	2
(dividend)		(divisor)		(quotient)

Division: Equal Groups (Partitive Division)

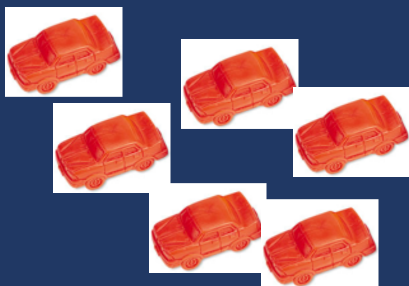
Show the dividend, divide equally among divisor, count quotient



$$6 \div 3 = 2$$

Division: Equal Groups (Measurement Division)

Show the dividend, make groups of the divisor, count groups

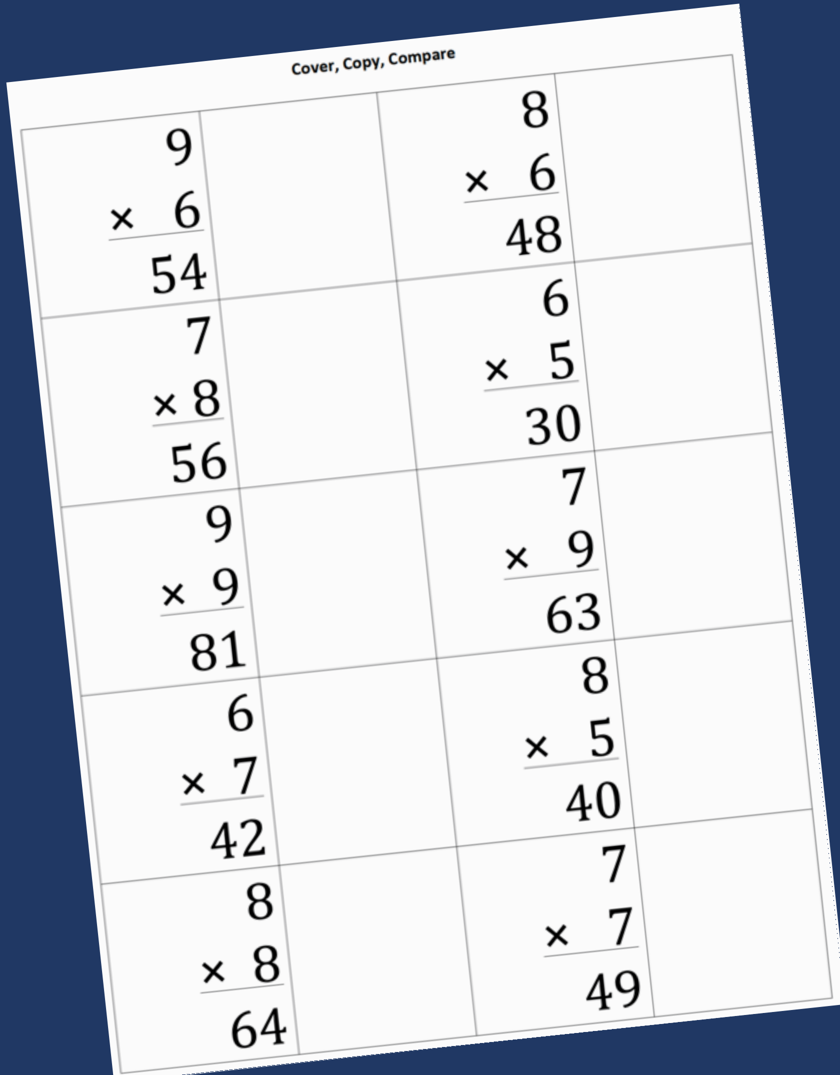


$$6 \div 3 = 2$$

Equal Groups

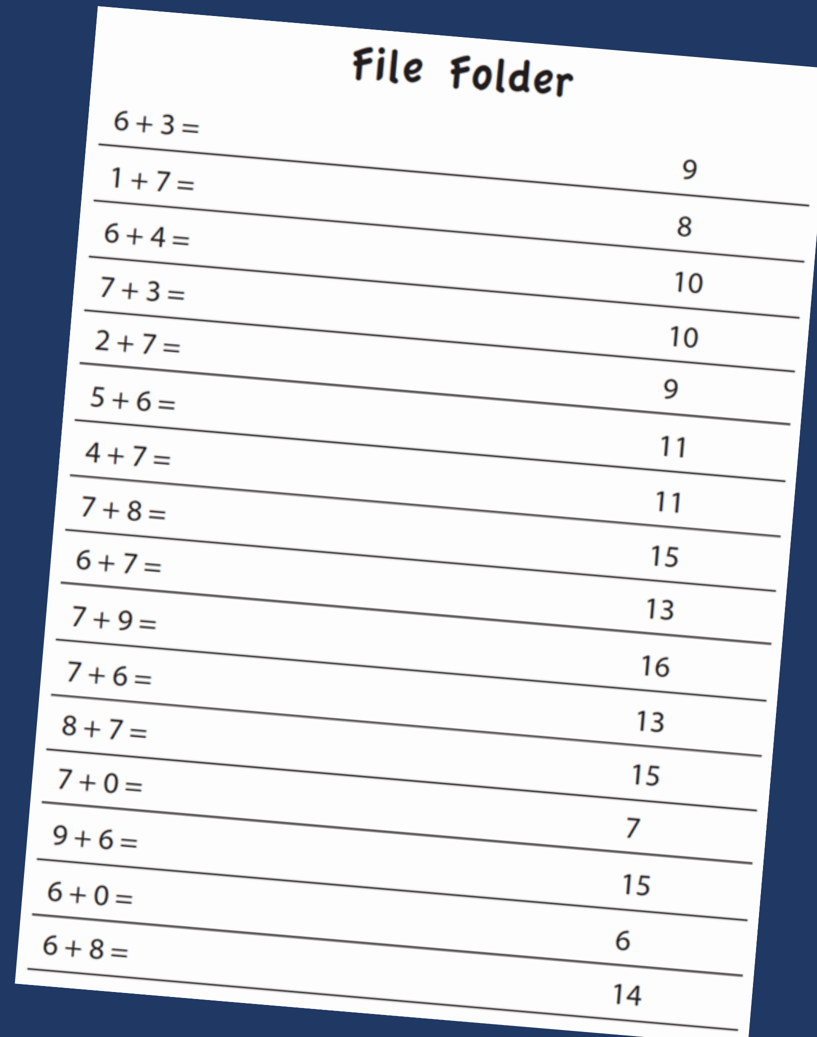
Groups multiplied by number in each group for a product

- Carlos has 12 apples. He wants to share them equally among his 2 friends. How many apples will each friend receive?
  - $2 \times ? = 12$
- Carlos has 12 apples. He put them into bags containing 6 apples each. How many bags did Carlos use?
  - $? \times 6 = 12$



Cover, Copy, Compare

$\begin{array}{r} 9 \\ \times\ 6 \\ \hline 54 \end{array}$		$\begin{array}{r} 8 \\ \times\ 6 \\ \hline 48 \end{array}$	
$\begin{array}{r} 7 \\ \times 8 \\ \hline 56 \end{array}$		$\begin{array}{r} 6 \\ \times\ 5 \\ \hline 30 \end{array}$	
$\begin{array}{r} 9 \\ \times\ 9 \\ \hline 81 \end{array}$		$\begin{array}{r} 7 \\ \times\ 9 \\ \hline 63 \end{array}$	
$\begin{array}{r} 6 \\ \times\ 7 \\ \hline 42 \end{array}$		$\begin{array}{r} 8 \\ \times\ 5 \\ \hline 40 \end{array}$	
$\begin{array}{r} 8 \\ \times\ 8 \\ \hline 64 \end{array}$		$\begin{array}{r} 7 \\ \times\ 7 \\ \hline 49 \end{array}$	



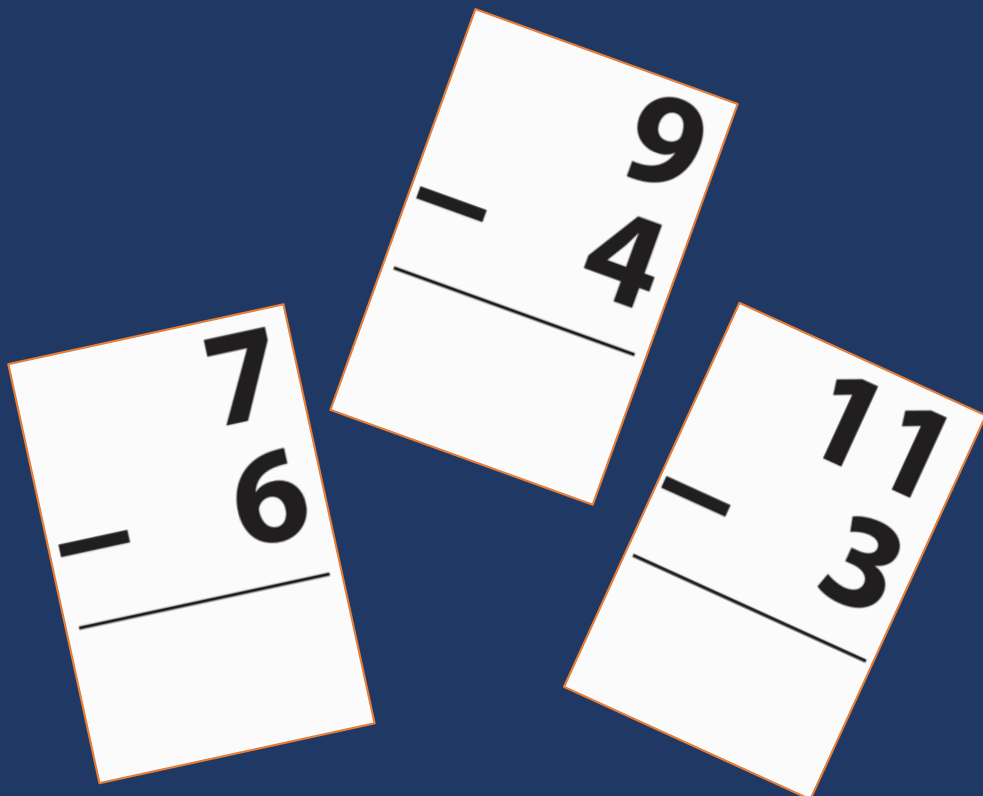
File Folder

6 + 3 =	
1 + 7 =	9
6 + 4 =	8
7 + 3 =	10
2 + 7 =	10
5 + 6 =	9
4 + 7 =	11
7 + 8 =	11
6 + 7 =	15
7 + 9 =	13
7 + 6 =	16
8 + 7 =	13
7 + 0 =	15
9 + 6 =	7
6 + 0 =	15
6 + 8 =	6
	14

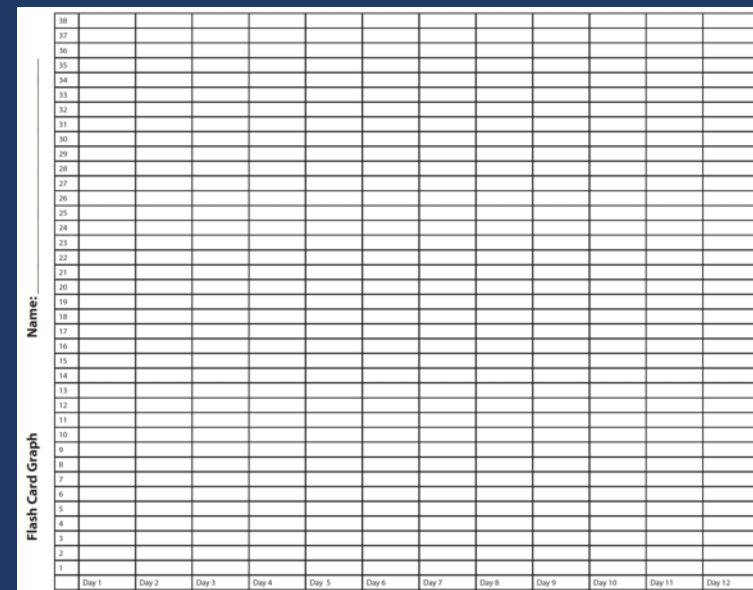
sarahpowellphd.com
Evidence-based mathematics resources for educators

Taped Problems

6×5	8×6	7×9	6×8
9×8	8×5	7×8	6×6
7×7	6×9	5×9	8×4
9×4	6×9	9×5	8×7
6×7	8×8	4×8	5×7



9
− 4
7
− 6
11
− 3



Flash Card Graph
Name:
38
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22
21
20
19
18
17
16
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14
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11
10
9
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7
6
5
4
3
2
1
Day 1
Day 2
Day 3
Day 4
Day 5
Day 6
Day 7
Day 8
Day 9
Day 10
Day 11
Day 12

		(place sum or product from baggie here)

5

4

12

8

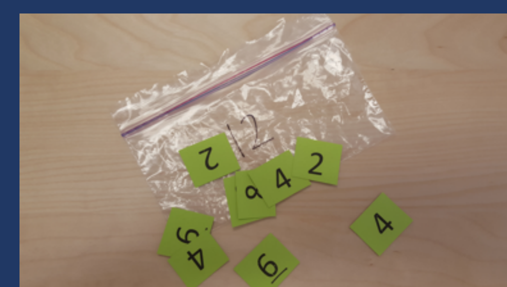
6

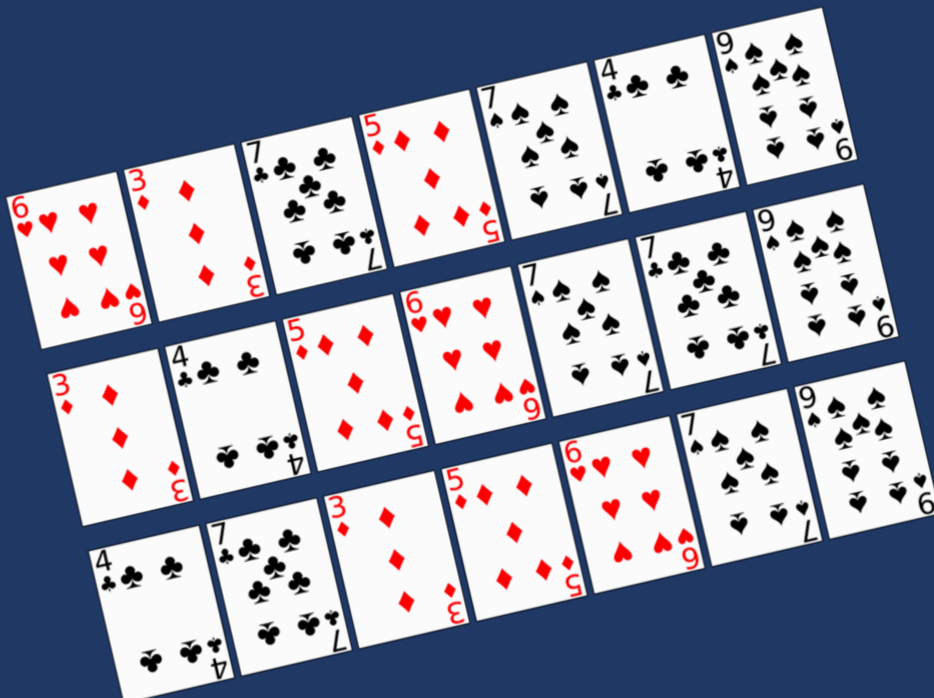
1

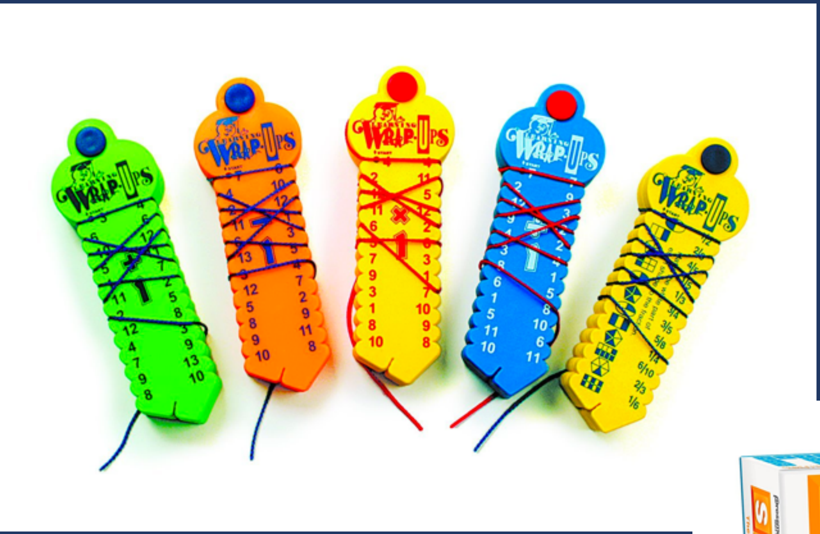
3

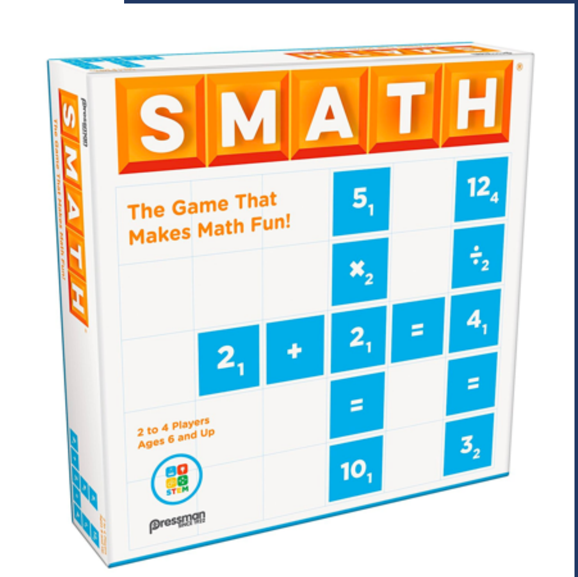
3

6

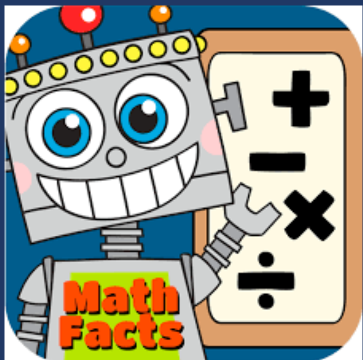






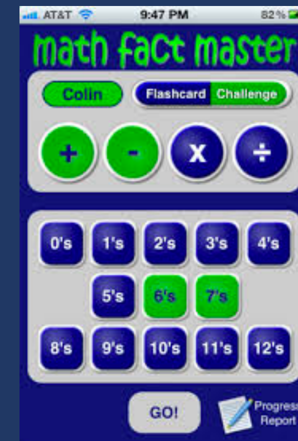

SMATH
The Game That Makes Math Fun!
2 to 4 Players
Ages 6 and Up
Pressman


möbi


Math
Facts


MATH
BINGO
PLAY
SCORE BOARD
HOW TO PLAY


AT&T
10:44 AM
9 + 8
=
19
16
17
15
Rnd


AT&T
9:47 PM
math fact master
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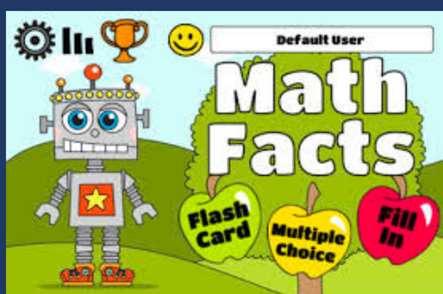
12's
GO!
Progress Report


Default User
Math Facts
Flash Card
Multiple Choice
Fill In


5+4=


Equal To 4
PRACTICE MATH

Addition

Subtraction

Multiplication

Division

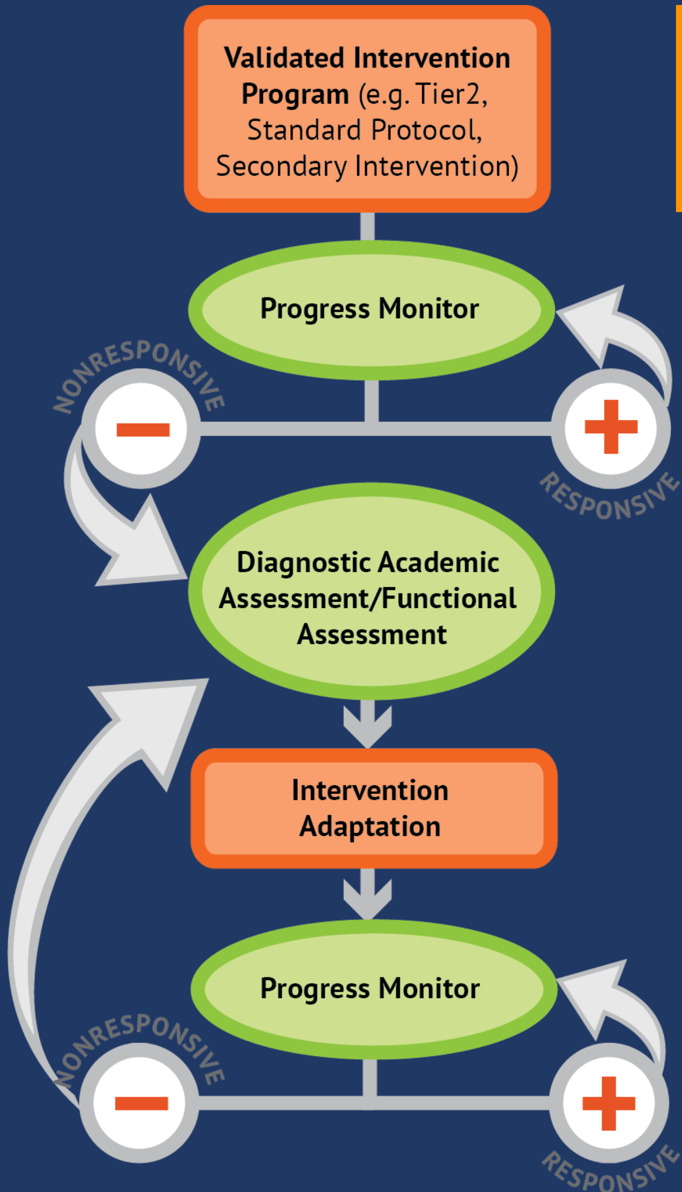
Design **fluency**

$18 \div 3$ $\frac{1}{2} + \frac{3}{4}$ $2x - 5 = 3$

How do you build fact fluency?

- ❑ Teach the *concepts* of the operations
- ❑ Teach *strategies* to understand how facts fit together
- ❑ Practice building *fluency* with a variety of activities and games

Instructional Platform



INSTRUCTIONAL DELIVERY

- Explicit instruction
- Precise language
- Multiple representations

INSTRUCTIONAL STRATEGIES

- Fluency building
- Problem solving instruction

Don’t tie key words to operations

Do have an attack strategy

Do teach word-problem schemas

SOLVE

Study the problem.
Organize the facts.
Line up the plan.
Verify the plan with computation.
Examine the answer.

SIGNS

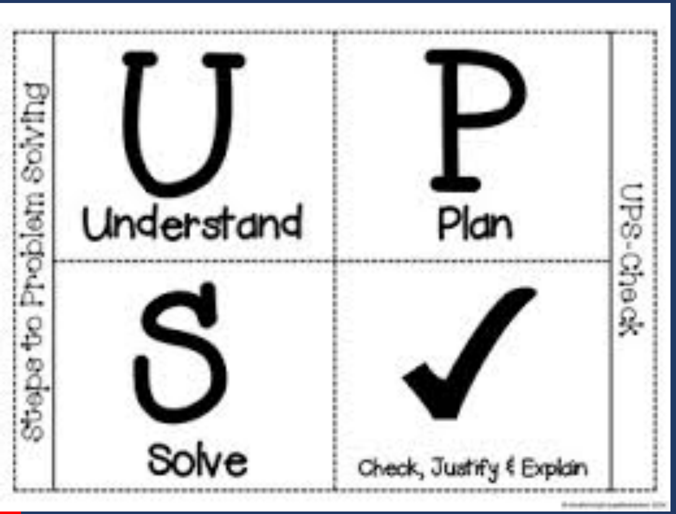
Survey questions

Identify key words

Graphically draw problem

Note operations

Solve and check



Schemas

When teaching about word problems, students should learn the *schema* of the word problem.

- Total
- Difference
- Change
- Equal Groups
- Comparison
- Ratios/Proportions

Don't tie key words to operations

Do have an attack strategy

Do teach word-problem schemas

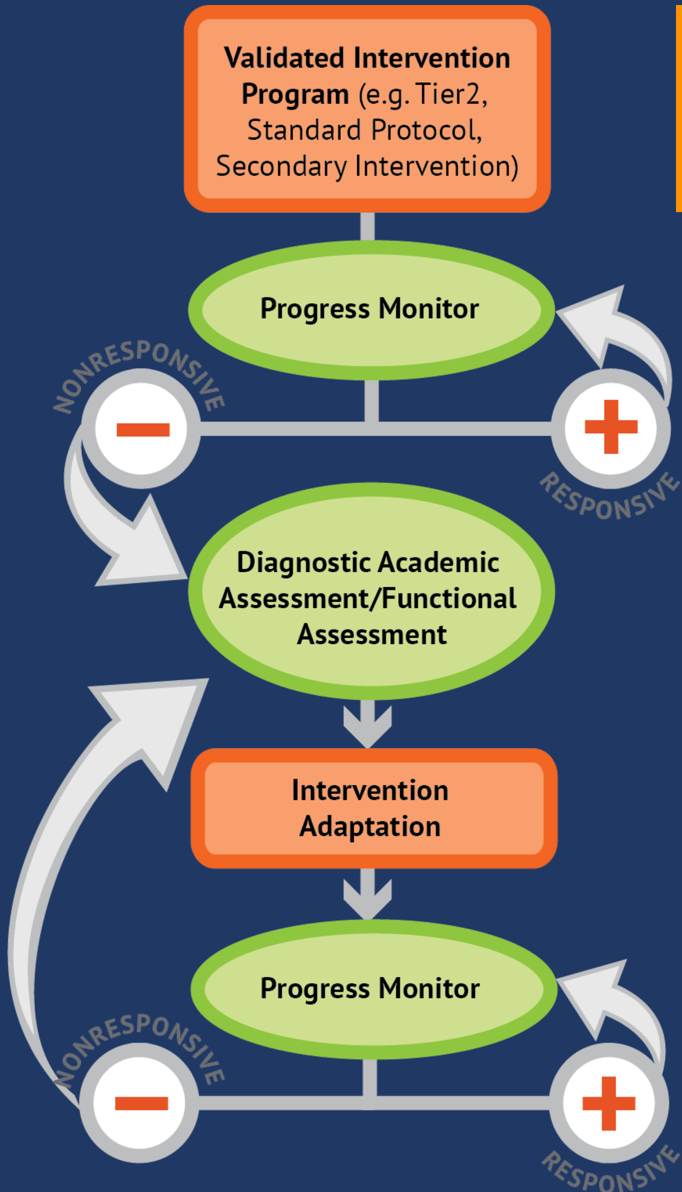
Design **problem solving**

$18 \div 3$ **$\frac{1}{2} + \frac{3}{4}$** **$2x - 5 = 3$**

How do you incorporate effective problem-solving strategies?

- ❑ **Don't** use key words tied to operations
- ❑ **Do** teach students an *attack strategy*
- ❑ **Do** teach students *schemas*

- ❑ **Do** explicitly teach problem solving
- ❑ **Do** provide problem-solving instruction regularly (i.e., several times a week)
- ❑ **Do** practice schemas that students will encounter regularly



Instructional Platform

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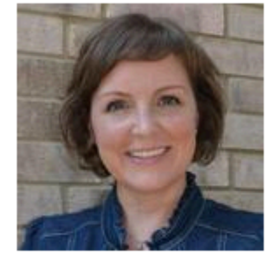
Sarahpowellphd.com

Evidence-based mathematics resources for educators



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