Blind Frequency Offset Estimation for PCC-OFDM with Symbols Overlapped in the Time Domain

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Abstract

This paper presents a blind frequency offset estimator for Polynomial Cancellation Coded Orthogonal Frequency Division Multiplexing (PCC-OFDM) signals with symbols overlapped in the time domain (Overlap PCC-OFDM). The estimation is carried out in the frequency domain. A frequency offset estimate is obtained by using a pair of demodulated subcarriers at the Discrete Fourier Transform (DFT) output. As no pilot tones are required for the estimation there is no loss in bandwidth efficiency. Simulations show that this estimator is an approximately linear function of frequency offset and has low variance.

1. Introduction

OFDM has become the modulation method of choice in many high speed data applications [1]. However OFDM is very sensitive to frequency errors [2]. Polynomial cancellation coded OFDM (PCC-OFDM) is a form of OFDM in which data is mapped onto weighted groups of subcarriers rather than single subcarriers [3]. PCC-OFDM has much reduced sensitivity to frequency errors and multipath transmission [4,5]. In its simplest form PCC-OFDM is not bandwidth efficient. By overlapping the PCC-OFDM symbols in the time domain, the good properties of PCC-OFDM are retained while bandwidth efficiency greater than OFDM with a cyclic prefix is achieved [5].

A number of frequency offset estimation schemes have been proposed in the literature for OFDM. Many of these exploit the redundancy of the cyclic prefix by calculating the correlation between the cyclic prefix and the symbol [6,7]. In a PCC-OFDM system no cyclic prefix is required [8]. In [9] it was shown that the frequency offset in PCC-OFDM could be accurately estimated from a pair of demodulated subcarriers. It is shown in this paper that the same form of the estimator can be used for Overlap PCC-OFDM. However in the overlap case the estimator has increased variance because of the additional components in the demodulated subcarriers due to the overlapping symbols. The estimator is based on the function

$$F(\Delta fT)=\mathrm{Re}\left(\frac{z_{2M+1,i}+z_{2M,i}}{z_{2M+1,i}-z_{2M,i}}\right) \quad (1)$$

where $\mathrm{Re}(\bullet)$ is the real part of a complex number. ΔfT represents the normalized frequency offset. $z_{2M,i}$ and $z_{2M+1,i}$ are a pair of the receiver DFT outputs in the $2M$th and $(2M+1)$th channel in the ith symbol period. When the frequency offset is zero, $z_{2M,i}$ and $z_{2M+1,i}$ contain balanced components with same amplitude and opposite polarities, the expected value of the estimator will be zero. A frequency offset interrupts the balance in a predictable way. For $|\Delta fT|<0.5$, the estimator is an approximately linear function of frequency offset. No training sequence, pilot tones or cyclic prefix are required.

2. Description of Overlap PCC-OFDM

Overlap PCC-OFDM is a data transmission scheme based on the PCC-OFDM [5]. Figure 1 shows how adjacent symbols are overlapped in the time domain.

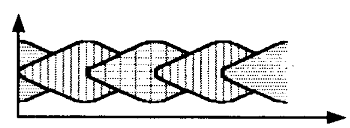

Fig. 1. PCC symbols overlapped in the time domain

Figure 2 shows the structure of an Overlap PCC-OFDM communication system, where $d_{0,i}\cdots d_{M-1,i}$ are M data values in the ith data block to be transmitted. They are mapped onto N subcarriers $a_{0,i},\cdots,a_{N-1,i}$. In this paper, the case where data is mapped onto pairs of subcarriers is considered [3], so $M=N/2$ and $a_{2M+1,i}=-a_{2M,i}$, where i represents the ith IDFT output. At the receiver, the received signal is sampled in such a way that the sampling window of length N moves in increments of $N/2$ samples rather than N samples [10]. The ith DFT output is vector $z_{0,i},z_{1,i},\cdots,z_{N-1,i}$. Therefore, the

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number of output vectors of the DFT demodulator is the same as input vectors of the IDFT modulator.

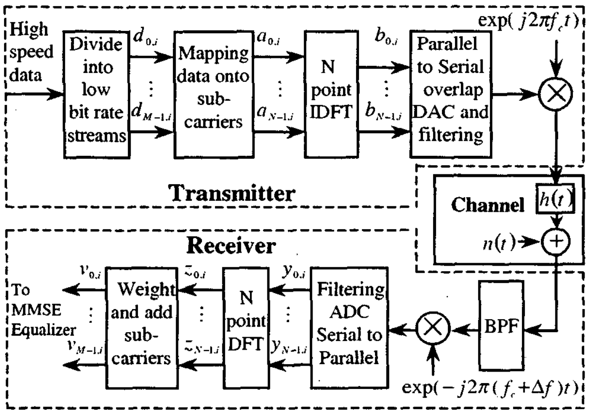

Fig. 2. Block diagram of an Overlap PCC-OFDM system

When there is no frequency offset, the transmitted data block $d_{0,i} \cdots d_{M-1,i}$ can be recovered from a sequence of the vectors $\mathbf{V}_i = [v_{0,i}, \cdots, v_{N-1,i}]$ by using an MMSE two-dimensional equalizer [8]. The frequency offset is therefore estimated before the equalization.

3. Performance of the estimator

To demonstrate the performance of the estimator, computer simulations were performed, where an AWGN channel was considered and 4-QAM was used to modulate 128 subcarriers. The number of symbols simulated was 10,000. The simulation results will be reported for two different cases. The first uses only a single pair of subcarriers in each symbol to estimate the frequency offset and the second averages over all subcarrier pairs in a symbol.

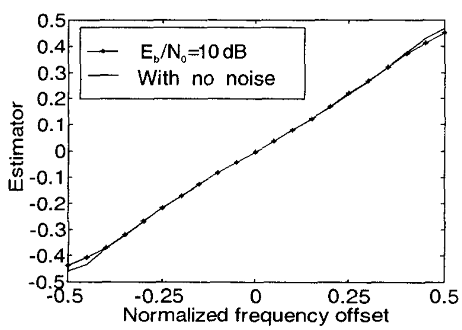

Fig. 3. Estimator as a function of frequency offset

Figure 3 and 4 show the results of simulations of the estimator based on a single subcarrier pair per symbol. Figure 3 shows the mean as a function of normalized frequency offset for the cases of no noise and for a bit energy to noise ratio E_b/N_0=10dB. There is an approximately linear relationship between the offset and the estimator. The mean of the estimator is insensitive to noise. The mean of the estimator is approximately linear even for low E_b/N_0. Figure 4 shows the variance of the estimator as a function of the frequency offset. The main component in the variance is due to the intersymbol interference (ISI) from the overlapping symbols. For non-zero frequency offsets interchannel interference (ICI) also contributes to the variance.

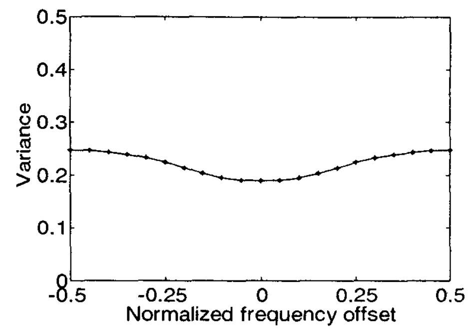

Fig. 4. Variance as a function of frequency offset for estimate from single subcarrier pair

A lower variance estimate can be obtained by averaging the values of (1) over all of subcarrier pairs in a symbol period. Figure 5 shows the variance of the second form of estimator. Note the change in scale from figure 4. The variance is much reduced because of the averaging over each symbol.

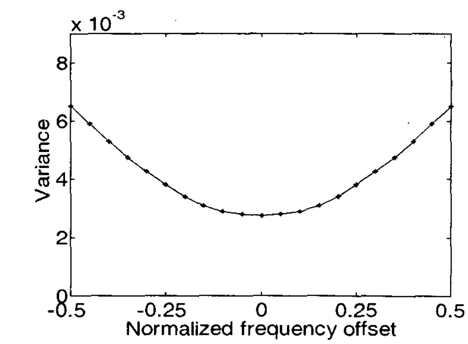

Fig. 5. Variance as a function of frequency offset for averaging all subcarrier pairs in a symbol

In a practical system, particular values of noise may cause the denominator of the estimator (1) to be very close to zero, therefore the magnitude values are very large. In this case a watch dog of either hardware or software could be designed to clip the amplitude of the estimator to unity or to simply discard the estimate. Because there are $N/2$ estimates in a symbol, an accurate estimate can be achieved by averaging over the rest of the estimates. Simulations show that a software watch dog can effectively eliminate the impact from those extreme values. Figure 6 is the variance as a function of E_b/N_0 from 0 to 20dB for the case of the averaging estimator. The variance is very small. In the simulations of Figure 5 and 6 extreme estimate values were discarded.

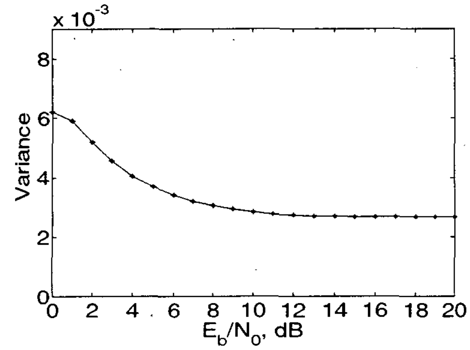

Fig. 6. Variance as a function of E_b/N_0 for averaging over all subcarrier pairs in a symbol

4. Statistical analysis of the estimator

To investigate the statistical properties of the estimator, an approximation will be derived. An expression for the $2M$th demodulated subcarrier in the ith DFT output is given in the Appendix.

$$z_{2M,i}=\exp(j\theta)\left\{\sum_{L=0}^{N/2-1}\left(c_{2(L-M)}-c_{2(L-M)+1}\right)d_{L,i}+\sum_{L=0}^{N/2-1}\left(c_{2(L-M)}+c_{2(L-M)+1}\right)d'_{L,i}\right\}+w_{2M,i} \tag{2}$$

where θ is the phase offset between the phase of the receiver local oscillator and the carrier phase at the start of the received symbol. $c_0,\cdots,c_{N-1}$ are weighting coefficients defined in the Appendix. $w_{2M,i}$ is Additive White Gaussian Noise (AWGN). The demodulated signal is not only a function of the transmitted data block $d_{L,i}$ but also a function of an overlapping component $d'_{L,i}$. Substituting (2) into (1) gives

$$F(\Delta fT)=\mathrm{Re}\left(\frac{\sum_{L=0}^{N/2-1}\left(c_{2(L-M)-1}-c_{2(L-M)+1}\right)d_{L,i}+\sum_{L=0}^{N/2-1}\left(c_{2(L-M)-1}+2c_{2(L-M)}+c_{2(L-M)+1}\right)d'_{L,i}}{\sum_{L=0}^{N/2-1}\left(c_{2(L-M)-1}-2c_{2(L-M)}+c_{2(L-M)+1}\right)d_{L,i}+\sum_{L=0}^{N/2-1}\left(c_{2(L-M)-1}-c_{2(L-M)+1}\right)d'_{L,i}}\right) \tag{3}$$

For simplicity, the noise terms are not considered in (3). Using $(c_{-1}-2c_0+c_1)d_{L,i}\neq 0$, rearrange (3) to give

$$F(\Delta fT)=\mathrm{Re}\left(\frac{A_0+A_t}{1+A_b}\right) \tag{4}$$

where $A_0=(c_{-1}-c_1)/(c_{-1}-2c_0+c_1)$ and A_t and A_b expressions are given by

$$A_t=\frac{\sum_{L=0,\,L\neq M}^{N/2-1}\left(c_{2(L-M)-1}-c_{2(L-M)+1}\right)d_{L,i}+\sum_{L=0}^{N/2-1}\left(c_{2(L-M)-1}+2c_{2(L-M)}+c_{2(L-M)+1}\right)d'_{L,i}}{\left(c_{-1}-2c_0+c_1\right)d_{M,i}} \tag{5}$$

$$A_b=\frac{\sum_{L=0,\,L\neq M}^{N/2-1}\left(c_{2(L-M)-1}-2c_{2(L-M)}+c_{2(L-M)+1}\right)d_{L,i}+\sum_{L=0}^{N/2-1}\left(c_{2(L-M)-1}-c_{2(L-M)+1}\right)d'_{L,i}}{\left(c_{-1}-2c_0+c_1\right)d_{M,i}} \tag{6}$$

It is shown in [9] that $\mathrm{Re}(A_0)\approx\Delta fT$ for large N. Under the assumptions that $d_{L,i}$ are uncorrelated with zero mean and finite variance and when the normalized frequency offset is less than 0.5, then $|A_b|<1$. Equation (4) can be expanded as a complex Taylor series. The series converges rapidly, using the three most significant terms gives

$$F(\Delta fT)\cong\Delta fT+I_M \tag{7}$$

where $I_M=\mathrm{Re}(A_t-A_0A_b)$. A comparison between the approximation and the simulation result is presented in figure 7. It is shown that the simulation result very closely follows the approximation.

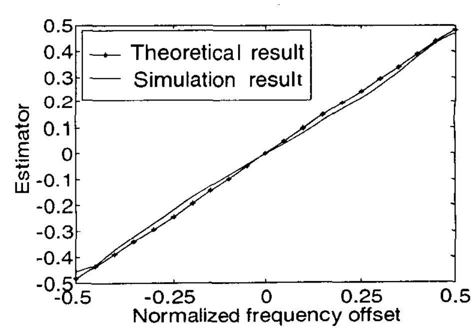

Fig. 7. Comparison between simulation and theoretical results of the estimator for the second form

5. Conclusion

A blind frequency offset estimator has been proposed for Overlap PCC OFDM systems. The estimation is carried out in the frequency domain using a pair of demodulated subcarriers. It is shown that the estimator is particularly robust to AWGN channel noise. To analyze the theoretical performance of the estimator, a linear approximation has been presented on the basis of a complex Taylor Series. It is shown that the theoretical and simulation results are closely matched. The variance of the estimator depends on the frequency offset and the noise level. Simulations show the estimator is an approximate linear function of frequency offset. A low complexity frequency offset synchronizer can be easily designed based on the estimator.

Appendix Expression of a demodulated subcarrier

The ith output of N point IDFT of a data sequence is given by

$$b_{k,i}=\frac{1}{N}\sum_{l=0}^{N-1}a_{l,i}\exp\left(\frac{j2\pi kl}{N}\right),\ k=0,1,\cdots,N-1 \qquad \text{(A1)}$$

Similarly, the second half of the $(i-1)$th and the first half of the $(i+1)$th IDFT outputs are given by

$$b_{N/2+k,i-1}=\begin{cases}\frac{1}{N}\sum_{l=0}^{N-1}(-1)^l a_{l,i-1}\exp\left(\frac{j2\pi kl}{N}\right) & 0\le k\le N/2-1\\ 0, & N/2\le k\le N-1\end{cases} \qquad \text{(A2)}$$

$$b_{k-N/2,i+1}=\begin{cases}0, & 0\le k\le N/2-1\\ \frac{1}{N}\sum_{l=0}^{N-1}(-1)^l a_{l,i+1}\exp\left(\frac{j2\pi kl}{N}\right) & N/2\le k\le N-1\end{cases} \qquad \text{(A3)}$$

Then the ith time domain overlapped symbol is therefore given by

$$\begin{aligned}b'_{k,i}&=b_{N/2-k,i-1}+\frac{1}{N}\sum_{l=0}^{N-1}a_{l,i}\exp\left(\frac{j2\pi kl}{N}\right)+b_{k-N/2,i+1}\\&=\frac{1}{N}\sum_{l=0}^{N-1}(a_{l,i}+a'_{l,i})\exp\left(\frac{j2\pi kl}{N}\right)\end{aligned} \qquad \text{(A4)}$$

where

$$a'_{l,i}=\begin{cases}(-1)^l a_{l,i-1}, & 0\le k\le N/2-1\\ (-1)^l a_{l,i+1}, & N/2\le k\le N-1\end{cases} \qquad \text{(A5)}$$

The demodulated subcarrier is therefore given by

$$z_{m,i}=\exp(j\theta)\sum_{l=0}^{N-1}c_{l-m}(a_{l,i}+a'_{l,i})+w_{m,i} \qquad \text{(A6)}$$

where $w_{m,i}$ is the DFT of AWGN, according to the result of [11], it is also AWGN. c_{l-m} is weighting coefficients defined by [3]

$$c_{l-m}=\frac{\sin(\pi(l-m+\Delta fT))}{N\sin(\pi(l-m+\Delta fT)/N)}\times\exp(j\pi(N-1)(l-m+\Delta fT)/N) \qquad \text{(A7)}$$

In a PCC-OFDM signal, $a_{2L}=-a_{2L+1}=d_L$, $L=0,1,\cdots,N/2-1$. Hence $a'_{2L+1,i}=a'_{2L,i}$ in (A5). Defining $a'_{2L+1,i}=a'_{2L,i}=d'_{L,i}$, then the $2M$th subcarrier in the ith demodulated vector can be represented by

$$z_{2M,i}=\exp(j\theta)\left\{\sum_{L=0}^{N/2-1}(c_{2(L-M)}-c_{2(L-M)+1})d_{L,i}+\sum_{L=0}^{N/2-1}(c_{2(L-M)}+c_{2(L-M)+1})d'_{L,i}\right\}+w_{2M,i} \qquad \text{(A8)}$$

Similarly, we can get the expression of the $(2M+1)$th demodulated subcarrier.

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