# Pros and Cons of the Analysis of Slope Stability by Various Methods of Slices or Columns

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## General Background

The history of slope stability analyses in Geotechnical Engineering is well documented in the textbook by Duncan, Wright and Brandon (2014) and elsewhere. The procedures used for slope stability analysis started out as hand or graphical methods, but with the introduction of limit equilibrium methods of analysis most calculations became computerized. Many engineers seem to believe that these computer programs automatically give the correct answer, but, in addition to the “garbage in – garbage out” rule still holding, these analyses are simplified and thus approximate at best. There are also specific features of the common methods of analysis that may limit their usefulness in numerous ways.

These limitations include unrealistic solutions where the line of thrust goes outside the assumed potential sliding mass and the possibility of developing unreal interslice tensile forces. Despite advice from a range of experts from Morgenstern and Price (1965) to Wright (2013) that the user always should check the solution to make sure that it is physically valid, many computer programs do not make this easy and in the writer’s experience many, perhaps even the majority, of computerized analyses are flawed. Other limitations include the failure to compute local factors of safety which might indicate progressive failure, the failure to include seepage forces, and the failure to consider 3D effects. While there is some published literature on the difference between 2D and 3D slope stability analyses, the truth is that, lacking suitable tools for routinely conducting 3D analyses, no-one really has known in the past how great the difference between 2D and 3D analyses might be. The same is true of the inclusion of seepage forces.

The ways that applied loads, including tie-back forces, and internal reinforcement are modelled, and the effect of that on stability calculations, is also unclear in the documentation for many computer programs.

This report seeks to clarify these issues making use of a new, inherently 3D computer program called TSLOPE (http://tagasoft.com) to make comparisons of the results obtained using two alternate simplified methods of analysis. The paper is confined to the use of the method of slices (or columns in 3D). The vast majority of all evaluations of slope stability to date have been carried out using one variation or another of the method of slices and this is likely to continue. The advantages and the limitations of attempting what might be considered more complete analyses using finite element or finite difference techniques are discussed at the end of the paper.

## Implications of the definition of the factor of safety

There are two ways that the factor of safety has been defined in the analysis of slope stability using the method of slices or columns.

The first is the simple definition that the factor of safety is the sum of the resisting forces around the failure plane divided by the sum of the driving forces. This was used in early analyses using the method of slices and many geotechnical engineers appear to still believe that this is the way the factor of safety is calculated.

However, most modern computer programs define the factor of safety differently, as a strength reduction factor. The factor of safety is that factor by which the assumed shear strengths must be reduced in order that the sums of the driving and resisting forces are equal.

A common argument in support of this definition is that the shear strengths around the failure plane are the greatest source of uncertainty in the analysis, so that it makes sense to factor the shear strengths. That is questionable. In practice, most geotechnical engineers adopt conservative values for the shear strengths or shear strength parameters, so that the uncertainty in these values is already considered. A better argument is that the methods that define the factor of safety this way force the factor of safety to be the same at the base of each slice and obscure the fact that some parts of the potential slip surface may be overstressed, even if the overall factor of safety is above 1.0. That is a good argument for normally requiring an overall factor of safety of 1.5 in practice. If the factor of safety is 1.5 or greater, then the local factors of safety are less likely to fall below 1.0 and the risk of progressive failure should be diminished. Also, as will be shown subsequently, the omission of seepage forces in limit equilibrium methods of analysis can cause the factor of safety to be overestimated by as much as 30 percent, so that is yet another reason for requiring a factor of safety of 1.5. On the other hand, 3D effects, while they can in some cases reduce the factor of safety, normally increase it, sometimes very significantly, so that requiring a 2D factor of safety of 1.5 may be excessive.

Using the second definition of the factor of safety, the sums of the driving and resisting forces are made equal, therefore the methods of analysis that use it are called “limit equilibrium analyses”. Some methods of analysis, such as Bishop’s Simplified Method, are limit equilibrium analyses but they do not “fully satisfy equilibrium”, meaning that force and moment equilibrium is not satisfied for each slice or column and thus for the potential sliding mass as a whole. Methods which do “fully satisfy equilibrium” such as those of Morgenstern and Price (1965) or Spencer (1967) are now generally preferred by both academics and practitioners.

The principal direct implication of how the factor of safety is defined is that with the first, simple definition one can calculate “local factors of safety” for each slice or column whereas in limit equilibrium analyses, the factor of safety at the base of each slice or column is by definition the same. Equations of equilibrium are set up and then solved for two unknowns – the factor of safety and a second unknown which usually has to do with the assumptions made regarding side forces acting on the slices of columns. In Spencer’s Method this unknown is the angle of inclination of the side forces, which is assumed to be constant for all slices or columns. In the Morgenstern and Price method it is a scale factor for the side forces whose varying angles of inclination are specified by the user.

With the second definition of the factor of safety there is only one factor of safety and it applies to each slice or column as well as the overall potential sliding mass. As noted previously, this obscures the fact that some segments of the potential slip surface are likely closer to failure than others, but it also forces an artificial distribution of the normal and shear stresses around the potential slip surface. The normal stresses will impact the shear strengths calculated for non-cohesive materials, that is, materials for which the strength is at least in part specified to be a function of the normal stress on the potential slip surface. This is demonstrated subsequently in several examples which show the normal stress distributions obtained using a limit equilibrium analyses and a simple method of analysis which is not a limit equilibrium analyses. It turns out that the difference in the normal stresses is the big contributing factor to any differences in the factor of safety that are computed by the two methods.

## Limitations of limit equilibrium methods of analysis

Given the previous discussion, one might then ask, "why do people generally prefer methods that fully satisfy equilibrium?" The basic answer to this question seems to be that engineers are taught in undergraduate classes that any analysis of the stresses in a rigid body should "fully satisfy equilibrium", and it certainly looks more elegant or sophisticated to do this. But is it correct for a potential sliding mass that is deformable and can't take tension?

The second definition of the factor of safety and the quest to fully satisfy equilibrium implies that the potential sliding mass acts as a rigid body. Leaving aside for the moment whether this is reasonable or not for real slopes, this forces the factor of safety to be the same for all slices, thus forcing an artificial distribution of the normal and shear stresses around the potential slip surface, and has other implications as well. These implications have to do with the development of tensile interslice forces and the calculated line of thrust, and also whether or not the solution converges and, further, whether or not it converges to the correct solution.

Solutions that "fully satisfy equilibrium" will tend to develop negative interslice forces wherever there is a hump in the potential slip surface and at the upper end of a shallow potential slip surface. The computed factors of safety in these cases may be quite unconservative because the assumed rigid body gets hung up. Thus, the user needs to insert tension cracks as necessary to eliminate any tensile interslice forces, since soil and rock masses generally have no tensile capacity. The user also then has to decide whether a model with perhaps artificially deep tension cracks is real or not.

More attention in the literature has been applied to the line of thrust, that is the locus of the points of application of the interslice forces, and this has generally been the principal recommended test for whether a solution is reasonable or not. Ideally the line of thrust should be located at something like the third point of the slices or columns but it should never travel outside the boundaries of the potential sliding mass, as it commonly does in problems with tensile interslice forces and sometimes does in pseudo-static seismic analyses.

The occurrence of tensile interslice forces and odd lines of thrust is illustrated using the example of a relatively simple embankment dam, but it is also helpful to compare the results using a method that "fully satisfies equilibrium", in this case Spencer's Method, with a method that uses the first, simple definition of the factor of safety, in this case the

Ordinary Method of Columns (OMC), a 3D implementation of the Ordinary Method of Slices (OMS). The OMC and some past criticisms of the OMS are described in more detail subsequently, but for this first example, it is sufficient to say that it uses the first definition of the factor of safety and that interslice, or intercolumn, forces are neglected. It is as if a bunch of square columns coated with Teflon can slide up and down as the overall slope deforms. Thus, the OMC implies that the potential sliding mass is deformable whereas Spencer's method implies that the potential sliding mass is rigid.

Figure 1 shows results for the stability of the downstream slope of a simple dam embankment analysed using both Spencer's Method and the OMC. In Figure 1(a) using Spencer's Method causes the development of tensile interslice forces, indicated by slices coloured red, and causes the line of thrust, shown as a red line, to swing outside the potential sliding mass.

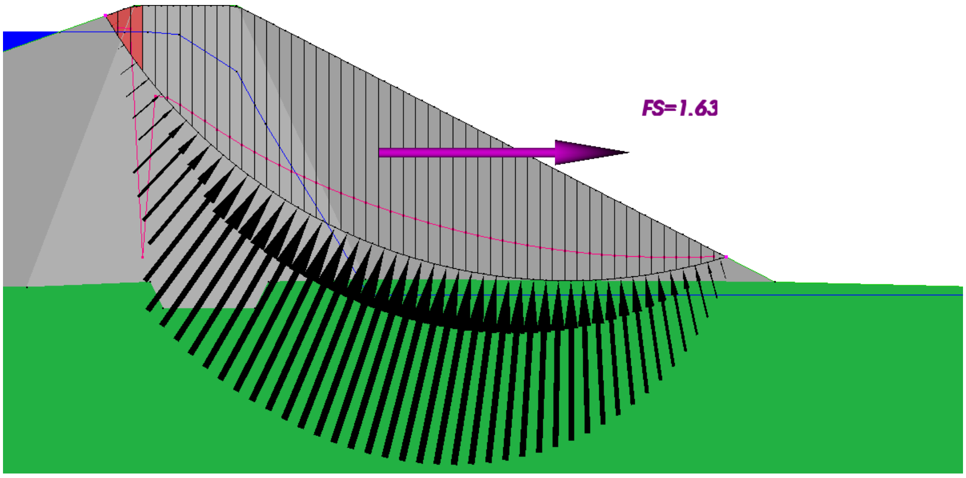


**Figure 1(a) – No Tension Crack, Spencer**

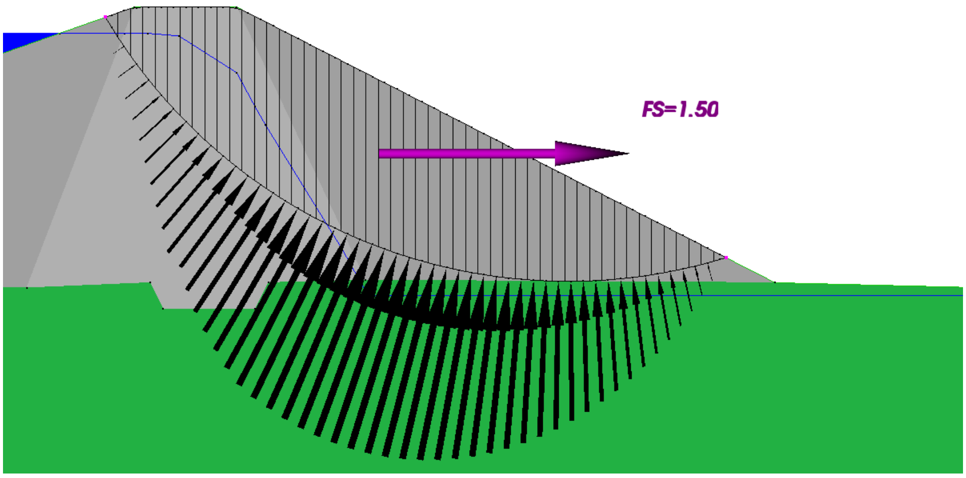


**Figure 1(b) – No Tension Crack, OMC**

In Figure 1(c), the tensile interslice forces in the solution by Spencer's method have been eliminated by inserting a tension crack, slightly lowering the computed factor of safety. However, instead of the line of thrust going way outside of the potential sliding mass at the top of the slope, it now has a hiccup at the toe.

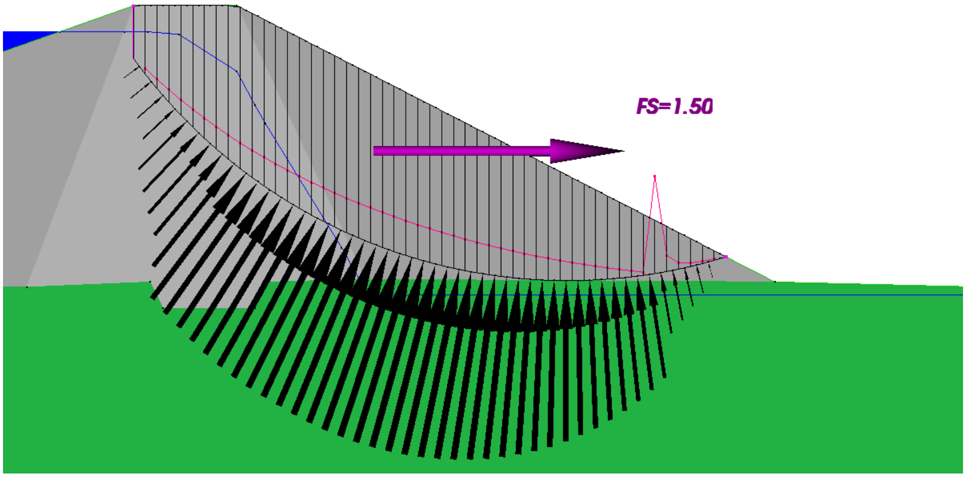


**Figure 1(c) – With Tension Crack, Spencer**

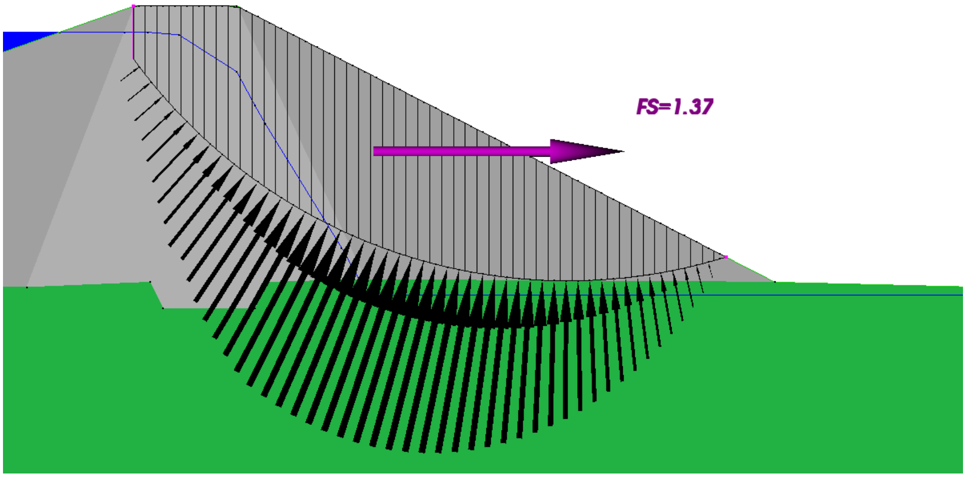


**Figure 1(d) – With Tension Crack, OMC**

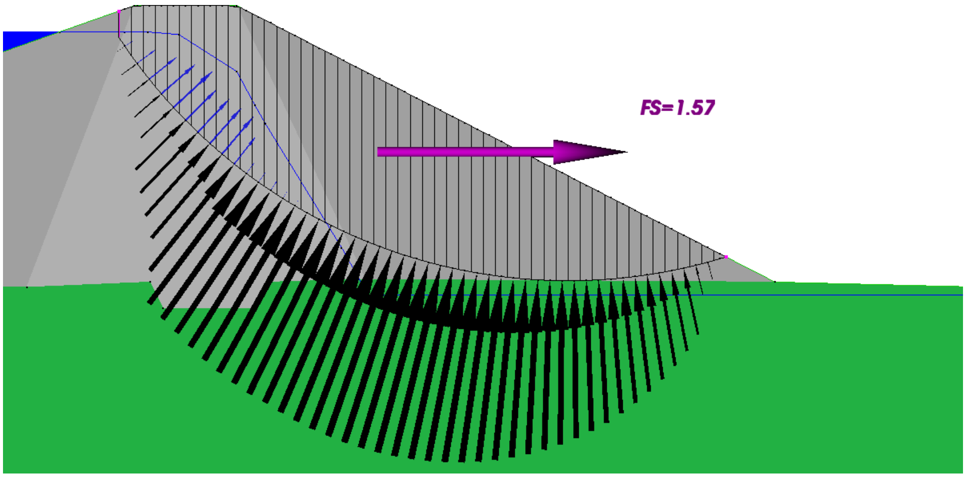


**Figure 1(e) – With Shorter Tension Crack, Spencer**

Note that in both Figures 1(b) and 1(d), the OMC gives a lower factor of safety than Spencer's method. This is partly because Spencer's method does not account for seepage forces, but the seepage forces in this problem are not very large relative to the gravity forces and most of the difference results from the difference in the distribution of the normal stresses on the bases of the columns. In this problem both the core and the downstream shell are specified to have shear strengths with both cohesive and non-cohesive components and the non-cohesive component is sensitive to the normal forces. The normal forces in the figures have different scales because the scale is set so that the maximum values have the same length, but the vector sums of the normal forces are equal. However, with Spencer's method more of the load is transferred towards the ends of the slip surface and, overall, this increases the shear strengths and the factor of safety. With the OMC there is no internal load transfer and the normal stress results solely from the weight of the column in question. The truth likely lies somewhere in between these two extremes.

If the engineer is troubled by the line of thrust in Figure 1(c) and wants to spend more time on the problem, it can be eliminated by halving the depth of the tension crack, as shown in Figure 1(e). The resulting factor of safety of 1.57 might be considered the "best answer" by many authorities, but the corresponding value of 1.44 by the OMC is a safer and likely more realistic value, given that the dam embankment is not rigid and must be subject to some seepage forces.

The difficulty of obtaining what Morgenstern and Price called a "physically acceptable" solution, without tension and with the line of thrust contained within the potential sliding mass, using Spencer's method illustrates the importance of the user being able to readily see the line of thrust and the occurrence of tension.

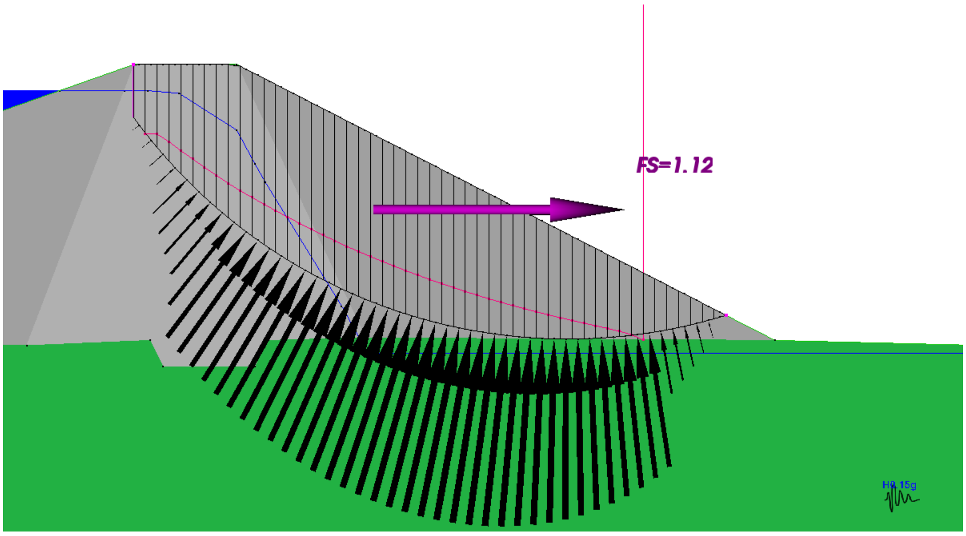


**Figure 1(f) – With Tension Crack and Seismic Coefficient, Spencer**

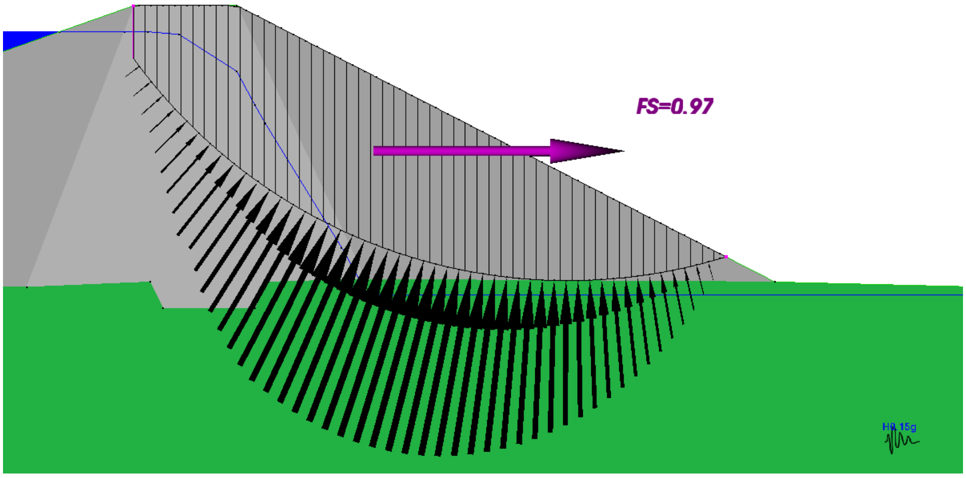


**Figure 1(g) – With Tension Crack and Seismic Coefficient, OMC**

Figures 1(f) and 1(g) illustrate how these problems can be compounded by the addition of external loads such as pseudo-static seismic forces. In Figure 1(f) the problem with the line of thrust seen in Figure 1(c) is now aggravated. It is not uncommon for the line of thrust in pseudo-static analyses using methods that fully satisfy equilibrium to come out of the slope and the engineer must decide whether he/she can live with that or not. In the corresponding analysis by the OMC, the normal stresses on the bases of the columns are not impacted by the added seismic loads so that a nicer looking distribution of the normal forces are obtained and the same shear strengths apply around the slip surface as were used in the static analysis. But, pseudo-static analyses are approximate anyway and these points are less important than whether standard static strength properties are used or whether adjustments, which might be considerable, are made for different drainage conditions and rates of loading. This latter point is illustrated in a subsequent example involving Treasure Island.

This question of the reasonableness of the results of slope stability analyses obtained using the various forms of the method of slices has been repeatedly addressed in the literature but, sadly, it is often ignored in practice. Morgenstern and Price (1965), in their very elegant paper which introduced the concept of a user-specified distribution of the angle of inclination, emphasized that there were multiple possible solutions and that the user should vary the assumed distribution of the angle of inclination so that a reasonable line of thrust was achieved, if possible. Whitman and Bailey (1967), who correctly took Morgenstern and Price to be the gold standard for analyses that fully satisfy equilibrium, said “the use of the Morgenstern-Price approach together with a computer does not free the engineer from making a judgment concerning the reasonableness of a solution.” Chin and Fredland (1983) noted some difficulties with methods that fully satisfy equilibrium, including the fact that they sometimes have trouble converging to a solution, and suggest some possible workarounds. Krahn (2003) discussed the limits of limit equilibrium analyses including convergence issues and difficulties with applying external forces. He suggested that the latter can best be addressed using a hybrid finite element - limit equilibrium analysis but that seems unwieldy for routine use. Wright (2013), in a “must watch” lecture, included several case histories that illustrate various problems with methods that fully satisfy equilibrium. Wright emphasized that there is no absolutely correct solution, and suggested that the engineer should always use at least two computer programs for any critical problem, in part because computer programs may include hidden assumptions and also may not show the intermediate results that are necessary to judge the reasonableness of the final result. Or, as an alternative, the engineer can use one program that offers two good methods of solution and makes all the key data visible.

**Seepage Forces**

Slope failures are often said to be due to water, or rather the failure to recognize the correct water conditions, and application of the pore pressures normal to the bases of the slices in conventional slope stability analyses may give the impression that this accounts for seepage forces in non-hydrostatic conditions. However, this is not correct. The seepage forces that one assumes might be applied by using total unit weights and specifying the pore pressures along the slip surface do not actually make their way into the analysis. This can easily be checked by running an analysis of a cohesive slope with varying phreatic surfaces. Regardless of how steep the phreatic surface, it will make no difference to the computed factor of safety. The reason that a cohesive slope, or a slope in which all the strengths are specified as fixed quantities, as with undrained shear strengths, must be used is that the strength of frictional material will vary with the normal effective stress so that changing the phreatic surface will make a difference, but it does not make a difference to the limit equilibrium problem.

This problem related to seepage forces was noted by King (1989) and is most simply explained by saying that if the seepage forces are pictured as boundary water pressures, the corresponding forces will be applied at the centre of the base of each slice and they make no difference to the standard equations of equilibrium. They make no difference to the moment because the moment arm is zero and they are not included in the solution for force equilibrium parallel to the base of the slice. They make no difference to force equilibrium normal to the base of the slice because the force due to the weight of the slice is fixed and increasing the pore pressure simply reduces the effective stress, which may change the calculated shear strength, but doesn't impact the solution of the equations of equilibrium. The shear strength will change if a frictional material is specified but it has no effect if the shear strength is specified as a fixed number. The writer and his then colleagues learnt this the hard way some years ago when trying to include excess pore pressures generated by earthquake loading in a second stage analysis. Once the programming was completed we found that it made no difference to the calculated factor of safety! King suggested a solution which involved calculating the distributed seepage forces and applying them at the appropriate height in each slice, but this is a little unwieldy and requires a companion seepage analysis, so that his proposed solution has never caught on. In the OMC, however, it is easy to specify the seepage forces as horizontally applied forces on each slice, as discussed by Pyke (2016).

The application in the OMC or non-application in Spencer's method of seepage forces is illustrated in Figure 2 which shows an idealized levee section that is based on real levees in the Sacramento - San Joaquin Delta of California. The soil properties are specified entirely as a cohesion, so that the shear strengths used in both Spencer's method and the OMC are the same, despite the difference in the distribution of the normal stresses which can be seen in Figures 2(a) and 2(b). In Figures 2(a) and 2(b) the phreatic surface is made flat and brought down below the levee so that there are no seepage forces. For this case Spencer's method and the OMC give identical factors of safety of 1.27, as shown on the figures and in Table 1.

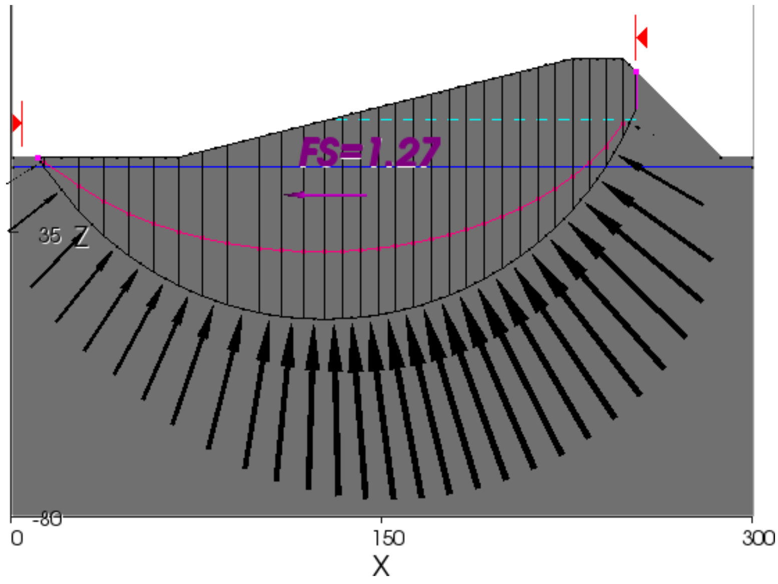


**Figure 2(a) – Flat Phreatic Surface, Spencer**

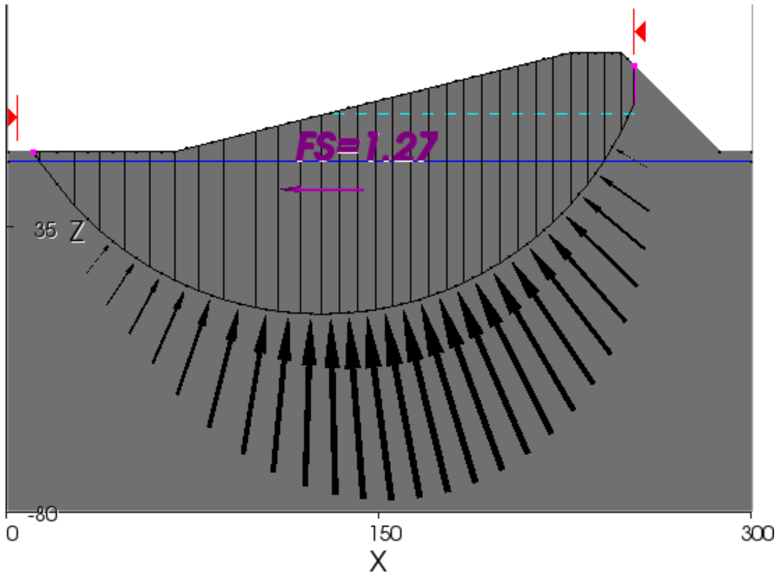


**Figure 2(b) – Flat Phreatic Surface, OMC**

In Figures 2(c), (d), (e) and (f) the phreatic surface is raised in two steps and it can be seen that the factor of safety by Spencer's Method does not change. Note that for the top phreatic surface there is a red flag on the first slice using Spencer's method. This is not due to interslice tension but is the result of a negative normal stress on the base of that slice as a result of the steep angle of inclination of the base, the same thing that standard implementations of the OMS has been criticized for. It can be eliminated by moving the tension crack to the left but in this case it makes no difference to the calculated factor of safety. The same issue does not arise in the OMC as implemented in

TSLOPE because the alternate method of calculating normal stresses is used, as explained further below.

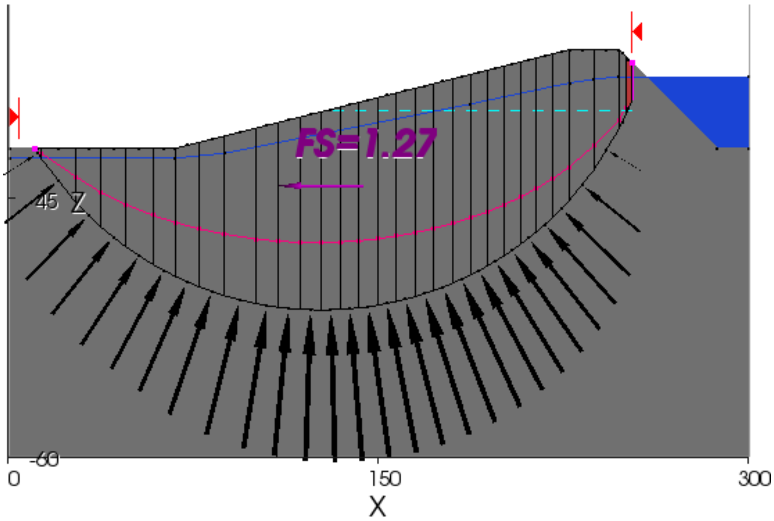


**Figure 2(c) – Middle Phreatic Surface, Spencer**

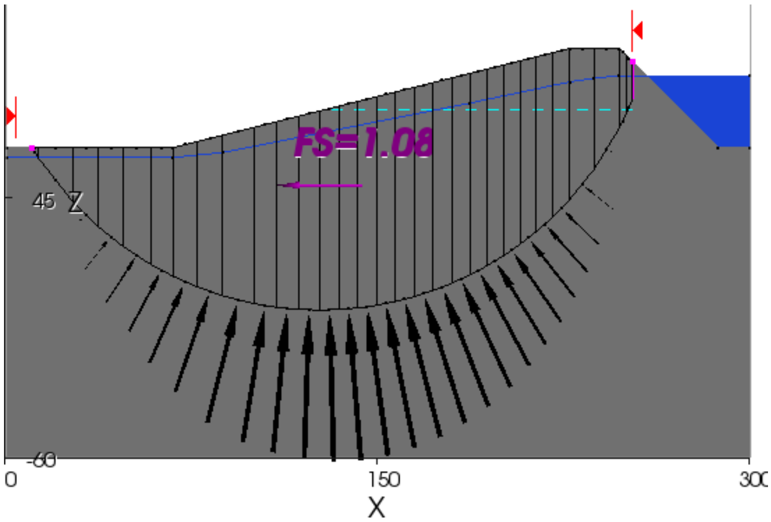


**Figure 2(d) – Middle Phreatic Surface, OMC**

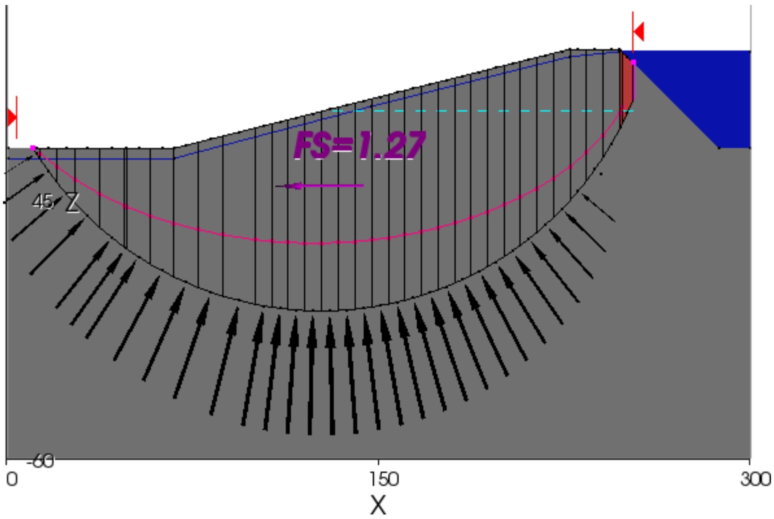


**Figure 2(e) – Top Phreatic Surface, Spencer**

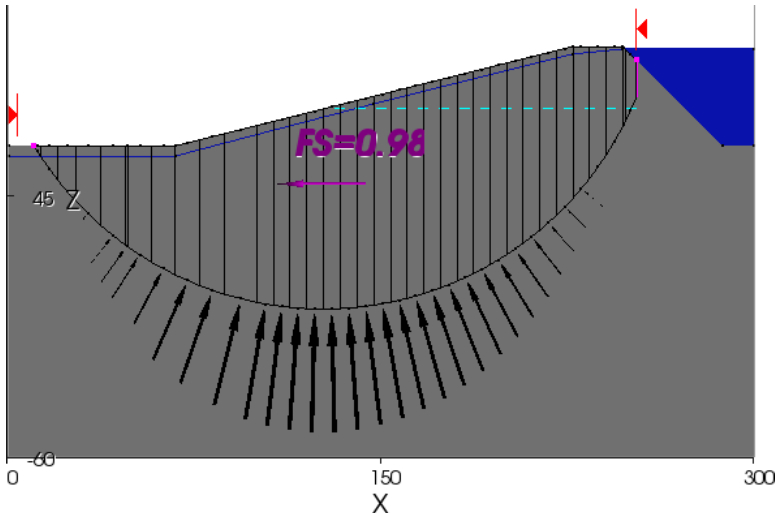


**Figure 2(f) – Top Phreatic Surface, OMC**

| Case | OMC | Spencer |
|---|---|---|
| Flat phreatic surface | 1.27 | 1.27 |
| Middle phreatic surface | 1.08 | 1.27 |
| Top phreatic surface | 0.98 | 1.27 |

**Table 1 – Factors of Safety, All Cohesion Case**

It may be seen that the OMC gives factors of safety which are 15 and 23 percent lower than the factors of safety by Spencer's method so that provides a measure of the importance of including seepage forces. For this levee example, the difference could be critical, but this example has been chosen to represent something of a worst case and often the seepage forces do not make this much difference because they are small relative to the driving forces due to gravity. But if you can't or don't make a check on the magnitude of the seepage forces, you will never know.

If the shear strength is specified to be entirely non-cohesive and is thus a function of the normal stresses at the bases of the columns, the effect of changing the phreatic surface becomes more dramatic because it includes two factors, the seepage forces and a change in the shear strengths. This can be seen in the following figures. Figures 2 (g) and 2(h) show the solution for the flat phreatic surface using both Spencer's Method and the OMC. It can be seen that the spreading out of the normal forces by Spencer's method results in higher average shear strengths and hence a higher factor of safety. Again, the truth probably lies somewhere in between these two limits but the engineer has to decide whether the potential sliding mass is more rigid or less rigid in assigning weights to the two answers.

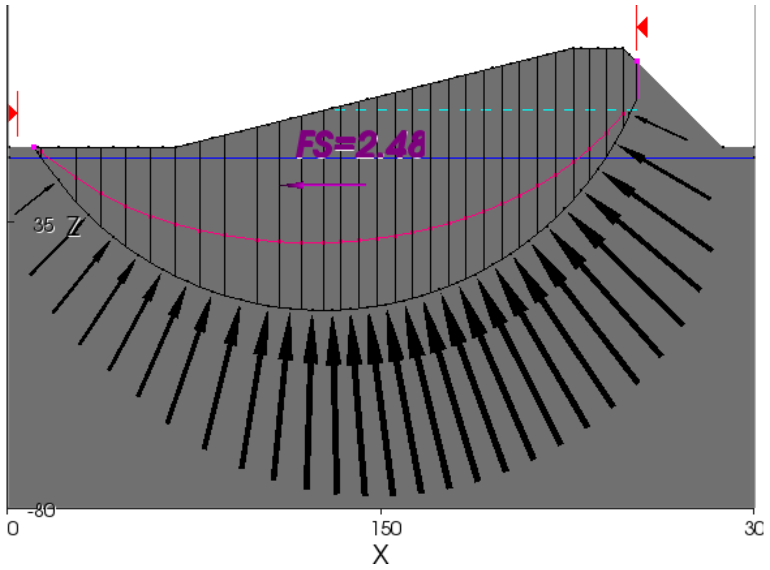


**Figure 2(g) – Flat Phreatic Surface, Spencer, Cohesionless**

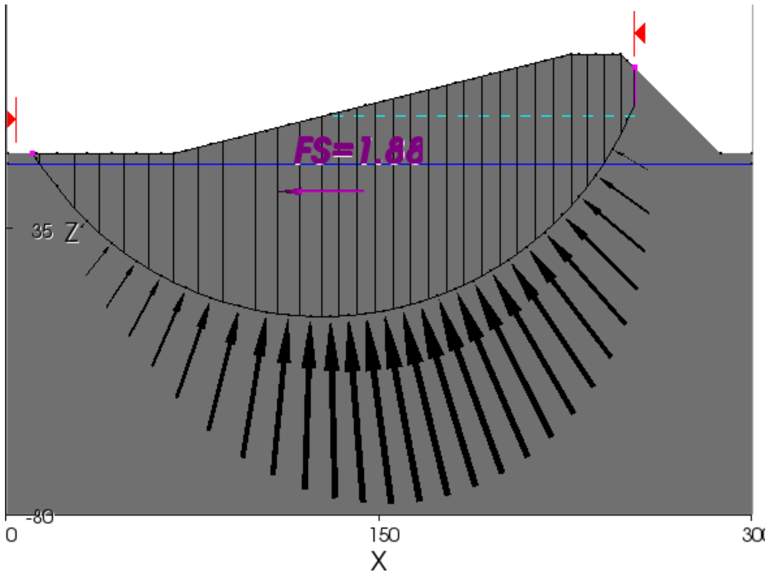


**Figure 2(h) – Flat Phreatic Surface, OMC, Cohesionless**

The results for the top phreatic surface are shown in Figures 2(i) and 2(j). It may be seen that the result of combining the lack of seepage forces in Spencer's Method and the effect of the different normal stress distribution is now quite large. The OMC factor of safety for the flat phreatic surface was 75% of the Spencer factor of safety but another 75% or so reduction due to the different normal stress distribution now means that the OMC factor of safety is only 55% of the Spencer factor of safety. Again, this might be something of an extreme case since both effects are maximized. In practice a levee would not be composed of solely cohesionless materials, nonetheless, even a factor of safety of 1.5 and a conservative choice of soil strength parameters might not provide the expected margin of safety for levees with significant cohesionless soil content if the analysis is done using standard limit equilibrium methods. Figures for the intermediate case are not shown but the calculated factors of safety are shown for all three cases in Table 2.

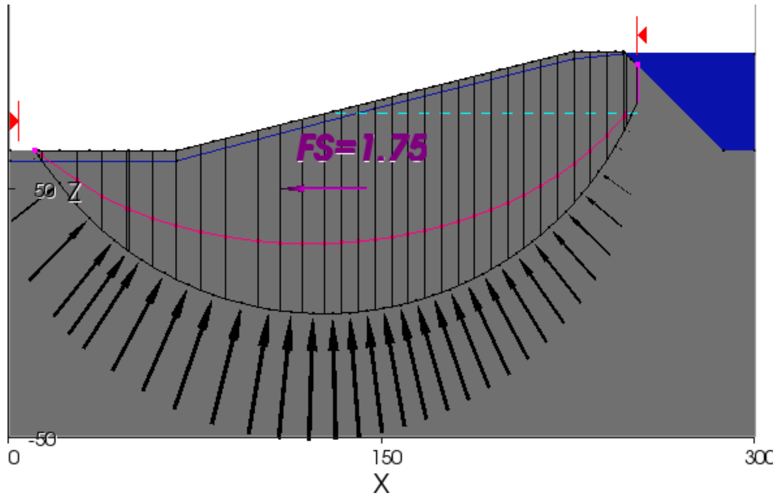


**Figure 2(i) – Top Phreatic Surface, Spencer, Cohesionless**

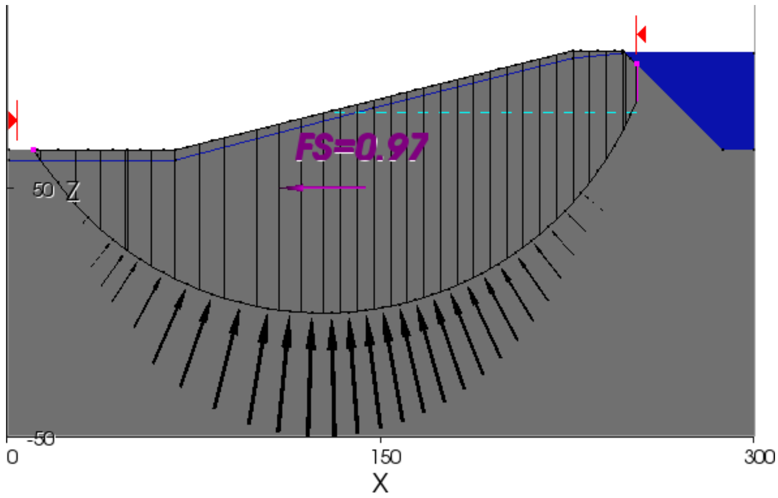


**Figure 2(j) – Top Phreatic Surface, OMC, Cohesionless**

| Case | OMC | Spencer |
|---|---|---|
| Flat phreatic surface | 1.88 | 2.48 |
| Middle phreatic surface | 1.22 | 1.97 |
| Top phreatic surface | 0.97 | 1.75 |

**Table 2 – Factors of Safety, Cohesionless Case**

Returning to just the question of seepage forces, since these will often not make that much difference, this suggests that an increase in the total weights, elimination of negative pore pressures in partially saturated soils, a change from drained to undrained loading conditions and more general softening of soils might be bigger factors in why landslides often appear to be triggered by water. But, in critical cases the engineer needs to use a method of analysis such as the OMC, which does include seepage forces, at least as a check on standard limit equilibrium calculations.

## The Ordinary Method of Columns as an Alternative

Because of the difficulties noted above with methods of analysis that "fully satisfy equilibrium" and limit equilibrium methods in general, it is worth re-examining the Ordinary Method of Slices (OMS) in which interslice forces are neglected – which is similar to assuming a frictionless contact between adjacent slices. The OMS does not require an iterative solution so that convergence and multiple solutions are no longer issues. The OMS, as described for instance by Duncan, Wright and Brandon (2014), and also sometimes referred to as the Fellenius or Swedish Circle Method, uses moment equilibrium about the centre of a circular slip surface and the factor of safety is defined as the sum of the resisting moments divided by the sum of the driving moments. A similar method can be extended to non-circular slip surfaces if the factor of safety is defined as the sum of the resisting forces around the failure plane divided by the sum of the driving forces. These driving and resisting forces are computed as stresses that are normal and parallel to the base of each slice. The sums of these forces make no sense if they are added arithmetically but if they are added as vector sums, they do. In the early days of slope stability analyses such vector additions were done graphically but now they can be done by computer using modern programming languages.

In addition to having the virtue of simplicity, the OMS also effectively accounts for the deformable nature of soil and rock masses and allowing the slices to slide up and down relative to each other, while still not physically correct, is generally more consistent with reality than assuming that the entire potential sliding mass is a rigid body. The assumption of a rigid body is really only realistic when the slip surface is either circular or a logarithmic spiral, but even then the potential sliding mass is likely deformable. For non-circular slip surfaces, which are not kinematically admissible, the assumption of a rigid body is unrealistic and may often lead to overly conservative results. Even wedges of rock mass, for which special analysis techniques have been developed in rock mechanics, and for which failures are kinematically admissible, are rarely if ever single unjointed and unfractured blocks of rock.

The OMS has been criticized and largely fallen out of favour in recent times because of a widely quoted example contained in the otherwise excellent paper by Whitman and Bailey (1967). This example consists of the analysis of the submerged upstream slope of an inclined core dam and is reproduced below. Whitman and Bailey, and Duncan, Wright and Brandon (2014), and others before them, correctly pointed out a problem, that actually applies to all analyses by the Method of Slices, but can be seen more clearly in the OMS. If the normal effective stress on the failure plane is computed by taking the component of total weight of the slice and any water above it that is normal to the potential slip surface and then subtracting the pore pressure that acts on the base of the slice, when either or both of the angle of inclination of the base of the slice and the pore pressure are large, the calculated normal effective stress can be less than zero. The problem is illustrated and discussed in more detail by Pyke (2016). However, Whitman and Bailey and Duncan, Wright and Brandon, and, again, others before them, also suggested a solution to this problem which is to use the buoyant unit weight of the slice in calculating the weight of the slice and the component normal to the base of the slice. Pyke (2016) explains that not only does this solve the problem but that it is the "more correct" solution, and that under non-hydrostatic conditions the buoyant unit weights need to be adjusted for any seepage forces in the vertical direction. This can most easily be done in practice by going back to using the total unit weights but applying the water pressures in the vertical direction that act on the base and the top of the slice.

The computer program TSLOPE offers only two methods of solution, the Ordinary Method of Columns (OMC), which is equivalent to the OMS in 2D, and a unique 3D solution for Spencer's Method. The solution for Spencer's method uses an optimization technique that always converges and the final imbalance in the moments and forces (if any) can be seen by the user. The user can also view a 3D surface that shows the variation in the factor of safety with respect to other variables so that it can be confirmed that the appropriate minimum value has been found. A not uncommon problem with Spencer's Method is that it might converge to a false minimum. The TSLOPE solution space analysis provides feedback on this issue.

Spencer's Method assumes that the angle of inclination of all the interslice forces is the same. This is obviously not correct, but Spencer's Method converges more reliably than the Morgenstern and Price Method, and even with the theoretically nicer Morgenstern and Price method, the user has to check that a valid line of thrust and a solution without tension is obtained, in addition to struggling with what distribution of interslice forces to use in the first place. Because the OMC provides a direct calculation of the factor of safety and has no convergence issues, even if Spencer's method is specified, TSLOPE initially calculates the factor of safety using the OMC and then uses this value and the direction of sliding (for a 3D analysis) as the starting point for the calculation by Spencer's Method.

The OMC, as implemented in TSLOPE, is generally similar to the method for 3D analysis of slopes described by Hovland (1977) in which inter-column forces are neglected and driving and resisting forces are computed parallel to the bases of the slices or columns. The factor of safety in TSLOPE is defined as the vector sum of the resisting forces divided by the vector sum of the driving forces. The normal effective stresses on the potential failure plane are calculated as discussed above.

The results obtained using TSLOPE for two variations of Whitman and Bailey's Example 4, shown in their Figure 11, are shown in Figure 3 and Table 3. Figures 3(a) and (b) show the original Whitman and Bailey problem with the pond level close to, but not quite at, the top of the core. Figures 3(c) and (d) have the pond level brought down to the top of the potential sliding mass. It can be seen from the figures that lowering the pond level makes no difference to the factor of safety, as should be the case for hydrostatic conditions and a fully submerged potential sliding mass. The factor of safety computed by Spencer's Method is 2.09. Because much of the potential failure surface passes through the core which includes significant cohesion and the distributions of normal stresses are not that different, the corresponding factors of safety by the OMC are only 8 percent lower at 1.93.

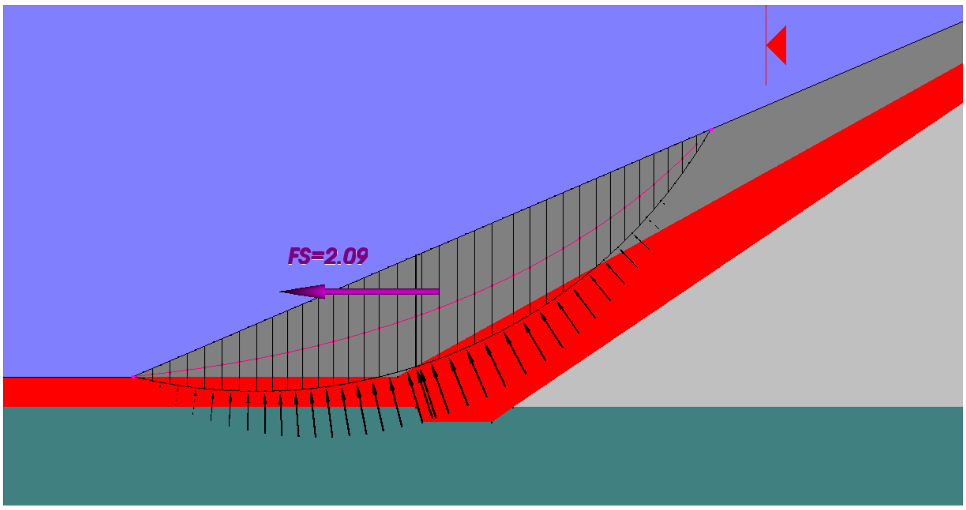


**Figure 3(a) – Original Whitman and Bailey Example 4, Spencer**

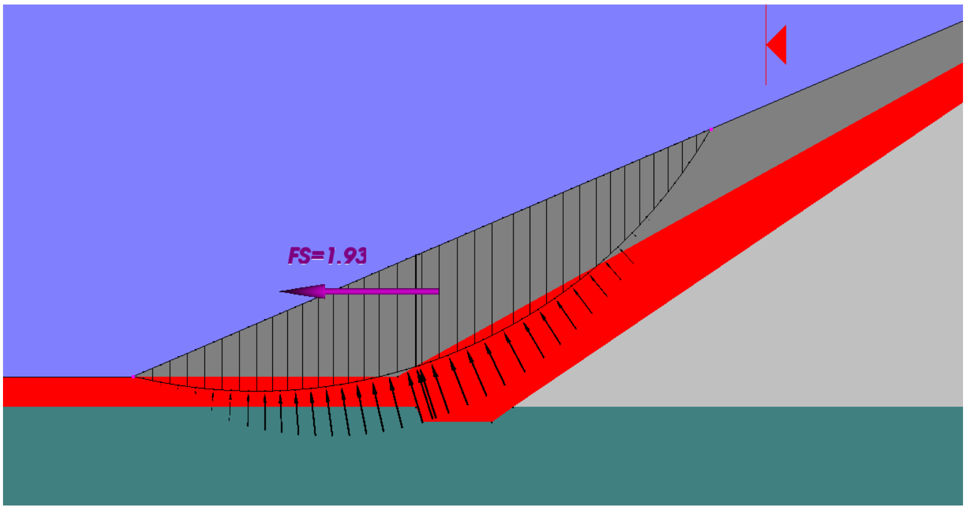


**Figure 3(b) – Original Whitman and Bailey Example 4, OMC**

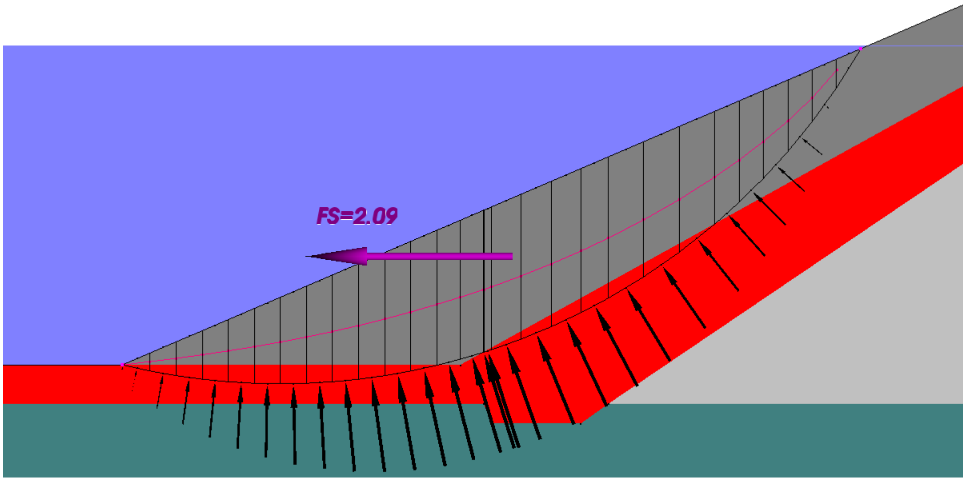


**Figure 3(c) - Whitman and Bailey Example 4 with Lowered Pond, Spencer**

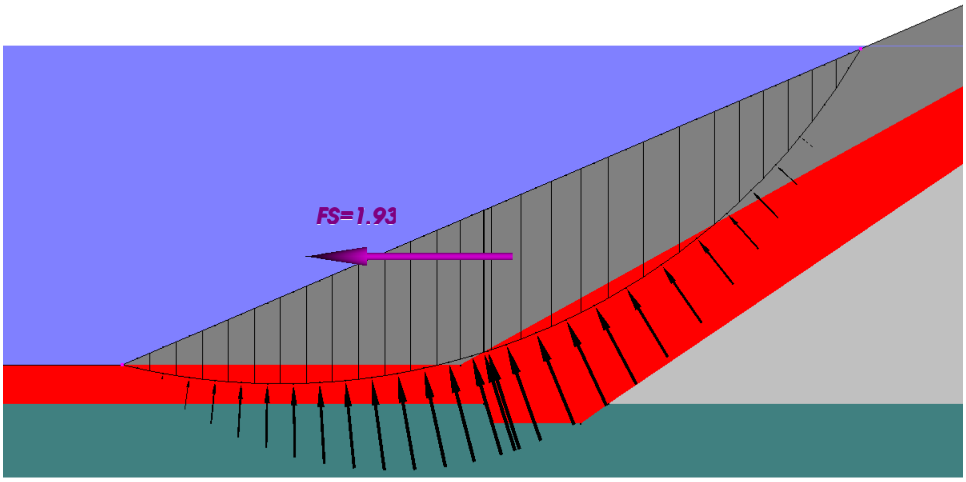


**Figure 3(d) - Whitman and Bailey Example 4 with Lowered Pond, OMC**

| Case | OMC | Spencer |
|---|---|---|
| Original problem | 1.93 | 2.09 |
| With lowered pond | 1.93 | 2.09 |

**Table 3 – Factors of Safety for Whitman and Bailey Example 4**

Whitman and Bailey obtained factors of safety ranging from 2.01 to 2.03 using the Morgenstern and Price method, which they considered to be the "most correct" solution. These values are consistent with the answers from TSLOPE given that the details of the geometry that we have read from their figure may not be precisely correct. It might also be noted that these results may not be for the critical slip circle as we are just using the circle adopted by Whitman and Bailey. For that circle, Whitman and Bailey obtained a factor of safety of 1.84 by the Fellenius Method, which is generally considered to be the same as the OMS, when using buoyant unit weights. This answer is not dissimilar to the answer given by TSLOPE and differs from their "most correct" solution by only 9 percent.

If Whitman and Bailey had emphasized the point that this is the "more correct" way of conducting a Fellenius or OMS analysis, the OMS would have been cast in an entirely different light. However, instead of doing that, they placed great emphasis on the factor of safety of 1.14 that they obtained using the Fellenius Method with total unit weights, which was only 57 percent of their "most correct" solution. The details of their programming are unknown and we have not been able to reproduce that number, but that is not critical. The critical point is that if Fellenius or the OMS is used with the "more correct" way of handing unit weights and pore pressures, it gives factors of safety that are not inconsistent with limit equilibrium methods, and that differ only because of the difference in the distribution of effective stresses around the slip surface, at least for problems without seepage forces, applied loads or internal reinforcing. These factors can introduce more significant differences depending on how they are handled.

## The difference between 2D and 3D analyses

The short answer to the question "what is the difference between 2D and 3D analyses of slope stability by the Method of Slices" is that it can be significant and varies in surprising ways. The examples below illustrate some of the differences, but these are just the tip of the iceberg. Results are given for analyses using both the OMC and Spencer's Method so that these examples also illustrate the differences between results obtained by these two methods.

**1. Hungr et al. Example No. 1**

This example, from the paper by Hungr, Salgado and Byrne (1989), has a spherical slip surface that cuts into a planar slope. The original problem had a homogeneous frictionless soil with a value of cohesion that was reported to give a factor of safety of 1.402 according to a “closed-form” solution. The problem and the solution obtained using the OMC is shown in Figure 4(a).

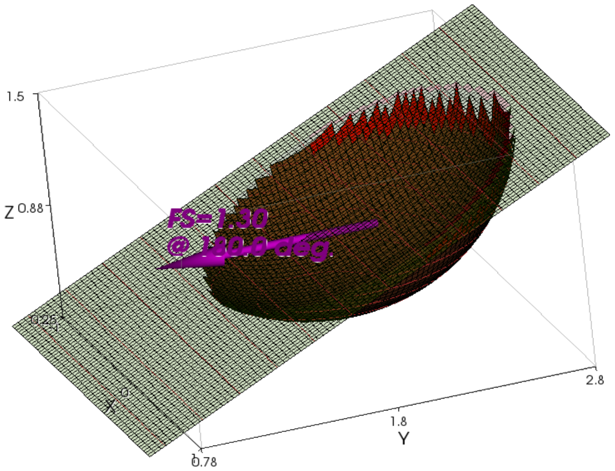


**Figure 4(a) – Hungr et al. Problem - OMC Solution**

The factor of safety of 1.35 is slightly less than the theoretical factor of safety of 1.40 because constant width columns that fit entirely within the spherical slip surface were used, leaving small patches of the spherical surface to which the cohesion was not applied. A factor of safety of 1.30 was obtained using Spencer’s Method.

In 2D both the OMC and Spencer give the same answer, as shown below in Figures 4(b) and (c), even though the computed stresses around the slip surface are different and the Spencer solution shows significant tension. This occurs because the strength is specified only as a cohesion and the factor of safety is close to one. But note the difference between the 2D and 3D answers. The 3D solution gives a factor of safety that is some 25 percent higher than the 2D solution. This should not come as a surprise because it is entirely consistent with the well-known technical note by Baligh and Azzouz (1975) on end effects, but it probably does come as a surprise to many engineers who would assume that a 2D analysis through a slope with a constant cross section will provide the correct answer. It turns out that it does not in all cases. In particular, significant errors can arise when trying to compute soil properties by using back analyses of failures because a 2D solution may give back-calculated properties that are too high or too low.

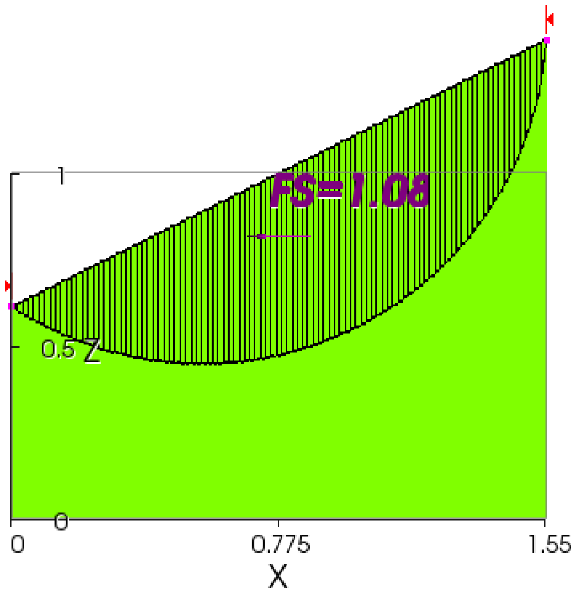


**Figure 4(b) – 2D Solution by OMC**

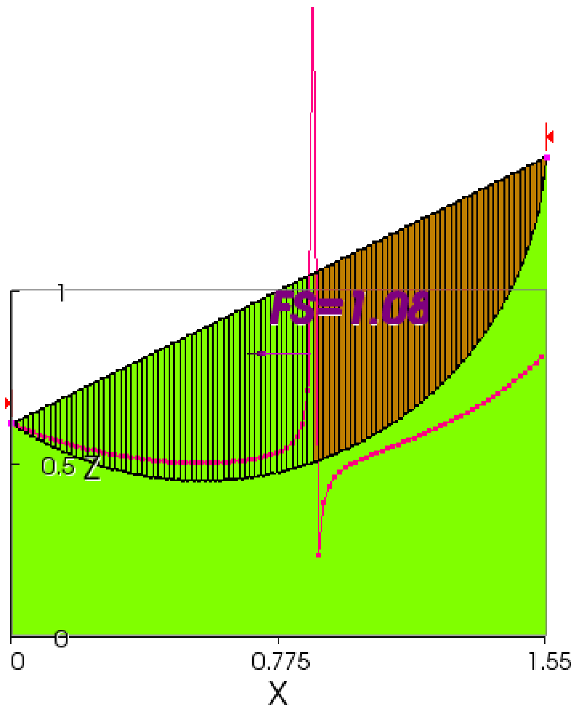


**Figure 4(c) – 2D Solution by Spencer's Method**

Back-calculated properties can be even more in error if the failure has an aspect ratio of less the one, as is more typically the case, rather than being spherical.

Figure 4(d) shows a top view of a family of three ellipsoids with the middle one being the same as the sphere in Figure 4(a) and the inner and outer ellipsoids having aspect ratios of 0.5 and 2.0 respectively.

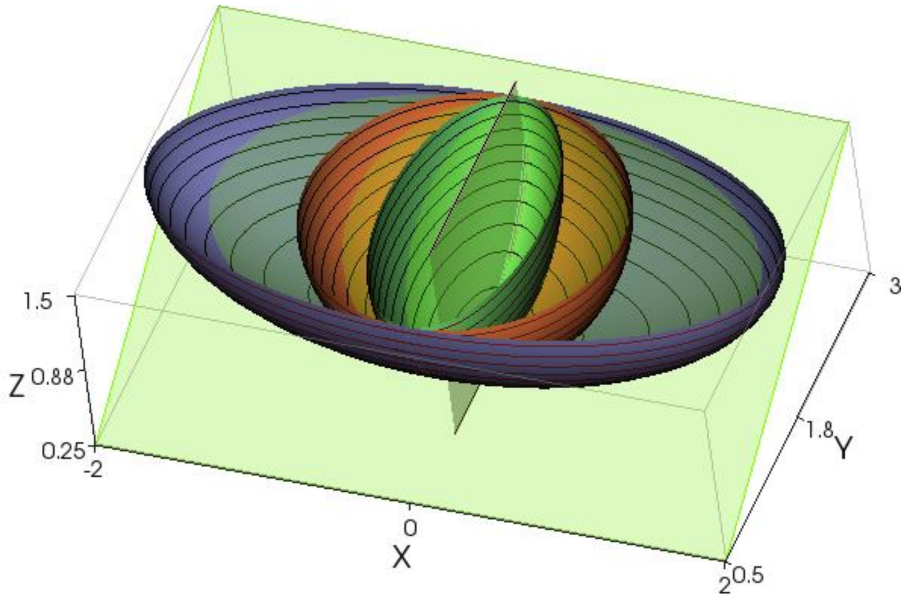


**Figure 4(d) – Family of Ellipsoids**

| Aspect Ratio | OMC | Spencer |
|---|---|---|
| 0.5 | 1.63 | 1.54 |
| 1.0 | 1.35 | 1.30 |
| 2.0 | 1.20 | 1.17 |
| 2D | 1.08 | 1.08 |

**Table 4 – Factors of Safety for Hungr et al. Example 1**

The computed factors of safety for these three ellipsoids along with the 2D case are shown in Table 4. It may be seen that for an aspect ratio of 0.5, the 3D factor of safety is now 50 percent greater than the 2D solution. As the aspect ratio increases, the "end effects" diminish and the factor of safety for an ellipsoid approaches the 2D factor of safety. Many, or even most, natural landslides have aspect ratios of less than one being controlled by local weaknesses in structure, material properties or water conditions and accurate reconstruction or prediction of failures requires a 3D analysis.

Going back to examine the 2D solutions in more detail, it can be seen that the interslice forces in the Spencer's Method solution are in tension in the upper half of the potential sliding mass. While not shown in these figures, in the OMC solution the local factors of safety are correspondingly less than one in the upper half of the potential sliding mass. The line of thrust in the solution by Spencer's method of course is not tenable, but that is of no great consequence in this limited instance. However, if the material is assumed to be cohesionless and to have shear strengths that vary with the normal effective stress on the slip surface, the load redistribution that takes place in Spencer's Method in order to force the factor of safety to be the same on the base of each slice, leads to a reasonable line of thrust but also a calculated factor of safety that diverges from that obtained by the OMC, as shown in Figures 4(e) and (f). The black arrows in these figures indicate the effective normal stresses on the base of each slice.

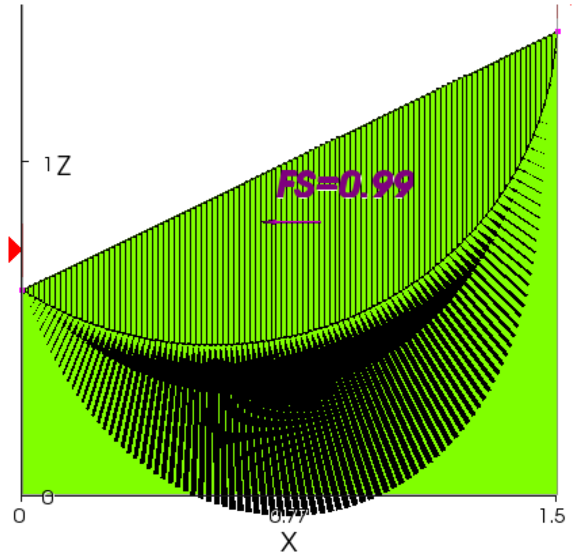


**Figure 4(e) – 2D Solution by OMC, c=0, ϕ=20 degrees**

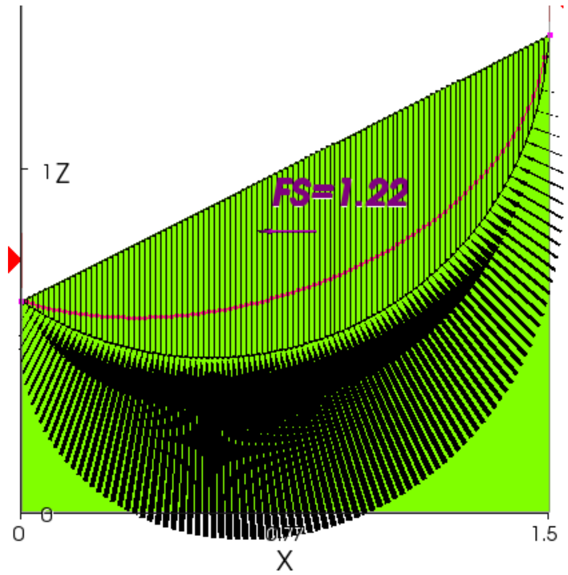


**Figure 4(f) – 2D Solution by Spencer's Method, c=0, ϕ=20 degrees**

It is not really possible to say which of the solutions shown in Figures 4(e) and (f) is the "more correct". If the potential sliding mass is more like a rigid body the Spencer's solution, which is kinematically admissible in this case, may be "more correct" and if the potential sliding mass is deformable, if for instance it is composed of sand particles, the OMC solution may well be "more correct".

The differences between both 2D and 3D analyses with an aspect ratio of one and the two solution methods are further illustrated in Tables 5(a) and 5(b). Table 5(a) shows the calculated factors of safety for three values of cohesion and Table 5(b) shows the calculated factors of safety for three values of the angle of friction.

| Cohesion | 2D OMC | 2D Spencer | 3D OMC | 3D Spencer |
|---|---|---|---|---|
| 0.1 | 1.08 | 1.08 | 1.35 | 1.30 |
| 0.2 | 2.17 | 2.17 | 2.69 | 2.59 |
| 0.3 | 3.25 | 3.25 | 4.04 | 3.89 |

**Table 5(a) – Factors of Safety for All Cohesion**

| Angle of Friction | 2D OMC | 2D Spencer | 3D OMC | 3D Spencer |
|---|---|---|---|---|
| 10 | 0.73 | 0.90 | 0.67 | 0.93 |
| 20 | 0.99 | 1.22 | 0.90 | 1.26 |
| 30 | 1.58 | 1.94 | 1.42 | 2.00 |

**Table 5(b) – Factors of Safety for All Friction**

Several interesting things can be seen in these two tables. In Table 5(a) it can be seen that when the shear strengths are specified as fixed numbers, the OMC and Spencer's method give essentially the same answer. And, for this geometry, when the shear strengths are specified as fixed numbers, the 3D factor of safety is about 25 percent greater than the 2D factor of safety. In Table 5(b) it can be seen that when the shear strengths are a function of the effective stresses on the bases of the slices or columns, the OMC gives a factor of safety that is about 25 percent less than Spencer's method in 2D analyses and 40 percent less in 3D analyses. 3D analyses give factors of safety by Spencer's Method that are essentially the same as those from 2D analyses and the 3D factors of safety by the OMC are about 10 percent less than those computed in 2D analyses. The reason for this last result is simply that the slices in the 2D section down the centre of the spherical slip surface have higher effective stresses at the base than do the columns in the 3D analysis which shorten as they move out to the perimeter of the potential sliding mass.

So, at least an interim conclusion that can be drawn from this example is that simple rules on the effects of different methods of analysis and 3D effects are likely to be misleading and the engineer both has to keep his or her wits about them and test alternate solutions to their particular problem. If you don't do that, you will never know what the possible errors might be.

This example was confined to simple circular, spherical or ellipsoidal slip surfaces. The next two examples illustrate the possible effects of natural and man-made 3D topography.

### 2. Kettleman Hills Landfill Failure

An early example of the analysis of 3D effects on real world problems was provided by the failure of the liner system at the Kettleman Hills hazardous waste landfill, reported by Mitchell et al. (1990) and Seed et al. (1990) and summarized and updated by Duncan, Wright and Brandon (2014, pp. 32 and 282).

As may be seen in Figure 5(a), the initial landfill was placed against liners on one side and one end of what was to be a completely lined basin. Because the liner had not been completed on the left hand side of the basin as seen in Figure 5(a), the landfill had a partially free face on that side, as well as on the front. When the landfill had reached a

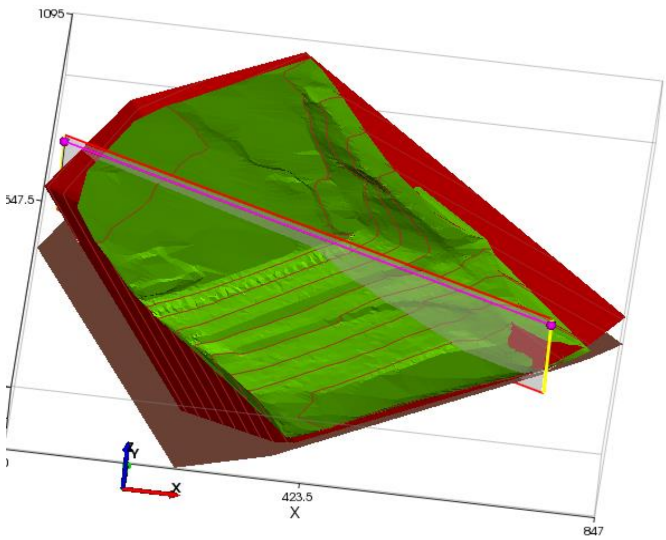


**Figure 5(a) – 3D View of Kettleman Hills Landfill**

maximum height of 90 feet, a slope failure occurred with horizontal and vertical movement of up to 14 and 35 feet. Subsequent investigations suggested that the basic failure occurred along a wetted HDPE liner compacted clay layer interface which resulted from the clay layer having been placed wet of optimum moisture content and then subsequent consolidation with drainage restricted by the HDPE liner. The extent of this wetted interface condition was not clear, thus Seed et al. (1990) conducted stability analyses for both partial and full wetting of the base. For a number of 2D cross sections, Seed et al. (1990) obtained factors of safety of 1.2 to 1.25 for the partial base wetting case and 1.10 to 1.15 for the full base wetting case. For the diagonal 2D cross-section that is shown in Figures 5(a) and (b), drawn in the direction of movement indicated by a 3D analysis, TSLOPE gives 2D factors of safety of 1.21 and 1.22 for the OMC and Spencer's Method respectively, assuming full base wetting.

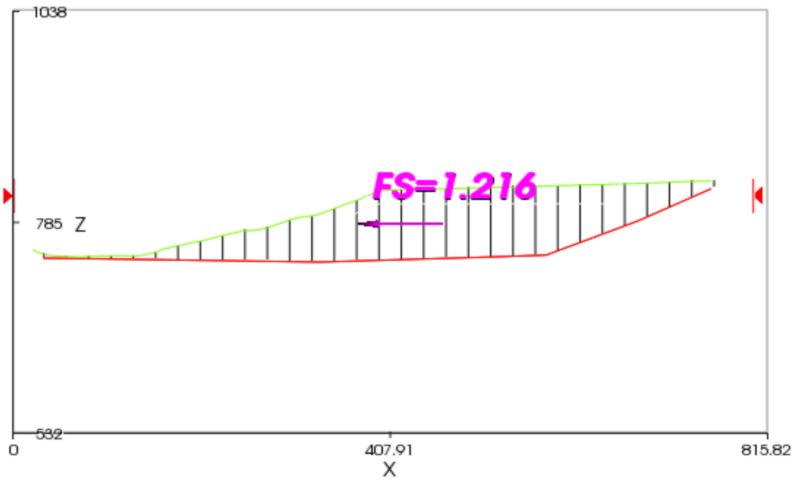


**Figure 5(b) – Analysis of 2D Section by Spencer's Method**

| Case | OMC | Spencer |
|---|---|---|
| 2D Section | 1.21 | 1.22 |
| 3D Problem | 1.00 | 1.05 |

**Table 6 – Calculated Factors of Safety**

Seed et al. then explored 3D effects by conducting what they described as a "force-equilibrium analysis" using five blocks and obtained 3D factors of safety of 1.08 and 1.01 for the partial and full base wetting cases. Assuming full base wetting, TSLOPE gives factor of safety of 1.00 and 1.05 by the OMC and Spencer's method respectively, as listed in Table 6. Thus, making normal judgments about the appropriate 2D section to analyse, the 3D factor of safety appears to be 10-20 percent below the 2D factor of safety. The reason for the lower 3D factor of safety can be explained in either of several ways. One way is to say that because of the longer back slope relative to the base area, there is more "push" from the slope relative to the resistance provided by the base. Alternately, one can view this as a problem where the "end effects" are less than they would be in a long slope with a constant 2D cross section. The 10 to 20 percent difference may or may not be a big deal from the design point of view because the failure to recognize the lower wetted interface strengths was a larger problem, but, again, it is significant in back calculating properties from the failure and understanding the failure mechanism.

Duncan et al. (2014) summarized subsequent studies which tended to play down the significance of the 3D effects pointing to other uncertainties and noting that it was possible to find 2D cross sections that showed factors of safety of less than one, suggesting that there might have been progressive failure. However, these cross sections had at least some “end effects” and could not fail on their own. And, at least some of the guess work can be eliminated by conducting a 3D analysis in the first place. If this is done using the OMC, a single analysis can also calculate the local factors of safety, as shown in Figure 5(c) indicating locations where progressive failure might start, and repeat analyses can be conducted as desired to follow progressive failure.

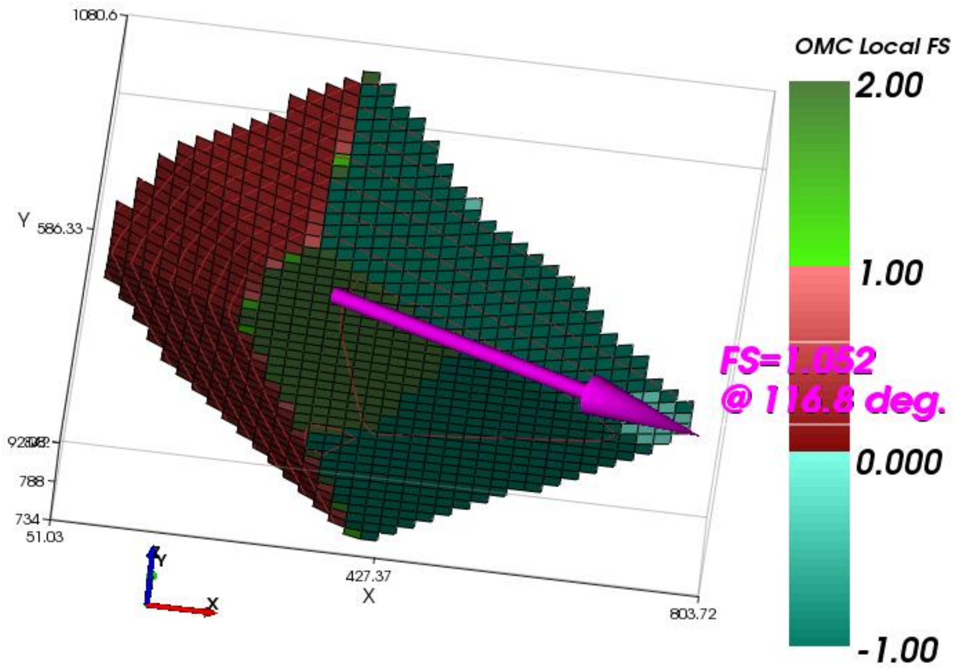


**Figure 5(c) – Analysis of 3D Problem Showing LFoS from OMC and FoS by Spencer**

However, the cases where 3D effects lead to lower factors of safety are not as dramatic and are probably limited in number compared to the cases where 3D effects increase the factor of safety.

## 3. Puente Hills Canyon 9 Design

A more graphic example of the positive effects of 3D geometry is provided by the Puente Hills Canyon 9 landfill of the Sanitation Districts of Los Angeles County, which happens to have triggered the development of the 3D approach used in the current version of TSLOPE. Canyon 9 represented an expansion of an existing landfill so that one side of the expanded facility consisted of existing compacted municipal solid waste (MSW) that sat on natural ground without a liner. However, new regulations required that both the floor and the slopes of the expansion be placed on a single HDPE liner. Over the floor the HDPE liner was placed on top of a compacted clay layer that had a lower strength than the interface of a roughened HDPE liner and the MSW, so that sliding along the floor was controlled by the undrained strength of the clay layer (c = 250 psf; ϕ = 13.5 degrees). However, on the slopes, a smooth HDPE liner was placed directly on the slopes excavated in the *in situ* soft rock and the weakest interface was judged to be the contact between the liner and the MSW (c = 0; ϕ = 10 degrees) because the liner was anchored into the *in situ* material on a number of benches. Thus, there were three zones of the base of the landfill that had different strengths for the purpose of analysis, although 2D analyses of sections that passed through the mouth of the canyon, such as shown in Figures 6(a) and (b), suggested that the critical 2D section involved sliding only on the floor and the slopes and did not involve the existing MSW (assumed at the time to have a shear strength of c = 0; ϕ = 30 degrees). However, when construction was well advanced, a leading geotechnical consultant who was brought in to perform the analyses of slope stability that were required by regulators, found, not surprisingly,

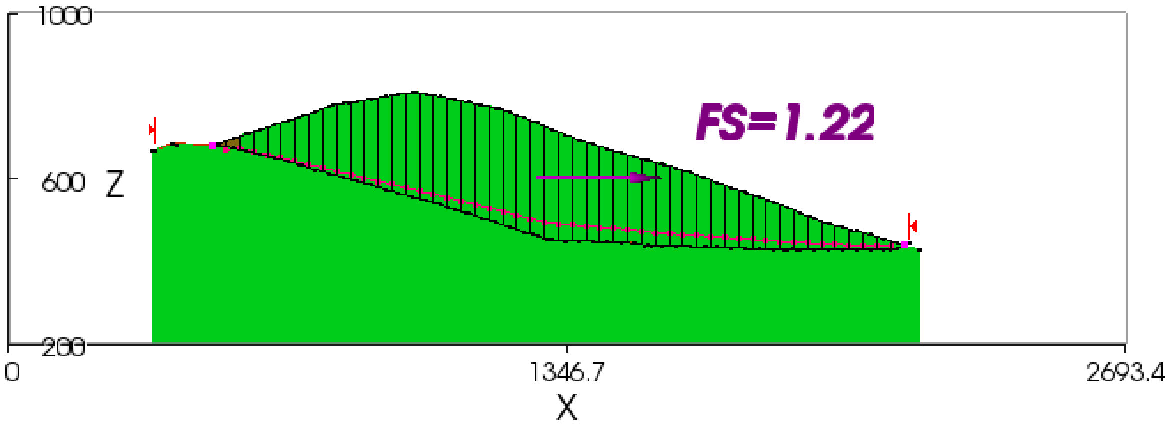


**Figure 6(a) – 2D Cross Section through Mouth of Canyon**

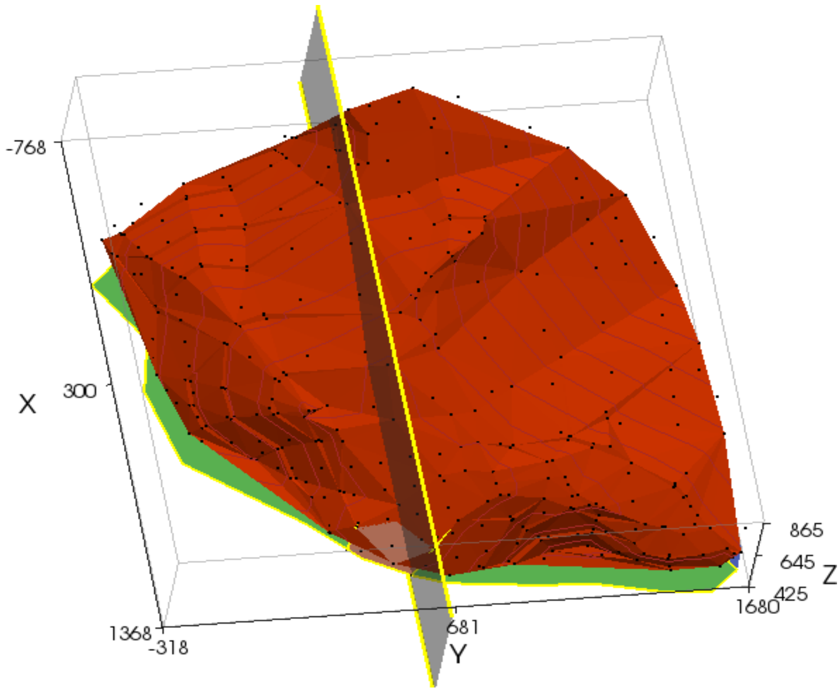


**Figure 6(b) – 3D View of Base of Canyon**

that the conventional 2D factor of safety of a section passing through the mouth of the canyon was only 1.22 – less than the required minimum – and, surprisingly, found that an early, commercially available 3D slope stability program gave similar results.

The owner, justifiably, threw a flag at this point because it was evident that in a "bottleneck" canyon like this, as shown in Figure 6(b), the 3D effects had to be significant with the mouth of a canyon acting like the abutments of a good arch dam site. The upshot of this was that another geotechnical consultant brought in the writer to develop and use a program that more properly modelled the 3D geometry and its effects. The original program, variously called TSLOPE3 or T3, used a horizontal force equilibrium solution which Los Angeles County had for some years required geotechnical consultants to perform by hand. The properties cited above were assigned over the basal surface as shown in Figure 6(c), where the red zone is the floor of the canyon, the green zone is slopes lined with HDPE, and the blue zone is the adjacent MSW. Using the original program, a factor of safety of 1.92 was obtained, more than satisfying the regulatory requirement of a factor of safety of 1.5. The updated program, now called just TSLOPE, gives a factor of safety or 1.70 using the OMC and, by chance, gives a factor of safety of 1.93 using Spencer's Method.

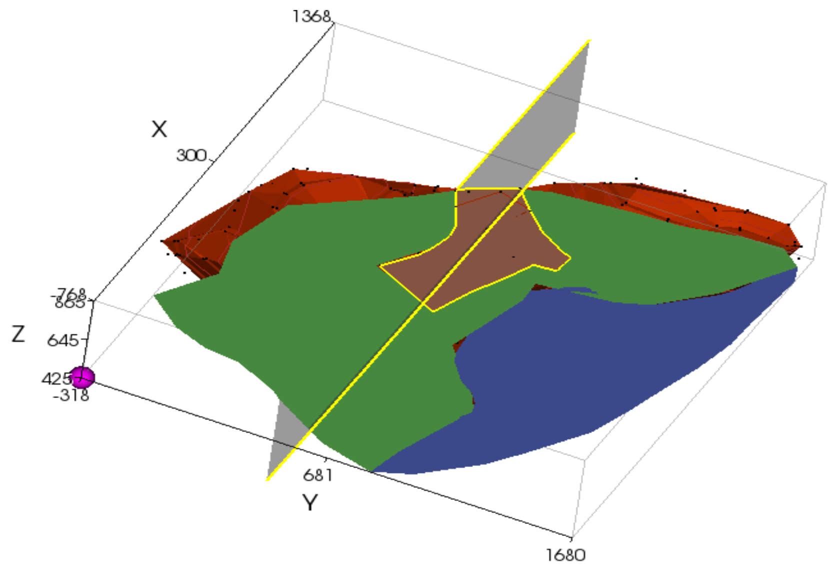


**Figure 6(c) – 3D View Showing Zones with Different Properties**

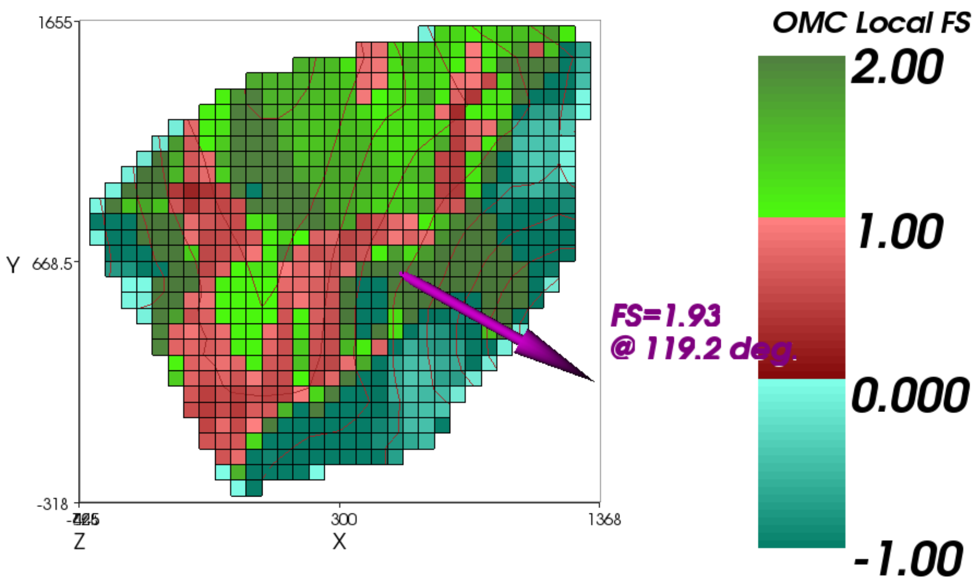


**Figure 6(d) – Local Factors of Safety in 3D Analysis**

As in other problems, the higher factor of safety by Spencer's Method is consistent with the implication of a rigid body which is restrained even more by the bottle-neck than is the OMC model which assumes that each column can move independently. The fact that the Los Angeles County horizontal force equilibrium method and Spencer's method give almost identically the same factor of safety for this problem is a fluke, rather than an indication of a fundamental truth. The local factors of safety computed using the OMC are shown in Figure 6(d). In this figure the blue colours indicate a negative factor of safety or, in other words, a reverse slope. Basically, the abutments and the floor of the canyon are holding the MSW up while it tries to slide down the back slope. The 2D factors of safety for the section shown in Figure 6(a) are 1.20 by the OMC and 1.22 by Spencer's Method, so that the 3D analyses show increases of 42 percent and 58 percent over the 2D analyses.

The fact that there is a significant 3D effect for this problem, or for a dam in a narrow canyon, should come as no surprise, but, again, what turns out to be surprising is that there can also be significant 3D effects in real life problems where it would normally be thought that a 2D analysis of a slope with a constant cross section suffices. This can happen where a 2D potential failure surface dives under a wall or a revetment whereas in reality the failure has to cut through the wall or revetment, as shown in the following example.

## 4. Treasure Island

Treasure Island, a man-made island in San Francisco Bay, was originally intended to serve as an airport, but, after the completion of the 1939 World's Fair, the island was taken over by the US Navy. It is presently being redeveloped for civilian use. The sand fill that was place to form the island will be densified to mitigate possible liquefaction and the final grades will be raised up to 5 feet to allow gravity flow of stormwater for the foreseeable future. Prefabricated vertical drains and surcharging will be used to limit future settlement of the underlying young Bay Mud. The cross section below and the soil properties are taken from publicly-released bid documents.

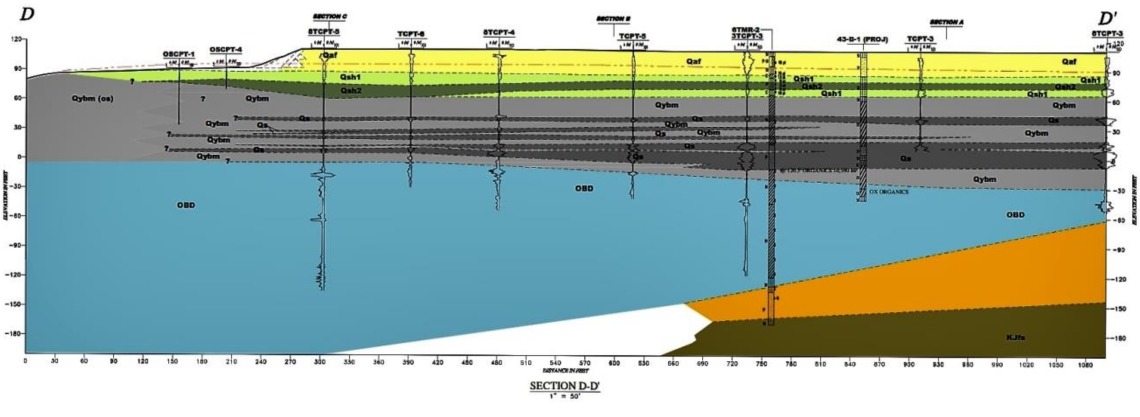


**Figure 7(a) - Section D-D'**

The shoal materials which underlie the sand fill are clayey sands that generally contain from 15 to 30 percent fines. These materials are not liquefiable in any conventional sense and they were very resistant to densification by vibratory loading in trials that were performed at the site. Thus, with the young Bay Mud consolidated not only under the weight of the existing fill but under additional surcharge loads and the rock revetment is composed of free-draining, competent rock, there is no obvious concern about shoreline stability at this site, even given its proximity to the San Andreas and Hayward faults.

Nonetheless, in the bid documents there were brief descriptions of work done by the project's geotechnical consultant using simplified methods of analysis which indicated a potential shoreline stability problem. This raised the question of whether there is any screening analysis that is appropriate for this site. The short answer is yes, there is. As explained by Harry Seed in his Rankine lecture (Seed, 1979), for materials that do not undergo a loss of strength and stiffness as a result of cyclic loading, pseudo-static analyses are not too bad. And pseudo-static analyses are also required to compute the yield acceleration (the seismic coefficient that reduces the factor of safety to unity – the factor of safety for a specified seismic coefficient can then be derived from this) for use in the various simplified methods to compute deformations.

TSLOPE was used to compute the static factors of safety and the yield acceleration for both 2D and 3D slip surfaces. For Section D-D', when a circular slip circle is transformed to a spherical or ellipsoidal slip surface, two things happen. One is that the slip surface now has to cut through the rock revetment, rather than diving under it – this will increase the factor of safety. The other is that relatively more of the slip surface will be in the young Bay Mud – this might either reduce or increase the factor of safety, depending on the strength of the Bay Mud relative to the other materials that are involved.

For the "seismic" loading case undrained strengths were used for all materials below the water table, except for the rockfill in the revetment. These strengths were also corrected for rate of loading effects in order to represent the short rise time of an earthquake pulse. The critical circular slip surfaces obtained using Spencer's Method and the "static" and "seismic" properties are shown in Figures 7(b) and (c). These are

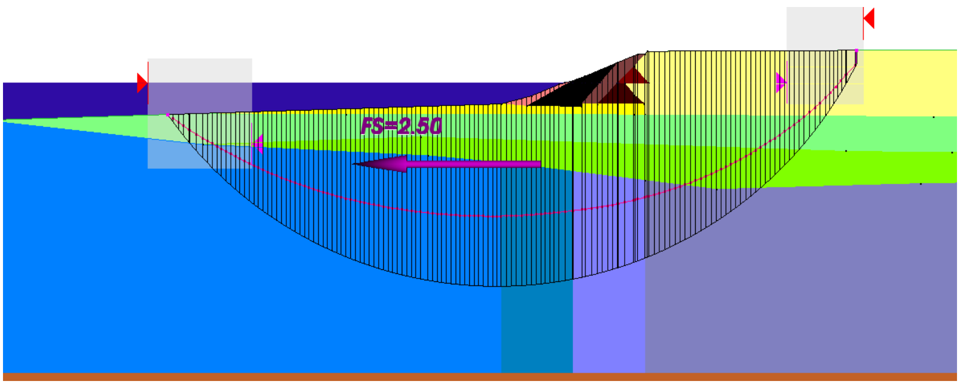


**Figure 7(b) - Section D-D' Static Analysis**

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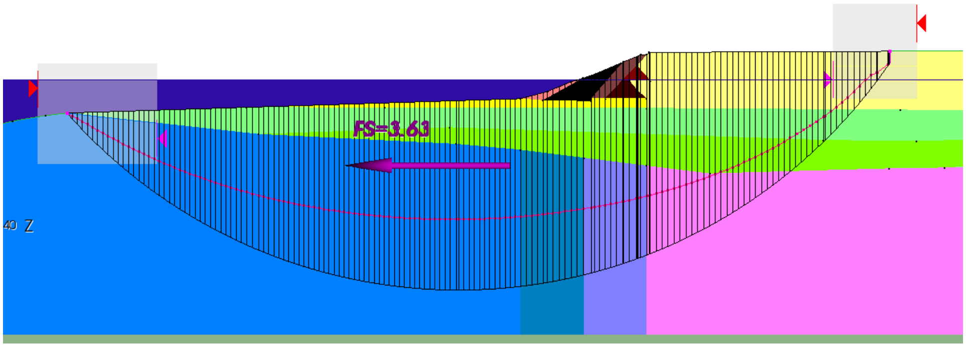


**Figure 7(c) - Section D-D' Static Analysis with "Seismic" Properties**

both for "static" analyses without the application of a seismic coefficient. The critical circular slip surface obtained in the "static" analysis with "seismic" properties was then used in subsequent searches for the yield acceleration.

The critical 2D failure surface was also used as the basis for constructing three 3D failure surfaces, as shown in Figure 7(d). The centre 3D slip surface is a sphere, which has an aspect ratio of 1.0. In addition, there are two further ellipsoids that have aspect ratios of 0.5 and 2.0. The larger the aspect ratio, the more the 3D solution approaches the 2 D solution.

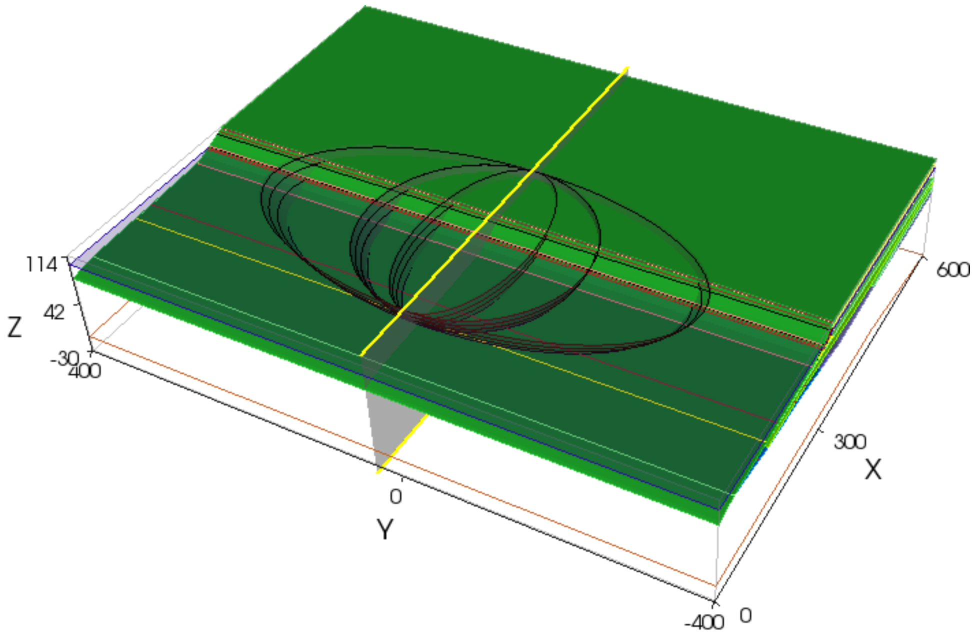


**Figure 7(d) – 3D Potential Failure Surfaces**

| OMC | Spencer | |
|---|---|---|
| **Static analyses** | | |
| **2.23** | **2.51** | **2D FoS** |
| **2.59** | **2.95** | **3D FoS aspect ratio = 2.0** |
| **2.44** | **2.97** | **3D FoS aspect ratio = 1.0** |
| **2.57** | **3.52** | **3D FoS aspect ratio = 0.5** |
| | | |
| **Seismic analyses** | | |
| **0.22g** | **0.26g** | **2D yield acceleration** |
| **0.27g** | **0.31g** | **3D yield acceleration – aspect ratio = 2.0** |
| **0.29g** | **0.33g** | **3D yield acceleration – aspect ratio = 1.0** |
| **0.35g** | **0.39g** | **3D yield acceleration – aspect ratio = 0.5** |

**Table 7 – 2D and 3D Factors of Safety and Yield Accelerations**

The results are shown in Table 7. Again, the reason that the 3D factors of safety are higher than the 2D is that in 3D you must cut through the revetment, rather than diving under it as happens in 2D. Of the four cases, the one with the aspect ratio of 0.5, which gives the highest factor of safety, is probably the most like a typical landslide.
As expected for a slope that has been stable for many years and would have been at greatest risk at the end of construction, the static factors of safety are healthy enough and the yield accelerations are great enough relative to the design peak ground acceleration of 0.46g to suggest any deformations under earthquake loadings would be quite small.

This example strongly suggests that simplified analyses using conventional procedures and 2D slope stability analyses can be unnecessarily conservative, and in this particular case suggest that there is a problem where no problem actually exists.

## 5. Greensteep Wall System

However, a slope retained by a wall does not always have higher factors of safety in a 3D analysis as the following example shows.

Figure 8(a) shows a 2D cross-section through a Greensteep wall system which has been designed to have a factor of safety greater when 1.1 when both a surcharge load and a seismic coefficient are applied. The static factor of safety without the surcharge and seismic loads is 1.9.

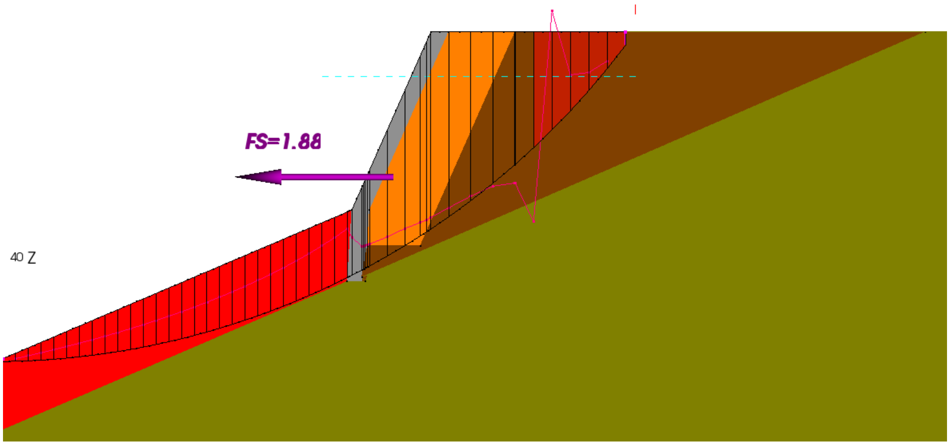


**Figure 8(a) – 2D Section Through Greensteep Wall System**

However, a 3D spherical slip surface through the wall, as shown in Figure 8(b), has a factor of safety of 2.5, so that a more economic design might be possible when a 3D analysis is used.

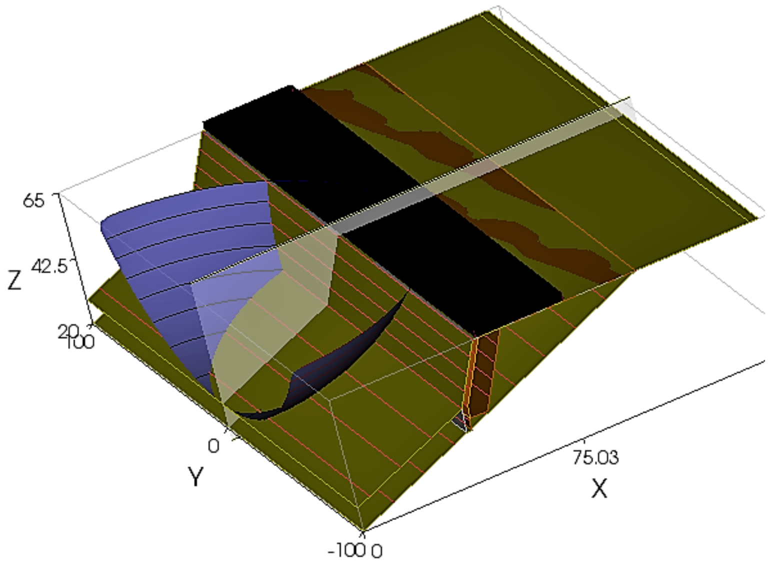


**Figure 8(b) - 3D Spherical Slip Surface Through Greensteep Wall System**

But, if a uniform surcharge load of 200 psf is applied behind the top of the slope, as shown in Figure 8(b), the 2D factor of safety only falls to 1.7, whereas the 3D factor of safety falls to 1.5. This happens because in the 3D analysis there is a relatively large increase in the driving forces for both the shorter columns and for the cohesive materials, such as the wall, which show no increase in strength.

Thus, without the surcharge, 3D effects increase the factor of safety, but with the surcharge, 3D effects reduce the calculated factor of safety.

| Case | OMC | Spencer |
|---|---|---|
| No surcharge 2D | 1.9 | 1.9 |
| No surcharge 3D | 2.5 | 2.5 |
| With surcharge 2D | 1.7 | 1.7 |
| With surcharge 3D | 1.4 | 1.5 |

**Table 8 – Computed Factors of Safety**

While the engineer should still be wary of taking the calculated factors of safety at precisely their face value, this is a great example of how an improved analysis can provide much greater insight into a problem.

## Applied Loads and Internal Reinforcing

These have some similarities but are not identical because applied loads are fixed quantities, but can be driving or resisting forces, while internal reinforcing is always a resisting force but also involves the question of whether the allowable forces or the ultimate forces should be specified.

**Applied Loads**

These might include uniform or non-uniform pressures loads, line loads and point loads. Pond pressures are a particular example of a non-uniform pressure load.

In limit equilibrium methods, when such forces impinge on the top of a slice or column, they are included[1] in the total horizontal and vertical forces acting on the slice. These forces also include the weight of the slice, any seismic loads and, maybe, internal reinforcement capacities. These forces will impact the normal stresses on the bases of the slices or columns and the shear strength when the angle of friction is non-zero. These values are determined in an iterative solution, but the impact on the normal stresses and the shear strength is not that easy to see. In the OMC it is both more

[1] Included or added or subtracted means as a vector sum in this note.

complicated and more simple. It is more complicated in the sense that more programming is required, but simpler for the user to see what is happening. Because applied loads will not automatically be reflected in the normal stresses on the bases of the slices or columns, the user has to specify the way in which the applied loads spread out and impinge on the potential failure surface. But also, because of the way the factor of safety is defined in the OMC, the user has to specify whether the applied load is added as a driving force or a resisting force. For instance, a uniform vertical load above a slope is clearly a driving force, but the same uniform vertical load beyond the toe of a slope could be thought of as a resisting force. Pond pressures applied to the upstream face of an impervious or lined embankment might be viewed resisting forces when the upstream face is being analysed, but when the user is analysing the stability of the downstream slope of the same embankment, they may be driving forces, separate from any seepage forces. Whether these loads are added to or subtracted from the numerator or added to the denominator in the equation for the factor of safety makes a difference. But there is generally no question of this class of loads being reduced by the factor of safety in the same way that the shear strength is in limit equilibrium methods. These loads are what they are. We will return to a recommended approach after discussing “internal reinforcement”.

**Internal Reinforcement**

The question of internal reinforcement is discussed by both Rocscience in their documentation of the program SLIDE and Duncan, Wright and Brandon (2014). Their treatments are similar (and they even use the same worked example) but their terminology is different. Rocscience talk about Active Support and Passive Support whereas Duncan et al. call these Method A and Method B. In either case the resistance provided by the reinforcement is treated as resisting force rather than as a driving force. Both explanations are a bit confusing because the discussion is made in terms of the factor of safety as if the solution calculates the factor of safety directly, which is true for the OMC but not for limit equilibrium methods in which the factor of safety is implicit and is obtained by iteration. Thus, how the two different methods should be applied in programming the equations of equilibrium is not entirely clear nor is how they are actually applied in most slope stability programs.

However, for the “Active” case, in Rocscience’s equation 2 or Duncan et al.’s equation 8.2, the resistance provided by the reinforcement is “subtracted” from the applied loads, and does not involve any factoring. In other words, the resistance provide by the reinforcement is just added vectorially to the total forces acting on the slice or column. For the “Passive” case, in Rocscience’s equation 3 or Duncan et al.’s equation 8.4, the resistance provided by the reinforcement is added to the strength of the soil or rock and factored by the factor of safety. Thus, in the "Active" case the user should specify the allowable loads whereas in the "Passive" case the user should specify something more like the ultimate load since the loads are going to end up being factored by some average factor of safety which applies to both the soil or rock strengths and the reinforcement capacities. But Duncan et al., as quoted by Rocscience, are quite clear that the "Active" approach is preferable because the desired factoring of the reinforcement capacities might well be different for the soil or rock and the reinforcement, and even for several types of reinforcement in the same problem. Thus, it is not at all clear why the "Passive" option should even be offered.

In the case of the OMC, the equivalent of the "Active" case is that the resistance providing by the reinforcing should be subtracted from the driving forces. The equivalent of the "Passive" case or Method B is that the resistance providing by the reinforcing should be added to the resisting forces. In either case the allowable capacities should be used unless the engineer is very bold. But now we have a choice between using the "Active" or the "Passive" cases and it would generally seem to be logical to use the "Passive" case both for internal reinforcing such as geogrid or soil nails whose strength will normally be only partly mobilized and for anchors which are tensioned and apply a known load from the outset. However, this would give numerically smaller factors of safety than equivalent limit equilibrium methods using the "Active" alternative, which is the preferred approach for those methods, so that, for consistency, the allowable capacities of internal reinforcing should be subtracted from the driving forces in the OMC. In all cases the user should take appropriate account of installation damage, creep and deterioration over time as discussed by Duncan et al. in coming up with the allowable capacities.

**Conclusion**

The safest course of action for consistency between methods of analysis is to treat all applied loads and internal reinforcement capacities as "Active" forces and to add or subtract them from the numerator of the equation for the factor of safety in the OMC. In both the OMC and limit equilibrium methods this means that internal reinforcement capacities should be factored separately from the soil or rock strengths.

**Examples (to be added).**

## To be completed:

## Wedge analyses

## Rapid drawdown analyses

## Why not finite element or finite difference or discrete element analyses?

Need reliable slip and gapping elements and some way to locate them. Normally too hard to assign the necessary properties, but might be worth it for rapid draw-down and other cases where there are transient seepage forces.

## Conclusions (to be expanded)

Methods of analysis.

Seepage forces and 3D effects.

Applied loads and internal reinforcement.

FE or FD or DE for special situations only.

Even with the best tools, the engineer still must exercise judgment!

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