AB and (half of) BC Calculus Must-Knows

Note: a, b, c, d, m, and n generally represent constants; f, g, h, u, v, x, y, and z, variables or functions.

Basic Derivatives

Memorize these 3:

$$\frac{d}{dx}x^n = nx^{n-1}, \quad \frac{d}{dx}\ln x = \frac{1}{x}, \quad \frac{d}{dx}e^x = e^x$$

Think slope of graph at $x = \pi/4$ for these:

$$\frac{d}{dx}\cos x = -\sin x, \quad \frac{d}{dx}\sin x = \cos x$$

Derive via quotient rule and identities:

$$\frac{d}{dx}\tan x = \sec^2 x, \quad \frac{d}{dx}\sec x = \sec x \tan x$$

Inverse Trig Derivatives

Derived via ***implicitly differentiating***
$\sin(y) = x$ or $\tan(y) = x$
and applying right triangle trig—
i.e., memorize them!

$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}, \quad \frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$$

Derive these via
logarithmic differentiation –
i.e., definitely memorize them!

$$\frac{d}{dx}a^x = a^x \ln a \qquad \frac{d}{dx}\log_a x = \frac{1}{x \ln a}$$

All of the above yield antiderivatives, too.

Differentiation Rules

Chain, Product, and Quotient Rules

$$\frac{d}{dx}\left(O\left(I\left(x\right)\right)\right) = O'\left(I\left(x\right)\right)\cdot I'\left(x\right).$$

$$\frac{d}{dx}(uv) = v\frac{du}{dx} + u\frac{dv}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v - v'u}{v^2}$$

Recognizing Definition of Derivative

$$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$

$$f'(c) = \lim_{x\to c}\frac{f(x) - f(c)}{x-c}$$

l'Hôpital's Rule

If both $f(x)$ and $g(x) \to 0$ (or ∞) as $x \to c$

then $\lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)}$

Omissions mainly include *pre*calculus concepts: concavity and monotonicity; local extrema and inflection points; asymptote; trig; even, odd, and periodic functions; slope fields; and local linearity.

A Big Idea of Differentiation—Optimization

Suppose $f'(c) = 0$. (See graphs at right.)

Second Derivative Test:

$f''(c) > 0 \Rightarrow f$ has local **min *value*** at $x = c$.

$f''(c) < 0 \Rightarrow f$ has local **max *value*** at $x = c$.

$f''(c) = 0 \Rightarrow$ *Use the …*

First Derivative Test:

f' changes from – to + at $x = c \Rightarrow$
f has a local **minimum** value at $x = c$;

f' changes from + to – at $x = c \Rightarrow$
f has a local **maximum** value at $x = c$.

Also consider the Extreme Value Theorem.

Implicit Differentiation

Vastly oversimplified, if not downright misleading.

Suppose $f(x) = g(y)$.
Then $f'(x) = g'(y)\cdot y'(x)$

One use: **Another Big Idea of Differentiation—Related Rates** of Change

Suppose $f(x) = g(y)$ and both x and y are a functions of time, t.
Then $f'(x)\cdot x'(t) = g'(y)\cdot y'(t)$

Continuity

Definition: f is ***continuous*** at $x = c \Leftrightarrow$

$$\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = f(c)$$

In the two theorems, f must be continuous on $[a,b]$.

Intermediate Value Theorem

If d is between $f(a)$ and $f(b)$ then there is a $c \in (a,b)$ where $f(c) = d$.
(A continuous function takes on all values between any two range values)

Extreme Value Theorem

Function f has both a **maximum** and a **minimum *value*** on any closed interval (maybe at an endpoint!).

Differentiability $\Rightarrow$ continuity so
Not continuous $\Rightarrow$ not differentiable.

Mean Value Theorem

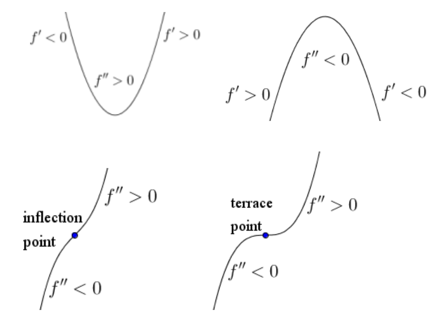
If f is ***differentiable*** on (a,b) and continuous on $[a,b]$,
then there is a $c \in (a,b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b-a}.$$

(slope of tangent = slope of secant)
(Instantaneous rate = average rate)

Shapes of Graphs

(Refer to First and Second Derivative Tests)



Fundamental Theorem #1

$$A(x) = \int_a^x f(t)dt \Rightarrow A'(x) = f(x)$$

Useful when techniques of integration fail:

$$\int f(x)dx = \int_a^x f(t)dt$$

Thus useful in solving IVP
$f'(x) = g(x)$, $f(a) = b$
when antiderivative of g can't be found:

$$f(x) = b + \int_a^x g(t)dt$$

The Big Idea of Integration

A ***Riemann sum*** $\sum_{k=1}^{n} f(t_k)\Delta t$

is an integral $\int_{x_0}^{x_n} f(x)dx$ if $n \to \infty$.

Fundamental Theorem #2

To compute ***accumulated change*** in the integrand function:

$$\int_a^b f'(x)dx = f(b) - f(a)$$

Position s, Velocity v, Acceleration, a

of object moving in one dimension

$v = \frac{ds}{dt}$ = slope of position graph

$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$ = slope of velocity graph

$$\Delta v = \int_{t_0}^{t_1} a(t)dt = v(t_1) - v(t_0)$$

$$\Delta s = \int_{t_0}^{t_1} v(t)dt = s(t_1) - s(t_0)$$