Intro to Stats + Probability
(Mathy edition)
(mostly probability)

$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial x}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial z}{\partial x} + \frac{\partial z}{\partial x}$

might get into multiple variable integration.

Mostly ln, log, polynomials, and integrating and deriving them. Lots of integration by parts in here.

Next one after this is 4305 stats.

Tests
- June 13
- June 24
- July 3

No/group ↓

Major	Freq	Relative freq
math	6	6/21 = .286
CS	10	10/21 = .476
→ physics	0	0 = 0
could put it in or get rid of engineering	1	1/21 = .048
exercise	4	4/21 = .190
tot	21	

categorical. 87% of this class is continuous variable stuff.

not usually useful to reduce fractions in here. 21 is the total number of people in here, it is usually easier.

Usually decimal is not better either, usually just leaving it in original ugly fraction is better.

between 0 and 1, not percents. Think in 0's and 1's, not percents.

Don't put a probability greater than 1 on a test, you fucked up.

Vocab

Random Experiment -

guesses on meaning of random
- no order
- not reproducable
- unpredictable
- (what happens next)

- a lot more order to random in a mathematical sense generally.

- can predict what will happen in the long term. (usually)

- free throws (random event, 50% and 90% person both may or may not make the next shot, but over the long term, the 90% guy will win most of the time).

- random does not mean we know nothing, might not know order, or short term, but in the long run, we usually know.

Random experiment - an experiment

where a particular outcome is unpredictable, but repeated trials follow a regular pattern.

Why use x? Dude who wrote (father of) algebra books in 1500s, in France, printing press guy said I can print faster if we use x for the variables, so he said sure.

Outcome Space / Sample Space -

Set of all possible outcomes.

aka S or $\mathcal{S}$

Random Variable -

We are in this class restricting this to numbers for simplicity.

def - a number assigned to outcomes of a random experiment

Distribution of a Random Variable

super important word on here →

- a description of the behavior of the variable

Need -

i) Description of Outcome Space

ii) need a way to assign probability to each outcome.

(Gaussian, Binomial, Poisson, ...)

(table, histogram, ...)

flipping coins - this or that kind of random event
rolling die - not as many as uses, but still a lot.
casinos

Examples of Random Experiments

- Toss a Six Sided Die

- $S = \{1, 2, 3, 4, 5, 6\}$

discrete random variables
(can map it to integers, this counts)

- distribution of random variable
  - for our six sided die, 1/6 seems reasonable

- different ways to express

is alive, it exists, we got that number

capital X is what we got in experiment
lower case x is the variable in general

a)

x	P(X=x)
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6

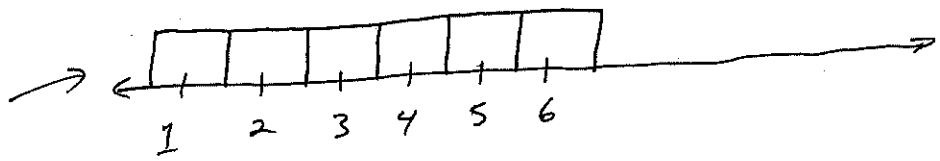
table

b) $P(X=x) = 1/6, \quad x \in S$ formula

c) Histogram graph
- like a bar graph, bars touch (cover everything, all of S)

- Density histogram ⇒ Area = ~~Propo~~ Probability

Center at #



So want area = to prob, so 1 wide and 1/6 tall

$1 \cdot 1/6 = 1/6$

d) name of Distribution, X follows the Discrete Uniform (Dist) with $n = 6$ ← basic # of outcomes

Why is it 1/6?
- Six options
- Is that enough?
- How do we know they are equal?
- don't know if it is a fair die
  - dimples could change likelihood of each outcome

Toss a "fair" six sided die.
⇒ means we know that it is uniform + good

20% chance of rain.

20% chance of what?

- have models with barometric pressure where this or that is, winds.
- 20% in the past, when it was like today, it rained.

roll the die 30 times

1	2	2
2	2	5
3	6	5
4	6	3
5	6	4
6	4	4
7	2	5
8	6	5
9	5	6
10	1	3
11	1	1
12	4	5
13	4	2
14	4	3
15	3	2

x	freq	%
1	3	3/30
2	5	5/30
3	4	4/30
4	6	6/30
5	6	6/30
6	5	5/30

everyone did not get the same number of stuff?
Why?
Random..

class avg	Rel. Freq.
1	20/120 = .167
2	18/120 = .15
3	22/120 = .183
4	25/120 = .208
5	17/120 = .142
6	18/120 = .15

not all the same.
more you do it, more likely
you are to get close.

Sample - collection of observations
from repeated trials
of a random experiment.

- what we actually observed happening

10,000 rolls to determine fairness of die.

Difference Between Probability And Statistics

Probability: Study how to predict what happens in the sample if we know the probability model

model to sample

Statistics: Estimate aspects of the probability model based on what we observe in the sample.

sample to model

Toss a coin until the first heads

sample space?

- $\{H, T\}$ (no)
- $\{H, TH, TTH, TTTH, \ldots\}$

or

X = # of tosses needed

or

$\{1, 2, 3, \ldots\}$ $X = \{x \mid x \in \mathbb{Z}\}$

Play a game

Toss a "fair" 2 sided coin 4 times

More heads: I win
More tails: You win
Tie: Tie

$2 \times 2 \times 2 \times 2$ outcomes

{ {TTTT, HTTT, THTT, TTHT, TTTH, HHHH, THHH, HTHH, HHTH, HHHT, HHTT, HTTH, THTH, ... }

or

{I win, You win, Tie}

↑ not all equally likely

or

# of heads { 0, 1, 2, 3, 4 }

↑ not all equally likely

$$\frac{a+b}{b} \qquad \frac{a}{b} + \frac{b}{b}$$

$$\frac{aa+ab}{ab} = \qquad \frac{\frac{a^2}{b} + \frac{ab}{b}}{a} =$$

Tails + Heads

Possibilities w/ four flips

Supposed to be equal H/T prob.

HHHH	HHHT	HHTT	HTTT	TTTT
	HHTH	HTHT	THTT	
	HTHH	THHT	TTHT	
	THHH	THTH	TTTH	
		TTHH		
		HTTH		
	(I win) 5/16	6/16	(You win) 5/16	

common kind of experiment (live or die, works/doesn't, made shot/did not, up/down)

four flips: $2 \times 2 \times 2 \times 2$ outcomes or 2^n n = flips

Outcome Space / Sample Space

- Set of all possible outcomes
- different ways to express them, best depends on what you're doing and personal preference
- Play a game:
  - flip a "fair" coin 4 times
    - More heads: I win
      More tails: You win
      Equal: Tie
    - Could look at the tails or heads

Fathom

great program for flipping imaginary coins and analyzing

Simpson's Paradox

- ~~was~~ compare 2 different Histograms
- interested in percentage frequency of something happened
- sometimes percentages lie to you

on time	Alaska Air		America West	
LA	495/559	.88	694/811	.86
Phoenix	221/233	.95	4840/5255	.92
San Diego	212/232	.91	383/448	.85
San Fran	503/605	.83	320/449	.72
Seattle	1841/2146	.86	201/262	.77
tot	3274/3775	.87	6438/7225	.89

above line Alaska won
tot, Amwest won

Statistics is a really honest thing if done well. and Depends on the creators proper use.

Probability

More Definitions

given: S = Outcome Space

Event: - subset of S
- something that can happen

Empty Set: subset with no elements

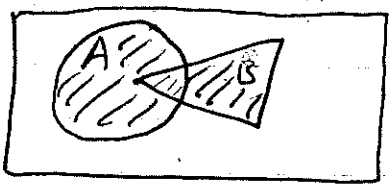
$\emptyset$

$A \subset B$: A is a proper subset of B

$A \subseteq B$ is a proper subset of B

$A \subset B$ but not all of B

$A \cup B$: $A \cup B$ is a A union B

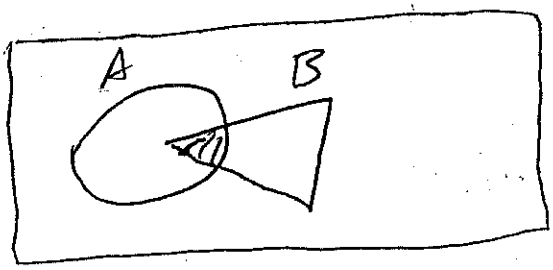


Sample Space

$A \cup B = 1111$

$A \cup B = A \underline{\underline{\text{or}}} B$

$A \cap B$: $A \cap B$ = A and B



Sample Space

Complement of A:

→ A' = all outcomes not in A

In this book (ten other notations in math at least)

$A_1, A_2, A_3, A_4, \ldots$ are mutually exclusive if

$A_i \cap A_j = \emptyset$ if $i \neq j$

$A_1, A_2, A_3, A_4, \ldots$ are

exhaustive if

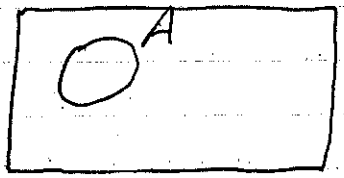
$S = A_1 \cup A_2 \cup A_3 \cup A_4 \cup \ldots$

Probability is much easier if it is mutually exclusive and exhaustive, but it is not, at all, by any means

Let $A \subset S$ be an event,

what does $P(A)$ mean?

- Probability that A occurs



- Probability can be seen as area

Probability of $S = 1$,

- Probability can be seen as counting / ratios / proportion

- Probability as opinion (subjective) not so useful

main focus in this class

- Probability as a limit (ε and stuff coming.)
as sample size approaches ∞

P(Tie ~~in the~~ H/T game) = $\frac{6}{16}$

$\frac{\text{number of times we tie}}{\text{total number of times we play}}$

that converges to P(Tie) as $n \to \infty$
(n = # of times we play)

Mathematical definition of Probability

US PS 1

given S = sample space
A ⊆ S is an event
P is a set function that assigns a number to subsets of S.

1) $P(A) \geq 0$

2) ~~[illegible]~~ & $P(S) = 1$

3) If $A_1, A_2, A_3, \ldots$

If one occurs, the can't occur

3) If $A_1, A_2, A_3, \ldots$ are mutually exclusive then

i) $P(A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_k)$, for $k \in \mathbb{Z}$,
$= P(A_1) \cup P(A_2) \cup P(A_3) \cup \ldots \cup P(A_k)$

ii) $P(A_1 \cup A_2 \cup A_3 \cup \ldots)$
$= P(A_1) \cup P(A_2) \cup \ldots$

keeps going to ∞
countably many A_i's

All of the probability stuff fits this math def

Theorems

(1) $P(A) = 1 - P(A')$

Proof:

Let A, A' be mutually exc and $P(S) = 1$.

$1 = P(S) = P(A \cup A') = P(A) + P(A')$

$\Rightarrow 1 - P(A') = P(A)$

Theorems.. cont.

2) $P(\emptyset) = 0$

$\emptyset' = S$

$P(S) = 1$

$P(\emptyset) = 1 - P(S) = 0$

3) if $A \subset B$, then $P(A) \leq P(B)$

4) $P(A) \leq 1$

5) $P(A \cup B)$

i) if $A \cap B = \emptyset$

then mutually exclusive

and $P(A \cup B) = P(A) + P(B)$

ii) if A, B are general events,

inclusion exclusion principle →

$$P(A \cup B) = [P(A) + P(B)] - P(A \cap B)$$

mutually exclusive is easy to prove, so for this, want to break into mutually exclusive parts.

$(A \cup B)'$, $(A \cap B)$, ~~$A \cup B$~~

$[A \setminus (A \cap B)]$ or $A \cap B'$, $A' \cap B$

↑ set minus

in fact - separate mut. excl. pieces, not too bad.

De Morgan's Law

↓

So $A' \cap B' = (A \cup B)'$

	A	A'	
B	$A \cap B$	$A' \cap B$	$= B$
B'	$A \cap B'$	$A' \cap B'$	$= B'$
	$= A$	$= A'$	$= S$

Probability Table

he returns... Prob + Stats

turns out this class is all

Probability

Uniform
Binomial (Yes/No)
Exponential (waiting times)

HW Posted On Tracs
due ~~tomorrow~~ tomorrow
due June 6

Post Stuff To forum
a couple are required
during semester

TI 83/84 other versions are okay

We study randomness, so...
he'll pick random problems to
grade HW

Quizzes, totally wrong, still 6/10

From Sechuan province
known for spicy food + pandas,
most of china's pandas live there, mountainous region.

If curved on test, your score stays same, and new cutoffs are given for each letter category

Normally he covers more than the proposed stuff on syllabus

Deck of Cards

- Standard deck
- 52 cards
- 1 card at random, observe what you get

random ~~event~~ experiment?
- don't know what we'll get, so yes.

Put All Possible Outcomes together
- Bam, = S

1st exp. $S = \{52 \text{ cards}\}$ ← can define events (subset of sample space

2nd exp. $S = \{51 \text{ cards}\}$

A B C D
king red 7 ♡

which events are mutually exclusive?
- A and C