Particle Swarm Optimization Method for the Design of Digital IIR High Pass Filter

Ishu Kamra[1], Darshan Singh Sidhu[2], Balraj Singh Sidhu[3]
[1, 2, 3]*GZS-PTU Campus, Bathinda, India*

Abstract- Digital signal processing is an area of engineering and science that has continued to have a major and increasing impact in many key areas of technology including media, digital television, digital audio, instrumentation, video recorders, hard disc drive controllers, mobile phone, biomedicine, CD players and modems. Digital filter plays an important role in today's world of computation and communication. Digital filtering is one of the most powerful and important tools of DSP. On the other hand to design a digital infinite impulse response (IIR) filter satisfying all the required conditions is a challenging one. Many techniques have been proposed by researchers from time to time to design optimal digital IIR filter. In this paper Particle Swarm Optimization (PSO) method is used to design the low order high-pass IIR filter. The proposed PSO offers many advantages like simplicity in implementation, ability to converge to a reasonably good solution and robustness against local minima. Our simulation results shows that the proposed approach is accurate and has a fast convergence speed and the results obtained illustrates that PSO offers a viable tool to design IIR filters.

Keywords- *Digital IIR filter, Particle Swarm Optimization, High-pass filter, gbest, pbest, weighting factors.*

I. INTRODUCTION

A DSP is a programmable device like a general purpose microprocessor chips that are capable of processing out millions of floating point operations per second to provide real time performance. The DSP presents greater flexibility, higher performance (in terms of attenuation and selectivity), better time, environment stability and lower equipment production costs than traditional analog signal processing system. Digital filtering is one of the most important and powerful tools of DSP. A filter is essentially a system or network that are widely used in communication circuit systems and signal processing in applications such as video processing, audio processing, radar, channel equalization, biomedical signal processing, analysis of economic, noise reduction and financial data. The main objective of filtering is given below:

1. To improve the signal quality.
2. To extract the important information from the signals.
3. To divide the signal into two or more sub-bands.
4. To modify the frequency spectrum of the transmitted signal in communication.
5. To confine the signal into desired frequency band as in low pass, high pass, band pass, band reject and all pass.

Filters are mainly comes in two flavors; analog filters and digital filters. Analog filters are the device that works on continous-time signals. Analog filters consists of active and passive basic elements of electronics like operational amplifiers, resistance, capacitors etc. and these are easy to develop and exceptionally cheap as compared with digital filters. Digital filters are consists of controller and DSP processors which plays an important role in DSP applications such as signal analysis and estimation. Digital filters are divided into two parts, finite impulse response (FIR) and infinite impulse response (IIR) on the basis of their impulse response.

Digital infinite impulse response (IIR) filter design principally follows two methods: optimization method and transformation method. In the transformation method, initially analog IIR filter is designed and then it is converted into digital IIR filter. Butterworth, Chebyshev and elliptic functions have been proposed using transformation method [1]. Optimization algorithms are becoming increasingly popular in engineering design activities. In this we seek values of the variables that lead to an optimal value of the function that is to be optimized. Optimization techniques have been applied whereby ripple magnitude of both pass band and stop band, magnitude error are to be measured.

Genetic algorithm is a one of the type of EAs. These EAs methods are having better control of parameters and the highest stop-band attenuation. The genetic algorithms are capable of searching multimodal and multi-dimensional spaces and also capable to optimize discontinuous and complex functions. Tang *et al.*[2] proposed design of digital IIR filters by using genetic algorithm. The only difficulty with genetic algorithm arises in terms of solution quality and convergence speed. Tsai et al. [3] applied hybrid Taguchi genetic algorithm for the design of optimal IIR filter. In hybrid Taguchi method, the conventional genetic algorithm is combines with Taguchi method. Tsai et al.[4] has introduced an approach by combining the immune algorithm and the Taguchi method named as Taguchi-immune algorithm (TIA).

In this paper, the benefits of designing the IIR high-pass digital filter using an evolutionary technique known as Particle Swarm Optimization (PSO) has been explored. In many aspects, PSO proves itself to be more efficient than other previously discussed techniques like Genetic Algorithm (GA), Differential Evolution (DE) etc. PSO developed by Eberhart and kennedy in 1995. PSO is a robust population based flexible optimization technique. It is a kind of swarm intelligence that is based on social psychological Principles.

The advantages of PSO lie in its simplicity to implement as well as its convergence speed can be controlled via few parameters. PSO has shown many advantages in searching speed and precision as compared with other EAs methods [5]. Irrespective of several advantages of PSO, it has some disadvantages. The convergence speed of PSO depends upon many parameters, so the proper selection of these parameters is an important factor for efficient working of PSO algorithm. If PSO parameters are wrongly chosen than this may results in divergent particle trajectories which cause trapping into local minimum value. In case PSO applied to high-dimensional optimization problems, the premature convergence problem may suffer which results in low optimization precision or sometimes even failure. For improving the performance of PSO various researches have been introduced by researchers either through mathematical analysis or by improving PSO algorithms [6-9].

The intent of this paper is to propose a particle swarm optimization method for the design of digital IIR high-pass filter. The values of the filter coefficients are optimized by using PSO to achieve magnitude error and ripples magnitude as objective functions for optimization problem. This paper is organized in five sections. Section 2 describes the IIR digital filter design problem statement. Section 3 introduces the underlying mechanism and details regarding PSO algorithm for designing the optimal digital IIR filters. In section 4, the performance of the proposed PSO has been evaluated and achieved results are compared with previous year results. Section 5 gives the conclusion and future scope of the proposed work.

II. IIR FILTER DESIGN PROBLEM

This section details the design strategy of digital IIR filter. IIR filter is classified as recursive type filter which means its output is depends on present and past inputs as well as on past outputs. This indicates that the system is prone to instability and feedback. A digital filter design problem determines a set of filter coefficients which meet performance specifications. These performance specifications are (i) width of the stop-band and attenuation, (ii) pass band width and its corresponding gain, (iii) band edge frequencies, and (iv) tolerable peak ripple in the pass band and stop-band. The traditional design of IIR digital filter is described by the following difference equation:

$$y(n)=\sum_{i=0}^{M} p_i x(n-i) - \sum_{k=1}^{N} q_k y(n-k) \quad (1)$$

Where, p_i and q_k are the coefficients of the filter. x(n) and y(n) are the input and output respectively. N and M gives order of the filter with $N \geq M$.

The transfer function of IIR digital filter is expressed as:

$$H(z)=\frac{\sum_{i=0}^{U} p_i z^{-i}}{1+\sum_{k=1}^{V} q_k z^{-k}} \quad (2)$$

The design of digital IIR filter involves the determination of a set of filter coefficients, p_i and q_k. The structure of the cascading type digital IIR filter is described as :

$$H(w,x)=A\left(\prod_{k=1}^{M}\frac{1+p_{1k}e^{-jw}}{1+q_{1k}e^{-jw}}\right)\times\left(\prod_{j=1}^{N}\frac{1+r_{1j}e^{-jw}+r_{2j}e^{-2jw}}{1+s_{1j}e^{-jw}+s_{2j}e^{-2jw}}\right) \quad (3)$$

Where:- $x=[p_{11},q_{11}.......p_{1M},q_{1M},r_{11},r_{12},s_{11},s_{12}......r_{1N},r_{2N},s_{1N},s_{2N},A]^T$ and' x ' represents the filter coefficients of first and second order sections of dimension $V\times 1$ with $V=2M+4N+1$.

The IIR filter is designed by optimizing some coefficients or parameters such as approximation error function in L_p-norm. In this paper, we are optimizing the absolute error L_1-norm of magnitude response and squared error L_2-norm of magnitude response. These above two parameters are represented as :

$$E(x)=\left\{\sum_{i=0}^{k}\left|H_d(w_i)-\left|H(w_i,x)\right|\right|^p\right\}^{\frac{1}{p}} \quad (4)$$

In IIR filter design we use fixed grid approach. For $p=1$ the error is called L_1 - norm error, $E_1(x)$ expressed as (5) and for $p=2$ the error is called as L_2-norm error. In above expression $H_d(w_i)$ is the desired response of the IIR filter.

$$E_1(x)=\sum_{i=0}^{k}\left|H_d(w_i)-\left|H(w_i,x)\right|\right| \quad (5)$$

$$H_d(w_i)=\begin{cases}1, & ;\text{for } w_i \in \text{passband}\\ 0, & ;\text{for } w_i \in \text{stopband}\end{cases} \quad (6)$$

The ripple magnitude of pass band and stop band is expressed as :

$$r_1(x)=\max\{|H(w_i,x)|\}-\min\{|H(w_i,x)|\};w_i\in \text{passband} \quad (7)$$

$$r_2(x)=\max\{|H(w_i,x)|\};w_i\in \text{stopband} \quad (8)$$

Where, $r_1(x)$ and $r_2(x)$ represents the ripple magnitude of pass band and stop band respectively. The inclusion of stability constraints is compulsory for the design of causal recursive IIR filter. Therefore, the stability constraints are given by following equations:

$$1+q_{1k}\geq 0\,(k=1,2,...,M) \quad (9)$$

$$1-q_{1k}\geq 0\,(k=1,2,....,M) \quad (10)$$

$$1-s_{2j}\geq 0\,(j=1,2,....,N) \quad (11)$$

$$1+s_{1j}+s_{2j}\geq 0\,(j=1,2,...,N) \quad (12)$$

$$1-s_{1j}+s_{2j}\geq 0\,(j=1,2,......,N) \quad (13)$$

PSO is applied and constraints are forced to satisfy by randomly updating them.

III. PARTICLE SWARM OPTIMIZATION

Optimization is a procedure used to make a system or design as effective or functional as possible, especially the mathematical techniques. An optimization algorithm is a procedure which is executed iteratively by comparing various solutions till the optimum or a satisfactory solution is found. The PSO algorithm for global optimization has been introduced by Eberhart and Kennedy in 1995. It is a kind of swarm intelligence that is based on social-psychological principles. People solve their problems by interacts or communicate with other people in this way the individuals moving toward one another in sociocognitive space. The population in PSO is called a swarm and each individual is called a particle. To search the global optimization each particle flies through the solution space. The particles iteratively evaluate the objective or fitness function at different locations and remember that location which they had their best fitness value. Each particle knows its best value so far (p best) or local best and each particle knows their best value so far in group or swarm (g best) among p bests. Each particle tries to modify their position using the following information:

1. The distance between the current position and the pbest.
2. The distance between the current position and the gbest.

In PSO the system is initialized with population of random solutions. A randomized velocity is assigned to each potential solution. These potential solutions are called particles. PSO has been utilized in many applications due to its easier implementation, simplicity and efficiency. It is similar to Genetic algorithm (GA) or we can say that it seems to lie in between Evolutionary and Genetic algorithm. Each particle is treated as a point in a N-dimensional space which adjusts its "flying" according to its own flying experience as well as the flying experience of other particles. Being easy to implement and yet so effective, PSO has been utilized in a wide variety of optimization applications like audio application and to design digital filters. There are two versions of the basic PSO algorithm viz. the continuous PSO algorithm and the binary PSO algorithm. The two can be briefly explained as follows:

1. The Continuous PSO:- This uses a real valued multidimensional space as belief space .
2. The Binary PSO:- This algorithm has attracted much lesser attention in previous work. In this version, the particle's position is not a real value, but a 0 or a 1.

Algorithm for Particle Swarm Optimization (PSO):-

Step 1:
The swarm (initial population) is generated where each particle in the swarm is a solution vector containing elements. Each particle assigned a random position in the problem search space.

Step 2:
Find the fitness function for each particle.

Step 3:
To obtain pbest compare each particle's fitness with the current particle's

Step 4:
Compare fitness evolution with the population's overall previous best to obtain gbest.

Step 5:
The position and velocities of all particles is update for further processing. The expression for update velocity and position is given as :-

$$v_i = v_i + 2*\text{rand}()*(\text{pbest} - x_i) + 2*\text{rand}()*(\text{gbest} - x_i)$$
$$x_i = x_i + v_i$$

Where, rand() is the random number having value in between 0 and 1. x_i and v_i are the position and velocity vector respectively,

Step 6:
Repeat step 2 to 5 until the stopping criterion is met.

The flowchart of PSO technique is shown as:

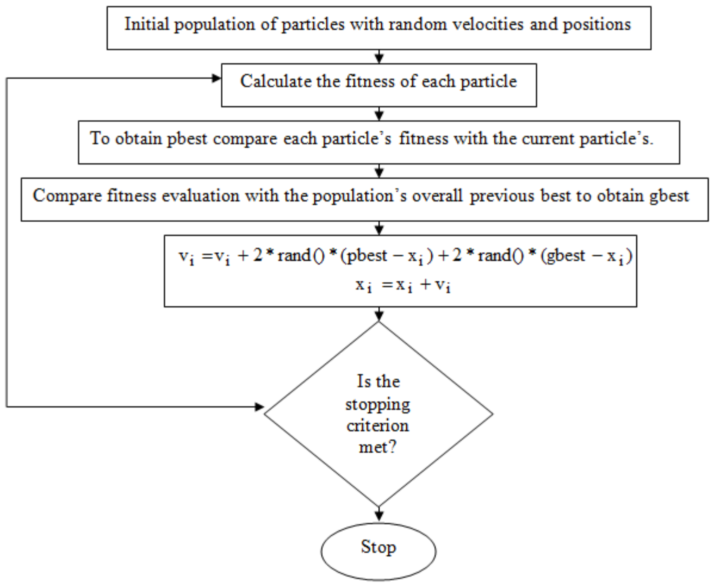

Fig.1: Flowchart of PSO algorithm

IV. SIMULATION AND RESULT ANALYSIS

In this section, the frequency response and stability response of high pass (HP) filter, whose coefficients are optimized by using Particle Swarm Optimization (PSO) are presented. For designing of low order High-pass IIR digital filter 200 equally spaced points are set within the frequency domain $[0, \pi]$. The order of the digital IIR filter is fixed throughout the optimization process. The main objective of this paper is to minimize the objective function with satisfying the stability constraints. For the comparison of results, the lowest order of the IIR filter is set exactly same as that given by Ranjit kaur *et al.*[11] and Tsai *et al.*[3]. The desired or prescribed design conditions for high-pass IIR filter is given in Table 1.

TABLE 1: DESIRED DESIGN CONDITION FOR HP IIR FILTER

Filter type	Pass-band	Stop-band	$\lvert H(w,x) \rvert$
High-Pass	$0.8\pi \le w \le \pi$	$0 \le w \le 0.7\pi$	1

The designed IIR digital filter model obtained by using PSO method for HP is expressed by following equations :

$$H_{HP}(z) = .046091 \frac{(z - 0.987076)(z^2 + 0.693326z + 1.002996)}{(z + 0.655395)(z^2 + 1.404628z + 0.749138)} \quad (14)$$

The obtained magnitude error, minimum and maximum ripples magnitude of passband and minimum ripple magnitude of stopband are given in Table 2.

TABLE 2: VALUE OF MAGNITUDE ERROR AND RIPPLES OF HP FILTER

Magnitude Error	Maximum Pass-band Ripple	Minimum Pass-band Ripple	Maximum Stop-band Ripple
3.9863	1.0192	0.8992	0.1137

The frequency response and pole zero plot using PSO is shown in Fig.2 and Fig.3 respectively.

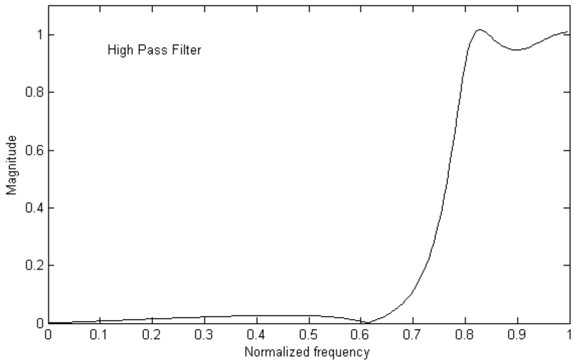

Fig.2: Frequency Response of HP filter with order 3.

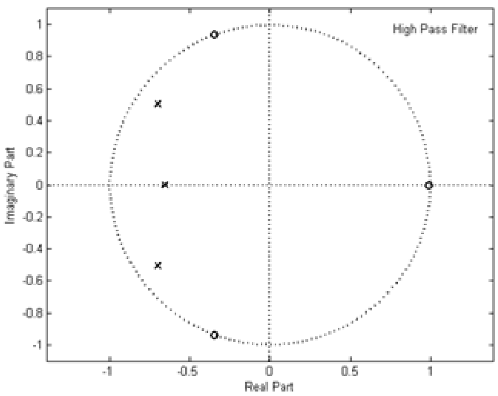

Fig. 3: Pole-Zero plot of HP filter with order 3.

From the above response of IIR HP filter by using PSO shows that in the frequency response of IIR LP filter the stopband attenuation is less and in pole-zero plot all the poles are lies in unit circle which indicates that the system is stable.

The computational results obtained by PSO technique and its comparison with other techniques on the basis of magnitude error and ripples magnitude of both passband and stopband are given in Table 3 for high-pass filter (HP).

TABLE 3: COMPARISON OF VARIOUS TECHNIQUES WITH ITS OWN SIMULATION RESULTS.

Method	Magnitude Error	Pass-band Performance	Stop-band performance
PSO	3.9863	$0.8992 \le \lvert H(e^{jw}) \rvert \le 1.019$ (0.1199)	$\lvert H(e^{jw}) \rvert \le 0.1137$
RCGA [11]	4.5296	$0.9677 \le \lvert H(e^{jw}) \rvert \le 1.018$ (.0508)	$\lvert H(e^{jw}) \rvert \le 0.1540$
HTGA [3]	4.8372	$0.9460 \le \lvert H(e^{jw}) \rvert \le 1.000$ (.0540)	$\lvert H(e^{jw}) \rvert \le 0.1457$
TIA[4]	4.1819	$0.9229 \le \lvert H(e^{jw}) \rvert \le 1.000$ (.0771)	$\lvert H(e^{jw}) \rvert \le 0.1424$

V. CONCLUSION AND FUTURE SCOPE

In this paper, particle swarm optimization (PSO) algorithm has been applied to design digital high-pass IIR filter. Therefore, the proposed PSO algorithm posses the merits of controllable convergence speed, easier implementation and robustness. The results obtained by the proposed PSO are either better or atleast comparable in magnitude error and ripple magnitude in passband and stopband. Therefore, the proposed PSO method is a powerful and useful tool for the design of digital IIR filter. The computational results shows that all the poles in their stability responses are lies inside a unit circle whch shows that the filter is stable one. In this paper, PSO is used to design digital IIR low-pass filter of low order (3^{rd} order) but it can be used to design filters of higher order. Any type of filter LP, HP, BP and BS can be easily or independently realized by using proposed PSO. On the basis of above computational results obtained for the design of digital IIR filter, it can be concluded that the proposed PSO method is superior to the RCGA [11], HTGA [3] and TIA [4]. PSO has shown many advantages in searching speed and precision as compared with other EAs techniqus The proposed PSO can be used to optimize the design problems having multi objective function. Based on this technique more research efforts are required for making long life of IIR filters.

VI. REFRENCES

[1] John G. Proakis and Dimitris G. Manolakis,”Digital Signal Processing”, Pearson, fourth edition, 2013.

[2] K.S. Tang, K.F. Man, S. Kwong and Z.F. Liu, “Design and Optimization of IIR Filter Structure Using Hierarchical Genetic Algorithms”, IEEE Transactions on Industrial Electronics, Vol. 45,no. 3,pp.481– 487, June 1998.

[3] Jinn-Tsong Tsai, Jyh-Horng Chou and Tung- Kuan Liu,“ Optimal Design of Digital IIR Filter By Using Hybrid Taguchi Genetic

Algorithm", IEEE Transactions on Industrial Electronics, Vol. 53, no. 3, pp.867- 879, June 2006.
[4] Jinn-Tsong Tsai and Jyh-Horng,"Optimal Design of Digital IIR Filters By Using An Improved Immune Algorithm ", IEEE Transactions on Signal Processing, Vol. 54, no. 12, pp. 4582–4596, December 2006.
[5] J.B. Park, K.S. Lee, J.R. Shin and K.Y. Lee, "A Particle Swarm Optimization For Economic Dispatch With Non-Smooth Cost Function",. IEEE Transactions on Power Systems 2005; Vol. 20, no.1, pp 34–42, 2005.
[6] Sangeeta Mandal, S.P. Ghoshal, Rajib Kar and Durbadal Mandal," Design of Optimal LinearPhase FIR HP Filter Using IPSO", ACEEE International Journal on Signal and Processing, Vol. 03,No. 01, Jan 2012.
[7] S.P. Ghoshal, Sangeeta Mondal, Rajib Kar and Durbadal Mandal," Novel Particle Swarm Optimization For Low Pass FIR Filter Design", WSEAS Transactions on Signal Processing, issue 3,Vol. 8, July 2012.
[8] J. Sun, B. Feng and W. Xu, "Particle Swarm Optimization With Particles Having Quantum Behavior",Proceedings Congress Evolutionary Computation, pp 325–331, 2004.
[9] H. Gao and W. Xu, "A New Particle Swarm Algorithms and Its Globally Convergent Modification" IEEE Transactions on Systems, Man,and Cybernetics-Part B, Vol. 41, no. 5, pp.1334-1351, 2011.
[10] Ranjit Kaur, Manjeet Singh Patterh and J.S.Dhillon, " Design of Optimal L1 Stable IIR Digital Filter Using Hybrid Optimization Algorithm",International Journal of Computer Applications,Vol. 38, no.2, Jan 2012.
[11]Ranjit Kaur, Manjeet Singh Patterh and J.S. Dhillon," Digital IIR Filter Design Using Real Coded Genetic Algorithm", International Journal of Information Technology and Computer Science, Vol. 7, pp 27-35, 2013.
[12] Ranjit Kaur, Manjeet Singh Patterh , Damanpreet Singh and J.S. Dhillon," Heuristic Search Method For Digital IIR Filter Design", WSEAS Transactions on Signal Processing, issue 3, vol 8,July 2012.
[13] Dawei Zhou , Xiang Gao, Guohai Liu , Congli Mei , Dong Jiang and Ying Liu," Randomization in Particle Swarm Optimization For Global Search Ability", ELSEVIER, Expert Systems With Application, 38, 2011.
[14]Ying-ping Chen and Pei Jiang," Analysis of Particle Interaction in Particle Swarm Optimization", ELSEVIER, Theoretical Computer Science, 411, 2010.