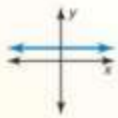
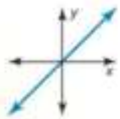
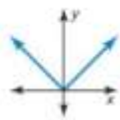
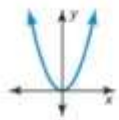
Core Concept

Parent Functions

Family	Constant	Linear	Absolute Value	Quadratic
Rule	$f(x) = 1$	$f(x) = x$	$f(x) = \lvert x \rvert$	$f(x) = x^2$
Graph				
Domain	All real numbers	All real numbers	All real numbers	All real numbers
Range	$y = 1$	All real numbers	$y \geq 0$	$y \geq 0$

THE GRAPH OF A QUADRATIC FUNCTION

The graph $y = ax^2 + bx + c$ is a parabola with these characteristics.

- The parabola opens up if $a > 0$ and opens down if $a < 0$. The parabola is wider than the graph of $y = x^2$ if $|a| < 1$ and narrower than the graph of $y = x^2$ if $|a| > 1$.
- The x-coordinate of the vertex is $-\frac{b}{2a}$.
- The axis of symmetry is the vertical line $x = -\frac{b}{2a}$.

Describing Transformations

A **transformation** changes the size, shape, position, or orientation of a graph. A **translation** is a transformation that shifts a graph horizontally and/or vertically but does not change its size, shape, or orientation.

A **reflection** is a transformation that flips a graph over a line called the *line of reflection*. A reflected point is the same distance from the line of reflection as the original point but on the opposite side of the line.

VERTEX AND INTERCEPT FORMS OF A QUADRATIC FUNCTION

FORM OF QUADRATIC FUNCTION	CHARACTERISTICS OF GRAPH
Vertex form $y = a(x-h)^2 + k$	The vertex is (h, k). The axis of symmetry is $x = h$.
Intercept form $y = a(x-p)(x-q)$	The x intercepts are p and q. The axis of symmetry is halfway between $(p, 0)$ and $(q, 0)$.

For both forms, the graph opens up if $a > 0$ and opens down if $a < 0$.

GRAPING ABSOLUTE VALUE FUNCTIONS

The graph $y = a|x-h| + k$ has the following characteristics.

- The graph has vertex (h, k) and is symmetric in the line x = h.
- The graph is V-shaped. It opens up if a > 0 and down if a < 0,
- The graph is wider than the graph of $y = |x|$ if $|a| < 1$.

  The graph is narrower than the graph of $y = |x|$ if $|a| > 1$.

GRAPHS OF RADICAL FUNCTIONS

To graph $y = a\sqrt{x-h} + k$ or $y = a\sqrt[3]{x-h} + k$, follow these steps.

STEP 1: Sketch the graph of $y = a\sqrt{x}$ or $y = a\sqrt[3]{x}$.

STEP 2: Shift the graph h units horizontally and k units vertically.

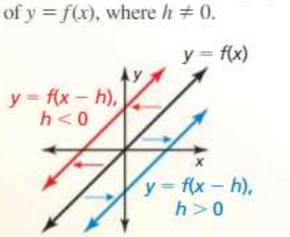
Translations and Reflections

You can use function notation to represent transformations of graphs of functions.

Core Concept

Horizontal Translations

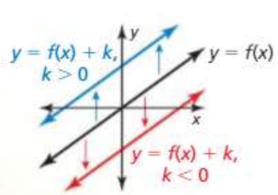
The graph of $y = f(x - h)$ is a horizontal translation of the graph of $y = f(x)$, where $h \neq 0$.



Subtracting h from the **inputs** before evaluating the function shifts the graph left when $h < 0$ and right when $h > 0$.

Vertical Translations

The graph of $y = f(x) + k$ is a vertical translation of the graph of $y = f(x)$, where $k \neq 0$.

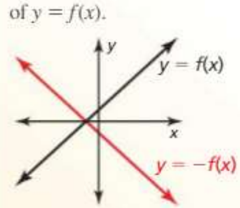


Adding k to the **outputs** shifts the graph down when $k < 0$ and up when $k > 0$.

Core Concept

Reflections in the *x*-axis

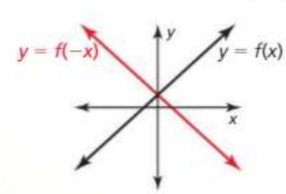
The graph of $y = -f(x)$ is a reflection in the x-axis of the graph of $y = f(x)$.



Multiplying the **outputs** by -1 changes their signs.

Reflections in the *y*-axis

The graph of $y = f(-x)$ is a reflection in the y-axis of the graph of $y = f(x)$.



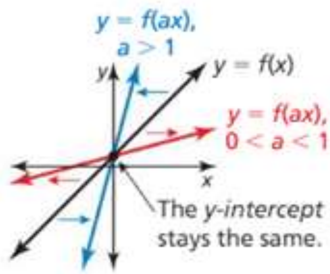
Multiplying the **inputs** by -1 changes their signs.

Core Concept

Horizontal Stretches and Shrinks

The graph of $y = f(ax)$ is a horizontal stretch or shrink by a factor of $\frac{1}{a}$ of the graph of $y = f(x)$, where $a > 0$ and $a \neq 1$.

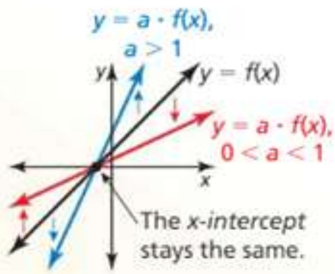
Multiplying the **inputs** by a before evaluating the function stretches the graph horizontally (away from the y-axis) when $0 < a < 1$, and shrinks the graph horizontally (toward the y-axis) when $a > 1$.



Vertical Stretches and Shrinks

The graph of $y = a \cdot f(x)$ is a vertical stretch or shrink by a factor of a of the graph of $y = f(x)$, where $a > 0$ and $a \neq 1$.

Multiplying the **outputs** by a stretches the graph vertically (away from the x-axis) when $a > 1$, and shrinks the graph vertically (toward the x-axis) when $0 < a < 1$.



Core Concept

Writing an Equation of a Line

Given slope m and y-intercept b — Use slope-intercept form:

$$y = mx + b$$

Given slope m and a point (x_1, y_1) — Use point-slope form:

$$y - y_1 = m(x - x_1)$$

Given points (x_1, y_1) and (x_2, y_2) — First use the slope formula to find m. Then use point-slope form with either given point.

Core Concept

Finding a Line of Fit

Step 1 Create a scatter plot of the data.

Step 2 Sketch the line that most closely appears to follow the trend given by the data points. There should be about as many points above the line as below it.

Step 3 Choose two points on the line and estimate the coordinates of each point. These points do not have to be original data points.

Step 4 Write an equation of the line that passes through the two points from Step 3. This equation is a model for the data.

Core Concept

Solving a Three-Variable System

Step 1 Rewrite the linear system in three variables as a linear system in two variables by using the substitution or elimination method.

Step 2 Solve the new linear system for both of its variables.

Step 3 Substitute the values found in Step 2 into one of the original equations and solve for the remaining variable.

When you obtain a false equation, such as $0 = 1$, in any of the steps, the system has no solution.

When you do not obtain a false equation, but obtain an identity such as $0 = 0$, the system has infinitely many solutions.