Math 1497 Calc 2

Trig integrals

Last class we considered

$$\int \sin^m x \cos^n x \, dx \qquad m, n \text{ integers}$$

Now we consider

$$\int \tan^m x \sec^n x \, dx$$

and the cases

(i) n even

(ii) m odd

(iii) $\tan^m x$ only

(iv) $\sec^n x$ only

Ex1 $\int \tan^2 x \sec^2 x \, dx$

let $u = \tan x$ $\quad du = \sec^2 x \, dx$

$$\int u^2 \, du = \frac{u^3}{3} + c = \frac{\tan^3 x}{3} + c$$

Ex2 $\int \tan^2 x \sec^4 x \, dx$

$$\int \tan^2 x \sec^2 x \sec^2 x \, dx$$

$u = \tan x$ $\quad 1 + \tan^2 x = \sec^2 x$

$du = \sec^2 x \, dx$ $\quad \Rightarrow \quad \sec^2 x = 1 + u^2$

$$\int u^2 (1 + u^2) \, du = \int u^2 + u^4 \, du$$

$$= \frac{1}{3} u^3 + \frac{1}{5} u^5 + c$$

$$= \frac{1}{3} \tan^3 x + \frac{1}{5} \tan^5 x + c$$

Case 2 m odd

$$\int \tan x \sec x \, dx = \sec x + c$$

Ex3 $\int \tan^3 x \sec^3 x \, dx$

$$\int \tan^2 x \sec^2 x \sec x \tan x \, dx$$

$u = \sec x$ $\qquad 1 + \tan^2 x = \sec^2 x$

$du = \sec x \tan x \, dx$ $\qquad \tan^2 x = \sec^2 x - 1$

$\qquad = u^2 - 1$

$$\int (u^2 - 1) u^2 \, du = \int u^4 - u^2 \, du$$

$$= \frac{u^5}{5} - \frac{u^3}{3} + c$$

$$= \frac{1}{5} \sec^5 x - \frac{1}{3} \sec^3 x + c$$

(iii) $\int \tan^m x \, dx$ only

$u = \cos x$
$du = -\sin x \, dx$

<u>Ex</u> $\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx$

$\int -\frac{du}{u} = -\ln|u| + c = -\ln|\cos x| + c$

<u>Ex</u> $\int \tan^2 x \, dx = \int (\sec^2 x - 1) dx$

$= \tan x - x + c$

<u>Ex</u> $\int \tan^3 x \, dx = \int \tan x \tan^2 x \, dx$

$= \int \tan x (\sec^2 x - 1) dx$

$= \int \underbrace{\tan x \sec^2 x}_{u = \tan x} dx - \int \tan x \, dx$

$= \frac{\tan^2 x}{2} - (-\ln|\cos x|) + c$

$$\int \tan^4 x \, dx = \int \tan^2 x \tan^2 x \, dx$$

$$= \int \tan^2 x (\sec^2 x - 1) \, dx$$

$$= \int \tan^2 x \sec^2 x \, dx - \int \tan^2 x \, dx$$

$u = \tan x$

$$\frac{1}{3} \tan^3 x - \int \tan^2 x \, dx$$

already done

$$\frac{1}{3} \tan^3 x - [\tan x - x] + C$$

(iv) $\int \sec^n x \, dx$ only

$$\int \sec x \, dx = + \ln|\sec x + \tan x| + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \sec^3 x \, dx$$

$$\int \sec x \sec^2 x \, dx \leftarrow \text{parts}$$

$u = \sec x \qquad v = \tan x$

$du = \sec x \tan x \, dx \qquad dv = \sec^2 x \, dx$

$$= \sec x \tan x - \int \sec x \tan^2 x \, dx$$

$$= \sec x \tan x - \int \sec x (\sec^2 x - 1) \, dx$$

$$= \sec x \tan x - \int \sec^3 x \, dx + \int \sec x \, dx$$

$$2 \int \sec^3 x \, dx = \sec x \tan x + \ln|\sec x + \tan x| + C$$

$$\int \sec^3 x \, dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln|\sec x + \tan x| + C$$