Math 2471 – Calc 3

Calc 1 Chain Rule

$y = \sqrt{x^2+1}$

$u = x^2+1 \quad y = u^{1/2}$

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = \frac{1}{\cancel{2}}u^{-1/2}\cdot \cancel{2}x = (x^2+1)^{-1/2}\cdot x$$

Calc 3 2 Types of chain Rule

(1) $z = f(x,y) \quad x = g(t),\ y = h(t)$

Ex $z = x^2 + y^3 \quad x = \sin t,\ y = t^2+1$

$z = (\sin t)^2 + (t^2+1)^3 \leftarrow$ fct t only

$$\frac{dz}{dt} = 2\sin t\,\frac{d(\sin t)}{dt} + 3(t^2+1)^2\frac{d}{dt}(t^2+1)$$

$$= 2\sin t\cos t + 3(t^2+1)^2\cdot 2t$$

In general

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

ex $\quad z = e^{x^2-y^2} \qquad x = \cos t, \quad y = \sin t$

$$\frac{\partial z}{\partial x} = e^{x^2-y^2}\cdot 2x \qquad \frac{\partial z}{\partial y} = +e^{x^2-y^2}(-2y)$$

$$\frac{dx}{dt} = -\sin t \qquad \frac{dy}{dt} = \cos t$$

$$\frac{dz}{dt} = 2x\, e^{x^2-y^2}(-\sin t) + e^{x^2-y^2}(-2y)\cos t$$

$$= -2\cos t\, e^{\cos^2 t-\sin^2 t}\sin t + e^{\cos^2 t-\sin^2 t}(-2\sin t)\cos t$$

$$= -4\sin t\cos t\, e^{\cos^2 t-\sin^2 t}$$

(2) $z = f(x, y) \quad x = g(r, s) \quad y = h(r, s)$

ex $\quad z = x^2 + y^3 \quad x = r + 2s \quad s = 2r - s$

$z = (r+2s)^2 + (2r-s)^3 \leftarrow$ fct r & s

$$\frac{\partial z}{\partial r} = 2(r+2s)\cdot 1 + 3(2r-s)^2(2)$$

$$\frac{\partial z}{\partial s} = 2(r+2s)\cdot 2 + 3(2r-s)^2(-1)$$

In general

$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial r}$$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial s}$$

generalize

\# 26 $\quad w = \sqrt{x^2+y^2+z^2}$

$x = st, \quad y = rs, \quad z = rt$

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial r} + \frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial r}$$

$$w_x = \frac{1}{2}(x^2+y^2+z^2)^{-1/2}(2x) = \frac{x}{\sqrt{x^2+y^2+z^2}}$$

$$w_y = \frac{y}{\sqrt{x^2+y^2+z^2}} \qquad w_z = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

$x_r = 0, \quad y_r = s, \quad z_r = t$

$$w_r = 0 + \frac{sy}{\sqrt{x^2+y^2+z^2}} + \frac{tz}{\sqrt{x^2+y^2+z^2}}$$

$$w_r = \frac{rs^2 + rt^2}{\sqrt{s^2t^2+r^2s^2+z^2t^2}}$$

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$z = \sin(2x+y) \qquad x = s-t,\ y = t^2$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial s}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t}$$

$z_x = \cos(2x+y)\cdot 2, \quad z_y = \cos(2x+y)\cdot 1$

$x_s = 1,\ y_s = 0 \qquad x_t = -1,\ y_t = 2t$

1) $$\frac{\partial z}{\partial s} = \cos(2x+y)\cdot 2\,(1) + 0$$

$$= \cos\left(2(s-t)+t^2\right)\cdot 2$$

2) $$\frac{\partial z}{\partial t} = \cos(2x+y)\cdot 2\,(-1) + \cos(2x+y)\,(1)\cdot 2t$$

$$= \cos\left(2(s-t)+t^2\right)\{-2+2t\}$$

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