Normal Dist. & Statistics

The time a mobile phone battery lasts before needing to be recharged is assumed to be normally distributed with a mean of 48 hours and a standard deviation of 8 hours.

(i) Find the probability that a battery chosen at random will last more than 60 hours.

(ii) Find the probability that the battery at least 35 hours.

$Z = \frac{X-\mu}{\sigma}$

(i) $P(X > 60)$

$P\left(Z > \frac{60-48}{8}\right)$

$P(Z > 1.5)$

$1 - P(Z < 1.5)$

$1 - 0.9332$

0.0668

(ii) $P(X < 35)$

$P\left(Z < \frac{35-48}{8}\right)$

$P(Z < -1.625)$

$1 - P(Z < 1.625)$

$1 - [0.9484]$

0.0516

Jars of honey are filled by a machine.
It has been found that the quantity of honey in a jar has a mean of 460.3 g with a standard deviation of 3.2 g.
It is believed that the machine controls have been altered in such a way that, although the standard deviation is unaltered, the mean quantity may have changed.
A random sample of 60 jars is taken and the mean quantity of honey per jar is found to be 461.2 g.

(i) State the null and alternative hypotheses.

(ii) Calculate the sample statistic for the mean.

(iii) Is there evidence, at the 5% level of significance, that the sample mean is different from the population mean?

(i) H_0: The mean has not changed

H_1: The mean has changed.

(ii) $Z = \frac{\bar{x}-\mu}{\left(\frac{\sigma}{\sqrt{n}}\right)} = \frac{461.2-460.3}{\left(\frac{3.2}{\sqrt{60}}\right)} = 2.18$

(iii) Critical region $Z < -1.96$ or $Z > 1.96$

Since $2.18 > 1.96$ then we reject H_0.

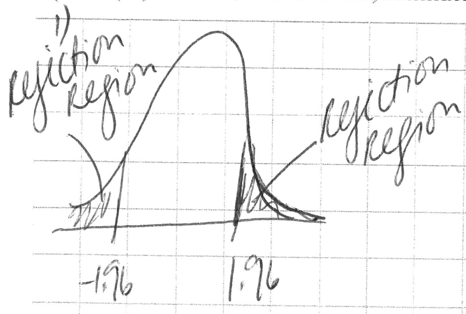
Draw a rough sketch of the normal curve showing the critical regions, at the 5% level of significance, of a hypothesis test.

(i) Clearly indicate the rejection regions.

(ii) What are the critical z-values for the limits of these rejection regions?

(iii) For a z-value of 1.6, estimate the corresponding p-value for this statistic.

i)



ii) $z < -1.96$ and $z > 1.96$

iii) $= 2 \times P(z > |\text{test statistic}|)$

$= 2 \times P(z > 1.6)$

$= 2 \times [1 - P(z < 1.6)]$

$= 2 \times [1 - 0.9452]$

$= 0.1096$.

A sample poll of 100 voters chosen at random from all voters in a given constituency indicated that 55% of them were in favour of candidate A.
Find the 95% confidence interval for the proportion of all the voters in the district in favour of this candidate.

$E = 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$E = 1.96\sqrt{\frac{0.55(1-0.55)}{100}}$

$E = 0.09750$

$0.55 - 0.09750 < p < 0.55 + 0.09750$

$0.453 < p < 0.648$

$45.3\% < p < 64.8\%$.

(i) Explain briefly what is meant by the term "95% confidence interval".

(ii) A car manufacturing company tested a random sample of 150 cars of the same model to estimate the mean number of kilometres travelled per litre of petrol consumption for all cars of that model.
The sample mean of kilometres travelled per litre consumed was 13.52 and the standard deviation was 2.23.

Form a 95% confidence interval for the mean number of kilometres travelled per litre of petrol consumed for all cars of that make.
Give all calculations correct to two places of decimal.

(i) We can be 95% Confident that the result for the actual population will lie in that interval. [It will happen 95 times out of 100].

(ii) $E = 1.96\frac{\sigma}{\sqrt{n}}$

$E = 1.96\left(\frac{2.23}{\sqrt{150}}\right) = 0.35687$

$13.52 - 0.35687 < \mu < 13.52 + 0.35687$

$13.163 < \mu < 13.877$

$13.16 < \mu < 13.88$

Four scatter diagrams are shown below:
For each of the following, choose the most appropriate of the scatter diagrams above:

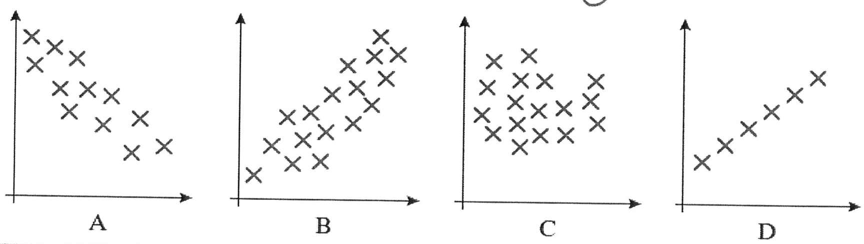
(i) Number of caps of an international rugby player and number of points scored B

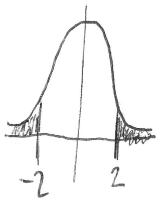
(ii) Distance (north) from equator and winter daylight hours. A

(iii) Men's percentage body fat and the time taken for them to solve a Rubik's Cube C

(iv) An example of negative correlation. A

(v) A scatter diagram showing a correlation coefficient of 0.7. B





Television tubes have a mean life of 4000 hours and a standard deviation of 500 hours. Assuming that their life can be modelled by a normal distribution, estimate:

(i) the percentage of tubes lasting less than 3000 hours

(ii) the **probability** that a tube will last for between 3000 and 5000 hours.

$P(Z<-\square)$

$1-P(Z<\square)$

or

$P(Z>\square)$

$1-P(Z<\square)$

(i) $P(X<3000)$

$P\left(Z<\frac{3000-4000}{500}\right)$

$P(Z<-2)=P(Z>2)$

$=1-P(Z<2)$

$=1-0.9772$

$=0.0228$

2.28%

$Z=\frac{x-\mu}{\sigma}$

(ii) $P(3000<X<5000)$

$P\left(\frac{3000-4000}{500}<Z<\frac{5000-4000}{500}\right)$

$P(-2<Z<2)$

$P(Z<2)-P(Z<-2)$

$0.9772-(1-P(Z<2))$

$0.9772-(1-0.9772)$

0.9544

The following table gives the number of employees and the units produced for a certain company over a 4-month period.

Month	Number of employees	Units produced
January	100	80
February	85	75
March	76	64
April	60	60

(i) Use your calculator to find r, the correlation coefficient.

(ii) Comment briefly on the value of r that you found.

(i) 0.9594944992 $\quad -1<r<1$

(ii) Very strong positive correlation between no. of employees & units produced.

If Z is a random variable having a standard normal distribution, find the value of k if $P(-k \leqslant Z \leqslant k) = 0.8438$.

$$P(Z \leq k) - P(Z \leq -k) = 0.8438$$

$$P(Z \leq k) - [1 - P(Z \leq k)] = 0.8438$$

$$P(Z \leq k) - 1 + P(Z \leq k) = 0.8438$$

$$2 \times P(Z \leq k) - 1 = 0.8438$$

$$+1 \qquad +1$$

$$\frac{\cancel{2} \times P(Z \leq k)}{\cancel{2}} = \frac{1.8438}{2}$$

$$P(Z \leq k) = 0.9219$$

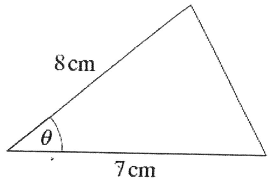
So: $P(Z \leq 1.42) = 0.9222$

closest!

So $k = 1.42$

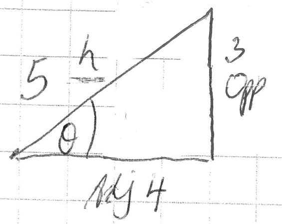
Trigonometry & Area & volume

If $\tan\theta = \frac{3}{4}$, find the area of the given triangle without using a calculator.



$A = \frac{1}{2}ab\sin C$

$\tan\theta = 3/4$



$\sin\theta = \frac{Opp}{Hyp} = \frac{3}{5}$

$A = \frac{1}{2}(8)(7)\sin\theta$

So find hypotenuse

$h^2 = 3^2 + 4^2$

$h^2 = 25$

$h = \sqrt{25}$

$h = 5$

$A = \frac{1}{2}(8)(7)\underbrace{\sin\theta}$

$A = \frac{1}{2}(8)(7)(3/5)$

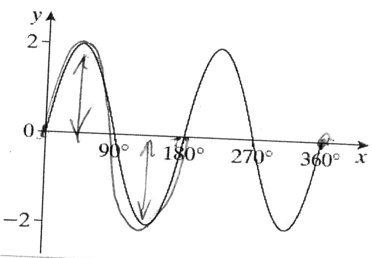
$A = 16\frac{4}{5}\,cm^2$

(i) Write down the period and range of the function graphed on the right.

(ii) If the function is

$$y = a\sin bx,$$

find the values of a and b.



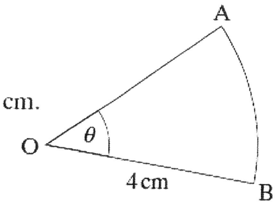
(i) Period 180°, Range [−2, 2]

(ii) $y = 2\sin 2x$

(i) Find all the solutions to the equation

$$\cos 2\theta = -\frac{\sqrt{3}}{2},\ \theta \in \mathbb{R} \text{ and } \theta \text{ in radians.}$$

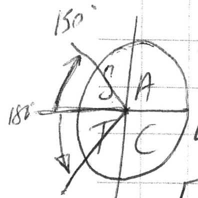
(ii) AOB is a sector of a circle of centre O and radius length 4 cm. If the area of AOB is 12 cm^2, express θ in radians.



i) $\cos \boxed{2\theta} = -\frac{\sqrt{3}}{2}$

$\boxed{2\theta} = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$\boxed{2\theta} = 150°$



$\boxed{2\theta} = 180° + 30° = 210°$

$\boxed{2\theta} = 150° + 360°n$ or $\boxed{2\theta} = 210° + 360°n$

$\theta = 75° + 180°n$ $\quad$ $\theta = 105° + 180°n$

$\times \pi/180$

$\boxed{\theta = 5\pi/12 + n\pi}$ $\quad$ $\boxed{\theta = 7\pi/12 + n\pi}$

ii) $A = \frac{1}{2} r^2 \theta$

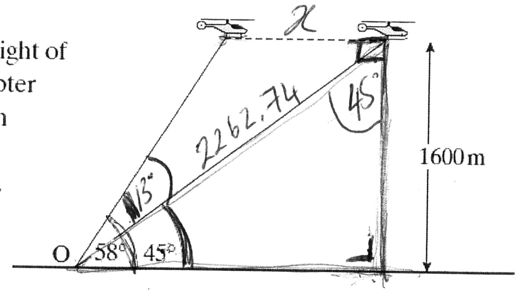
$12 = \frac{1}{2}(4)^2 \theta$

$12 = 8\theta$

$\frac{12}{8} = \frac{8\theta}{8}$

$1.5 = \theta \longrightarrow \theta = 1.5$ Radians.

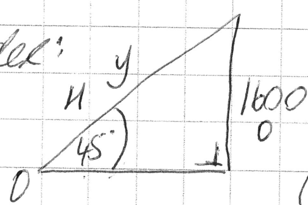
The diagram shows two sightings, made from a point O, of a helicopter flying at a height of 1600 metres. At the first sighting, the helicopter was due East of O and the angle of elevation was 58°. One minute later it was still due east of O, but the angle of elevation was 45°. Calculate the speed of the helicopter in kilometres per hour, correct to the nearest whole number.



$\frac{D}{S \times T}$

$S = \frac{D}{T} = \frac{x}{1\text{min}}$

Consider:



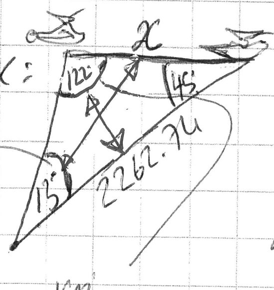
$\sin 45° = \frac{1600}{y}$

$y \times \sin 45° = \frac{1600}{y} \times y$

$y \times \sin 45° = 1600$

$y = \frac{1600}{\sin 45°} = 2262.74$

Now Consider:



$58° - 45° = 13°$

$180° - 45° - 13° = 122°$

$\frac{x}{\sin 13°} = \frac{2262.74}{\sin 122°}$

$x = \frac{2262.74}{\sin 122°} \times \sin 13°$

$x = 600.209\text{m}$

$S = \frac{600.209\text{m}}{1\text{min}} = \frac{0.600209\text{km}}{(1/60)\text{hrs}}$

$= 36.01254$

36 km/hr.

At 8 a.m. in the morning a computer starts collecting data from two sensors one inside and the other outside the glasshouse. On a particular day the temperature inside is given by $T_1(°C) = 22 + 3\sin\left(\frac{\pi t}{12}\right)$ while the temperature outside is given by $T_2(°C) = 19 + 8\boxed{\sin\left(\frac{\pi t}{12}\right)}$ where $t = 0$ corresponds to 8 a.m.

(i) Find the temperature inside and outside the glasshouse at 10 a.m. $t=2$

(ii) Find the temperature outside the glasshouse when the temperature inside is at its highest.

(iii) Draw a sketch T_1 and T_2 on the same axes in the domain $0 \le t \le 24$.

(iv) Find the times on this day when the temperatures inside and outside are equal.

(i) $t=2$, $T_1 = 22 + 3\sin\left(\frac{\pi(2)}{12}\right)$

$T_1 = 23.5°$

$t=2$, $T_2 = 19 + 8\sin\left(\frac{\pi(2)}{12}\right)$

$T_2 = 23°$

(ii) 25° is highest temp. inside

$25 = 22 + 3\sin\left(\frac{\pi t}{12}\right)$

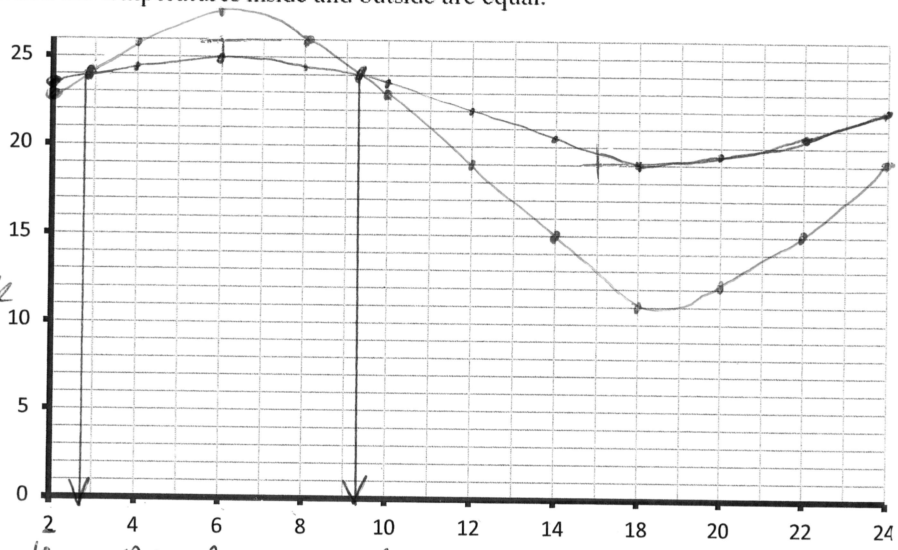
$-22 \quad -22$

$3 = 3\sin\left(\frac{\pi t}{12}\right)$

$\frac{3}{3} \quad \frac{3}{3}$

$1 = \boxed{\sin\left(\frac{\pi t}{12}\right)}$

$\therefore T_2 = 19 + 8(1) = 27°$



(iii) 10am 12pm 2pm. 4pm 6pm

(iv) 2.4

0.4x2hrs = 0.8hrs x 60 = 48mins

10:48am

8.7

0.7 x 2hrs = 1.4 x 60 = 84mins

1hr 24mins

4pm + 1hr 24mins = 5:24pm

(i) Show that $\tan\theta \sin\theta + \cos\theta = \sec\theta$.

(ii) If θ is an acute angle and $\cos\theta = \frac{5}{13}$, find the value of $\sin 2\theta$.

(i) $\tan\theta \sin\theta + \cos\theta = \sec\theta$

$\frac{\sin\theta}{\cos\theta} \cdot \sin\theta + \cos\theta = \frac{1}{\cos\theta}$

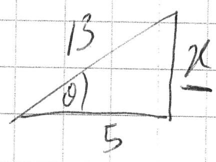
$\frac{\sin^2\theta}{\cos\theta} + \cos\theta = \frac{1}{\cos\theta}$

$\cancel{\cos\theta}\left(\frac{\sin^2\theta}{\cancel{\cos\theta}}\right) + \cos\theta(\cos\theta) = \cancel{\cos\theta}\left(\frac{1}{\cancel{\cos\theta}}\right)$

$\sin^2\theta + \cos^2\theta = 1$ ✓

$1 = 1$

(ii)



$13^2 = 5^2 + x^2$

$169 = 25 + x^2$

$-25 \quad -25$

$144 = x^2$

$\sqrt{144} = x$

$12 = x$

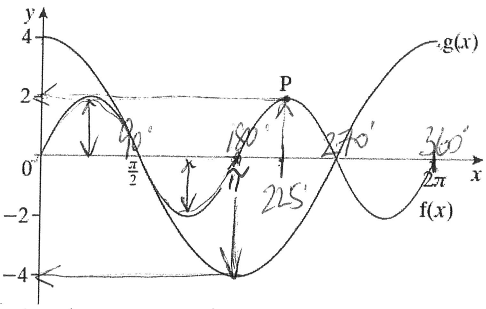
So $\sin\theta = \frac{12}{13}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\sin 2\theta = 2\left(\frac{12}{13}\right)\left(\frac{5}{13}\right) = \frac{120}{169}$

Two trigonometric functions f(x) and g(x) are graphed below:

(i) What is the range of g(x)?
(ii) What is the period of f(x)?
(iii) Write down the value of g(π)
(iv) Write down the equation of each function
(v) Write down the coordinates of the point P.



(i) $[-4,4]$

(ii) π

(iii) $g(\pi) = -4$

(iv) $g(x) = \boxed{0} + \boxed{4}\cos\boxed{1x}$

$g(x) = 4\cos x$

$f(x) = \boxed{0} + \boxed{2}\sin\boxed{2x}$

$f(x) = 2\sin 2x$

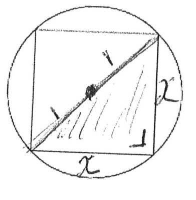
(v) when $x = 225°$, $y = 2\sin(2(225°))$

$y = 2$

$(225°, 2)$

or $(5\pi/4, 2)$ is P.

A square is inscribed in a circle, as shown.
If the area of the circle is π square units, find the area of the square.



$A = \pi r^2$

$\pi = \pi r^2$

$\frac{\pi}{\pi} = \frac{\pi r^2}{\pi}$

$1 = r^2$

$1 = r$

$(2)^2 = x^2 + x^2$

$4 = 2x^2$

$\frac{4}{2} = \frac{\cancel{2}x^2}{2}$

$2 = x^2$

$\sqrt{2} = x$

Area of $\square = x \times x$

$= (\sqrt{2}) \times (\sqrt{2})$

$= 2$ units squared.

If $\tan 75° = a + b\sqrt{3}$, find the values of a and b, where $a, b \in \mathbb{Z}$.

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(45°+30°) = \frac{\tan 45° + \tan 30°}{1 - \tan 45° \tan 30°}$$

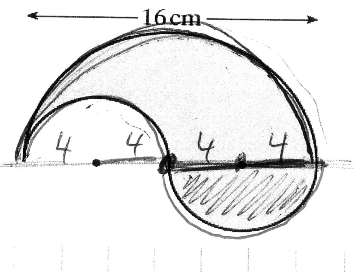
$$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - (1)\left(\frac{1}{\sqrt{3}}\right)}$$

$$= 2 + \sqrt{3}$$

A badge is to be made from a series of connected circles as shown. Find

(i) the length of the perimeter

(ii) the area of the composite figure.



(i) $l = 2\pi r$ $\quad$ $l = 2\pi r \times (1/2)$

$l = 2\pi(4)$ $\quad$ $l = 2\pi(8) \times (1/2)$

$l = 8\pi$ $\quad$ $l = 8\pi$

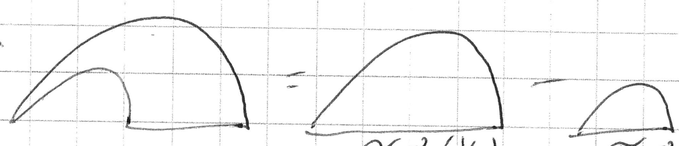
Total: $8\pi + 8\pi = 16\pi$ cm or 50.27 cm

(ii) $A = \pi r^2 \times (1/2)$

$A = \pi(4)^2 \times (1/2)$

$A = 8\pi \text{ cm}^2$

&



$\pi r^2(1/2) - \pi r^2(1/2)$

$\pi(8)^2(1/2) - \pi(4)^2(1/2)$

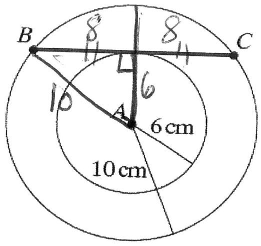
$32\pi - 8\pi$

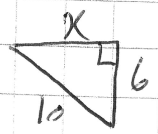
$24\pi \text{ cm}^2$

Total: $8\pi + 24\pi = \underline{32\pi \text{ cm}^2}$ or $\underline{100.53 \text{ cm}^2}$

Geometry

Two concentric circles with centre A have radii of 6 cm and 10 cm respectively. Find the length $|BC|$ of the chord that is a tangent to the smaller circle.





$(10)^2 = (6)^2 + x^2$

$100 = 36 + x^2$

$-36 \quad -36$

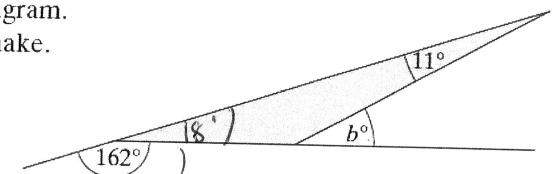
$64 = x^2$

$\sqrt{64} = x$

$8 = x$

$\therefore |BC| = 8 + 8 = 16\text{cm}.$

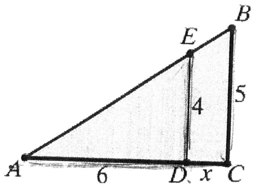
Find the value of the angle $b°$ in this diagram.
Give a reason for each deduction you make.



$180° - 162°$

So $b = 11 + 18° = 29°$

In the triangle ABC, $|\angle ADE| = |\angle ACB| = 90°$.
Find the value of x.



$$\frac{6+x}{6} = \frac{5}{4}$$

$$(6)(4)\left(\frac{6+x}{6}\right) = \left(\frac{5}{4}\right)(6)(4)$$

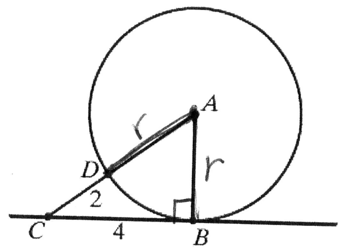
$$4(6+x) = (5)(6)$$

$$24 + 4x = 30$$
$$-24 \qquad -24$$

$$4x = 6$$

$$x = \frac{6}{4} = \frac{3}{2}$$

$\overleftrightarrow{CB}$ is a tangent to circle centre A at the point B.
If $|CD| = 2$ and $|CB| = 4$, find the length of the radius $|AB|$.



$$(r+2)^2 = (r)^2 + (4)^2$$

$$(r+2)(r+2) = r^2 + 16$$

$$r^2 + 2r + 2r + 4 = r^2 + 16$$

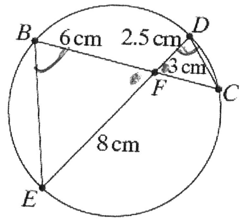
$$\cancel{r^2} + 4r + 4 = r^2 + 16$$
$$\cancel{-r^2} \qquad -r^2$$

$$4r + \cancel{4} = 16$$
$$\cancel{-4} \qquad -4$$

$$\frac{4r}{4} = \frac{12}{4}$$

$$r = 3.$$

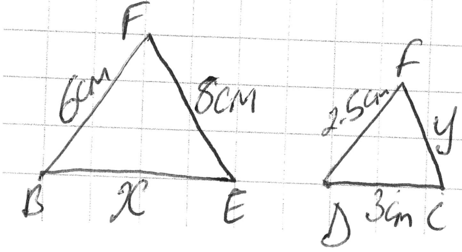
Prove that triangles BEF and DCF are similar.
(Give a reason for each deduction made).
Given that $|BF| = 6$ cm, $|EF| = 8$ cm, $|FD| = 2.5$ cm and $|CD| = 3$ cm, find the lengths of the sides $[BE]$ and $[FC]$.



$|\angle EBC| = |\angle EDC|$ angles standing on same arc

$|\angle BFE| = |\angle DFC|$ opposite angles are equal

$\therefore$ $\triangle BEF$ and $\triangle DCF$ are similar!



$$\frac{x}{3} = \frac{6}{2.5}$$

$$(2.5)(3)\left(\frac{x}{3}\right) = \left(\frac{6}{2.5}\right)(3)(2.5)$$

$$2.5x = 6(3)$$

$$\frac{2.5x}{2.5} = \frac{18}{2.5}$$

$$x = 7.2\text{cm}$$

$$\frac{y}{8} = \frac{2.5}{6}$$

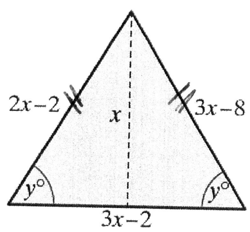
$$(6)(8)\left(\frac{y}{8}\right) = \left(\frac{2.5}{6}\right)(6)(8)$$

$$6y = (2.5)(8)$$

$$\frac{6y}{6} = \frac{20}{6}$$

$$y = \frac{10}{3} \text{ or } 3\tfrac{1}{3}\text{cm}$$

Find the area of this isosceles triangle.



$$2x - 2 = 3x - 8$$
$$-2x \qquad -2x$$
$$-2 = x - 8$$
$$+8 \qquad +8$$
$$6 = x$$

$$A = \tfrac{1}{2} \times b \times h$$

$$A = \tfrac{1}{2} \times (3x-2) \times x$$

$$A = \tfrac{1}{2} \times (3(6)-2) \times (6)$$

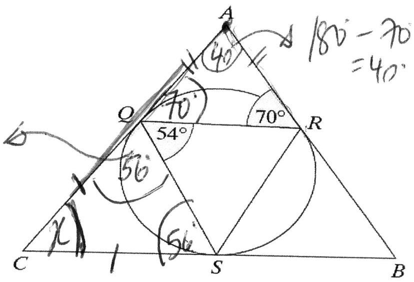
$$A = \tfrac{1}{2} \times 16 \times 6$$

$$A = 48 \text{ square units.}$$

The given circle is inscribed in the triangle ABC.
Q, R and S are the points of contact.
$|\angle ARQ| = 70°$ and $|\angle RQS| = 54°$.
Find $|\angle ACB|$.

$180° - 70 - 70°$
$= 40°$

$180°$
$-70°$
$-54°$
$\overline{56°}$



$\angle$: $180° - 56° - 56° = x$

$68° = x°$

$68° = |\angle ACB|$.