Chapter 9
Trigonometric Ratios and Functions

Section 9-8
Using Sum and Difference Formulas

Using Sum and Difference Formulas

In this lesson, you will study formulas that allow you to evaluate trigonometric functions of the sum or difference of two angles.

Core Concept

Sum and Difference Formulas

Sum Formulas	Difference Formulas
$\sin(a + b) = \sin a \cos b + \cos a \sin b$	$\sin(a - b) = \sin a \cos b - \cos a \sin b$
$\cos(a + b) = \cos a \cos b - \sin a \sin b$	$\cos(a - b) = \cos a \cos b + \sin a \sin b$
$\tan(a + b) = \dfrac{\tan a + \tan b}{1 - \tan a \tan b}$	$\tan(a - b) = \dfrac{\tan a - \tan b}{1 + \tan a \tan b}$

In general, $\sin(a + b) \neq \sin a + \sin b$. Similar statements can be made for the other trigonometric functions of sums and differences.

EXAMPLE 1 **Evaluating Trigonometric Expressions**

Find the exact value of (a) $\sin 15°$ and (b) $\tan \frac{7\pi}{12}$.

EXAMPLE 2 Using a Difference Formula

Find $\cos(a - b)$ given that $\cos a = -\frac{4}{5}$ with $\pi < a < \frac{3\pi}{2}$ and $\sin b = \frac{5}{13}$ with $0 < b < \frac{\pi}{2}$.

EXAMPLE 3 Simplifying an Expression

Simplify the expression $\cos(x + \pi)$.

Find the exact value of the expression.

2. $\cos 15^\circ$

5. Find $\sin(a - b)$ given that $\sin a = \frac{8}{17}$ with $0 < a < \frac{\pi}{2}$ and $\cos b = -\frac{24}{25}$ with $\pi < b < \frac{3\pi}{2}$.

Simplify the expression.

6. $\sin(x + \pi)$

Solving Equations and Rewriting Formulas

EXAMPLE 4 **Solving a Trigonometric Equation**

Solve $\sin\left(x + \frac{\pi}{3}\right) + \sin\left(x - \frac{\pi}{3}\right) = 1$ for $0 \le x < 2\pi$.

EXAMPLE 5 **Rewriting a Real-Life Formula**

The *index of refraction* of a transparent material is the ratio of the speed of light in a vacuum to the speed of light in the material. A triangular prism, like the one shown, can be used to measure the index of refraction using the formula

$$n = \frac{\sin\left(\frac{\theta}{2} + \frac{\alpha}{2}\right)}{\sin\frac{\theta}{2}}.$$

For $\alpha = 60^\circ$, show that the formula can be rewritten as $n = \frac{\sqrt{3}}{2} + \frac{1}{2}\cot\frac{\theta}{2}$.

Section 9-8 Homework #3-31odd