

# How to Help Students with Mathematics Difficulties Become Expert Problem Solvers

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RME

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# Thank You

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**8**

**Tickets for a play were sold on Monday, Tuesday, and Wednesday.**

**Tickets Sold**

<b>Day of the Week</b>	<b>Number of Tickets Sold</b>
Monday	197
Tuesday	364
Wednesday	?

**If a total of 900 tickets were sold for the play, how many tickets were sold on Wednesday?**

- Ⓐ 300
- Ⓑ 339
- Ⓒ 449
- Ⓓ 461

**How would you teach this problem?**

**What do students need to know to solve this problem?**

**What might cause difficulty for students?**



# Problem Solving Difficulties

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Reading problems

Understanding vocabulary

Identifying relevant  
information

Ignoring irrelevant information

Interpreting charts and graphs

Identifying appropriate  
operation(s)

Performing the computation(s)

# How to Help Students Become Expert Problem Solvers

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## Three Takeaways About Word Problems

1

2

3



Don't describe using **key words** or **operations**



Have an **attack strategy**



Teach word-problem  
**schemas**



# key words

subtract decrease fewer  
remain take away minus  
less than how many more...

division split quotient  
equal groups divide  
half shared equally each

average

distribute

## Key Words Used in Math Word Problems

### Addition Words

+ add  
+ all together or altogether  
+ and  
+ both  
+ combined  
+ how many in all  
+ how much  
+ in all  
+ increased by  
+ plus  
+ sum  
+ together  
+ total



### Subtraction Words

- change  
- decreased by  
- difference  
- fewer or fewer than  
- how many are left (or have left)  
- how many did not have  
- how many (or much) more  
- how much longer (shorter, taller, heavier, etc.)  
- less or less than  
- lost  
- minus  
- need to  
- reduce  
- remain  
- subtract  
- take away



### Multiplication Words

x by (dimension)  
x double  
x each group  
x every  
x factor of  
x increased by  
x multiplied by  
x of  
x product  
x times  
x triple



### Division Words

÷ as much  
÷ cut up  
÷ each  
÷ equal  
÷ half  
÷ how many in all  
÷ how much  
÷ in all  
÷ increased by  
÷ plus  
÷ sum  
÷ together  
÷ total

## Key Words Used in Math Word Problems

### Addition Words

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### Multiplication Words

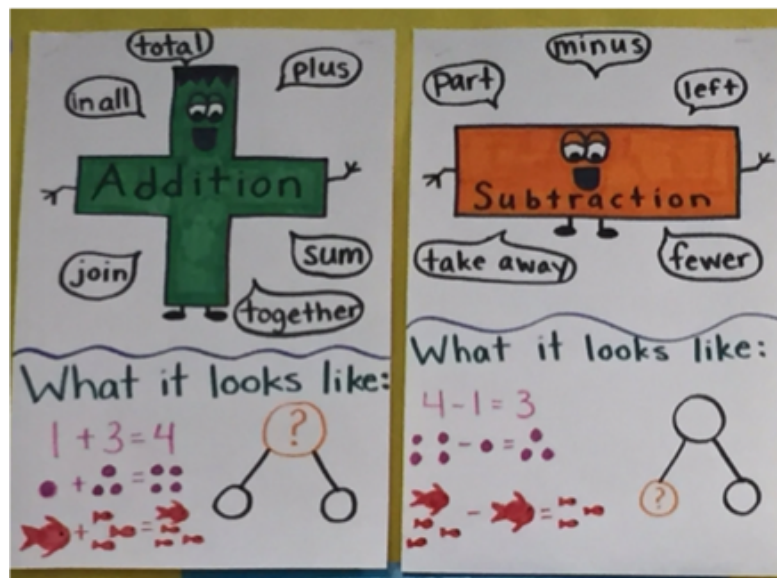
• by (dimension)  
• double  
• each group  
• every  
• factor of  
• increased by  
• multiplied by  
• of  
• product  
• times  
• triple



### Division Words

• as much  
• cut up  
• each group has  
• equal (sharing)  
• half (or other fractions)  
• how many in each  
• parts  
• per  
• percent  
• quotient of  
• ratio of  
• separated  
• share something equally



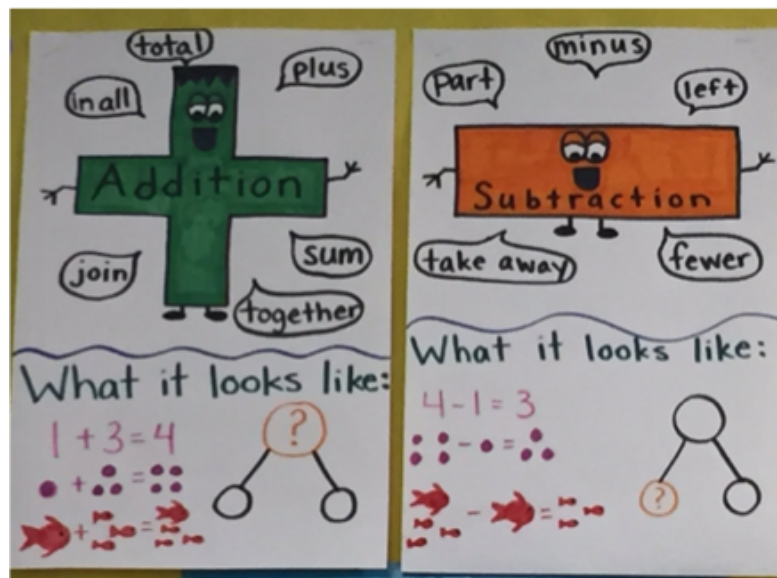


Kasey made \$42, and Mandy made \$37. How much money did they make *in all*?

Kasey and Mandy made \$79 *in all*. If Kasey made \$42, how much money did Mandy make?

Kasey mowed 12 lawns on Monday. Then, she mowed 10 *more* on Tuesday. How many lawns has Kasey mowed?

Kasey mowed 22 lawns and Mandy mowed 7 lawns. How many *more* lawns did Kasey mow than Mandy?

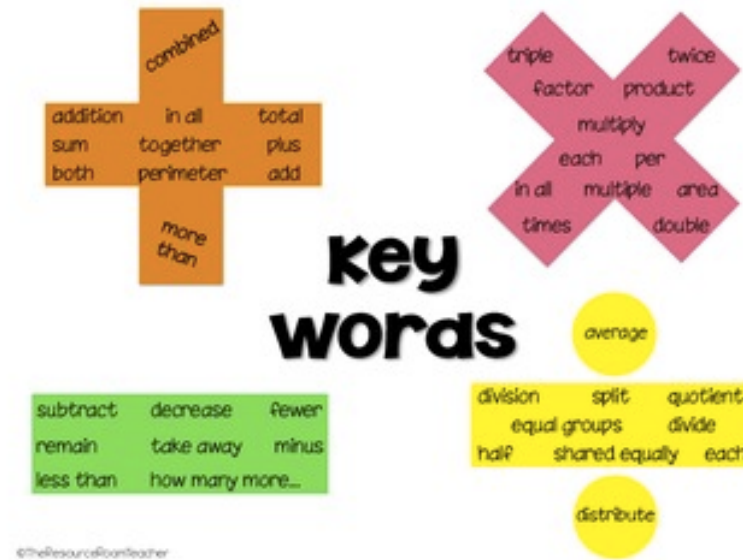


Becky has \$70 *more than* Perla. If Becky has \$120, how much money does Perla have?

Becky has \$70 *more than* Perla. If Perla has \$50, how much money does Becky have?

Becky had 9 dinosaurs and then her sister *took away* 4 of them. How many dinosaurs does Becky have now?

Becky had some dinosaurs and then her sister *took away* 4 of them. Now Becky has 5 dinosaurs. How many dinosaurs did she start with?



Matt baked 18 cookies. His brother baked *twice* as many. How many cookies did his brother bake?

Matt's brother baked *twice* as many cookies as Matt. If Matt's brother baked 36 cookies, how many did Matt bake?

Reece has 7 bags with 3 apples in *each* bag. How many apples does Reece have?

Reece had 21 apples and placed 3 apples *each* in several bags. How many bags does Reece need?





Rachel wants to *share* 36 brownies with 6 friends. How many cookies will *each* friend receive?

Rachel *shared* brownies with 6 friends. *Each* friend ate 6 brownies. How many brownies did Rachel have to start with?

Brent made 12 cupcakes. His brother made *half* as many cupcakes. How many cupcakes did Brent's brother bake?

Brent made 12 cupcakes. He cut each cupcake into *half*. How many pieces of cupcake does Brent have?





Michelle made 17 paper airplanes. Dante made 24 paper airplanes. How many airplanes did they make ***altogether***?

Michelle and Dante made 41 paper airplanes ***altogether***. If Dante made 24 paper airplanes, how many did Michelle make?

Michelle made 4 paper airplanes using 2 pieces of paper for each airplane. How much paper did Michelle use ***altogether***?

Dante and Michelle made 40 paper airplanes ***altogether***. Dante made 24 of the paper airplanes. If Michelle gave 7 of her paper airplanes to her friend Nicole, how many planes does Michelle have now?

### Key Words Used in Math Word Problems

Addition Words	Subtraction Words
<ul style="list-style-type: none"><li>• add</li><li>• all together or altogether</li><li>• and</li><li>• both</li><li>• combined</li><li>• how many in all</li><li>• how much</li><li>• in all</li><li>• increased by</li><li>• more</li><li>• plus</li><li>• sum</li><li>• together</li><li>• total</li></ul>	<ul style="list-style-type: none"><li>- change</li><li>- decreased by</li><li>- difference</li><li>- fewer or fewer than</li><li>- how many are left (or have left)</li><li>- how many did not have</li><li>- how many (or much) more</li><li>- how much longer (shorter, taller, heavier, etc.)</li><li>- less or less than</li><li>- lost</li><li>- minus</li><li>- need to</li><li>- reduce</li><li>- remain</li><li>- subtract</li><li>- take away</li></ul>
	
Multiplication Words	Division Words
<ul style="list-style-type: none"><li>x by (dimension)</li></ul>	<ul style="list-style-type: none"><li>÷ as much</li></ul>

Students need to understand *key words*. But,  
key words should not be directly tied to  
*operations*.

## Word Problems: Subtraction

Name \_\_\_\_\_ Date \_\_\_\_\_

Subtraction

Read each problem. Write a number sentence and solve.

1. Mrs. Smith has 33 poodles and 18 boxers. How many more poodles does Mrs. Smith have?



\_\_\_\_\_

2. The kennel holds 91 dogs. Mr. Glass has 67 dogs in the kennel now. How many spaces does he have left?



\_\_\_\_\_

3. Mr. Kelly has 44 beagles. 26 of them are puppies. How many adult beagles does Mr. Kelly have?



\_\_\_\_\_

4. Mrs. Green has 60 terriers. 25 of them are boys. How many terriers are girls?



\_\_\_\_\_

5. There were 58 kittens at the pet shop on Friday. 29 of them were sold on Saturday. How many kittens were left?



\_\_\_\_\_

6. Pat counted 22 lizards in the tank at the pet shop. 8 were sold later that day. How many lizards were left in the tank?



\_\_\_\_\_



Don't describe using **key words** or **operations**



Have an **attack strategy**



Teach word-problem **schemas**

# For every word problem

Regardless of problem type, students need an **attack** strategy for working through the problem

This strategy should work for any problem type

## Routine Word Problems

A library has 126 books about trees.

### 24. Part A

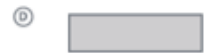
The library has 48 fewer books about rivers than about trees.

What is the number of books the library has about rivers and what is the total number of books the library has about trees and rivers?

- Ⓐ 78 and 126
- Ⓑ 48 and 204
- Ⓒ 48 and 126
- Ⓓ 78 and 204

## Instructional Word Problems

7. Which **three** shapes are quadrilaterals?



# RIDGES

**R**ead the problem.

**I** know statement.

**D**raw a picture.

**G**oal statement.

**E**quation development.

**S**olve the equation.

## RIDE

**R**ead the problem.

**I**dentify the relevant information.

**D**etermine the operation and unit for the answer.

**E**nter the correct numbers and calculate, then check the answer.

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**S** Stop and read the problem carefully.  
What is the problem asking?  
Find the important information.  
Is there any information you don't need?

**T** Think about your plan and the strategy you will use.  
Guess and Check      Draw a Picture  
Make an Organized List      Find a Pattern  
Make a Table or Chart      Use Logical Reasoning  
Use Manipulatives      Work Backwards

**A** Act - follow your plan and solve the problem.  
Be ready to explain how you solved the problem.

**R** Review your answer.  
Did you answer the question?  
Explain how you know your answer is correct.

**R**ead and Record the problem.

**I**llustrate your thinking.

**C**ompute.

**E**xplain your thinking.



**P**  
roblem



Read your problem.



**A**  
nalyze

Underline key words. Cross out information you do not need. Decide which operation you will need to use.



**W**  
ork it out

Use the operation to work the problem out. Show your work.



**S**  
olve



Write your answer. Ask yourself, "Does my answer make sense?"



**S**

lowly read the story problem twice.



**U**

nderline the question and circle the numbers you need.



**P**

icture it! Draw the scenario to show what is happening.



**E**

xplain the problem with a number sentence.

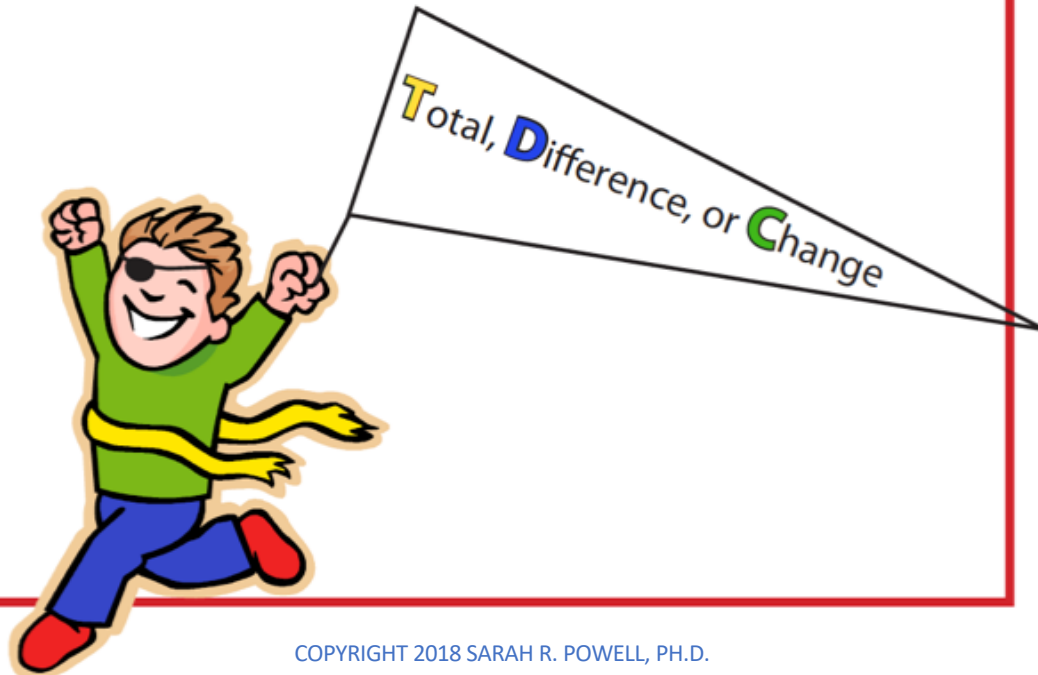
**R**

ewrite the answer in a sentence.



# RUN!

1. Read the problem.
2. Underline the labels.
3. Name the problem type.





S	slowly and carefully <b>READ</b> the problem.
H	highlight or underline <b>key information</b> .
I	<b>identify the question</b> by drawing a circle around it.
N	now <b>solve the problem</b> with numbers, pictures, and words. Show your work.
E	<b>examine your work</b> for precision, accuracy, and clarity.
S	share your answer by <b>writing a sentence</b> .



**SOLVING Word Problems** LET'S S.K.A.T.E.

**S** Survey the word problem and identify the question.

"The skateboard store had a total of 11 skateboards on Monday. By Friday, there were only 4 skateboards left. How many skateboards were sold?"

**K** Keep and highlight Important information.

"The skateboard store had a total of 11 skateboards on Monday. By Friday, there were only 4 skateboards left. How many skateboards were sold?"

**A** Attempt to estimate the answer.

USE THE FACTS TO MAKE A REASONABLE GUESS!

**T** Take time to solve the problem.

$11 - ? = 4$

By Friday, there were a total of 7 skateboards sold.

**E** Examine your answer carefully.

7 is a reasonable answer.

**GRASS**

**G** → What is important in the question?  
What information is delivered to help you solve the question?

**R** → What is the problem?  
What are you being asked to do?

**A** → What problem solving strategy will you use?  
What approach can you take to find the solution?

☐ Use Manipulatives ☐ Act it out ☐ Make a model  
☐ Make a table ☐ Draw a picture ☐ Work backwards  
☐ Look for patterns ☐ Use a Formula ☐ Process of Elimination  
☐ Make a graph ☐ Guess and Check

**S** → Time to do the math! Follow through on your strategy and find the answer!

**S** → A summary in words, of what you did.

I used \_\_\_\_\_ because \_\_\_\_\_  
I think \_\_\_\_\_ didn't work because \_\_\_\_\_  
\_\_\_\_\_ is why I chose my second application.  
I know that \_\_\_\_\_  
I found \_\_\_\_\_  
I have used \_\_\_\_\_ to solve something like this before.  
\_\_\_\_\_ worked better than \_\_\_\_\_



## SIGNS

Survey questions

Identify key words

Graphically draw problem

Note operations

Solve and check

## SOLVE

**Study the problem.**

**Organize the facts.**

**Line up the plan.**

**Verify the plan with computation.**

**Examine the answer.**



## Word Problem Steps



**C**ircle the Key <sup>1 2 3 4 5</sup>Numbers  
<sub>6 7 8 9 10</sub>



**U**nderline the question  
??????  
??????



**B**ox any Math "action" words & Write the operational sign  $+$   $-$   $\div$   $\times$



\* Eliminate (get rid of) Unnecessary Information  
\* Evaluate & Draw



**S**olve and Check ✓  
• Show your work  
• Does my answer make sense?  
• How can I double check?

**C**ircle Important Information

**U**nderline the Question

**B**ox the Word "NOT"

**E**liminate Unnecessary Information

## R-CUBES

Read the problem.  
Circle key numbers.  
Underline the question.  
Box action words.  
Evaluate steps.  
Solve and check.



# Steps to Solve a Problem: UPS Check!

## UNDERSTAND

- Read the problem carefully.
- Highlight or circle necessary data, key words, and labels.
- Identify the question.

## PLAN

- Record the information needed to solve the problem.
- Draw a picture or diagram of the situation and label all parts.
- Choose the appropriate strategy, tool, or operation.

## SOLVE

- Write an appropriate equation for the situation.
- Use your plan and data to solve.
- Write your solution with units if applicable.

## CHECK

- Check your math (substitute the value(s) into your equation).
- Did you answer the question?
- Is your answer reasonable?

## Understand

 Read the problem.



What am I looking for?

[Bracket the question]



What do I know?

Circle the important numbers

Underline labels

## Plan



Choose a strategy.

• Draw a Picture or Make a Table

Step Degree?	1	2	3	4	5	6
Working Pattern?	JUG	SUG	CUG	JEG	SEG	SEG
Operation?	+	-	-	X	÷	÷

## Solve

Show all your work.

Label your answer.



$$\begin{array}{r} 3 \\ +5 \\ \hline 8 \text{ crayons} \end{array}$$

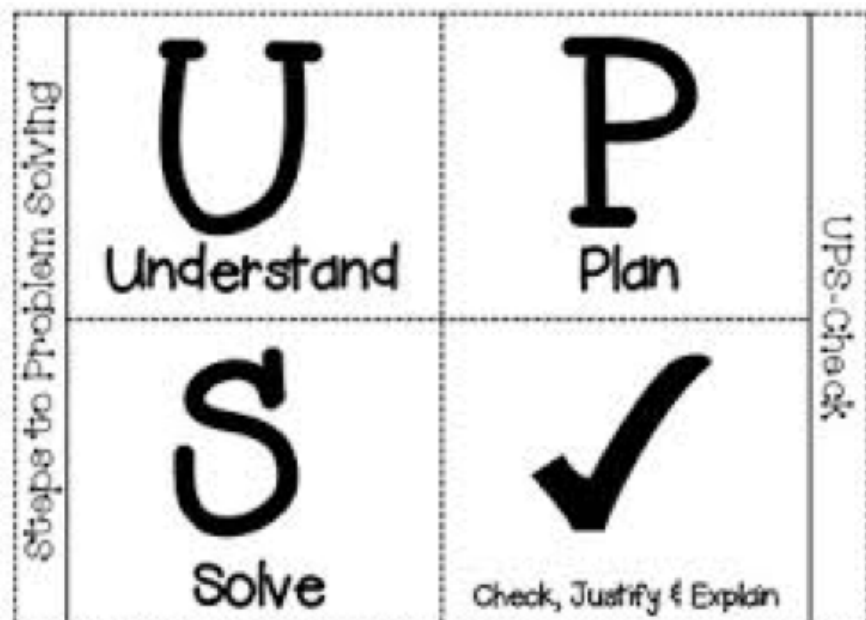
## Check



Explain & Justify

Is your answer is  
REASONABLE?

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Don't describe using **key words** or **operations**



Have an **attack strategy**



Teach word-problem  
**schemas**

# Additive Schemas

---

Problem type	Definition	Examples			Equation	Graphic organizer
<b>Total</b>		Total unknown	Part unknown			
<b>Difference</b>		Difference unknown	Greater unknown	Lesser unknown		
<b>Change (increase)</b>		End unknown	Start unknown	Change unknown		
<b>Change (decrease)</b>		End unknown	Start unknown	Change unknown		

Additive Word Problems

# Total

---

**Parts** put together into a **total**

- Emily saw **4** cardinals and **5** blue jays. How many birds did Emily see?  
◦ **4 + 5 = ?**
- Emily saw **9** birds. If **4** of the birds were cardinals, how many were blue jays?  
◦ **4 + ? = 9**
- Emily saw **9** birds. **5** of the birds were blue jays, how many were cardinals?  
◦ **5 + ? = 9**



Total

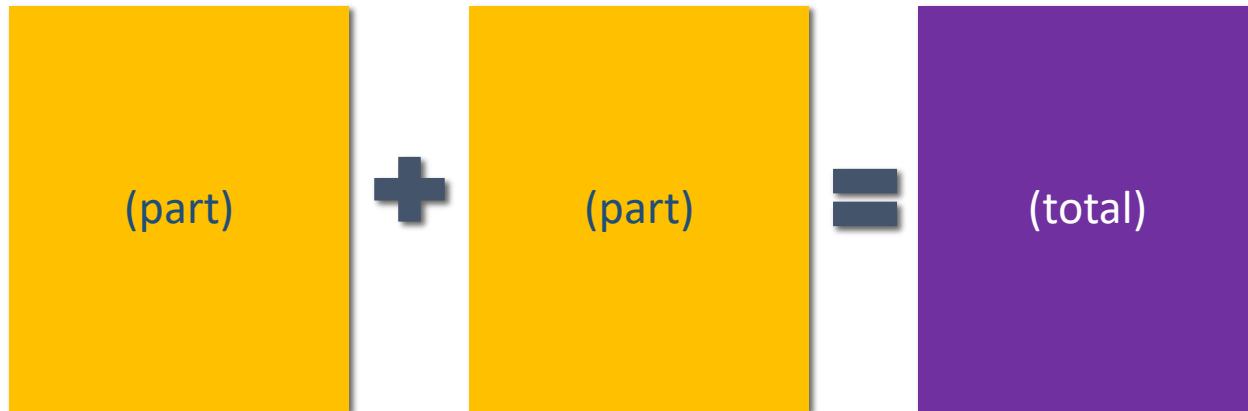
P1

+

P2

=

T



Total	
?	
Part	Part
20	16

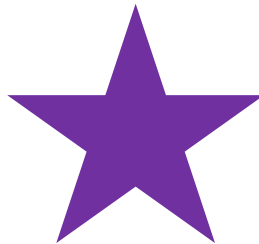
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# Total

## Additive Word Problems

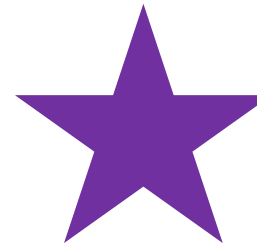
A.

Megan baked 28 sugar cookies and 24 chocolate chip cookies. Enter the total number of cookies Megan baked in all.



B.

A banana farm received a total of 12 millimeters of rain in March and April. If 11 millimeters of rain fell on the farm in March, how many millimeters of rain fell on the farm in April?



C.

Jana has 107 wooden beads and 68 glass beads. How many more wooden beads than glass beads does Jana have?

D.

Farmer Hank has 6 more cows than horses. He has 4 horses. He also has 9 chickens. How many cows does he have?

# Total

Megan baked 28 sugar cookies and 24 chocolate chip cookies.  
Enter the total number of cookies Megan baked in all.

✓

$$P1 + P2 = T$$

✓

$$28 + 24 = ?$$

✓

$$28 + 24 = 52$$




✓

$$X = 52 \text{ cookies}$$

Total

“Are parts put together for a total?”

# Total

 A banana farm received a total of  12 millimeters of rain in March and April. If  11 millimeters of rain fell on the farm in March, how many millimeters of rain fell on the farm in April?

12	
11	?

$$\begin{array}{r} 11 \\ + 1 \\ \hline 12 \end{array} \qquad \begin{array}{r} 12 \\ - 11 \\ \hline 1 \end{array}$$

? = 1 millimeter

Problem type	Definition	Examples			Equation	Graphic organizer
<b>Total</b>		Total unknown	Part unknown			
<b>Difference</b>		Difference unknown	Greater unknown	Lesser unknown		
<b>Change (increase)</b>		End unknown	Start unknown	Change unknown		
<b>Change (decrease)</b>		End unknown	Start unknown	Change unknown		

Additive Word Problems

# Difference

---

**Greater** and **less** amounts compared for a **difference**

- Shinead has **9** apples. Amanda has **4** apples. How many more apples does Shinead have? (How many fewer?)
  - $9 - 4 = ?$
- Shinead has **5** more apples than Amanda. If Amanda has **4** apples, how many does Shinead have?
  - $? - 4 = 5$
- Amanda has **5** fewer apples than Shinead. Shinead has **9** apples. How many apples does Amanda have?
  - $9 - ? = 5$

# Difference

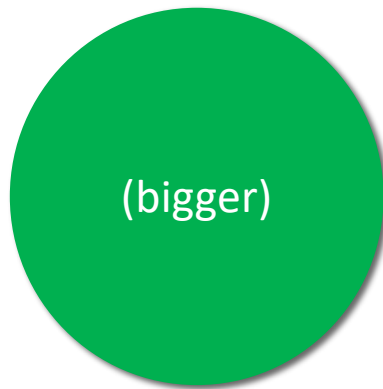
G

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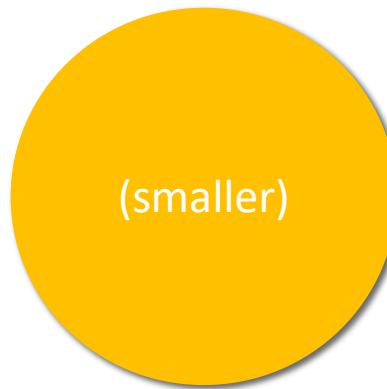
L

=

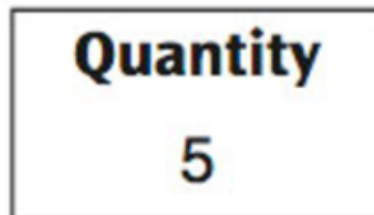
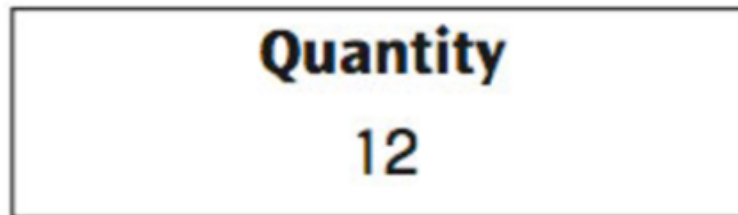
D



—



=



?

**Difference**



# Difference

Jana has 107 wooden beads and 68 glass beads. How many more wooden beads than glass beads does Jana have?

$$G - L = D$$

$$107 - 68 = B$$

$$107 - 68 = 39$$

$$\begin{array}{r} 107 \\ - 68 \\ \hline \end{array}$$

$$\begin{array}{r} 68 \\ + ? \\ \hline 107 \end{array}$$

B = 39 more beads

# Difference

## Additive Word Problems

A.

Megan baked 28 sugar cookies and 24 chocolate chip cookies. Enter the total number of cookies Megan baked in all.

B.

A banana farm received a total of 12 millimeters of rain in March and April. If 11 millimeters of rain fell on the farm in March, how many millimeters of rain fell on the farm in April?

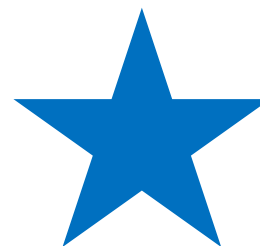
C.

Jana has 107 wooden beads and 68 glass beads. How many more wooden beads than glass beads does Jana have?



D.

Farmer Hank has 6 more cows than horses. He has 4 horses. He also has 9 chickens. How many cows does he have?



Total

“Are parts put together for a total?”

Difference

“Are amounts compared for a difference?”

# Difference

---

Farmer Hank has 6 more cows than horses. He has 4 horses. He also has 9 chickens. How many cows does he have?

Problem type	Definition	Examples			Equation	Graphic organizer
Total		Total unknown	Part unknown			
Difference		Difference unknown	Greater unknown	Lesser unknown		
Change (increase)		End unknown	Start unknown	Change unknown		
Change (decrease)		End unknown	Start unknown	Change unknown		

Additive Word Problems

# Change

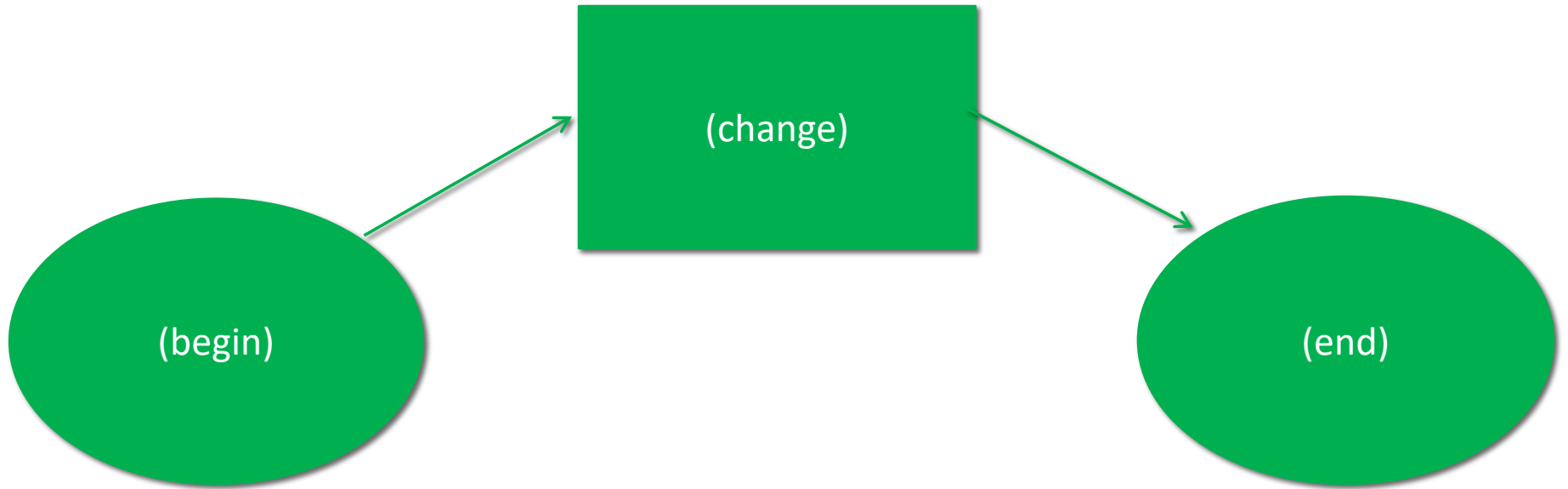
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An amount that **increases** or decreases

- Shannah had \$**4**. Then she earned \$**3** for cleaning her room. How much money does Shannah have now?
  - **4 + 3 = ?**
- Shannah has \$**4**. Then she earned money for cleaning her room. Now Shannah has \$**7**. How much money did she earn?
  - **4 + ? = 7**
- Shannah had some money. Then she made \$**3** for cleaning her room. Now she has \$**7**. How much money did Shannah start with?
  - **? + 3 = 7**

Change

$$ST + / - C = E$$



# Change

## Additive Word Problems

E.

A bus had 13 passengers. At the next stop, more passengers got on the bus. Now, there are 28 passengers. How many passengers got on the bus?



F.

Martina had some money. Then, she spent \$42 on a sweater. Now, she has \$13. How much money did she have to start with?



G.

Ramon has a total of 815 sheep in two fields. He has 348 sheep in one of the fields. How many sheep does Ramon have in the other field?

H.

Angelina looked in her closet and saw a container of markers. She took 42 markers out of the container and counted 88 left. How many markers were in the container when she found it in the closet?



# Change

A bus had ~~13~~ passengers. At the next stop, more passengers got on the bus. Now, there are ~~23~~ passengers. How many passengers got on the bus?



$$ST + C = E$$

$$13 + ? = 28$$

$$13 + 15 = 28$$

$$\begin{array}{r} 28 \\ - 13 \\ \hline \end{array}$$

$$\begin{array}{r} 13 \\ + ? \\ \hline 28 \end{array}$$

$$? = 15 \text{ passengers}$$

Total

“Are parts put together for a total?”

Difference

“Are amounts compared for a difference?”

Change

“Does an amount increase or decrease?”

# Change

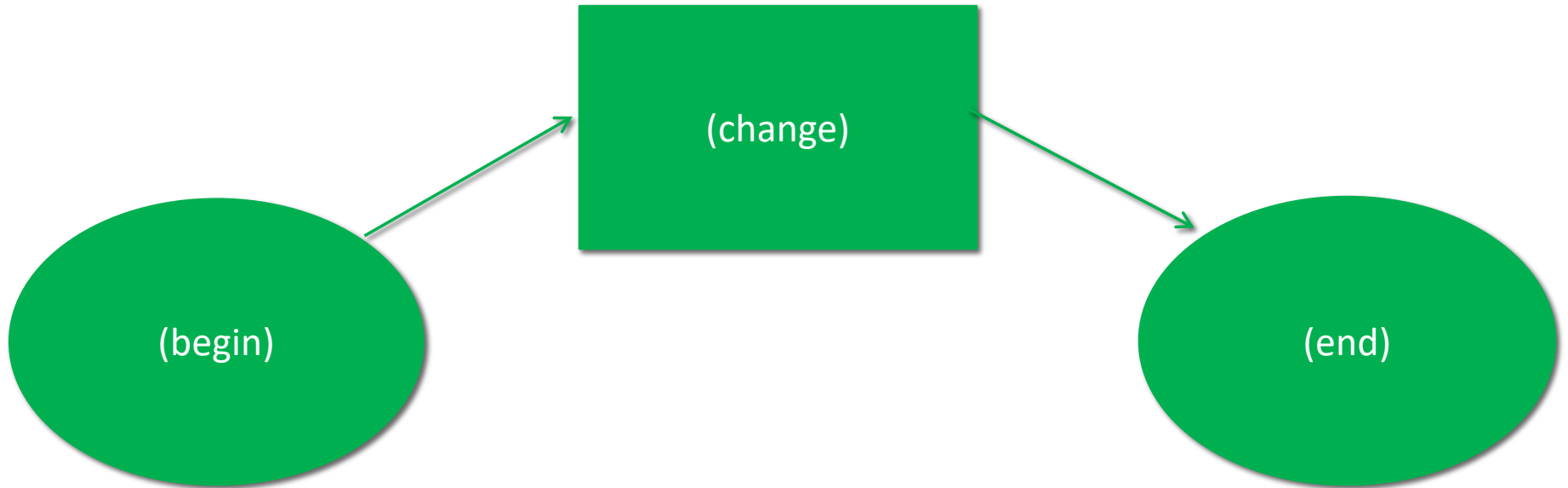
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An amount that increases or **decreases**

- Reece baked **9** cookies. Then, she ate **2** of the cookies. How many cookies does Reece have now?
  - **$9 - 2 = ?$**
- Reece baked **9** cookies. Then, she ate some of the cookies. Now, she has **7** cookies. How many cookies did Reece eat?
  - **$9 - ? = 7$**
- Reece baked some cookies. She ate **2** of the cookies and has **7** cookies left. How many cookies did Reece bake?
  - **$? - 2 = 7$**

Change

$$ST + / - C = E$$



# Change

---

Martina had some money. Then, she spent \$42 on a sweater. Now, she has \$13. How much money did she have to start with?

Problem type	Definition	Examples			Equation	Graphic organizer
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**Additive Word Problems**

### Additive Word Problems

E.

A bus had 13 passengers. At the next stop, more passengers got on the bus. Now, there are 28 passengers. How many passengers got on the bus?

F.

Martina had some money. Then, she spent \$42 on a sweater. Now, she has \$13. How much money did she have to start with?

G.

Ramon has a total of 815 sheep in two fields. He has 348 sheep in one of the fields. How many sheep does Ramon have in the other field?



H.

Angelina looked in her closet and saw a container of markers. She took 42 markers out of the container and counted 88 left. How many markers were in the container when she found it in the closet?



### Additive Word Problems

I.

The grocery store had 517 jars of crunchy peanut butter and 434 jars of creamy peanut butter. How many more jars of crunchy peanut butter were there?



J.

The animal park has 12 zebras, 25 monkeys, and some giraffes. If the total number of zebras, monkeys, and giraffes at the park is 50, how many giraffes are there?

K.

Mrs. Lanier saved \$617 in January. In February, she spent \$249 of the money she saved. She saved \$291 more in March. How much has Mrs. Lanier saved by the end of March?

NOTES:

# Total

---

Ramon has a total of 815 sheep in two fields. He has 348 sheep in one of the fields. How many sheep does Ramon have in the other field?



# Change

---

Angelina looked in her closet and saw a container of markers. She took 42 markers out of the container and counted 88 left. How many markers were in the container when she found it in the closet?

# Difference

---

The grocery store had 517 jars of crunchy peanut butter and 434 jars of creamy peanut butter. How many more jars of crunchy peanut butter were there?

## Additive Word Problems

I.

The grocery store had 517 jars of crunchy peanut butter and 434 jars of creamy peanut butter. How many more jars of crunchy peanut butter were there?

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NOTES:



# Total

The animal park has 12 zebras, 25 monkeys, and some giraffes. If the total number of zebras, monkeys, and giraffes at the park is 50, how many giraffes are there?

$$P1 + P2 + P3 = T$$

# Change

Mrs. Lanier saved \$617 in January. In February, she spent \$249 of the money she saved. She saved \$291 more in March. How much has Mrs. Lanier saved by the end of March?

$$ST - C + C = E$$

# Let's Review

---

What's a **Total** problem?

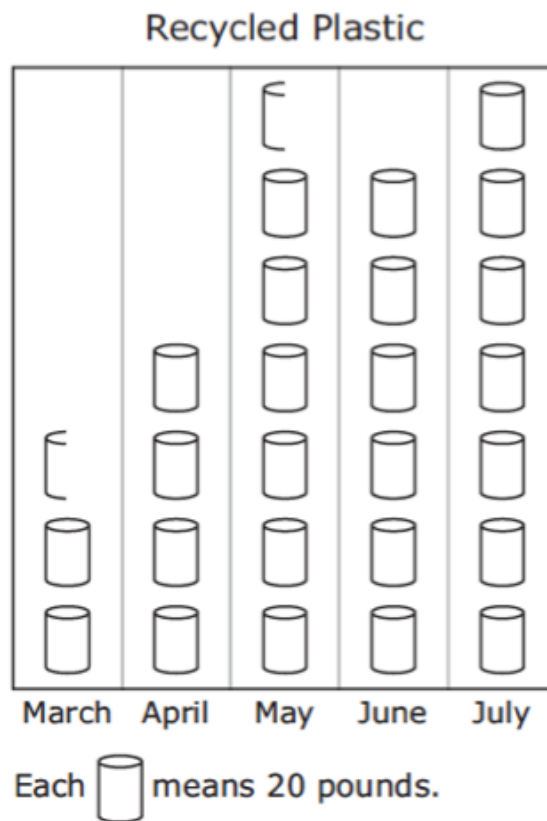
What's a **Difference** problem?

What's a **Change** problem?

# Schema Quiz Time!

# Difference

The graph below shows the number of pounds of plastic the Keller family recycled for five months.



Based on the graph, how many more pounds of plastic did the family recycle in July than in April?



# Total

Roland's family drove  $4\frac{6}{10}$  kilometers from their home to the gas station.

They drove  $2\frac{30}{100}$  kilometers from the gas station to the store.

Which expression can be used to determine the number of kilometers Roland's family drove altogether?

# Change

At the beginning of June, a bean plant was  $3\frac{4}{5}$  inches tall.

By the beginning of July, the plant was  $6\frac{2}{5}$  inches tall.

How many inches did the plant grow during June? Enter your answer in the response box.

# Let's Look Back

- 8** Tickets for a play were sold on Monday, Tuesday, and Wednesday.

**Tickets Sold**

<b>Day of the Week</b>	<b>Number of Tickets Sold</b>
Monday	197
Tuesday	364
Wednesday	?

**If a total of 900 tickets were sold for the play, how many tickets were sold on Wednesday?**

- Ⓐ 300
- Ⓑ 339
- Ⓒ 449
- Ⓓ 461

**How would you teach this problem?**



Don't describe using **key words** or **operations**



Have an **attack strategy**



Teach word-problem  
**schemas**

# Multiplicative Schemas

---

Problem type	Definition	Examples			Equation	Graphic organizer
Equal Groups						
Comparison						
Combinations						
Ratios and Proportions						

**Multiplicative Word Problems**

# Equal Groups

---

**Groups** multiplied by **number in each group** for a **product**

- Mark has **2** bags of apples. There are **6** apples in each bag. How many apples does Mark have altogether?
  - $2 \times 6 = ?$
- Mark has **12** apples. He wants to share them equally among his **2** friends. How many apples will each friend receive?
  - $2 \times ? = 12$
- Mark has **12** apples. He put them into bags containing **6** apples each. How many bags did Mark use?
  - $? \times 6 = 12$

# Equal Groups

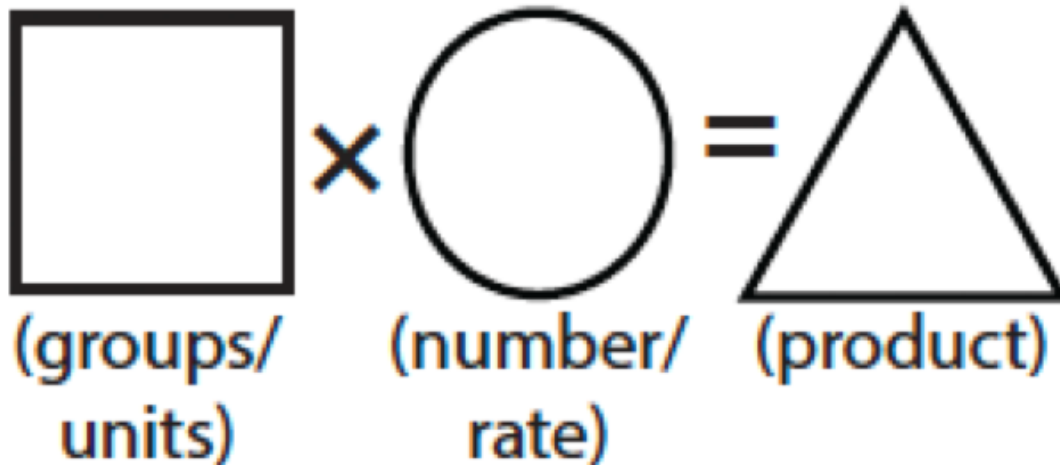
G

×

N

=

P





## Multiplicative Word Problems

A.

Ms. Thompson sold 6 cartons of cherries at the Farmers' Market. Each carton holds 25 cherries. How many cherries did she sell?



B.

Jane bought 24 light bulbs. The light bulbs come in packs of 4. How many packs of light bulbs did Jane buy?



C.

Isabella has 2 times as many DVDs as Emma. Emma has 6 DVDs. How many DVDs does Isabella have?

D.

There are 176 slices of bread in 8 loaves. If there are the same number of slices in each loaf, how many slices of bread are in 5 loaves?

A.

Ms. Thompson sold 6 cartons of cherries at the Farmers' Market. Each carton holds 25 cherries. How many cherries did she sell?



$$G \times N = P$$



$$6 \times 25 = ?$$



$$6 \times 25 = 150$$



? = 150 cherries

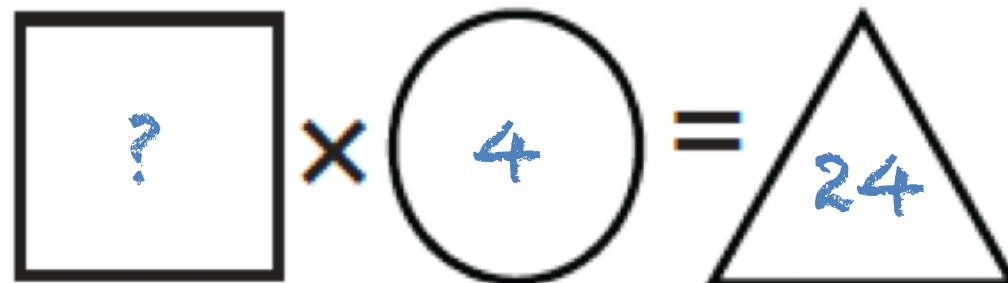
# Equal Groups

“Are there groups with an equal number in each group?”

# Equal Groups

Jane bought 24 light bulbs. The light bulbs come in packs of 4.

How many packs of light bulbs did Jane buy?



$\square \times \bigcirc = \triangle$

(groups/  
units)      (number/  
rate)      (product)

$$\begin{array}{r|l} ? \times \frac{4}{4} = \frac{24}{4} & \end{array}$$

$$? = 6 \text{ packs}$$

# Comparison

---

**Set** multiplied by a number of **times** for a **product**

- Jill picked **6** apples. Mark picked **2** times as many apples as Jill. How many apples did Mark pick?
  - $6 \times 2 = ?$
- Mark picked **12** apples. He picked **2** times as many apples as Jill. How many apples did Jill pick?
  - $? \times 2 = 12$
- Mark picked **12** apples, and Jill picked **6** apples. How many times as many apples did Mark pick as Jill did?
  - $6 \times ? = 12$

# Comparison

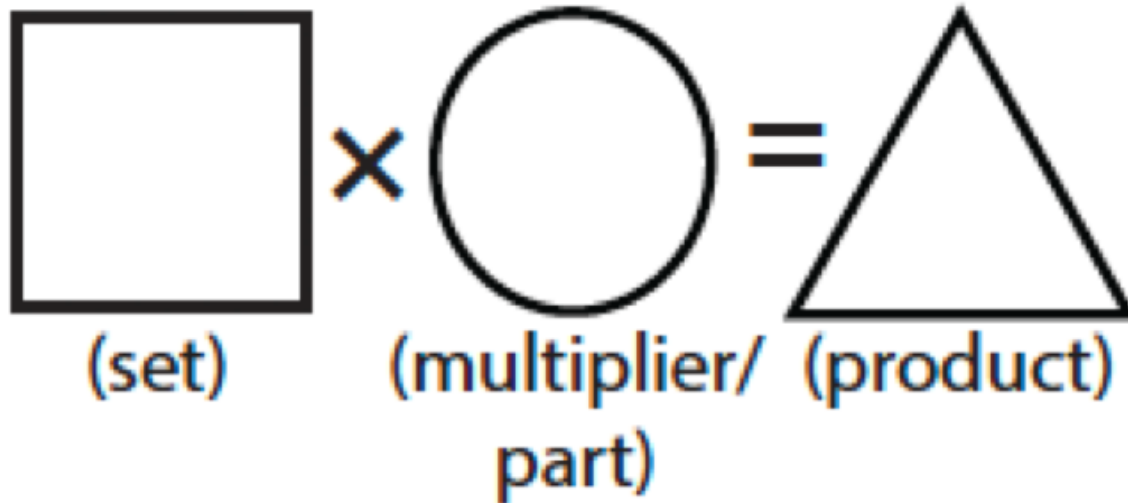
S

×

T

=

P



## Multiplicative Word Problems

A.

Ms. Thompson sold 6 cartons of cherries at the Farmers' Market. Each carton holds 25 cherries. How many cherries did she sell?

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Isabella has 2 times as many DVDs as Emma. Emma has 6 DVDs. How many DVDs does Isabella have?

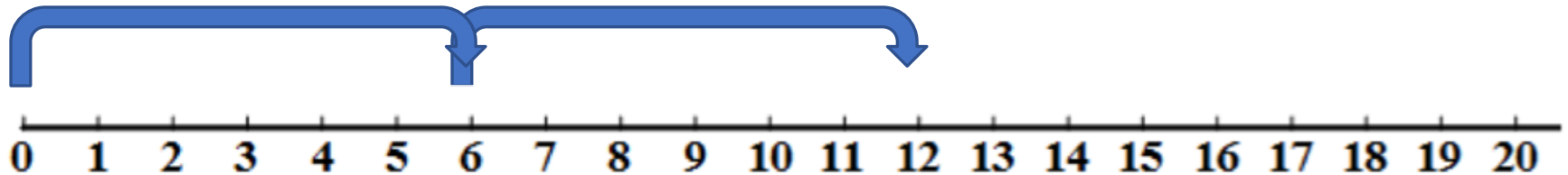


D.

There are 176 slices of bread in 8 loaves. If there are the same number of slices in each loaf, how many slices of bread are in 5 loaves?

# Comparison

Isabella has 2 times as many DVDs as Emma. Emma has 6 DVDs.



A hand-drawn multiplication equation is shown. On the left, a square contains the number 6, with the label "(set)" below it. This is followed by a multiplication symbol (×). To the right of the symbol is a circle containing the number 2, with the label "(multiplier/ part)" below it. This is followed by an equals sign (=). To the right of the equals sign is a triangle containing a question mark (?), with the label "(product)" below it.

? = 12 DVDs



## Equal Groups

“Are there groups with an equal number in each group?”

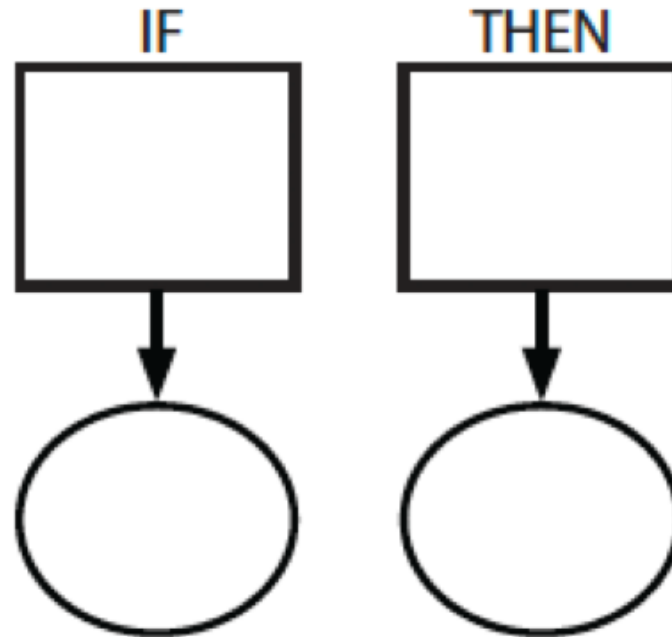
## Comparison

“Is a set compared a number of times?”

# Ratios/Proportions

---

Description of relationships among quantities



## Multiplicative Word Problems

A.

Ms. Thompson sold 6 cartons of cherries at the Farmers' Market. Each carton holds 25 cherries. How many cherries did she sell?

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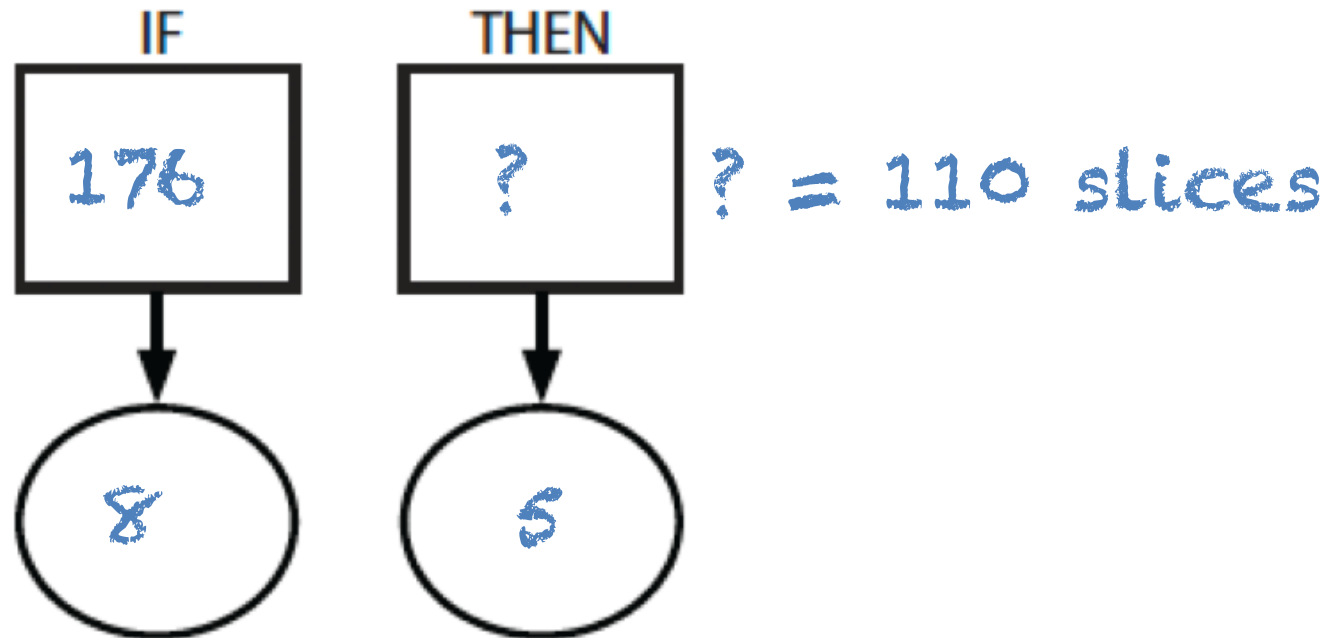
D.

There are 176 slices of bread in 8 loaves. If there are the same number of slices in each loaf, how many slices of bread are in 5 loaves?



# Ratios/Proportions

There are 176 slices of bread in 8 loaves. If there are the same number of slices in each loaf, how many slices of bread are in 5 loaves?



## Multiplicative Word Problems

E.

A sea turtle made 460 dives in 12 hours. At this rate, how many dives did the sea turtle make in 3 hours?



F.

Isaiah put 301 floor tiles in 7 rows. Each row had the same number of tiles. How many tiles did Isaiah put in each row?

G.

On average, thunder is heard in Tororo, Uganda, 251 days each year. What is the probability that thunder will be heard in Tororo on any day?

H.

Susan has 3 times as many books as Mary. Mary has 18 books. Which equation can be solved to figure out how many books Susan has?

# Ratios/Proportions

A sea turtle made 460 dives in 12 hours. At this rate, how many dives did the sea turtle make in 3 hours?

## Equal Groups

“Are there groups with an equal number in each group?”

## Comparison

“Is a set compared a number of times?”

## Ratios/Proportions

“Are there relationships among quantities - if this, then this?”

## Multiplicative Word Problems

E.

A sea turtle made 460 dives in 12 hours. At this rate, how many dives did the sea turtle make in 3 hours?

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Isaiah put 301 floor tiles in 7 rows. Each row had the same number of tiles. How many tiles did Isaiah put in each row?



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Susan has 3 times as many books as Mary. Mary has 18 books. Which equation can be solved to figure out how many books Susan has?





## Equal Groups

Isaiah put 301 floor tiles in 7 rows. Each row had the same number of tiles. How many tiles did Isaiah put in each row?

# Ratios/Proportions

On average, thunder is heard in Tororo, Uganda, 251 days each year. What is the probability that thunder will be heard in Tororo on any day? (1 year = 365 days)

# Comparison

Susan has 3 times as many books as Mary. Mary has 18 books.  
Which equation can be solved to figure out how many books  
Susan has?

# Let's Review

---

What's an **Equal Groups** problem?

What's a **Comparison** problem?

What's a **Ratios/Proportions** problem?

# Schema Quiz Time!

# Ratios/Proportions

Ethan correctly answers 80% of the total questions on his history test.  
He correctly answers 32 questions.

# Equal Groups

Ryan makes 6 backpacks. He uses  $\frac{3}{4}$  yard of cloth to make each backpack. What is the total amount of cloth, in yards, Ryan uses to make all 6 backpacks?

**A.**  $1\frac{1}{2}$

**B.**  $2\frac{1}{4}$

**C.**  $4\frac{1}{2}$

**D.**  $6\frac{3}{4}$

# Comparison

Danielle's full-grown dog weighs 10 times as much as her puppy. The puppy weighs 9 pounds.

Enter the number of pounds the full-grown dog weighs.



# Schemas

Total

Difference

Change

Equal Groups

Comparison

Ratios/Proportions

# An Example

---

# Pirate Math Intervention

---

**16 weeks**

**3 times a week**

- **30 min sessions**

Weeks 1-2	Introduction
Weeks 3-6	Total
Weeks 7-12	Difference (with Total review)
Weeks 12-14	Change (with T and D review)
Weeks 15-16	Review

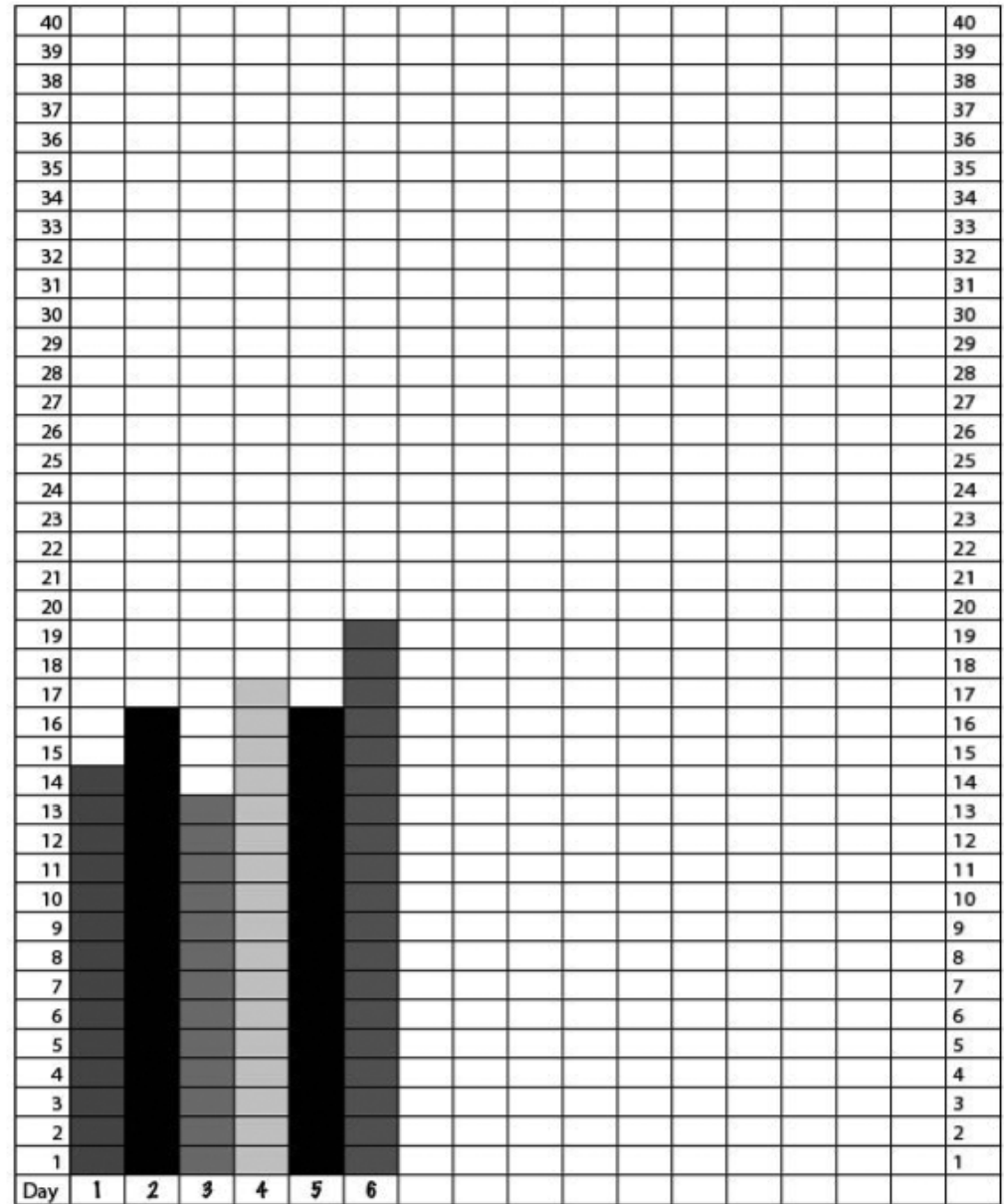


# Math Fact Flashcards

3 min

Math Fact Flashcard Graph

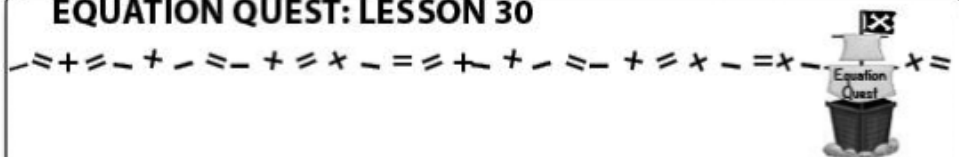
Student: Amaya



# Equation Quest

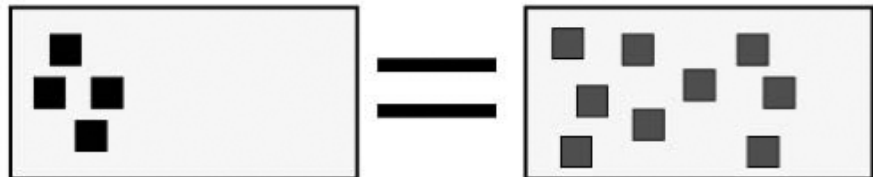
3-5 min

## EQUATION QUEST: LESSON 30

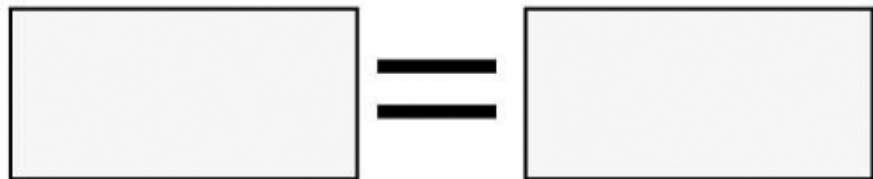


**equal sign:** *the same as*

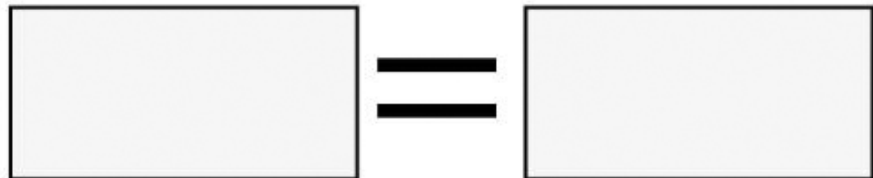
A.  $4 + X = 9$



B.  $8 - X = 2$



C.  $4 = 9 - X$

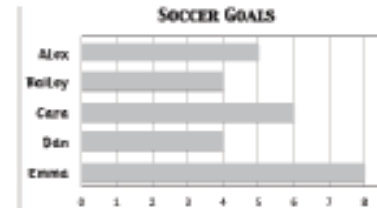


# Buccaneer Problems

15 min

## RUN

If needed, number the graph



1. Read the problem
2. Underline the label and ~~cross out irrelevant info~~
3. Name the problem type

Total

Difference

Change

# Buccaneer Problems

Total example

## BUCCANEER PROBLEMS: LESSON 11



A. Tanner spent \$27 on snacks and drinks. ~~He bought 5 kinds of snacks.~~ If Tanner spent \$19 on snacks, how much money did he spend on drinks? \$

$$P1 + P2 = T$$

$$\begin{array}{r} 19 + \textcircled{X} = 27 \\ \underline{-19} \quad \quad \underline{-19} \\ 0 \quad \quad \quad 8 \\ X = \$8 \end{array}$$

# Buccaneer Problems

Difference example

D.

Sports Games Dan Played	
Basketball	 20
<del>Baseball</del>	<del> 25</del>
Soccer	?

Each ball stands for 5 games.

[Dan played 10 <sup>G</sup>more <sup>L</sup>soccer games than basketball games] How many soccer games did he play?

$$G - L = D$$

$$\textcircled{X} - 20 = 10$$

$$\begin{array}{r} +20 \\ \hline \end{array}$$

$$\begin{array}{r} 0 \\ \hline \end{array}$$

$$X = 30 \text{ soccer games}$$



# Buccaneer Problems

Change example

B. Marta planted 34 lettuce plants in her garden. Then, she planted 13 more lettuce plants. One night a rabbit ate 22 of her lettuce plants. How many lettuce plants does Marta have left?

$$ST + C - C = E$$

$$34 + 13 - 22 = X$$

$X = 25$  lettuce plants

$$\begin{array}{r} 34 \\ + 13 \\ \hline 47 \\ - 22 \\ \hline 25 \end{array}$$

# Shipshape Sorting

2 min

## Shipshape Sorting

**T**

**D**

**C**

**?**

# Jolly Roger Review

1 min



## JOLLY ROGER REVIEW: LESSON 41

A. 
$$\begin{array}{r} 285 \\ - 86 \\ \hline \end{array}$$

D. 
$$\begin{array}{r} 188 \\ - 86 \\ \hline \end{array}$$

B. 
$$\begin{array}{r} 94 \\ + 99 \\ \hline \end{array}$$

E. 
$$\begin{array}{r} 84 \\ + 38 \\ \hline \end{array}$$


C.  $14 - 8 = \underline{\quad}$

# Jolly Roger Review

2 min

## JOLLY ROGER REVIEW: LESSON 44

Total Birds Seen at Camp on the First and Second Days

Robins	    
Crows	
Bluebirds	 
Woodpeckers	   

Each  stands for 5 birds.

On the first day of camp, campers saw 10 woodpeckers. On the second day, they saw some more. How many woodpeckers did the campers see on the second day?



Don't describe using **key words** or **operations**



Have an **attack strategy**



Teach word-problem  
**schemas**

# Contact Information

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