Designing extended warranties in supply chains: Game theory approaches

K. Shahanaghi
Academic member of School of Industrial Engineering, Iran University of Science and Technology
Shahanaghi@iust.ac.ir

M. Keyvan Rad
Ph.D. student of School of Industrial Engineering, Iran University of Science and Technology
Keyvanirad@iust.ac.ir

Abstract: This study proposes the design of extended warranties in a supply chain. Supply chain consists of a seller and buyer. The seller sells a single product exclusively through the buyer. The extended warranty could, however, be offered by the seller or by the buyer. The policy for best variables is decided by the party offering the extended warranty. Game theoretic models are used for exploring the interactions between the supply chain members. There is a competition between them for offering extended warranty. So that it is compared which member is better to presents the extended warranties? Then decentralized and centralized models of providing extended warranty are compared. Finally the extension of basic model with the presence of third party is presented. The models also illustrated by numerical example.

Key words: supply chain management, seller, buyer, extended warranty, stackelberg.

1- Introduction

Selling extended warranties is a rapidly growing business. In past, extended warranties were only for special items. But now, extended warranties are used for many categories. Providing extended warranties is profitable and there is a competition for offering it. The profit margins that extended warranties provide are much bigger than those from retailer’s price. Sometimes the extended warranty offer services that are more than which base warranty provides. Selling extended warranties are separated from the base products and actually with extra cost. An extended warranty can be sold and covered by different parts of supply chain.

In this study at first we consider a supply chain contain of a seller and a buyer. The seller provides a single product and sells that to the buyer and then the buyer sell those products to the end costumer. Both of the seller and the buyer could offer the extended warranty. So the policy of offering extended warranty is decided by which provides it. So, the benefit achieved is influenced in making decisions. In this case, game theoretic models are used to solve the questions such as who and in which situation of cooperation the extended warranty presents to catch the highest profit and also how the members are satisfied to let
other members to offer extended warranty. Then providing extended warranty by a reseller is compared.

For tangible our study consider that GE Appliances (a leading manufacturer), firms like Ford, GM, JVC, and Apple Many (Padmanabhan1995) and IBM (Li et al.2005) are the examples of manufacturer which offer extended warranties directly to the end consumers. About the retailers who directly provide extended warranties we can call Sears. Some Retail stores such as Best Buy, Circuit City, and Home Depot are the reseller of extended warranty and they are offering and promoting warranties underwritten by insurers.

Term of “service plan” is mostly used for the services and the extended warranty policy that is sold by the retailer. Sometimes these services has a longer period, and may offer additional benefits comparing with manufacturer’s original warranty but in this study we consider all these services same.

In this paper five scenarios are presented, centralized model and when a seller or a buyer being an extended warranty provider (model S; model B) and seller or a buyer being an extended warranty reseller for a third party provider (model 3p S; model 3p B) ((Li et al.2005)). Then the game theoretic models are used to analyze. To this step our paper is same with (Li et al.2005). But actually the difference is the demand function in marketing variables were not consider in their paper but we consider that in this paper and pay special focus on the price, duration and marketing.

This paper examines to which pricing decision, warranty price and period, and marketing expenditure is members should decide.

Also, we compare the performance of the decentralized and centralized models. Also it is compared whether the EWP or EWR models are better.

The remainder of this paper is as follows. The next section reviews the literature. The basic models (models S and B) are presented and the results are analyzed then the developed models are explained. The next Section summarizes and concludes the paper.

2. Literature review

There are many papers and studies about the warranty but notice that in this paper we just review the new ones about extended warranty mostly with supposing game theory.

(Bian and et al 2015) considered two competing retailers which purchase the substitute products from the same manufacturer, and then they sell those to the market. Manufacturer offer the base warranty and also the retailers offer extended warranty. Final demand depends on the retail price and the length of whole warranties. Their model solved by using game theoretic models. They examined whether no retailer and both the two retailers provide the extended warranty.

(Modak 2015) considered a supply chain composed of a manufacturer and a retailer. The demand was a function of price, quality level, and warranty period. Actually proposing
higher quality needs higher cost of technology. They considered that cost of raw material depends on the quality level. They presented decentralized and centralized model.

Jiang and Zhang (2011) and Heese (2012) presented model where the retailer’s extended warranties was an optional service and also they didn’t focus on take product prices.

Li et al. (2011) consider extended warranties in supply chain where it could only offer by the manufacturer or the retailer. They explained that when the retailer offers it, a longer warranty length and a higher supply chain profit can be gained.

Chen et al. (2012) investigated a model with one manufacturer and two competing retailers that offered the warranties and demand depends on warranty length. The influences of strategies of manufacturer’s pricing on the supply chain were checked. Sinha and Sarmah (2012) studied three scenarios where retailers compete on price, warranty duration and both the price and warranty duration. Also the warranties were provided by the retailers and demand depends on both the prices and warranty period.

Cardenas-Barron and Sana have (2014) studied a coordination model with a one manufacturer and one retailer. Esmaeili, Shamsi Gamchi, and Asgharizadeh (2014) have presented warranty service contracts in supply chain with game theory approach.

3- Methodology

3-1- The models of extended warranty provider

In this section, our base model was introduced. Both of seller or the buyer could separately offer the extended warranty. Note that the product is sell by the seller to the buyer and the buyer sell that directly to the consumers and the member that offer the extended warranty is called EWP were incurs all associated costs and bring all the revenue. In this paper these two cases and also centralized model were briefly explain and compared with each other and the total supply chain profit, each member profit and decision variables are calculated.

As (Li et al.2005) our models are consist of a supply chain with a seller and a buyer. The seller (manufacture) produces a product and sells it to the buyer (retailer) at a wholesale price V. Then the buyer resells that to the consumer at a retail price P. The base warranty was offered by the seller with the product and this paper just focus on the extended warranty not the base warranty.

Firstly two models of offering extended warranty are explained. Model S and model B were respectively presented the case were the seller and the buyer provides the extended warranty directly to the consumers. In both models, the wholesale price is set by the seller and the retail price set by the buyer. Such as (Li et al.2005 and Li and Chhajed 2011) our
Designing extended warranties in supply chains: Game theory approaches


models extended warranty demand is a function of warranty price and warranty length which denote by $P_e$ and $W_e$. And our product demand is a function of retail price and marketing expenditure which denote by $P$ and $M$. Also, our models have a different specification with these two recent papers were have notice to the marketing expenditure for extended warranty and also the demands are not linear.

As (Li et al.2005) presented in their paper Figure 1 schematically describes the three models along with their relevant decision variables. The solid line is associated with product decisions, while the dotted line is for extended warranty decisions.

![Figure 1: The EWP models](image_url)

### 3.2 Notation

In this section object functions and input parameters of models are explained

**Object functions**
- $\pi_C$: the centralized system supply chain profit.
- $\pi_s$: the seller profit.
- $\pi_b$: the buyer profit.
- $\Pi_{3p}$: the third party profit.

**Decision variables**
- $V$: the price charged by the seller to the buyer. ($/unit)
- $P$: selling price charged by the buyer. ($/unit)
- $Q$: lot size determined by the seller. (units)
- $M$: marketing expenditure incurred by the buyer demand. ($/unit).
Designing extended warranties in supply chains: Game theory approaches


M_e: marketing expenditure incurred by the buyer extended warranty demand. ($/unit).
W_e: the length of extended warranty. (time/unit).
V_e: extended warranty selling price charged by the third party. ($/unit).
P_e: extended warranty selling price charged by the seller or buyer. ($/unit).

**Input parameters**

A_s: seller’s setup. (Ordering cost) ($/setup)
C_s: seller’s production cost including purchasing cost. ($/unit)
k: scaling constant for demand function. (k>0)
i: percent inventory holding cost per unit per year.
α: price elasticity of demand function. (α≥1)
β: marketing expenditure elasticity of demand.
A_b: buyer’s ordering cost. ($/order)
A_s: seller’s setup cost. (Ordering cost) ($/setup)
C_s: seller’s production cost including purchasing cost. ($/unit)
D(P,M): total annual demand: for notational simplicity we let. D=D(P,M)
C_e: seller’s providing and managing extended warranty cost.
C_{eb}: buyer’s providing and managing extended warranty cost.
C_{ep}: third party’s providing and managing extended warranty cost.
k_e: scaling constant for extended warranty demand function. (k_e>0)
α_e: price elasticity marketing expenditure incurred by the buyer demand function. (α_e≥1)
β_e: marketing expenditure elasticity of extended warranty demand.
δ_e: marketing expenditure elasticity of extended warranty demand.
D_e(P_e,M_e,W_e): total annual extended warranty demand: for notational simplicity we let. D_e=D_e(P_e,M_e,W_e)
C_m(L_a): supplier’s production cost including purchasing cost C_m(L_a); for notational simplicity we let. C_m(L_a)=C_m

**The Costs**

Such as (Esmaeili et al. 2009) models the problems we assume that for each seller, the cost is contains of production cost and Setup cost. And the total cost of buyer is also consisting as the sum of purchase cost, market cost, ordering cost and holding cost. These costs would be explained briefly later.

The member offering the extended warranty chooses the extended warranty length W_e, its price P_e, and responsible repair costs during the extended warranty length.

**Demand Functions**

Our models have two demand functions, first the product demand and then the extended warranty demand. The First one is the product demand. Such as (Esmaeili et al. 2009) we
assume that the product demand is decreasing in price and increasing by marketing expenditure and is given by:

\[ D = kP^{-\alpha}M^\beta \quad (1) \]

Then the demand function for the extended warranty is derived. It’s important to notice that just the consumers who bought the product able to purchase the extended warranty so the extended warranty demand is smaller or equal to the demand of product demand. The demand for the extended warranty, is denoted by \( D_e \). The demand of extended warranty should be decreasing in extended warranty price and marketing expenditure, and increasing in extended warranty length and also change along the product demand.

Note that, if no extended warranty is offered or accepted, its related demand is zero. So, the demand for the extended warranty is as below

\[ D_e = \begin{cases} 
K.P^{-\alpha}.M^\beta.K_e.P_e^{-\alpha_e}.M_e^\beta_e.W_e^{\delta_e} & \text{if } W_e > 0 \\
0 & \text{if } W_e = 0 
\end{cases} \quad (2) \]

The above expression presented that the maximum allowable demand for the extended warranty is as \( D = kP^{-\alpha}M^\beta \), and also \( 1 \geq K_e.P_e^{-\alpha_e}.M_e^\beta_e.W_e^{\delta_e} \) and Note that \( P_e \leq P \) and \( D_e \geq 0 \).

The main assumption in our models is that there is no information asymmetry in the supply chain and the seller acts as a Stackelberg leader. So we solve the models with game theory. Here, the seller first determines its variables, and then the buyer simultaneously determine its variable to maximize own profits.

### 3-3 Model C: Centralized model

In centralized system the total supply chain profit is maximized by considering both of the product and extended warranty demands.

\[ \text{Max} \pi_c = [P.D-C_s.D-A_s.D/Q] + [-M.D-A_b.D/Q-0.5i.V.D] + [P_e.D_e^{-1}C_es.W_e^{2}.D_e] \quad (3) \]

Model C can be solved and maximized by using standard optimization techniques.

### 3-4 Model B: Buyer offer Extended Warranty

In model B, the buyer decides the retail price \( P \) and also the extended warranty policy by verifying \( P_e \) and \( W_e \). In this model at the first level, the seller chooses the wholesale price \( V \) and then maximizes her profit by the optimization problem of the seller is given by:

\[ \text{Max} \pi_s = [V.D-C_s.D-A_e^{-1}D/Q] \quad (4) \]

In the second level, by using the wholesale price as given above, the buyer maximizes his own profit by the problem is as follows.

\[ \text{Max} \pi_b = [P.D-M.D-A_b^{-1}D/Q-0.5i.V.D-V.D] + [P_e.D_e-C_{eb}.W_e^{2}.D_e] \quad (5) \]
The models solving is starting with the buyer’s problem as a backward problem. Note that as (Li et al.2005) other bi-level problems the second order conditions for the maximization problems are satisfied and solving for the buyer’s best responses as a function of the wholesale price $V$.

3-5- Model S: Seller offer Extended Warranty

The seller is the stackelberg leader in the game and the buyer’s pricing decision is anticipated by her and her profit is maximized then. At the first level of the game, the wholesale price $V$ is choosing by the seller, and accordingly the extended warranty policy $(P_e, W_e)$is determined. In the second level, the buyer uses the seller’s decisions as determined in first level and sets the retail price $P$. The seller’s optimization problem in this model is as follows.

\[
\text{Max}_S = [V \cdot D - C_s \cdot D - A_s \cdot \frac{D}{Q}] + [P_e \cdot D_e - C_{es} \cdot W_e^2 \cdot D_e]
\]  

(8)

In the second level, using the seller’s decisions variables, the buyer solves for $P$:

\[
\text{Max}_B = [P \cdot D - M \cdot D - A_p \cdot \frac{D}{Q} - 0.5i \cdot V \cdot D - V \cdot D]
\]  

(9)

The solution procedure for model $S$ is same as model $B$. solving is start backwards by with the buyer’s problem.

4- Result:

4-1- EXTENSION: Reseller and Extended Warranty Models

In previous discussion we introduced the extended warranty where either the seller or the buyer provide the extended warranty and the provider was the owner of the benefit gain from it. In this section the extension of basic models are introduce, where the third party sell extended warranty to the seller or the buyer and they resells that. These models are called the extended warranty reseller models (EWR). As (Li et al.2005), Best Buy, Circuit City and other buyers are often the resellers of the extended warranties from third party underwriters such as AIG, GE, and American Home Shield.

As (Li et al.2005) in analyzing the EWR models, the basic structure of our model were kept unchanged and the seller produces a single product and sells it through the buyer and that the seller sets the product wholesale price the retail price is sets by the buyer.

However, in the EWR models the seller or the buyer buys the extended warranty from a third party, and resells that to the end consumer. Managing the extended warranty is done by third party, and it decides its variables in her best interest. Also the retail price $P_e$ is set by the seller or the buyer. Two EWR models are considered here. Model 3R and model 3M are respectively present the buyer resells and the seller resells.

As (Li et al.2005) presented, Figure 2 schematically describes the two models, their decision-making sequence, and their relevant decision variables.
EWR scenarios are modeled as a three-level game. In the first level, seller is the Stackelberg leader so decides the product wholesale price \( V \). In the second level, after evaluating the average repair cost, the extended warranty decisions are made by the third party. Other variables are defined later. Retail price are calculated by the buyer and the retail price of the extended warranty is determined by either the main member of supply chain. We the model formulations are given below.

4.2 Model 3p B: Third party offer Extended Warranty and buyer resell it

At the first level, the seller maximizes her profit:

\[
\text{Max} \pi_S = [V \cdot D - C_e \cdot D - A_e \cdot \frac{D}{Q}]
\]  \hspace{1cm} (10)

In the second level, the third party optimizes her profit by finding the wholesale price \( V_e \) and the length \( W_e \) of the extended warranty

\[
\text{Max} \pi_{3p} = [V_e \cdot D_e - C_{e3p} \cdot W_e^2 \cdot D_e]
\]  \hspace{1cm} (11)

In the third level, taking variables in previous level as given, the buyer decides the retail price of the product \( P \) and also the extended warranty retail price to optimize the profit. The buyer’s problem is given below

\[
\text{Max} \pi_b = \left[ P \cdot D - M \cdot D - A_{b} \cdot \frac{D}{Q} - 0.5i \cdot V \cdot D - V \cdot D \right] + [P_e \cdot D_e - V_e \cdot D_e]
\]  \hspace{1cm} (12)

These models are solved by working backwards.
4.3 Model 3p S: Third party offer Extended Warranty and seller resell it

As explained above for model 3p B the formulation and the solution procedure of Model 3p S are as below.

Maxπs = \left[ V \cdot D - C_s \cdot D - A_s \cdot \frac{D}{Q} \right] + \left[ P_e \cdot D_e - V_e \cdot D_e \right] \quad (14)

Maxπ3p = \left[ V_e \cdot D_e - C_{e3p} \cdot W_e^2 \cdot D_e \right] \quad (15)

Maxπb = \left[ P \cdot D - M \cdot D - A \cdot \frac{D}{Q} - 0.5i \cdot V \cdot D - V \cdot D \right] \quad (16)

5 Conclusion

In this study, game-theoretic models had developed to design extended warranties schemes in a supply chain. We considered a supply chain consisting a seller, a buyer and a customer. Both of the seller and the buyer can provide or resell the extended warranty. At first two EWP models with the buyer and the seller studied. Our models consider the new demand for extended warranty that as we know none of the previous study notices that.

For determining optimal selling price of the product, selling price of the extended warranty, and length of warranty period and marketing expenditure for both of the main product and extended warranty in a two-echelon supply chain our model were briefly explained. These variables didn't study together in literature of profit maximization.

Also two models of seller and buyer and also centralized model explain completely.

Then two EWR models studied where a third party provide the extended warranty, and sell it to the seller or buyer and then the seller or the buyer resells it. These cases are modeled briefly.

These model solved by using a stackelberg games by backward approach. Also In our model no information asymmetry is present.

References


