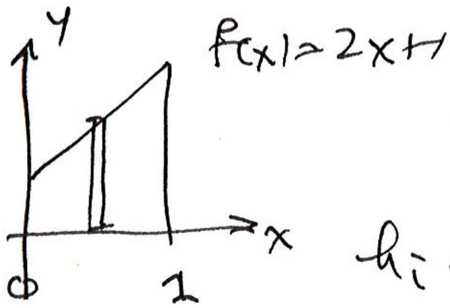
Math 1496 – Calc I

Area Using Rectangles



$\Delta x = \frac{2}{n}$ $\quad x_i = \frac{2i}{n}$

$h_i = 2\left(\frac{2i}{n}\right) + 1 \quad A_i = \left[2\left(\frac{2i}{n}\right)+1\right]\frac{2}{n}$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \left[2\left(\frac{2i}{n}\right)+1\right]\frac{2}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{8i}{n^2} + \frac{2}{n}\right)$$

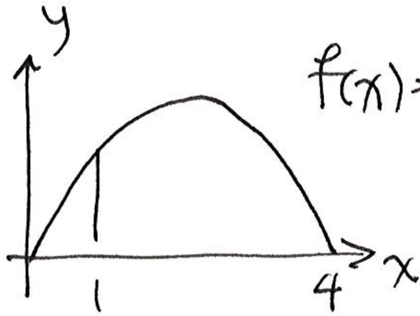
$$= \lim_{n\to\infty} \left(\frac{8}{n^2}\sum_{i=1}^{n} i + \frac{2}{n}\sum_{i=1}^{n} 1\right)$$

$$= \lim_{n\to\infty} \frac{8}{n^2} \cdot \frac{n(n+1)}{2} + \frac{2}{n} \cdot n$$

$$= \lim_{n\to\infty} 4\left(\frac{n+1}{n}\right) + 2$$

$$= \lim_{n\to\infty} 4 + \frac{4}{n} + 2 = 6$$

Ex 2



$f(x) = 4x - x^2$

$\Delta x = \frac{4-1}{n} = \frac{3}{n}, \quad x_i = 1 + \frac{3i}{n}$

$h_i = 4\left(1+\frac{3i}{n}\right) - \left(1+\frac{3i}{n}\right)^2$

$A_i = \left[4\left(1+\frac{3i}{n}\right) - \left(1+\frac{3i}{n}\right)^2\right]\frac{3}{n}$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \left[4\left(1+\frac{3i}{n}\right) - \left(1+\frac{3i}{n}\right)^2\right]\frac{3}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(4+\frac{12i}{n} - 1 - \frac{6i}{n} - \frac{9i^2}{n^2}\right)\frac{3}{n}$$

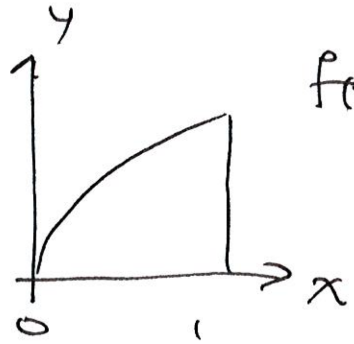
$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{9}{n} + \frac{18i}{n^2} - \frac{27i^2}{n^3}\right)$$

$$= \lim_{n\to\infty} \left(\frac{9}{n}\sum 1 + \frac{18}{n^2}\sum i - \frac{27}{n^3}\sum i^2\right)$$

$$= \lim_{n\to\infty} \frac{9}{n}\cdot n + \frac{18}{n^2}\frac{n(n+1)}{2} - \frac{27}{n^3}\frac{n(n+1)(2n+1)}{6}$$

$$9 + 9 - \frac{27(2)}{6} = 9+9-9 = 9$$

Ex 3



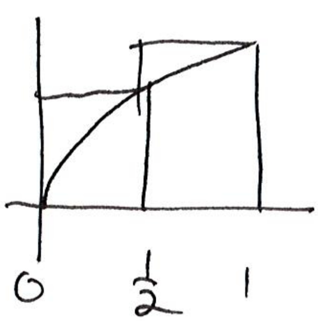
$f(x) = \sqrt{x}$

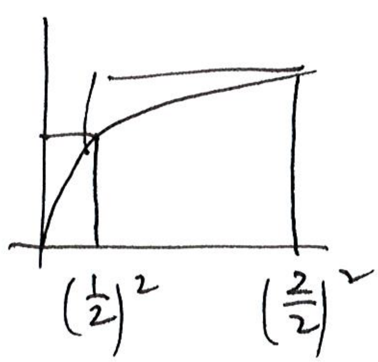
$\Delta x = \frac{1}{n}$ $\quad x_i = \frac{i}{n}$ $\quad h_i = \sqrt{\frac{i}{n}}$

$A_i = \sqrt{\frac{i}{n}} \cdot \frac{1}{n}$

$A = \lim_{n \to \infty} \sum_{i=1}^{n} \frac{\sqrt{i}}{n^{3/2}}$ $\quad$ we don't have a formula for $\sum \sqrt{i}$?

Instead





why h is easier to find

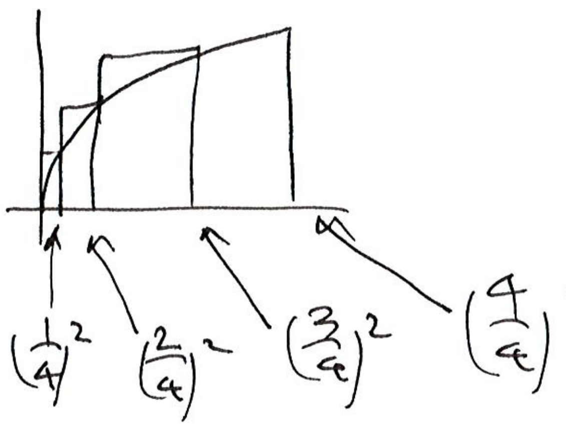
$h_1 = \sqrt{(\frac{1}{2})^2} = \frac{1}{2}$ $\qquad h_2 = \sqrt{(\frac{2}{2})^2} = 1$

$\Delta x_1 = \frac{1}{4}$ $\qquad \Delta x_2 = 1 - \frac{1}{4} = \frac{3}{4}$

$A_1 = \frac{1}{2} \cdot \frac{1}{4}$ $\qquad A_2 = \frac{3}{4} \cdot 1$ $\quad$ Then add

14)

4 rectangles



1st $\Delta x_1 = \frac{1}{16}$ $\quad A_1 = \frac{1}{16}\cdot\frac{1}{2}$

$h_1 = \sqrt{\frac{1}{4}} = \frac{1}{2}$

2nd $\Delta x_2 = \frac{4}{16} - \frac{1}{16} = \frac{3}{16}$

$h_2 = \frac{2}{4}$

$A_2 = \frac{2}{4}\cdot\frac{3}{16}$

$\underline{A_3}$

$\Delta x_3 = \frac{9-4}{16} = \frac{5}{16}$ $\quad A_3 = \frac{5}{16}\cdot\frac{3}{4}$

$h_3 = \sqrt{\left(\frac{3}{4}\right)^2} = \frac{3}{4}$

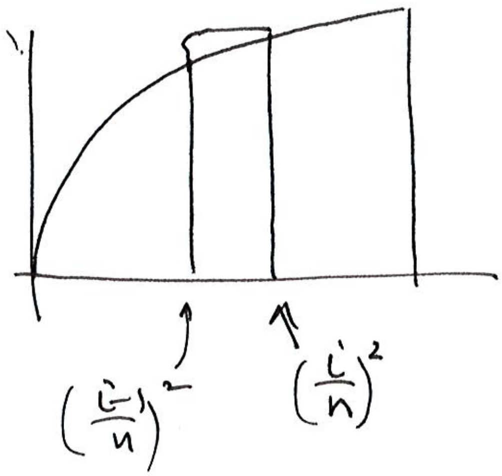
$\underline{A_4}$

$\Delta x_3 = \frac{4^2-3^2}{16} = \frac{16-9}{16} = \frac{7}{16}$ $\quad h_3 = \sqrt{\left(\frac{4}{4}\right)^2} = 1$

$A_4 = \frac{7}{16}\cdot\frac{4}{4}$

$A_1 + A_2 + A_3 + A_4 = \frac{1}{16}\cdot\frac{1}{4} + \frac{3}{16}\cdot\frac{2}{4} + \frac{5}{16}\cdot\frac{3}{4} + \frac{7}{16}\cdot\frac{4}{4}$

In general



$$\Delta x_i = \frac{i^2 - (i-1)^2}{n^2} = \frac{i^2 - (i^2 - 2i + 1)}{n^2} = \frac{2i - 1}{n^2}$$

$$h_i = \sqrt{\left(\frac{i}{n}\right)^2} = \frac{i}{n}$$

$$A_i = \frac{i}{n} \cdot \frac{2i - 1}{n^2}$$

$$A = \lim_{n \to \infty} \sum_{i=1}^{n} \frac{2i^2 - i}{n^3}$$

$$= \lim_{n \to \infty} \frac{2}{n^3} \sum i^2 - \frac{1}{n^3} \sum i$$

$$= \lim_{n \to \infty} \frac{2}{n^3} \frac{n(n+1)(2n+1)}{6} - \frac{1}{n^3} \frac{n(n+1)}{2}$$

$$\to \frac{4}{6} = 2/3$$