EE 221L Circuits II Laboratory #6

RC Circuits

By

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Background

Circuits with time-constants formed by combining resistors and capacitors are found in countless electronic circuit applications. The simplest and most convenient filters are formed using RC circuits. Many analyses of frequency response of complex electronic devices are often simplified down to single time-constant RC circuits. This is why having a fundamental understanding of RC circuits is important; most frequency dependent effects can be modeled by RC circuits. In this lab we will use RC circuits for filtering applications, understanding limits of our test equipment and for pulse-width modulation (PWM).

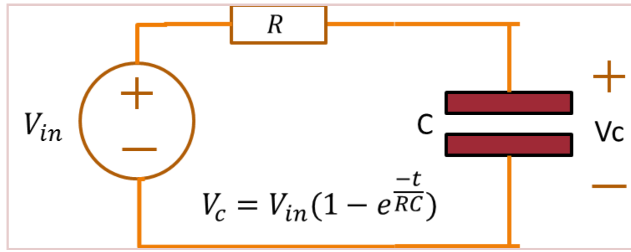


Figure 1. RC Circuit and Charging Equation

Consider a capacitor being charged by a voltage source through a resistor as shown in Fig. 1. Without considering any math, it is intuitive that the capacitor cannot charge instantaneously to the input voltage. Instead, the rate at which the capacitor charges will be dependent on the size of the resistor and capacitor. A water analogy can be used to represent this circuit. A bucket can represent the capacitor and a pipe can represent the resistor. The time to fill up the bucket will depend on the size of the bucket and pipe. The formula for the voltage across the capacitor as a function of time for the circuit in Fig. 1 is,

$$V_c = V_{in}(1 - e^{\frac{-t}{RC}})$$

Where RC is the time constant which is the time it takes for the capacitor voltage to reach 63% of its maximum value. We will be measuring the time-constant in this lab. In this example the voltage rises to the value of the input voltage. If the components were reversed as in Fig. 2 the formula for the voltage across the resistor is,

$$V_R = V_{in} e^{\frac{-t}{RC}}$$

In this case, the voltage starts at the input voltage and falls to zero. These equations represent a time-domain view. In electronics every circuit can be viewed in both the time domain and the frequency domain. Both are equally valid ways of examining circuits. You will learn about the formal relationship between the time and frequency domains in your signals and systems courses. In this lab we will only discuss applications in the two domains.

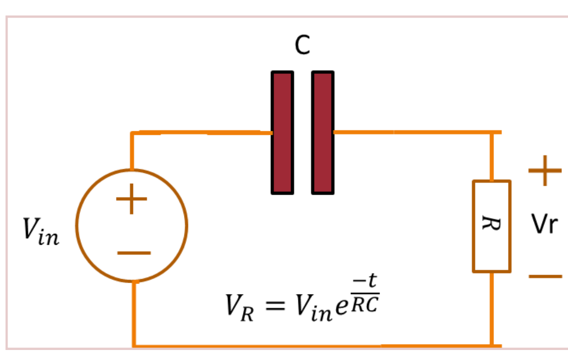


Figure 2. Flipping the resistor and capacitor.

Two time-domain applications of these circuits are as timers and for pulse shaping. For example the popular 555 timer uses an RC time constant as the basis for its timing. It is important to note that the RC circuit is only to provide a reference, if any sort of load is connected to it, the accuracy will be affected. Most digital logic switches in the middle of its supply voltage. A 5V CMOS logic device will have a 2.5V switch point. Let's say we wanted to make a timer for 15 seconds. The equation for the RC circuit can be solved for RC,

$$V_c = V_{in}(1 - e^{\frac{-t}{RC}})$$

$$\frac{1}{2}V_{in} = V_{in}\left(1 - e^{\frac{-t}{RC}}\right) \approx 0.7RC \rightarrow RC = 21.4$$

Any combination of R and C that satisfies this equation is valid theoretically, but in the lab there are limits due to practical values. Generally, there is less variety of available capacitors. A good capacitor value to pick would be 68uF. This results in a resistor value of 314k. This value should be rounded to the closest standard value which is 330k. The schematic of this timing circuit is shown in Fig. 3. This type of timing circuit is appropriate for general purpose applications but should not be used for an application that needs accuracy to within 5% or better. This type of circuit is also used to introduce a set delay in logic circuits. In this lab we will use a 555 timer which uses a different equation so be sure to not use the equation given above.

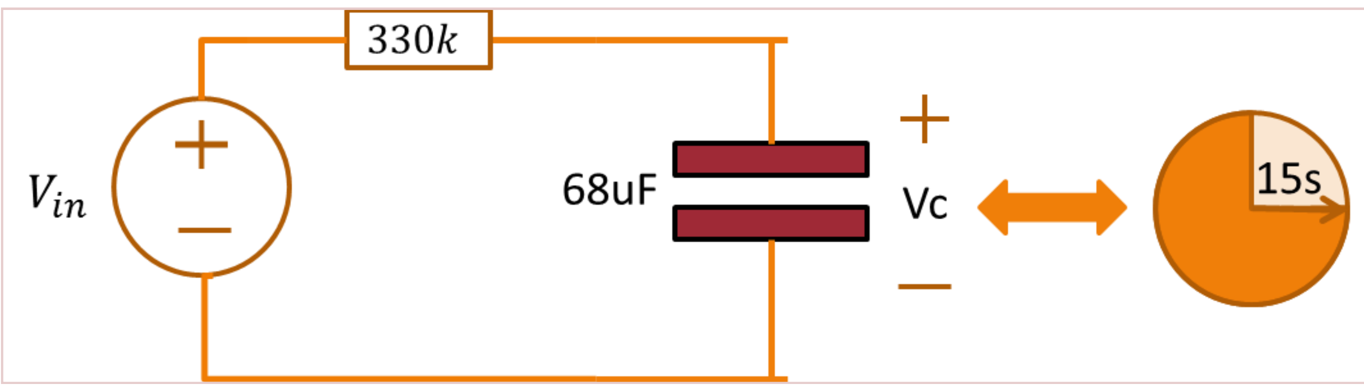


Figure 3. Using an RC circuit for timing.

The next application for this circuit is pulse shaping. Pulse shaping is a term that refers to modifying the properties of a pulse or square-wave. This can involve slowing down or speeding up the rise-time; changing the length of the pulse; or changing the shape itself. Why would one want to do this? There are a plethora of reasons but we'll consider two here. The first is slowing down the rise-time of a square-wave. An ideal square wave should transition between its low and high states instantly. The downside of a very fast transition is high frequency transients. The faster a waveform changes in the time domain, the more high frequency components there are. These high frequency components can cause interference to RF devices such as cell-phones and may violate FCC regulations. This is a major challenge for switching power supplies, which can be a major source of radiated noise. To solve this, some power supplies slow down the edges of their square-waves to reduce interference and comply with regulations.

Every electronic device has an associated rise-time. The function generators and oscilloscopes in this lab are limited in the upper limit of their frequency range directly by the rise-time. We will examine the time-frequency relationship later. For electronics engineering the commonly accepted definition of the rise-time is the time it takes for an input step to change from 10% to 90% of its final value. This can be derived using the equation for the charging RC circuit,

$$V_c = V_{in}(1 - e^{\frac{-t}{RC}})$$

Setting the output to 10% of the input

$$0.1V_{in} = V_{in}\left(1 - e^{\frac{-t}{RC}}\right) \rightarrow t \approx 0.105RC$$

Setting the output to 90% of the input

$$0.1V_{in} = V_{in}\left(1 - e^{\frac{-t}{RC}}\right) \rightarrow t \approx 2.302RC$$

Subtracting the two time periods

$$2.302RC - 0.105RC \approx 2.2RC$$

$$RiseTime = T_R \approx 2.2RC$$

Using this equation, we could slow down an ideal square-wave's rise-time to any desired value. This would be accurate if the rise time of the input square wave is extremely fast compared to the desired value. However this is rarely the case and another equation needs to be introduced for accurate results. For a system with multiple parts, each portion affects the total rise-time of the system. For example, a system with a function generator, oscilloscope probe and oscilloscope will have a total rise-time that is a function of the rise-times of each component. This total rise time for a system with n distinct rise times is given by the equation,

$$T_{rise\_total} = \sqrt{T_{r1}^2 + T_{r2}^2 \ldots + T_{rn}^2}$$

This equation is basically the same as that for RMS quantities of voltage or current. An example of slowing down the rise time of a square-wave is shown in Fig. 4. The generator has a 10 ns rise time and it is desired to slow it down to 100 ns. To solve this equation for RC,

$$100\ ns = \sqrt{(10\ ns)^2 + T_{rRC}^2}$$

$$(100\ ns)^2 = (10\ ns)^2 + T_{rRC}^2$$

$$10\ fs - 0.1\ fs = T_{rRC}^2$$

$$T_{rRC} = \sqrt{9.9\ fs} = 99.4\ ns$$

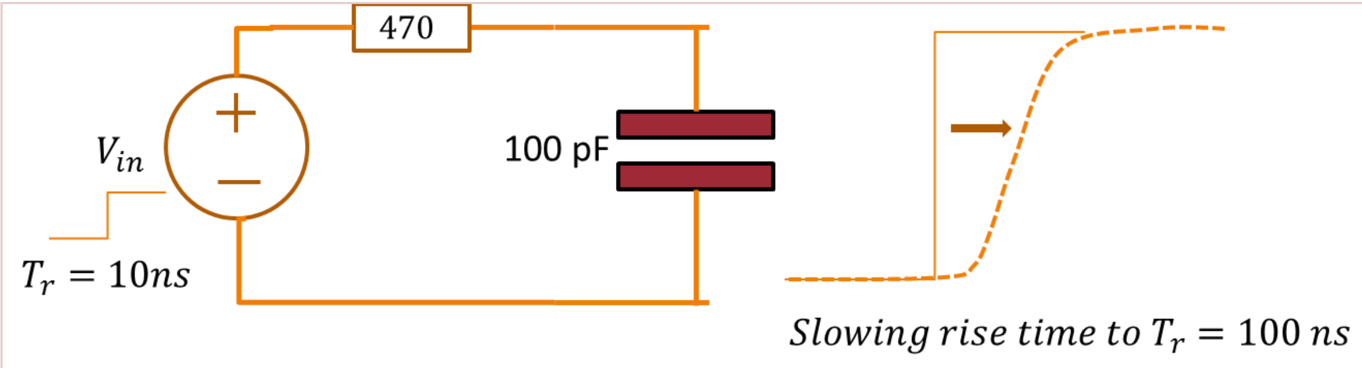


Figure 4. Slowing down rise time. In this case, the rise time of the signal generator can be ignored.

From this result it seems that the 10 ns rise time is fast enough that it can be disregarded. Generally as a rule of thumb if one of the devices has a rise time that is 10 times faster than the other, this calculation is not required. Using the equation $T_R \approx 2.2RC$ is sufficient. Setting the capacitor to be 100 pF, results in a resistor of 454 ohms or 470 ohms for the nearest practical value. Of course, adding this resistor will limit the driving ability of the square wave source and will need to be considered for the application.

Let's repeat this example for an example where the rise time of the function generator is 20 ns. If the rise time needs to be slowed down by 50% to 30 ns,

$$30\ ns = \sqrt{(20\ ns)^2 + T_{rRC}^2}$$

$$(30\ ns)^2 = (20\ ns)^2 + T_{rRC}^2$$

$$0.9\ fs - 0.4\ fs = T_{rRC}^2$$

$$T_{rRC} = \sqrt{0.5\ fs} = 22.3\ ns$$

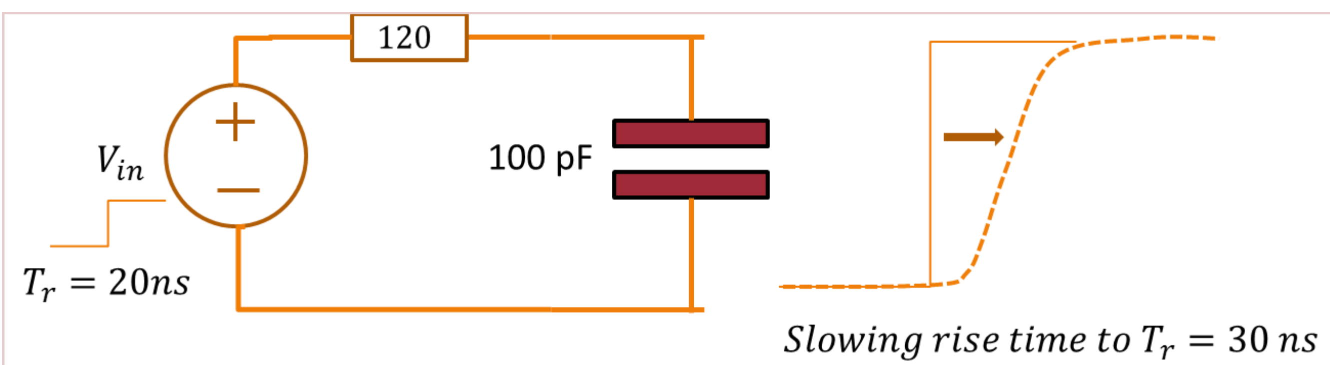


Figure 5. Slowing down rise time. In this case, the rise time of the signal generator needs to be taken into account.

This example shows the importance of using this equation when the rise times are close in value. If only the single RC equation was used the total desired rise time would be 8 ns off target. Although this may not seem like a large amount, in some applications it could be the difference between a product meeting FCC regulations or not. As stated before, resistors come in a wider range of values than capacitors. Let's rewrite the rise time equation to solve for the resistor and set the capacitor to 100 pF,

$$R = \frac{T_R}{2.2C} = \frac{30\ ns}{2.2 * 100\ pF} = 136\ ohms$$

Resulting in a resistor of 136 ohms or 120 ohms for the nearest available value as shown in Fig. 5. These simplified examples don't take the output impedance of the signal generator into account. Ideally it is best to do a SPICE simulation and then fine-tune values in the lab.

RC circuits can also be characterized in the frequency domain. The RC circuit can be characterized with a transfer function. The transfer function describes how the input and output amplitude and phase are related. The output equation for a low-pass RC circuit is

$$V_{out} = V_{in} \frac{1}{1 + RC \cdot s}$$

$$s = j\omega$$

Rewriting this equation, the transfer function is

$$\frac{V_{out}}{V_{in}} = \frac{1}{1 + RC \cdot s}$$

Since the transfer function is complex, the magnitude is an absolute value,

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

The phase is given by,

$$\angle\left(\frac{V_{out}}{V_{in}}\right) = tan^{-1}(-\omega RC)$$

These equations allow the amplitude and phase to be calculated for any frequency for a low pass RC circuit. We will not be using these equations at all in this lab and they are only provided for your reference. Rather we will define the most important frequency for an RC circuit, the "corner" frequency. The corner frequency is so called because it looks like a corner on a frequency response graph. It is also called the -3dB frequency because the magnitude is 3dB lower than the reference point. This corresponds to the frequency where the output power is half of the input power and where the output voltage is 0.707 of the input voltage. The corner frequency is the same for both low-pass and high-pass RC circuits. This is illustrated in Fig. 6. For a low-pass RC circuit, the frequency decreases steadily at 20 dB per decade after the corner frequency. For a high-pass RC circuit the frequency decreases at 20 dB per decade before the corner frequency. Note that frequency response graphs are logarithmic for both amplitude and frequency. This is done so that large variations in quantities can be plotted on the same graph. Also many natural phenomena exhibit a logarithmic response such as human hearing. A decade is a fixed distance whose last point is ten times higher than its first point. For example, 1 Hz to 10 Hz is a decade, 10 Hz to 100 Hz is the next decade, 100 Hz to 1000 Hz is the next decade and so on.

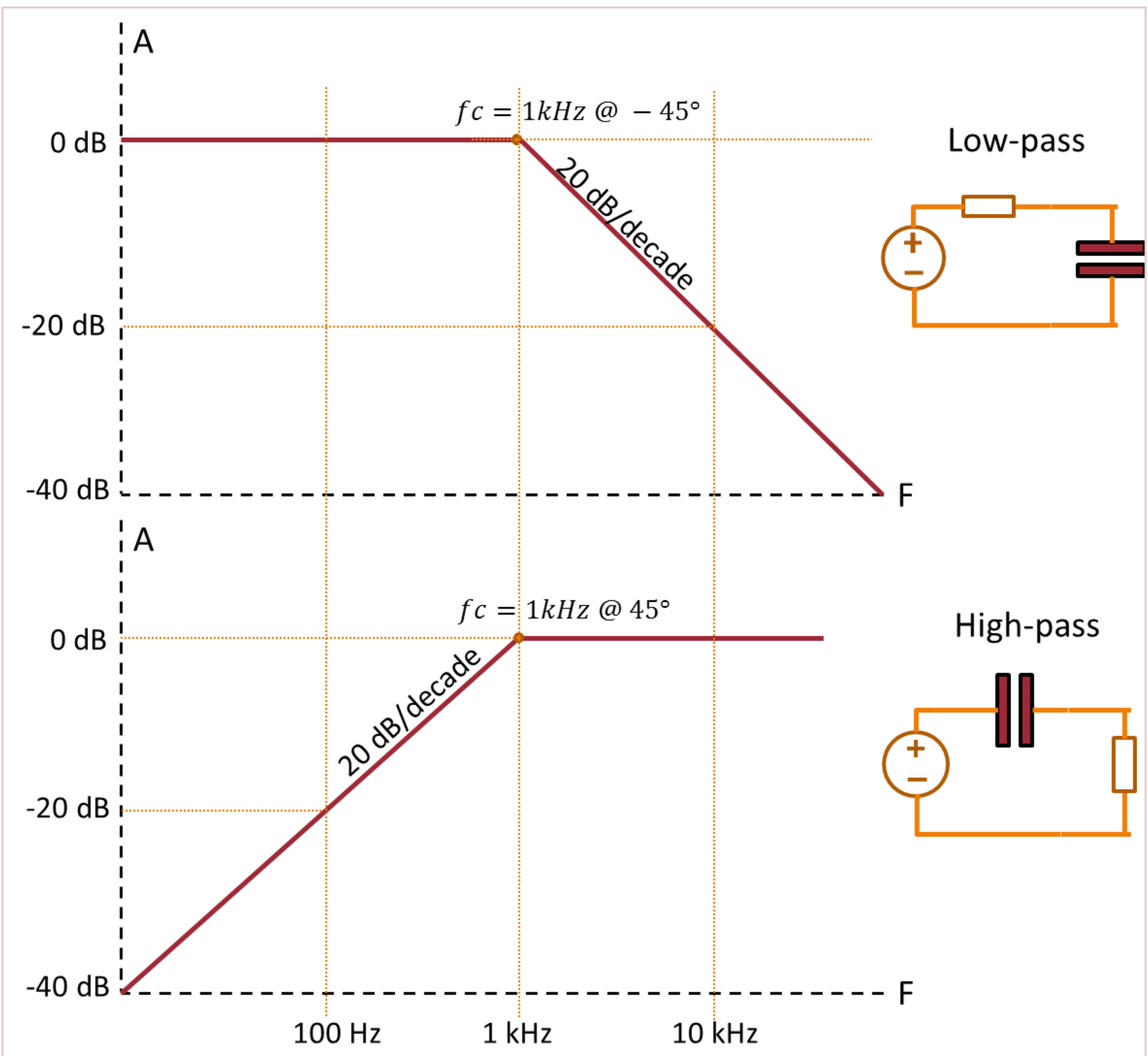


Figure 6. Frequency response plots for low –pass and high-pass RC circuits.

Phase can be displayed on the frequency response plot as well but has been omitted in Fig. 6. for clarity. Instead, the phase shift at the corner frequency is written next to it. For a low-pass RC circuit, the output phase is shifted -45 degrees at the corner frequency. On an oscilloscope, this means the output would appear to lag the input. For a high-pass RC circuit, the output phase is shifted +45 degrees at the corner frequency. On an oscilloscope this means the output would appear to lead the input. This is illustrated for clarity in Fig. 7 so that you will know what you're seeing on an oscilloscope. The corner frequency for both a low-pass and high-pass RC circuit is given by

$$f_c = \frac{1}{2\pi RC}$$

As in previous examples, it is best to select the capacitor value first since there is a more limited selection when designing for a chosen corner frequency. This equation assumes that the source driving the RC circuit has zero output impedance and that there is no loading on the circuit. For accurate analysis these effects should be taken into account.

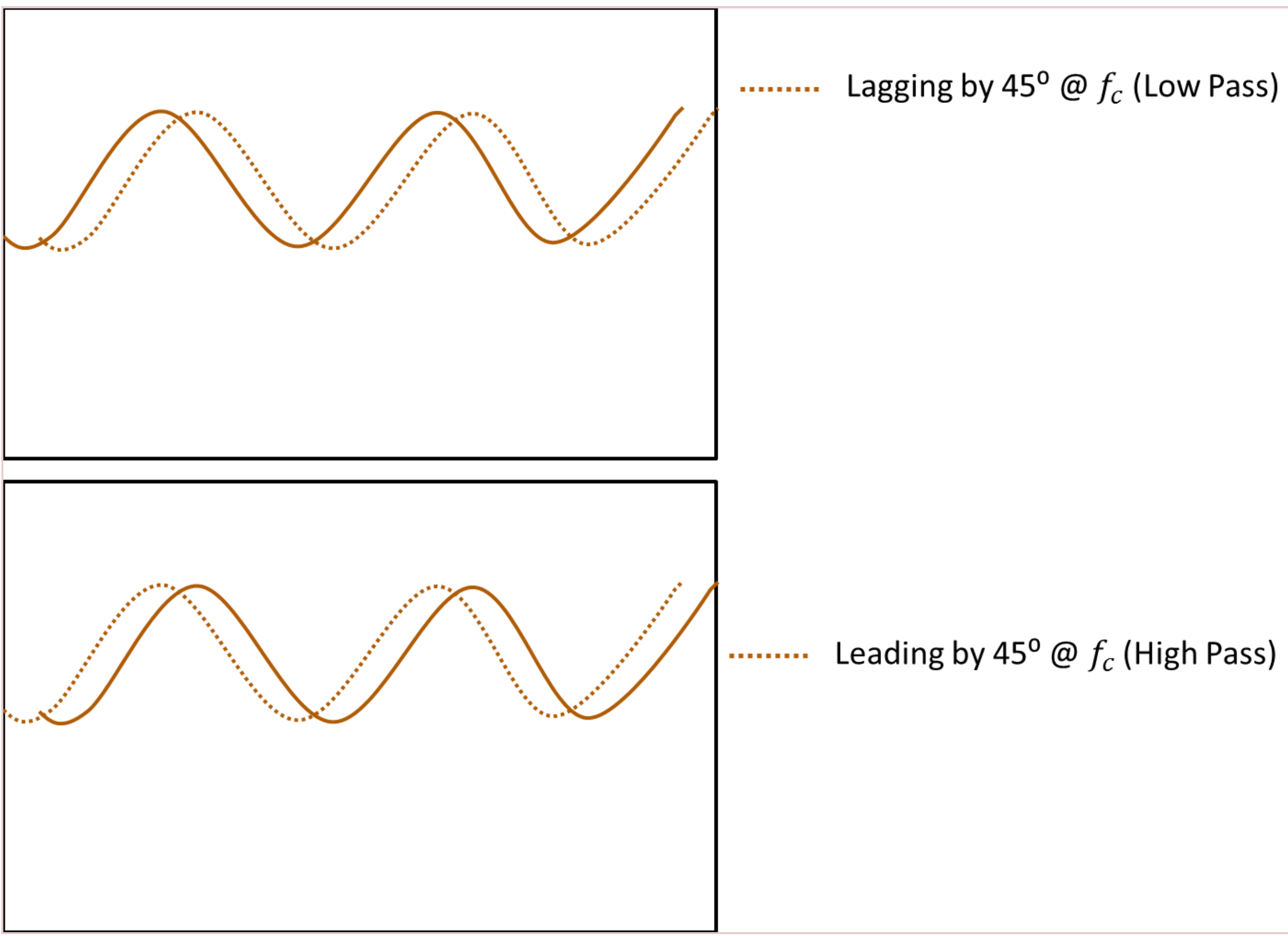
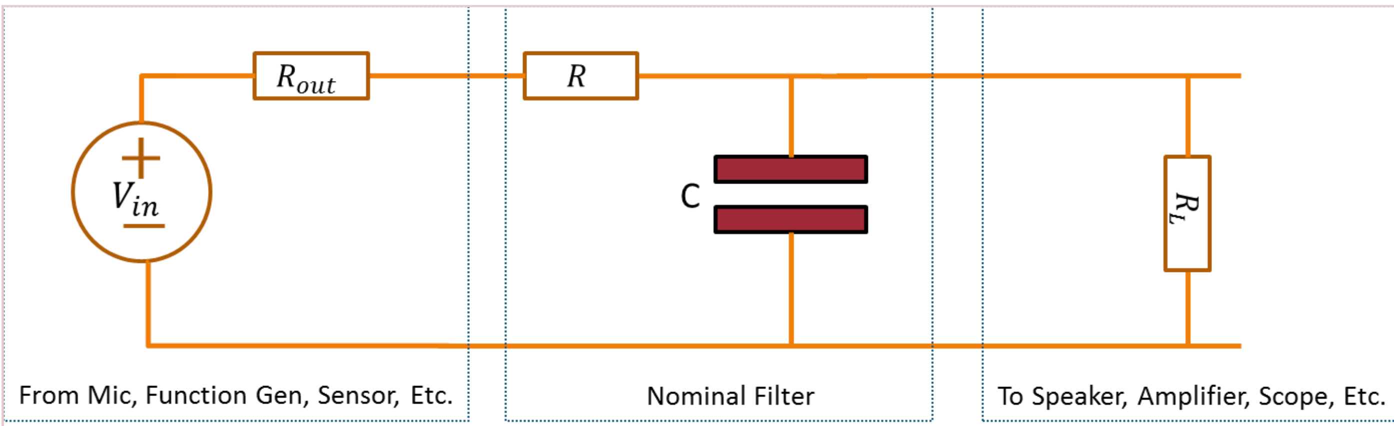


Figure 7. Phase shift on an oscilloscope.

The output impedance of the source (function generators in the lab in our case) should be added to the "R" in the equation if it is 1/10th or greater of the desired calculated R value or if high accuracy is desired. This is straightforward since it is in series with the resistor and can be added. If there is a significant load however, it may not immediately obvious what should be done. The case for finite source impedance (output impedance of the source) in addition to a load resistance is illustrated in Fig. 8. The equation for the corner frequency is changed to account for these differences. This equation was developed by posing the question: What resistance does the capacitor see? Shorting the voltage source and finding the Thevenin equivalent resistance results in the nominal "R" resistor in series with the source resistance of the function generator with the sum of the two in parallel with the load resistance. For a high-pass RC circuit, the modified equation is still valid.



$$f_c = \frac{1}{2\pi((R + R_{out})||R_L)C}$$

Figure 8. Taking output impedance and load resistance into account.

We have examined the low-pass RC circuit in both the time and frequency domains and the high-pass RC circuit in the frequency domain so far. Next we will design a time domain application for the high-pass RC circuit but using the equation for corner frequency. This will illustrate the close relationship between the time and frequency domains. Suppose we wanted a very short pulse, but our function generator has limits on how short the pulse duration can be or it only has a square wave function with duty cycle adjustment. In this case, we can create a pulse duration that is the sum of the rise time and the fall time (we'll assume that they are equal for this example). Nominally, the actual "on" time (flat portion of square wave) is zero but the finite rise and fall times result in some flat portion. For simplicity we will only look at the total duration of the pulse. These types of pulses are used for timing extraction, counters, triggering and many other uses especially in test and measurement.

There is a relationship between the rise time and the maximum bandwidth of a circuit. This is given by the equations,

$$Bandwidth\ in\ Hz = \frac{0.35}{T_R}\ or\ Rise\ time\ in\ seconds = \frac{0.35}{BW}$$

This set of equations allows for conveniently finding the bandwidth or rise time of any circuit given the other. It is a good idea to memorize this relationship because it will be useful in the future. Consider a function generator with a 10 ns rise time. This corresponds to a bandwidth of 35 MHz. This would be the maximum output frequency of a sine wave for this function generator. This is also the reason why a 20 MHz function generator can output a 20 MHz sine wave but not a 20 MHz square wave. Generally a bandwidth of 5 to 10 times greater than the fundamental frequency is needed to create a good quality square wave.

Since the corner frequency is the same as the bandwidth for an RC circuit, we can use the equation for the corner frequency to determine our pulse duration. If the square-wave output from a function generator with a 10 ns rise time is passed through an RC circuit with a corner frequency of 35 MHz, the output will be a 20 ns pulse comprised of the 10 ns rise and fall times. The peak level of the output pulse will be 70% of the input level due to the 3 dB attenuation at the corner frequency. This is illustrated in Fig. 9. An example output is shown in Fig. 10. The initial input is a 1 MHz square-wave and the output is distinct pulses every time the direction of the square wave changes. Note that when the square wave switches from high to low, the output is a negative pulse instead of a positive pulse. This is why a high-pass filter is also called a differentiator. The amplitude is proportional to the rate of change in the input signal. In this case, the circuit is taking the derivative of the square wave.

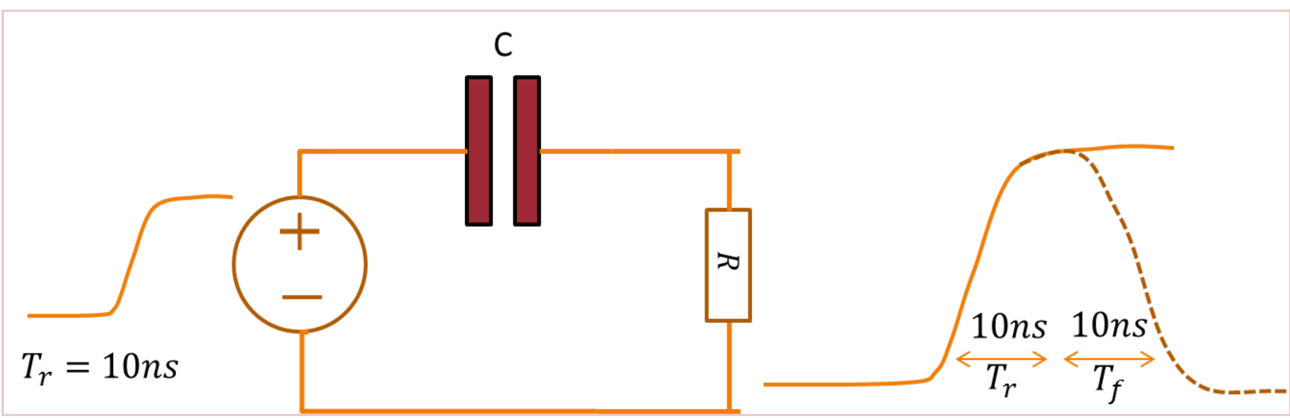


Figure 9. Differentiator circuit.

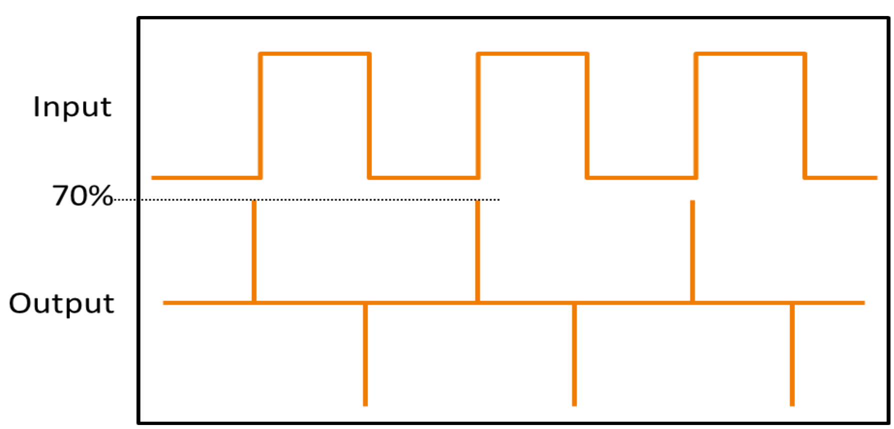


Figure 10. Example output.

Prelab Summary

You will simulate basic RC circuits using .tran and .ac commands. You will also do simple PCB layouts.

Prelab Tasks

Prelab #1: PCB and Netlist Required
Prelab #2: PCB and Netlist Required
The other prelabs use identical circuits so just two PCB layouts are fine.

Prelab

This prelab will extensively use LTSpice. It is assumed that the student has some familiarity with LTSpice and will not go over basic functions. More complex functions will be detailed however. There are many LTSpice tutorial resources available on the web.

Prelab #1

Follow the steps below. Deliverables are in bold.

1. Calculate the time constant for the circuit shown below in Fig. 11. **(Hand Calc)**
2. Simulate the schematic shown in Fig. 11. Verify the calculated time constant. **(Schematic, Plot)**
3. Simulate the schematic shown in Fig. 12. Verify the calculated time constant. **(Schematic, Plot)**

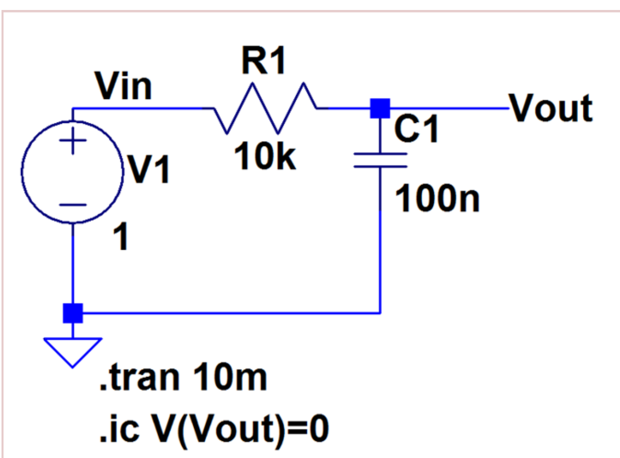


Figure 11. Low-pass RC circuit.

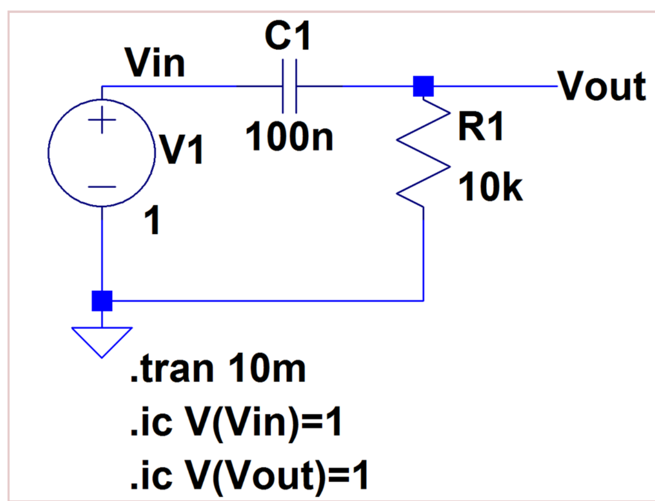


Figure 12. High-pass RC circuit.

Prelab #2

Follow the steps below. Deliverables are in bold.

Simulate the schematic shown in Fig. 13. Note the time that it takes for the circuit to charge to 2/3rd of the input voltage. **(Schematic, Plot)**

Redesign the circuit so that it charges up to 2/3rd of the input voltage in 15 seconds. Solve this equation,

$$V_c = V_{in}(1 - e^{\frac{-t}{RC}})$$

First, leaving everything in variable form to get an approximate expression for Vc equal to 2/3rd of Vin. Your answer should be in the form $time = constant * RC$. **(Hand Calc)**

Simulate your modified circuit and verify that the time to reach 2/3rd input voltage is close to your calculations. **(Schematic, Plot)**

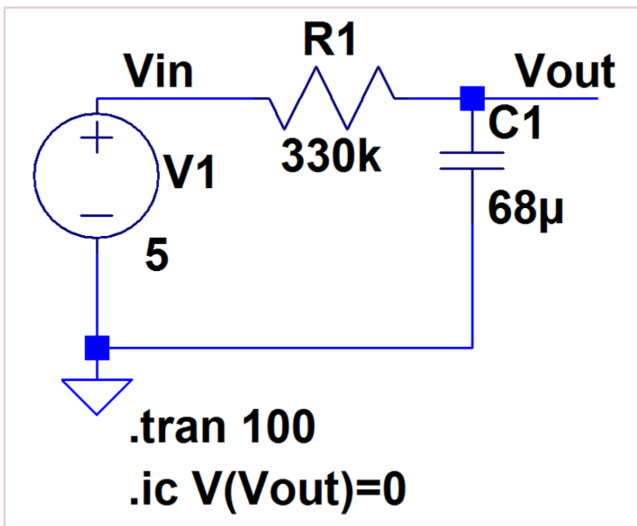


Figure 13. RC timer.

Prelab #3

Follow the steps below. Deliverables are in bold.

Calculate the corner frequency for the circuit shown in Fig. 14. **(Hand calc)**
Simulate the schematic shown in Fig. 14. Be sure to set the AC voltage equal to 1. **(Schematic, Plot)**
Simulate the schematic shown in Fig. 15. Be sure to set the AC voltage equal to 1. **(Schematic, Plot)**

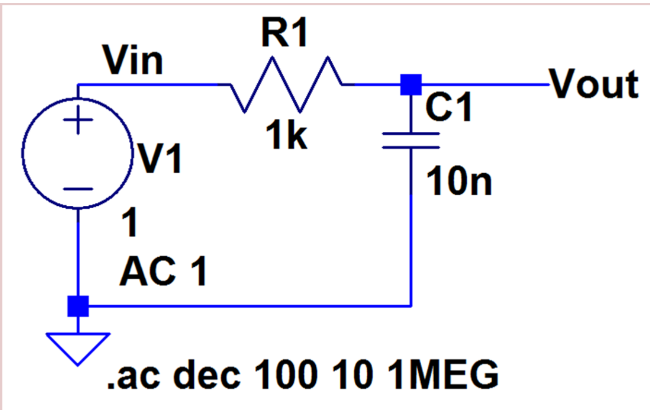


Figure 14. Low-pass RC circuit for AC analysis.

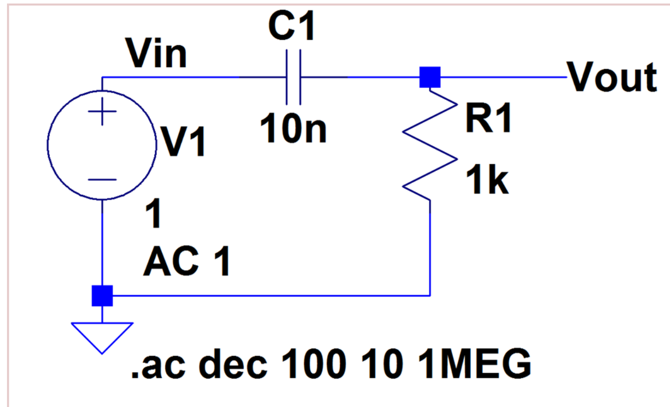


Figure 15. High-pass RC circuit for AC analysis.

Prelab #4

Follow the steps below. Deliverables are in bold.

Calculate the corner frequency for the circuit shown in Fig. 16. Use this to calculate the equivalent rise-time. **(Hand calc)**
Simulate the schematic shown in Fig. 16. **(Schematic, Plot)**

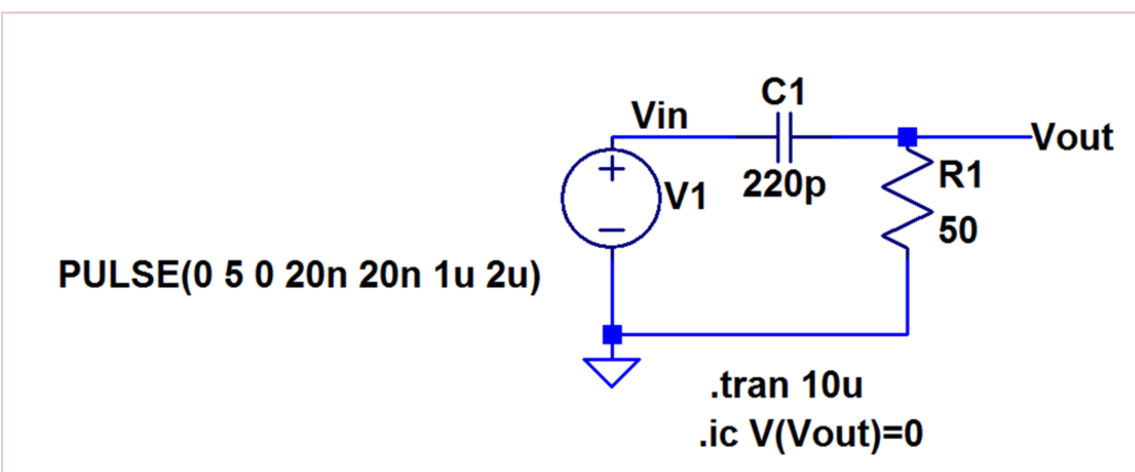


Figure 15. Differentiator circuit.

Required Materials and Equipment

1. Power Supply
2. Oscilloscope
3. Function Generator
4. LEDS, Resistors, Capacitors, Wire
5. 555 Timer
6. Banana Jack Cables
7. Scope Probes
8. BNC to Alligator

Postlab Tasks

Task 1: Rise-time adjustment
Task 2: RC Low-pass
Task 3: 555 Timer
Task 4: Differentiator

Measuring Equipment Basics

In this lab, we will be using the oscilloscope extensively and we will need to properly compensate our scope probes. Using uncompensated scope probes will result in significant errors when measuring rise time. Probe compensation is a way of canceling out the capacitance of the probe cable so that it does not influence the measurement. It is good practice to always compensate a scope probe when using it for the first time and periodically as the properties of the probe may drift over time.

Connect the probe tip and ground clip to the scope's probe compensation terminals. This is a calibrated square wave signal generator inside the oscilloscope. Adjust the vertical and horizontal controls to view the square wave signal. The probe adjustment is on the BNC connector end of the probe. Look for the screw adjustment hole. Undercompensated, compensated and over compensated probes are shown below in Fig. 16, Fig. 17, and Fig. 18 respectively. Adjust the screw until your scope screen looks like Fig. 17. In the undercompensated case, the probe overshoots and will lead to a shorter rise-time measurement. In the overcompensated case, the probe undershoots and will leader to a long rise-time measurement. When properly compensated the oscilloscope can be used to make accurate rise-time measurements.

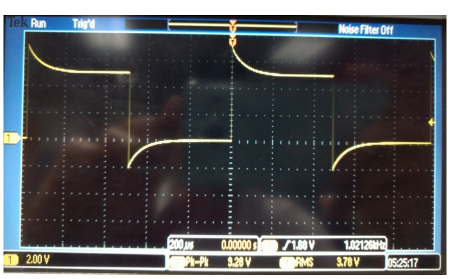


Figure 16. Undercompensated

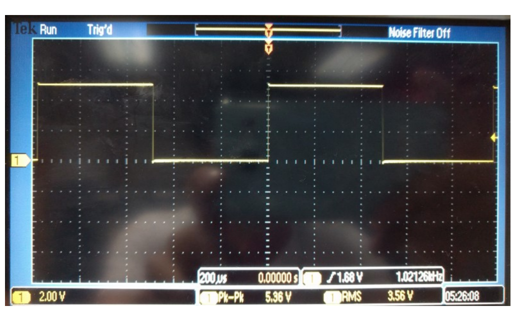


Figure 17. Properly Compensated

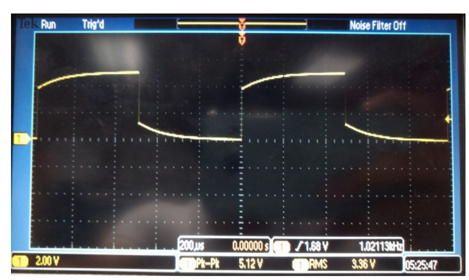


Figure 18. Overcompensated

Postlab #1: Rise-Time Adjustment

Follow the steps below. Deliverables are in bold. All work must be typed.

1. Compensate your scope probes correctly.
2. Measure the rise-time of the function generator itself. Connect a BNC tee to the output of the function generator. Connect a 50 ohm terminator to one end of the tee and connect a BNC cable between the remaining end and the oscilloscope. Set the function generator to output a 100 KHz 1 Vpp square wave with a 0.5V DC offset. The actual amplitude is not critical as long as it's easy to see on the scope. Zoom in on the rising edge of the square wave. Be sure the scope attenuation is set to 1X. Measure the rise time using the oscilloscope cursors. Refer to the manual if you need help with the cursors. Place one cursor at roughly 10% of the waveform amplitude and the other cursor at roughly 90% of the waveform amplitude. The "delta" or difference between these two points is the rise time. **(Photo, Measured Value)**
3. Multiply the rise time by 1.5. We want to slow down the rise time by 50%. Use the examples provided earlier in the background section (pg. 4 and pg. 5.) and calculate the required RC circuit. Be sure that the R value is not too small (should be greater than 100 ohms). You may have to do a few iterations depending on the available components in the lab. Conversely, do not use a resistor that is too large or a capacitor that is too small. A 10X scope probe presents a 10 MEG load resistance to the circuit. If a large resistor with a value in the region of mega-ohms is used, then the scope probe's impedance will significantly affect the result. Another issue that arises is the capacitor value has to scale down accordingly into the picofarads. The scope probe has around 13 pF of capacitance which will also affect the measurement. The circuit will be sensitive to capacitive coupling and the waveform on the scope will be prone to interference. In short, start with resistor values in the kilo-ohms to hundreds of kilo-ohms. **(Hand calc)**
4. Build this RC circuit on the breadboard. Connect a BNC to alligator cable to the input of this circuit and a scope probe to the output. It is best to use a 10X scope probe and be sure to set the attenuation setting to 10X on the oscilloscope. Measure the change in rise time and compare to your hand calculations. **(Photo, Measured Value)**

Postlab #2: RC Low-pass Filter

Follow the steps below. Deliverables are in bold. All work must be typed.

1. Use the same set-up as the previous lab exercise but remove the 50 ohm terminator. Also use a si-ne-wave output since you will be examining the circuit in the frequency domain.
2. Design a low-pass filter with a cut-off (corner) frequency of 100 KHz. Use a 75 ohm load resistor and use the circuit and equation in Fig. 8 to assist in designing the filter. Remember the output imped-ance of the function generator is 50 ohm. You don't have to get an exact frequency match since the component selection in the lab may be limited. Be sure to note this if you had to chose a different frequency for your design. Show that the output amplitude drop at the corner frequency is –3 dB or 0.707. For your reference point measure the voltage with the load connected and no filter in place. If you get your reference point without a load connected then your calculations will be wrong. **(Hand Calc, Measured Value, Photos)**
3. Measure the rise-time of the filter as in the previous lab. Be sure to use a square wave output. Based on this rise-time calculate the cut-off frequency using the rise time to bandwidth relation-ship. Compare this result to the measured cut-off frequency. **(Measured Value, Hand Calc, Photo)**

Postlab #3: 555 Timer

Follow the steps below. Deliverables are in bold. All work must be typed.

1. This lab exercise will introduce you to your first active circuit, meaning a circuit that requires a DC power source to operate. Skim through the LM555 timer datasheet. There are many variants of the 555 timer but most are interchangeable. It is a good idea to check the datasheet though to look for differences. For this lab, almost all 555 timer chips are suitable. You do not need to understand the internal operation of the circuit to use it.
3. The pin-out for the 555 timer is shown below in Fig. 19. The circuit you will be constructing is shown in Fig. 20. Build this circuit on a breadboard and try to keep connections short and neat. The 10 nF capacitor value is not critical and you can substitute the next higher value if you can't find 10 nF.
4. Use a Vcc voltage of 5V. Select R and C based on this equation, $t = \ln(3) \cdot RC \approx 1.1RC$
5. Set the time for 10 seconds or a different value that is convenient based on available components. Connect a high value resistor from the "Trigger" pin to Vcc. The value is not critical but be sure to select one that is over 100k. Connect a wire to the trigger connection and leave the other end floating for now. When this pin goes low or connects to ground, the circuit is triggered and the output goes to Vcc for the set time interval as shown in Fig. 21.
6. Let's connect an LED to the output so that we can see the output pulse duration. The voltage at the output pin will be 5 V. Using this and the LED's forward voltage as described in "Lab #1: DC Circuits", select a resistor that limits the current to 20 mA. Finally, to trigger the circuit, plug the floating end of the trigger wire into a ground connection for a moment and pull it out. The LED should light up and then turn off after the time interval. Measure the time duration using a clock or timer. **(All hand calcs in all steps, Photo of circuit)**

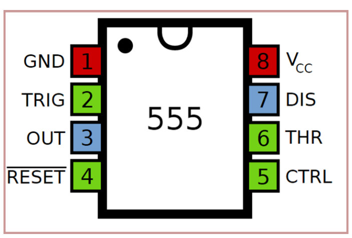


Figure 19. Pin out of 555 timer.

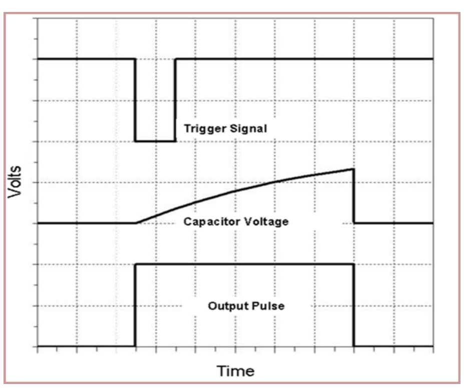


Figure 21. Operation of 555 timer.

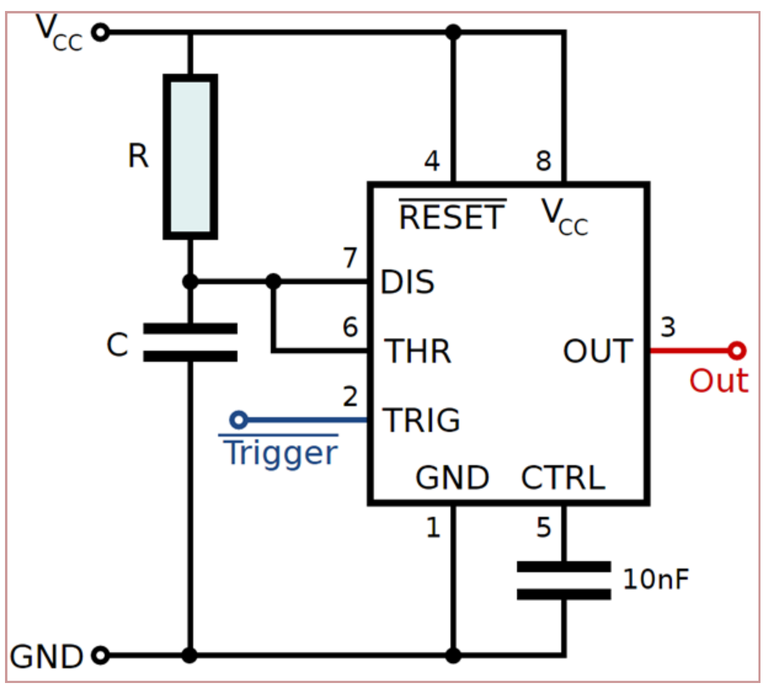


Figure 20. Schematic of monostable application of 555 timer.

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Postlab #4: Differentiator

Follow the steps below. Deliverables are in bold. All work must be typed.

1. Use the measured rise-time of the function generator from the first lab exercise and calculate the bandwidth of the function generator using the equations for time to bandwidth conversion. **(Hand Calc)**
2. Design a high-pass filter with a cut-off frequency equal to the bandwidth. For best results select R to be 51 ohms and you have to pick an appropriate capacitor. If you can't find the exact value round up or down to the next available value. **(Hand Calc)**
3. Use the same test setup as Postlab #2. Measure the peak to peak output amplitude of the function generator without the differentiator circuit to get a reference measurement. Use a 100 KHz or 1 MHz square wave. For the reference measurement use the 50 ohm terminator to get a correct reading. Otherwise your final values will be incorrect.
4. Disconnect the 50 ohm terminator and connect the differentiator circuit. Note that the output amplitude of the pulse should be around 70% of the reference value. Due to variations in components and test set-up you may be off by 10% or more. Note the rise time and pulse duration. An example of a differentiator circuit output is shown below in Fig. 22. **(Photos)**

Figure 22. Output of differentiator circuit.