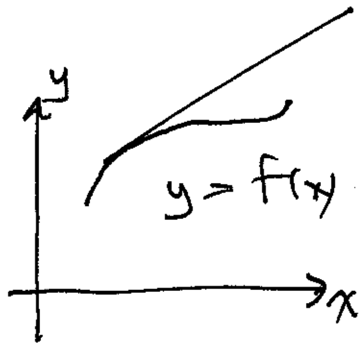
Math 2471- Calc 3

Tangent Plane

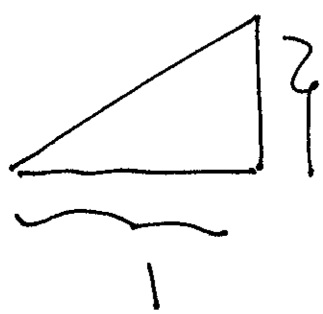
Consider



slope of tangent

f'

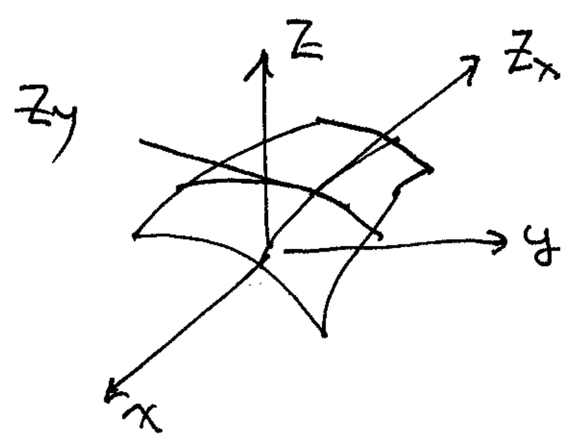
we associate a vector here



slope is $\frac{f'}{1}$

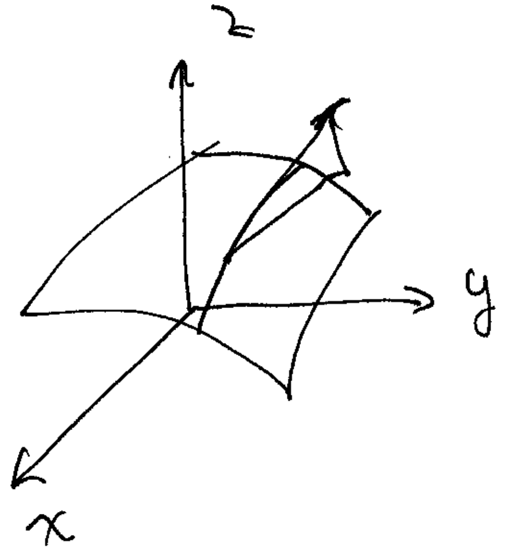
So tangent vector is $\langle 1, f' \rangle$

Now we go to 3-D

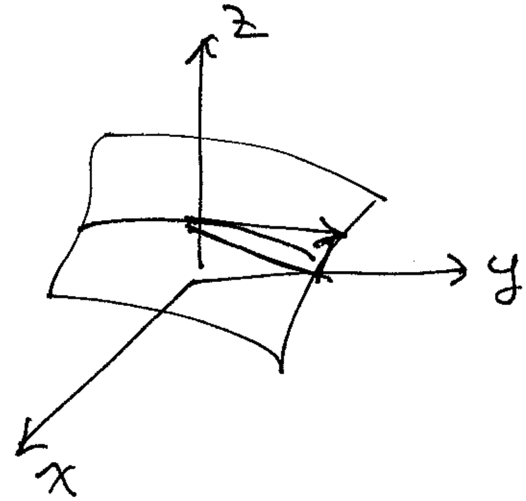


we will identify 2 tangent vector

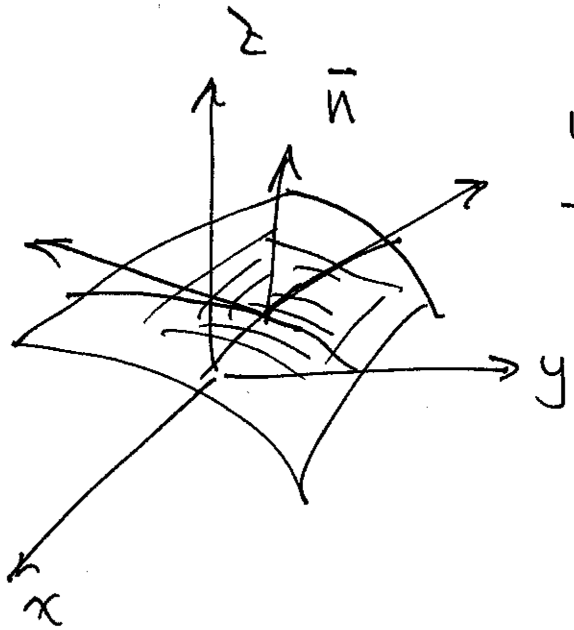
$\langle 1, 0, z_x \rangle$ $\langle 0, 1, z_y \rangle$



no variation in y direction



no variation in x direction



if we cross these ~~to~~ we get a normal vector so $\bar{n} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & 0 & z_x \\ 0 & 1 & z_y \end{vmatrix}$

$$= \begin{vmatrix} 0 & z_x \\ 1 & z_y \end{vmatrix} \bar{i} - \begin{vmatrix} 1 & z_x \\ 0 & z_y \end{vmatrix} \bar{j} + \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \bar{k}$$

$$= -z_x \vec{i} - z_y \bar{j} + \bar{k}$$

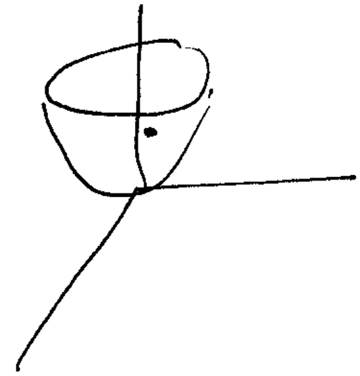
$$= \langle -z_x, -z_y, 1 \rangle \Big| \text{ at same pt.}$$

so $P(x_0, y_0, z_0)$

$$-z_x\big|_p (x - x_0) - z_y\big|_p (y - y_0) + 1\,(z - z_0) = 0$$

or $z_x\big|_p (x - x_0) + z_y\big|_p (y - y_0) - (z - z_0) = 0$

ex $z = x^2+y^2$



at $P(1,2,5)$

$z_x = 2x, \quad z_y = 2y$

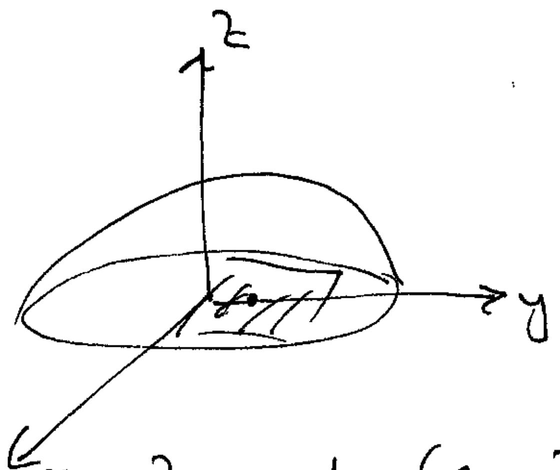
$z_x|_P = 2 \quad z_y|_P = 4$

so TP $2(x-1)+4(y-2)-(z-5)=0$

$$2x-2+4y-8-z+5=0$$

$$\boxed{2x+4y-z=5}$$

Ex $x^2+y^2+z^2=9$ $\quad P(2,2,1)$



$$z = +\sqrt{9-x^2-y^2}$$
$$= (9-x^2-y^2)^{1/2}$$

$$z_x = \frac{1}{2}(9-x^2-y^2)^{-1/2}(-2x)$$

$$= \frac{-x}{\sqrt{9-x^2-y^2}}$$

$$z_y = \frac{-y}{\sqrt{9-x^2-y^2}}$$

at $P(2,2,1)$ $\quad z_x\Big|_P = \frac{-2}{\sqrt{9-4-4}} = -2, \quad z_y = -2$

T.P. $\quad -2(x-2) - 2(y-2) - 1(z-1) = 0$

$$-2x+4 - 2y+4 - z+1 = 0$$

$$2x+2y+z = 9$$

Note: $z = \sqrt{9-x^2-y^2}$

$$z_x = \frac{-x}{\sqrt{9-x^2-y^2}} = -\frac{x}{z}$$

$$z_y = -\frac{y}{z}$$

so maybe I could go directly to

$$z_x = -\frac{x}{z}, \quad z_y = -\frac{y}{z}$$

from $x^2+y^2+z^2=9$

Implicit Differentiation!

HW pg 935 # 9, 17, 19, 21, 23
1st pt only