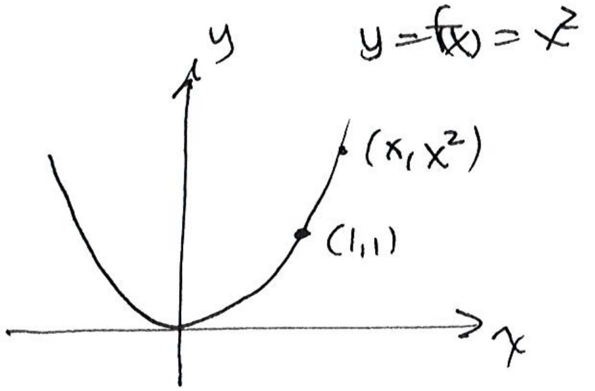
Math 1496 Calc 1

9/19/22

1-4

Earlier we considered



slope of secant line → tangent

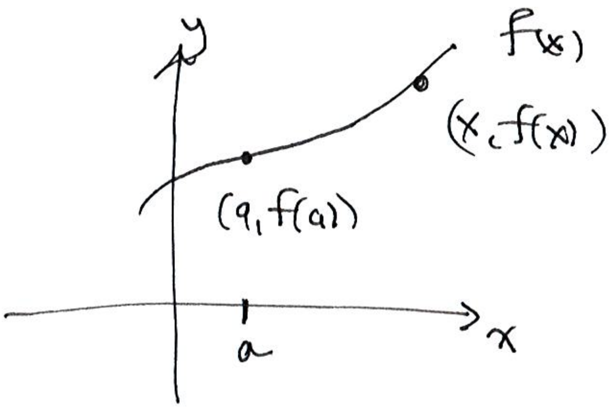
$$\lim_{x\to 1}\frac{x^2-1}{x-1}=\lim_{x\to 1}x+1=2$$

So slope of tangent is $m=2$

tangent line $y-y_0=m(x-x_0)$

so here $y-1=2(x-1)$

So can we do this in general?



$$\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

we call this the derivative at the pt $x=a$

notation $f'(a)$

Ex 1 $f(x) = \sqrt{x}$ $x = 4$ $(a = 4)$

$$\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4}$$

$$\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2} = \lim_{x \to 4} \frac{\cancel{x - 4}}{\cancel{(x-4)}(\sqrt{x}+2)}$$

$$\lim_{x \to 4} \frac{1}{\sqrt{x}+2} = \frac{1}{2+2} = \frac{1}{4}$$

tangent $y - 2 = \frac{1}{4}(x-4)$

~~Ex~~ Def^n If

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$ exists we say

f is differentiable at $x = a$

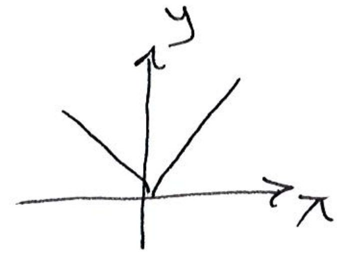
if it diff. ~~on~~ at each pt in (a, b)

we say it is diff on the entire interval ex 2 $\quad f(x) = \sin x$ at $x = 0$

$$\lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0} \frac{\sin x}{x} = 1$$

so yes f is diff at $x = 0$

ex 3 $\quad f(x) = |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$



corner at $x = 0$

so it depends on which side we approach

$$\lim_{x \to 0^-} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^-} \frac{-x}{x} = -1$$

$$\lim_{x \to 0^+} \frac{f(x) - 0}{x - 0} = \lim_{x \to 0^+} \frac{x}{x} = 1$$

Note

$f'_-(0) = -1$

$f'_+(0) = 1$

since these 2 are not the same

f is not diff at $x = 0$

Ex 3 Given

$$f(x) = \begin{cases} 5-2x & x \geq 1 \\ 4-x^2 & x < 1 \end{cases}$$

(1) Is f cont^s at $x=1$

(2) Is f diff at $x=1$

Cont^s

(1) $\lim_{x \to 1^-} 4-x^2 = 3$

$\lim_{x \to 1^+} 5-2x = 3$

so limits exists $\lim_{x \to 1} f = 3$

(2) $f(1) = 3$

(3) $\lim_{x \to 1} f = f(1)$ $\quad$ f is cont^s at $x=1$

Diff $\lim_{x \to 1^-} \frac{f(x)-f(1)}{x-1} = \lim_{x \to 1^-} \frac{4-x^2-3}{x-1} = \lim_{x \to 1^-} \frac{1-x^2}{x-1}$

$$= \lim_{x \to 1^-} -\frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}} = -2$$

$$\lim_{x \to 1^+} \frac{5-2x-3}{x-1} = \lim_{x \to 1^+} \frac{2-2x}{x-1} = \lim_{x \to 1^+} \frac{-2\cancel{(x-1)}}{\cancel{x-1}} = -2$$

$\because$ these are the same f is diff at $x=1$

$$f'(1) = -2$$