Multidimensional-multicomponent inversion of transient electromagnetic data: Synthetic and field data applications

R. Martin[1,2], and B. Tezkan[1]
[1] Institute of Geophysics and Meteorology, University of Cologne, Germany
[2] Zentrum für Geoinformationswesen der Bundeswehr, Euskirchen, Germany

Summary

A 3D inversion algorithm was developed for the interpretation of long offset (LOTEM) and short offset transient electromagnetic (SHOTEM) data, taking into account an explicit sensitivity calculation in the time domain. The computation of sensitivities is carried out by using the adjoint Green functions approach which is realized by convolution of the background electric field, originating from the primary signal, with the impulse response of the receiver acting as a secondary source. Maxwell's equations are solved directly in the time domain by using the high efficient Krylov-subsurface technique. The 2.5D inversion problem is solved by a regularized Gauss-Newton optimization, whereas a gradient based inversion scheme is used for the 3D inversion. The 3D inversion algorithm was tested and verified using synthetic LOTEM and SHOTEM data. To test the algorithm, two simple models were considered: First, we successfully carried out a 3D inverse modeling of a conductive cube at a depth of 300 m using a LOTEM configuration. Secondly, a thin conductive anomaly within 5 m depth was resolved using a SHOTEM setup. Both conductivity anomalies are embedded in a resistive homogeneous half-space. In the case of LOTEM, two galvanic coupled square-wave transmitter signals were considered and their electric and magnetic field responses were calculated at 168 receiver locations. For the 3D inversion of SHOTEM data an innovative and unusual field structure was used. It consists of six loop wires with overlapping areas. Three orthogonal magnetic field components were considered at different positions within the loop, allowing for multisite and multi-component 3D inversion. Until now, our 3D TEM inverse modelling was only tested by using simple conductivity models. However, the results of the 3D inversion of synthetic LOTEM and SHOTEM are very promising. The spatial dimensions of the conductive bodies as well as their specific resistances were clearly resolved within the LOTEM and SHOTEM examples. The inversion algorithm was also successfully applied to a SHOTEM field data set observed on a waste deposit resulting in a reliable 3D conductivity model.

Keywords: LOTEM, SHOTEM, 3D inversion, synthetic data, SHOTEM field application

Formulation of the regularized inverse problem for transient electromagnetics

The general formulation of the inversion problem for transient electromagnetics can only be solved as regularized optimization due to nonlinearity and ambiguity. Thus, first of all the minimization of the cost function

$$\begin{aligned} \Phi &= \Phi_d + \lambda \Phi_m \\ &= \left|\left|\mathbf{C}_d^{-\frac{1}{2}} \left(\mathbf{d} - \mathbf{f}\right)\right|\right|_2^2 + \lambda \left|\left|\mathbf{C}\left(\mathbf{m} - \hat{\mathbf{m}}\right)\right|\right|_2^2 \end{aligned} \quad (1)$$

of the measured data $\mathbf{d} \in \mathbb{R}^N$, of the model $\mathbf{m} \in \mathbb{R}^M$, and of the model response $\mathbf{f(m)} : \mathbb{R}^M_{>0} \rightarrow \mathbb{R}^N$ is formulated. $\mathbf{C}_d$ describes the data-covariance matrix, $\hat{\mathbf{m}}$ is our a priori model (i.e. a homogenous halfspace). Φ_d is the cost function of the data and Φ_m is the cost function of the model which is weighted by the Lagrange parameter $\lambda \in \mathbb{R}_{>0}$. Eq. (1) is minimized by solving the normal equations for a global regularization.

$$\begin{aligned} &\left(\mathbf{S}_n^T \mathbf{C}_d^{-1} \mathbf{S}_n + \lambda_n \mathbf{C}^T \mathbf{C}\right) \delta \mathbf{m}_n \\ &\quad = \mathbf{S}_n^T \mathbf{C}_d^{-1} \left(\mathbf{d} - \mathbf{f}(\mathbf{m}_n)\right) - \lambda_n \mathbf{C}^T \mathbf{C} \Delta \hat{\mathbf{m}}_n \end{aligned} \quad (2)$$

In Eq.(2), $\mathbf{S}_n$ denotes the sensitivity matrix, $\delta \mathbf{m}_n$, the n-th model update, and $\delta \hat{\mathbf{m}}_n = \mathbf{m}_n - \hat{\mathbf{m}}$ our model update. The gradient based minimization scheme (e.g. Hestenes & Stiefel, 1952) is used to solve the ill-posed and non-linear inverse problems. The computation of sensitivities is carried out by using the adjoint Green functions according to (Hördt, 1998, Martin, 2009) which is realized after each inversion step. In this way, the information content of the calculated inversion models can be better studied. In addition to the calculation of sensitivities, the forward calculation plays an important role for the implementation of the inverse problem. The high efficient Krylov-subspace technique of Druskin and Knizhnerman(1988, 1994) was used to solve the Maxwell equations directly in time domain. The regularization term was determined using the discrepancy principle as proposed by Constable *et al.*, 1987.

TEM measurements over 3D structures often involve sign reversals. Different methods exist to take both large amplitude variations and different signs into account. We use the Area-Sinus-Hyperbolicus-transformation (e.g. Hördt,

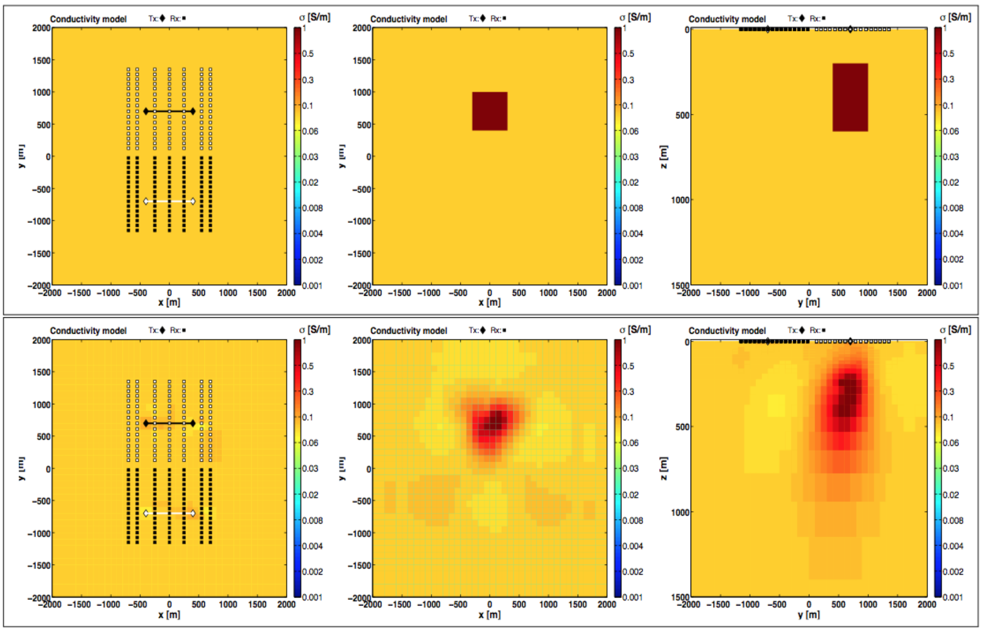

Figure 1: Comparison between the true model (upper part) and the 3D inversion result (bottom) for a LOTEM field set-up. The left row shows the plane view of the model at the surface with the LOTEM field setup. It consists of two dipole transmitters with a length of 1 km. Corresponding receivers and transmitters are color coded respectively.

1992) for the model responses and sensitivities within the inversion procedure (Martin, 2009).

3D Inversion of the synthetic transient electromagnetic data

The developed 3D inversion algorithm was tested for galvanic coupled dipole transmitters (LOTEM) and inductive coupled sources (SHOTEM) by using synthetic data.

LOTEM example

The upper part of Fig. 1 shows the unconventional LOTEM set-up and the conductivity model used in this study. Two in x-direction oriented dipole transmitters with a length of 1 km and spatially distributed receivers were necessary to resolve the good conductive box (1 Ωm) with an edge length of 500x500x400 m^3 in a depth of 300 m. The conductivity anomaly is embedded in a homogenous half-space (10 Ωm). At the spatially distributed receiver-sites Ex, $\dot{H}_y$, and $\dot{H}_z$ are calculated between 0.5 ms and 0.83 s at 21 time points using the forward algorithm of Druskin and Knizhnerman (1994). 5 % Gaussian noise was added to the data before the inversion procedure. A homogenous 10 Ωm half-space was used as a starting model. The model consists of 40572 model unknowns which are discretized in the x, y, and z directions. The lower part shows the result of the inversion. A good fitting between model response and synthetic data was achieved after 10 iteration steps (Martin, 2009).

SHOTEM example

To test the 3D inversion algorithm for the SHOTEM synthetic data, an uncommon in-loop configuration was used: In addition to the conventional vertical component we use two orthogonal horizontal components to increase small scale sensitivity. Also, the three components of the magnetic field were measured within different sites within the transmitter loops. Last not least, the whole set-up consists of several overlapping transmitters covering a larger area. A similar configuration was used for the field data. Fig. 2 shows our unconventional SHOTEM setup as well as the conductivity model. The setup consists of six loop wires with overlapping areas. Each transmitter comprises of 25 receiver sites. The distance between the receiver sites is 12.5 m in each direction. At the receiver sites, the $\dot{H}_x, \dot{H}_y$ and $\dot{H}_z$ magnetic field responses due to transmitter current shut-offs are calculated between 1 μs and 0.1 ms. The required model is thin conductivity anomaly (2 Ωm) with an extension of 110x70x10 m, embedded in a 100 Ωm homogenous half-space. The model discretization consists of 35200 unknowns. The lower part of Fig. 2 shows the results of the 3D inversion using a homoge-

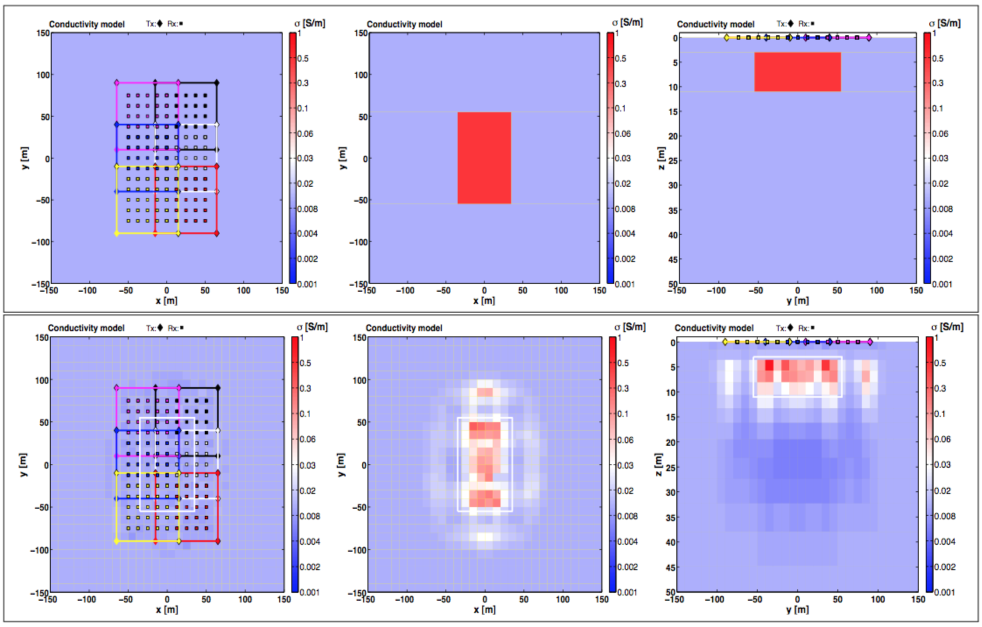

Figure 2: Comparison between the true model (upper part) and the 3D inversion (bottom) for a SHOTEM setup. The left row shows the plane view of the model at the surface and the SHOTEM setup. It consists of 6 in-loop-setups within an area of 80x80 m^2. The 25 multicomponent receivers are color-coded according to their transmitters.

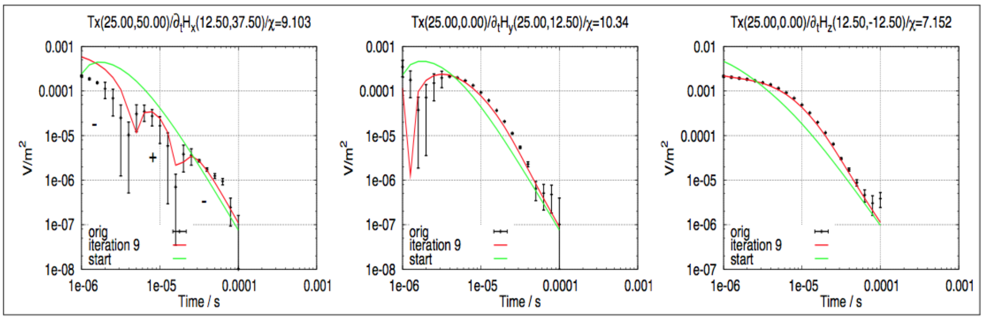

Figure 3: Comparison of selected synthetic data of the final model of the inverse solution with the original data for different receiver stations.

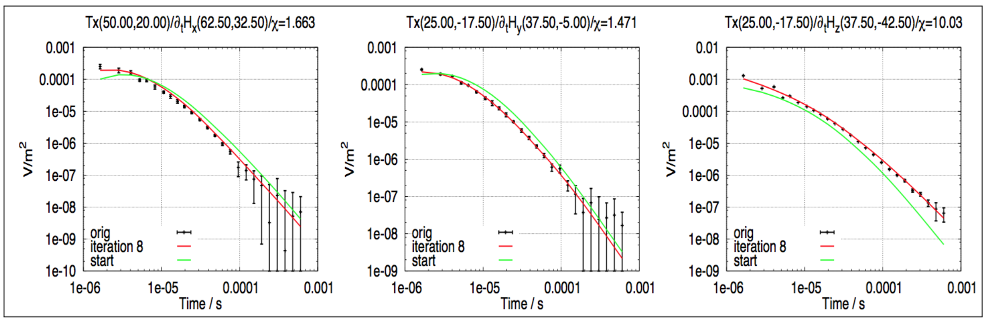

Figure 4: The comparison between data and model responses (start and final model response after eight iterations).

nous half-space as the starting model. As you can see, the good conductive anomaly is recovered after 10 iterations indicating the good resolution of the conductivity structure.

In Fig. 3 $\dot{H}_x$ (left), $\dot{H}_y$ (middle), and $\dot{H}_z$ (right) synthetic and 3D model responses are displayed for selected, different transmitter receiver locations. The response of the starting model is also shown. A relatively good fitting between synthetic and the 3D inverse modelling was achieved. In particular, the sign reversals in the $\dot{H}_x$ components, indicated by the notches in Fig. 3, could be reproduced well by the 3D inversion.

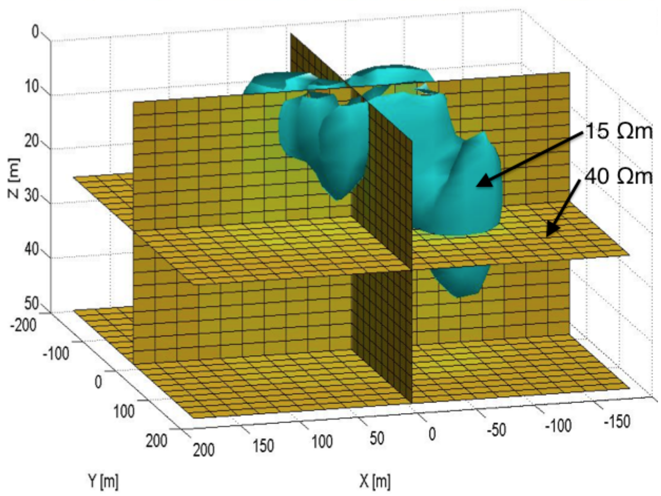

Figure 5: Results of our 3D inverse modelling of the SHOTEM field data observed on the waste deposit in Düren, Germany.

Field example

SHOTEM measurements were carried out on a waste site near Düren in Germany. Five overlapping transmitters with 80 m x 80 m loops were used and $\dot{H}_x, \dot{H}_y, \dot{H}_y$ were measured at 25 stations within the transmitters. This is a similar field setup as shown in Fig. 2 for the synthetic SHOTEM 3D inversion example. In order to have a good prior model, we carried out individual 1D inversions using the dHz/dt components. The derived 1D conductivity models are interpolated at different stations and used as a starting model for the 3D inversion. A 15 Ωm iso surface of the 3D conductivity model of our inverse modeling is shown in Fig. 5. The landfill could be characterized as a very inhomogeneous mixture of well and weakly conducting anomalies in a resistive environment (gravel and sand). The comparison between the measured data and the results of the model responses at some selected stations indicate a relatively good fitting by our inverse modelling (Fig. 4). The fitting error was 65 % for the starting model, and a 10 % error level was obtained after eight iterations, proving the fast overall convergence of the algorithm.

Conclusions

A multidimensional inversion algorithm is presented allowing the explicit calculation of sensitivities. The algorithm was tested on two synthetic data sets using innovative field setups. The results indicating, that a) spatial borders of the conductivity structures and b) electrical conductivities can be resolved by our 3D TEM inverse model. Horizontal components were important for the resolution of the near surface anomaly for our 3D inversion of the SHOTEM field data. The 3D inverse modelling was also successfully applied on the SHOTEM field data observed on a waste site near Düren, Germany.

REFERENCES

Constable, S. C., Parker, R. L., & Constable, C. G. (1987). Occamâs inversion: a practical algorithm for generating smooth models from em sounding data. *Geophysics*, *52(3)*, 289–300.

Druskin, V. L., & Knizhnerman, L. A. (1988). Spectral differential-difference method for numeric solution of three-dimensional nonstationary problems of electric prospecting. *Earth Physics*, *24(8)*, 641â648.

Druskin, V. L., & Knizhnerman, L. A. (1994). Spectral approach to solving three-dimensional maxwellâs diffusion equations in the time and frequency domains. *Radio Science*.

Hestenes, M. R., & Stiefel, E. (1952). Methods of conjugate gradients for solving linear systems. *J. Res. Nat. Bur. Stand.*, *49(6)*.

Hördt, A. (1998). Calculation of electromagnetic sensitivities in the time domain. *Geophysical Journal International*, *133*(3), 713–720. doi: 10.1046/j.1365-246x.1998.00533.x

Hördt, A., Jödicke, H., Strack, K.-M., Vozoff, K., & Wolfgram, P. A. (1992). Inversion of long-offset TEM soundings near the borehole münsterland 1, germany, and comparison with MT measurements. *Geophysical Journal International*, *108*(3), 930–940. doi: 10.1111/j.1365-246x.1992.tb03481.x

Martin, R. (2009). Development and application of 2d and 3d transient electromagnetic inverse solutions based on adjoint green functions: A feasibility study for the spatial reconstruction of conductivity distributions by means of sensitivities. university of cologne. *doctoral dissertation*.