

Mathematics Manipulatives Mania!



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Introduce yourself.

Describe your strengths in supporting mathematics.

Describe an opportunity for growth.



Operations and Place Value

Fractions and Decimals

Integers and Algebra

Geometry



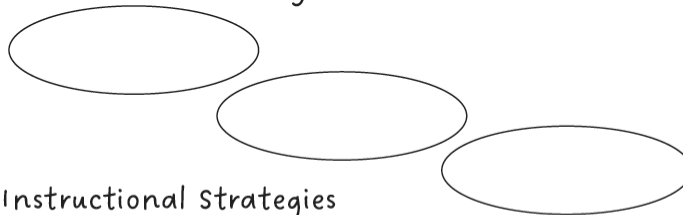


Math Manipulatives Mania!

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Instructional Platform

Instructional Delivery



Instructional Strategies



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Instructional Platform

INSTRUCTIONAL DELIVERY

Vocabulary

Representations

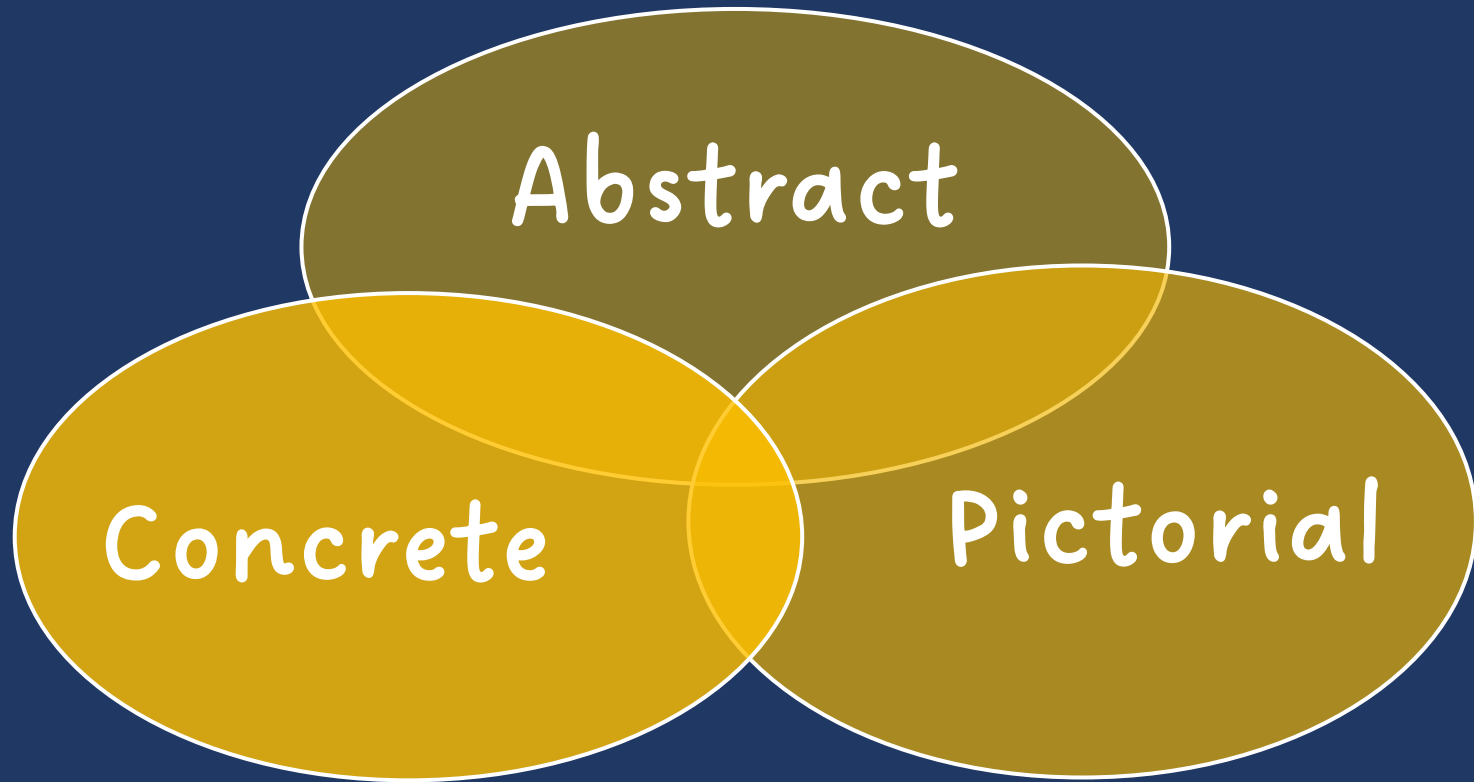
Model and
Practice

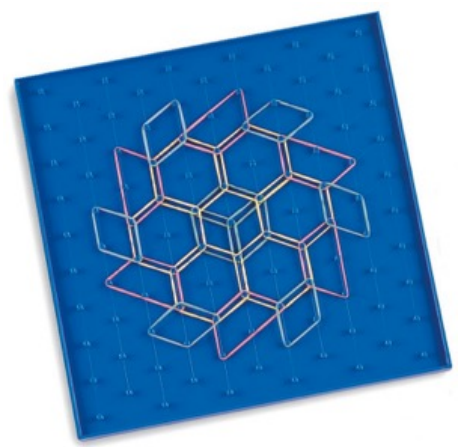
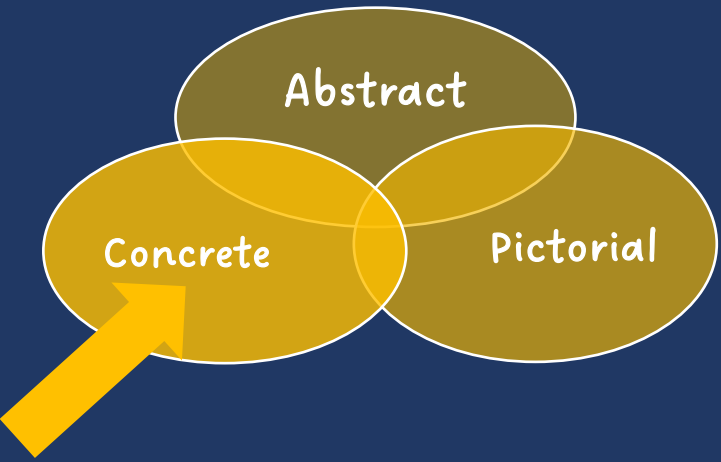
INSTRUCTIONAL STRATEGIES

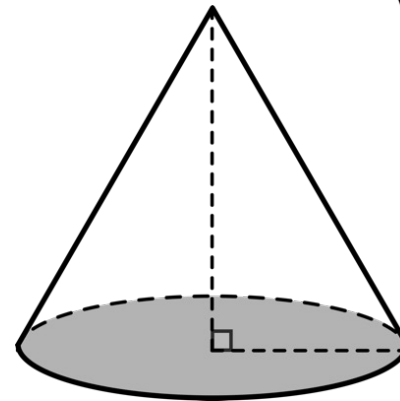
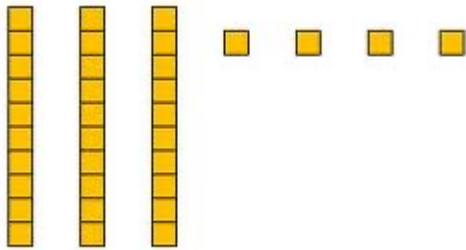
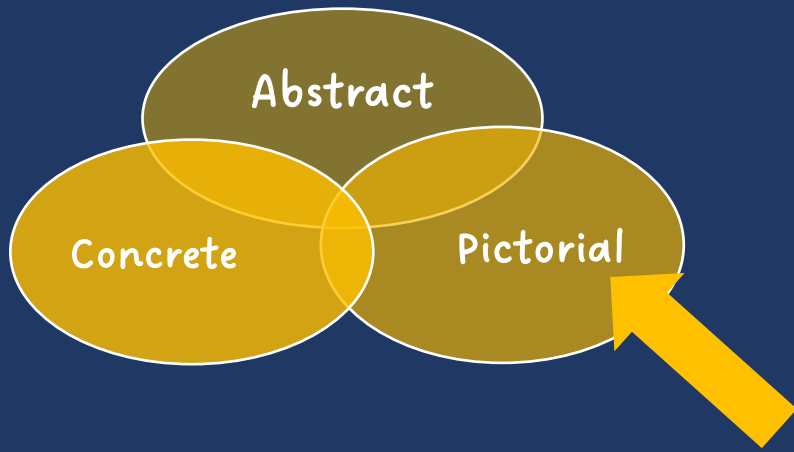
Fluency

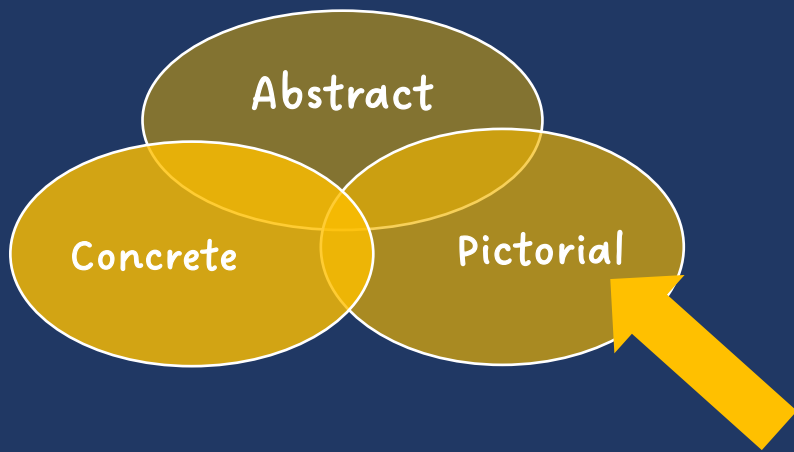
Word Problems









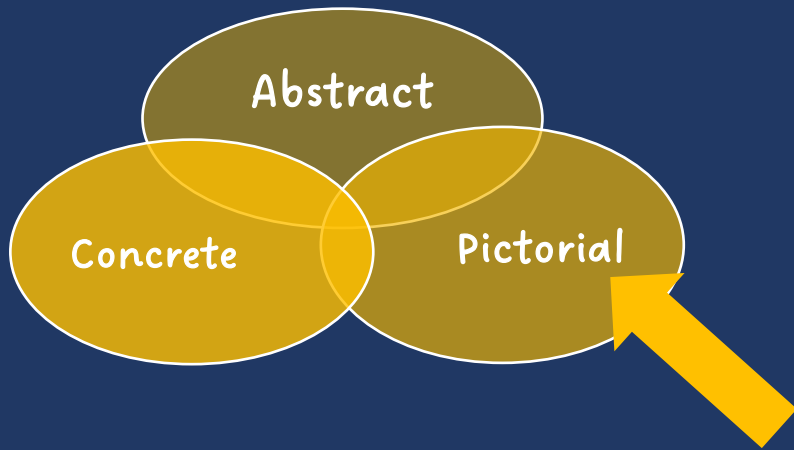


Modeling Fractions with Cuisenaire Rods

A screenshot of a digital interface for modeling fractions with Cuisenaire rods. On the left is a vertical palette of rods in various colors and lengths. In the center is a light green grid with a horizontal row of four red rods placed on it. On the right is a control panel with four buttons: 'View Hint' (lightbulb icon), 'Clear' (circular arrow icon), 'View Help' (question mark icon), and 'Trash Can' (trash bin icon).

A screenshot of a dot grid interface. A triangle is drawn on a black grid with white dots. The triangle's vertices are at grid points. The left vertical side is orange, the bottom horizontal side is red, and the right diagonal side is yellow. At the bottom of the grid is a toolbar with various icons for drawing and editing, including a trash can, a grid, a dot grid, a circle, a square, a rectangle, a triangle, a line, a plus sign, a minus sign, a pencil, and an information icon.



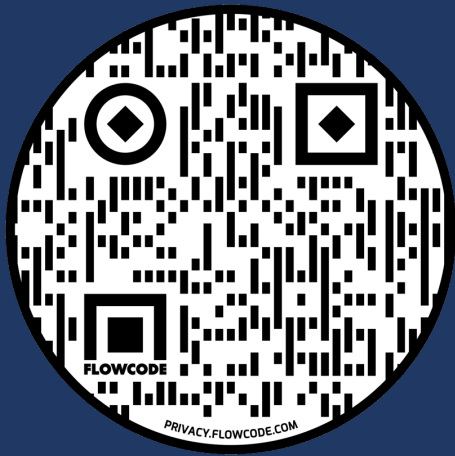


Virtual Manipulatives

Help students see and learn math using different tools!

Number & Operations	Place Value
Fractions & Decimals	Integers & Algebra
Geometry	Time & Money
Data & Probability	Extras

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Fractions & Decimals	fraction strips	fraction strips	fraction strips	Cuisenaire rods
	fraction circles	geoboard	geoboard	geoboard
	pattern blocks	two-color counters	decimal strips	place value disks
			percentage strips	house icon



Operations and Place Value

Fractions and Decimals

Integers and Algebra

Geometry





Unifix cubes
Snap cubes



Math links



Mini motors



Two-color counters



Dice



Dominoes

Addition Concepts

Blank rectangular box for notes on Addition Concepts.

Blank rectangular box for notes on Addition Concepts.

Subtraction Concepts

Blank rectangular box for notes on Subtraction Concepts.

Blank rectangular box for notes on Subtraction Concepts.



100 addition facts

Single-digit addends sum to a single- or double-digit number

$$\begin{array}{r} 5 \\ + 4 \\ \hline 9 \end{array}$$

(addend)
(addend)
(sum)



Total

Addition

Count one set, count another set, put sets together, count sum



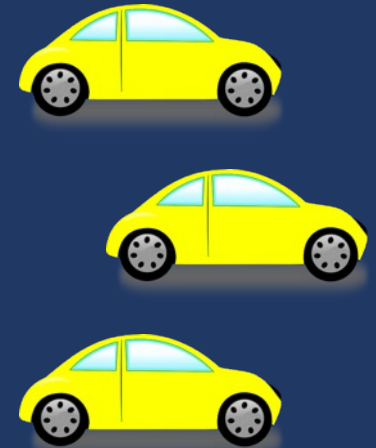
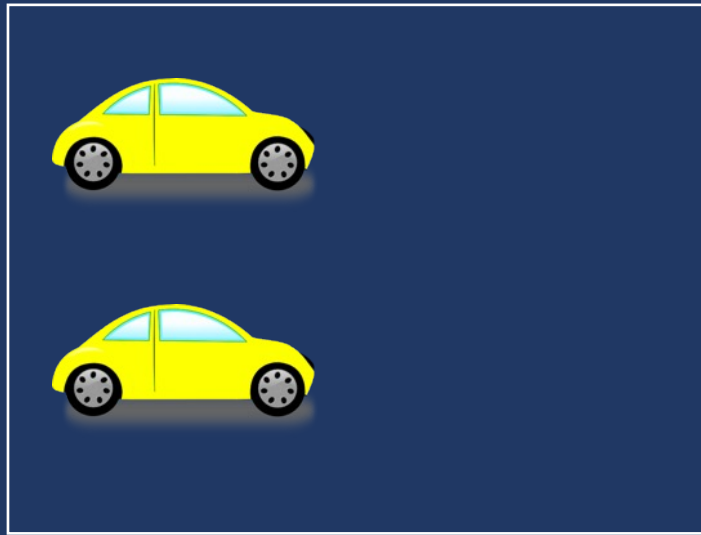
$$2 + 3 = 5$$



Change

Addition

Start with a set, add the other set, count sum



$$2 + 3 = 5$$



Total

Addition

Parts put together into a **total**

Karly saw **4** cardinals and **5** blue jays. How many birds did Karly see?



Change

Addition

An amount that increases or decreases

Premila had \$4. Then they earned \$5 for cleaning their room.
How much money does Premila have now?



Total

Change

$$7 + 4 = \underline{\quad}$$

$$5 + 8 = \underline{\quad}$$

$$9 + 2 = \underline{\quad}$$



Which representations would you use to help students understand addition?

100 subtraction facts

Subtrahend and difference are single-digit numbers and minuend is single- or double-digit number

$$\begin{array}{r} 16 \\ - 8 \\ \hline 8 \end{array}$$

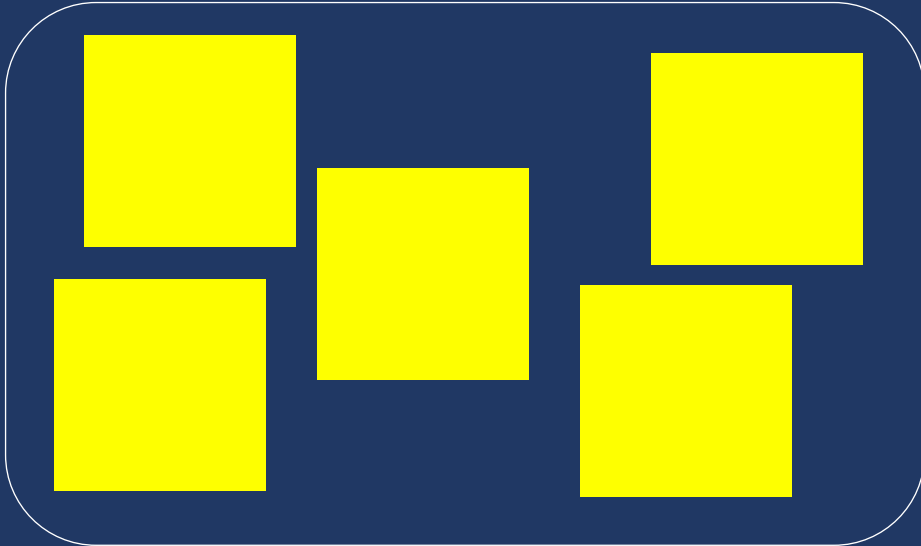
(minuend)
(subtrahend)
(difference)



Change

Subtraction

start with a set, take away from that set, count difference

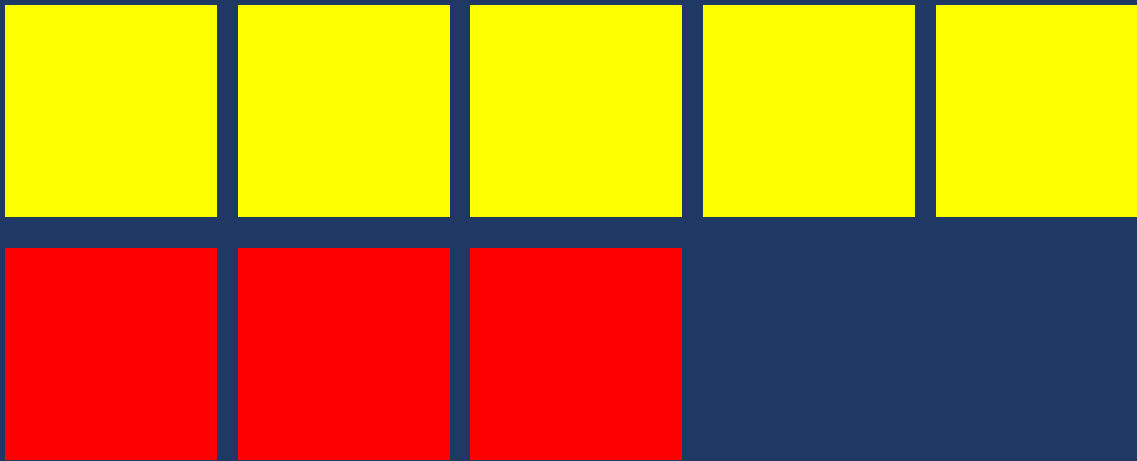


$$5 - 3 = 2$$

Difference

Subtraction

Compare two sets, count difference



$$5 - 3 = 2$$

Change

Subtraction

An amount that increases or decreases

Bronwyn had 9 cookies. Then they ate 2 of the cookies. How many cookies does Bronwyn have now?



Difference

Subtraction

Greater and lesser amounts compared for a difference

Rachel has 9 apples. Jodie has 2 apples. How many more apples does Rachel have? (How many fewer does Jodie have?)



Change

Difference

$$13 - 5 = \underline{\quad}$$

$$12 - 9 = \underline{\quad}$$

$$15 - 8 = \underline{\quad}$$



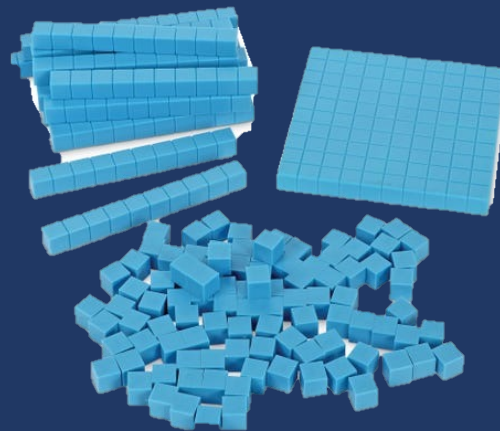
Which representations would you use for subtraction?



Unifix cubes
Snap cubes



Math links



Base-10 Blocks



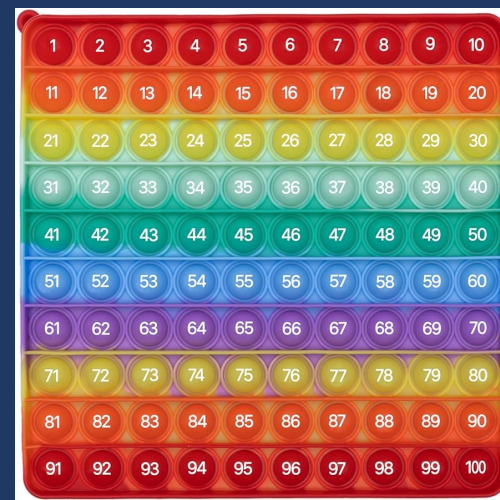
Place Value Disks

HUNDRED BOARD

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

hand2mind
hand2mind.com

Hundred Chart



Hundred Pop It

Ten Frame



Tens and Ones

Count sets with 0-19 items in the set using a ten frame.

Determine how many sets of ten.
Determine how many ones.

Read as:
14 is 1 ten and 4 ones.



Tens and Ones

Count sets with 0-19 items in the set using items that can be linked or connected.

Determine how many sets of ten.
Determine how many ones.

Read as:
14 is 1 ten and 4 ones.



Tens and Ones

Use a hundred chart to identify patterns with tens and ones.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



Standard Form and Expanded Notation

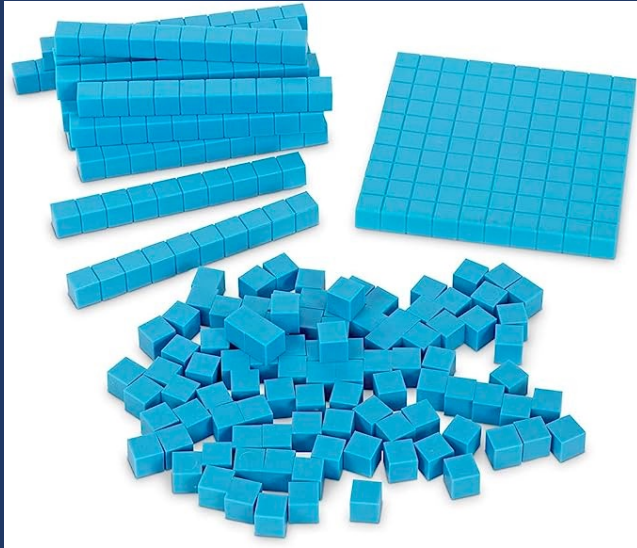
Roll dice to create numbers with tens and ones.

Write in standard form and expanded form.



1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____
11. _____
12. _____
13. _____
14. _____
15. _____
16. _____
17. _____
18. _____
19. _____
20. _____

Hundreds, Tens, and Ones



Proportional
materials



Non-proportional
materials

Hundreds, Tens, and Ones

Ones

Tens

Hundreds



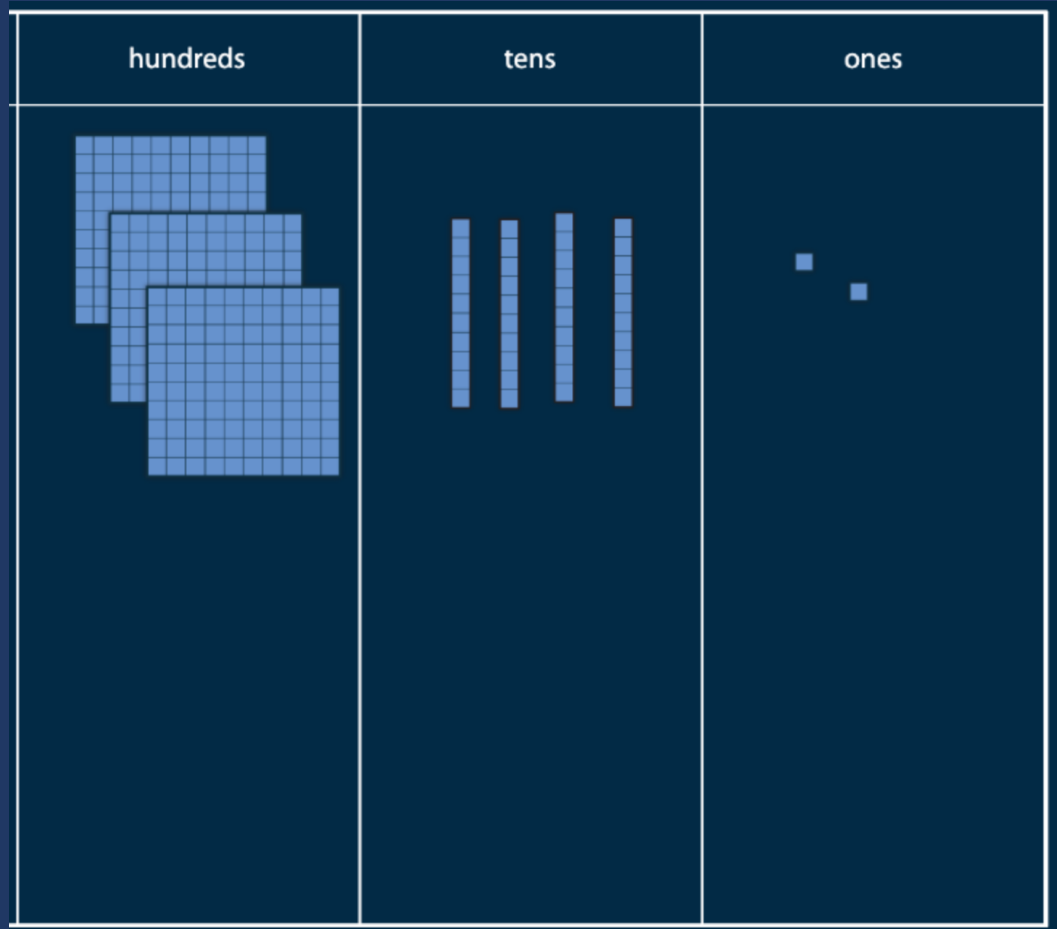
Hundreds, Tens, and Ones

Show numbers 0-999.

Determine how many hundreds, tens, and ones.

Read as:

342 is 3 hundreds, 4 tens, and 2 ones.



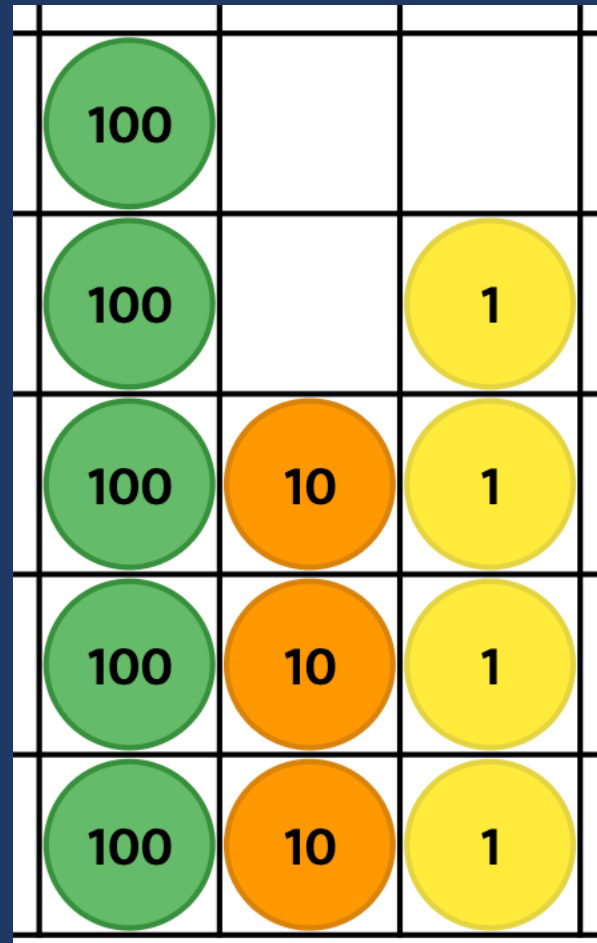
Hundreds, Tens, and Ones

Show numbers 0-999.

Determine how many hundreds, tens, and ones.

Read as:

534 is 5 hundreds, 3 tens, and 4 ones.



Show:

14

41

163

596



Which representations would you use for place value?

Addition Computation

$$24 + 35 =$$

$$64 + 29 =$$



Standard

A.

$$\begin{array}{r} 1 \\ 74 \\ + 18 \\ \hline 92 \end{array}$$

B.

$$\begin{array}{r} 1 1 \\ 725 \\ + 365 \\ \hline 1,090 \end{array}$$

Partial Sums

A.

$$\begin{array}{r} 74 \\ + 18 \\ \hline 80 \\ + 12 \\ \hline 92 \end{array}$$

B.

$$\begin{array}{r} 725 \\ + 365 \\ \hline 1,000 \\ 80 \\ + 10 \\ \hline 1,090 \end{array}$$

Opposite Change

A.

$$\begin{array}{r} 74 \\ + 18 \\ \hline \end{array} \xrightarrow{+4} \begin{array}{r} 70 \\ + 22 \\ \hline 92 \end{array}$$

B.

$$\begin{array}{r} 725 \\ + 365 \\ \hline \end{array} \xrightarrow{-5} \begin{array}{r} 730 \\ + 360 \\ \hline 1,090 \end{array}$$

Show:

$$24 + 35$$

$$64 + 29$$



Which representations would you use for addition computation?

Subtraction Computation

$$75 - 42 =$$

$$61 - 38 =$$



Standard

A.

$$\begin{array}{r} 5 \\ \cancel{6}12 \\ - 17 \\ \hline 45 \end{array}$$

B.

$$\begin{array}{r} 29 \\ \cancel{3}1\cancel{0}5 \\ - 96 \\ \hline 209 \end{array}$$

Partial Differences

A.

$$\begin{array}{r} 62 \\ - 17 \\ \hline + 50 \\ - 5 \\ \hline 45 \end{array}$$

B.

$$\begin{array}{r} 305 \\ - 96 \\ \hline + 300 \\ - 90 \\ - 1 \\ \hline 209 \end{array}$$

Same Change

A.

$$\begin{array}{r} 62 \\ - 17 \\ \hline \end{array} \xrightarrow{+3} \begin{array}{r} 65 \\ - 20 \\ \hline 45 \end{array}$$

B.

$$\begin{array}{r} 305 \\ - 96 \\ \hline \end{array} \xrightarrow{+4} \begin{array}{r} 309 \\ - 100 \\ \hline 209 \end{array}$$

Add Up

A.

$$\begin{array}{r} 62 \\ - 17 \\ \hline \end{array}$$
$$\begin{array}{r} 17 \\ 20 \\ 60 \\ \hline 62 \end{array}$$
$$\begin{array}{r} 3 \\ 40 \\ 2 \\ \hline 45 \end{array}$$

B.

$$\begin{array}{r} 305 \\ - 96 \\ \hline \end{array}$$
$$\begin{array}{r} 96 \\ 100 \\ 300 \\ \hline 305 \end{array}$$
$$\begin{array}{r} 4 \\ 200 \\ 5 \\ \hline 209 \end{array}$$

Show:

$$75 - 42$$

$$61 - 38$$



Which representations would you use for subtraction computation?

Multiplication Concepts

Blank box for multiplication concepts.

Blank box for multiplication concepts.

Division Concepts

Blank box for division concepts.

Blank box for division concepts.



100 multiplication facts

Multiplication of single-digit factors results in a single- or double-digit product

$$\begin{array}{r} 2 \\ \times 3 \\ \hline 6 \end{array}$$

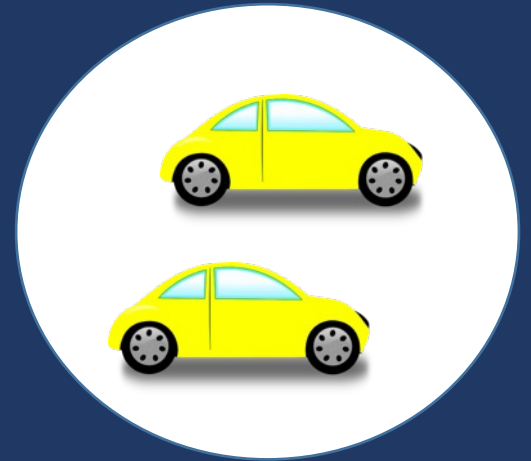
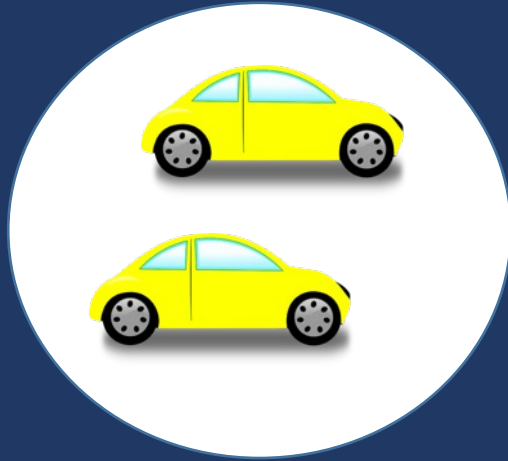
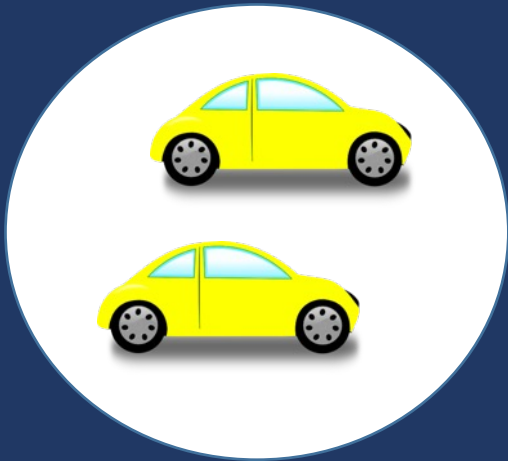
(factor)
(factor)
(product)



Equal Groups

Multiplication

Show the groups, show the amount for each group, count product

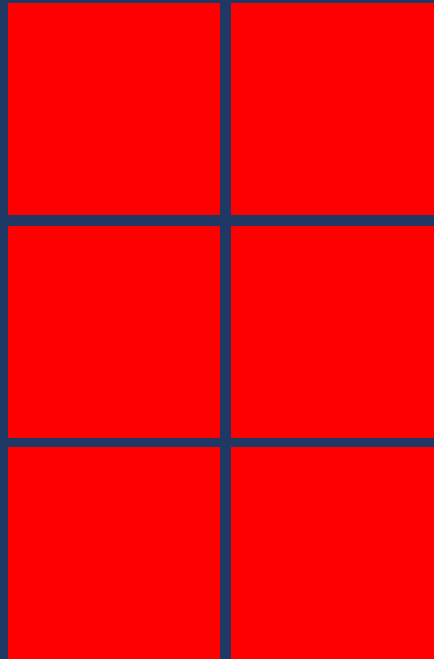


$$3 \times 2 = 6$$

Equal Groups

Multiplication

Show the groups, show the amount for each group, count product

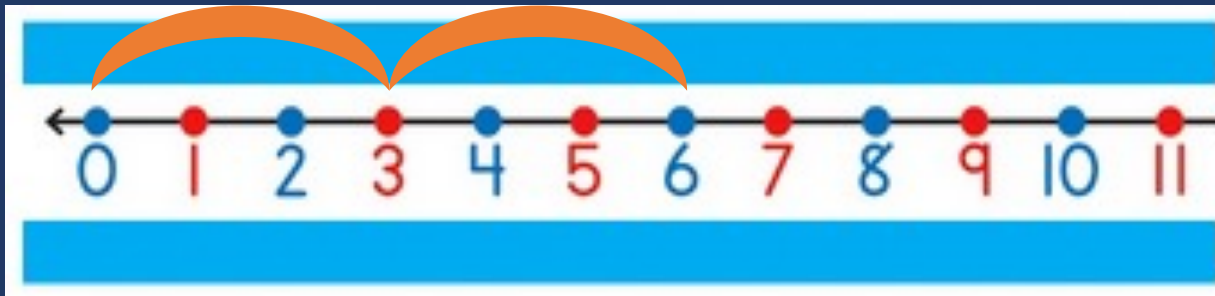


$$3 \times 2 = 6$$

Comparison

Multiplication

Show a set, then multiply the set



$$3 \times 2 = 6$$

Equal Groups

Multiplication

Groups multiplied by **number in each group** for a **product**

Rhiannon has **2** boxes of crayons. There are **12** crayons in each box. How many crayons does Rhiannon have altogether?



Comparison

Multiplication

Set multiplied by a number of times for a product

Vivienne had 12 stickers. Jessica had 2 times as many stickers as Vivienne. How many stickers did Jessica have?



Equal Groups

$$2 \times 5 = \underline{\quad}$$

$$3 \times 4 = \underline{\quad}$$

$$6 \times 2 = \underline{\quad}$$

Comparison



Which representations would you use to help students understand multiplication?

90 division facts

Divisor and quotient are single-digit numbers and dividend is single- or double-digit number

$$8 \div 4 = 2$$

(dividend) (divisor) (quotient)

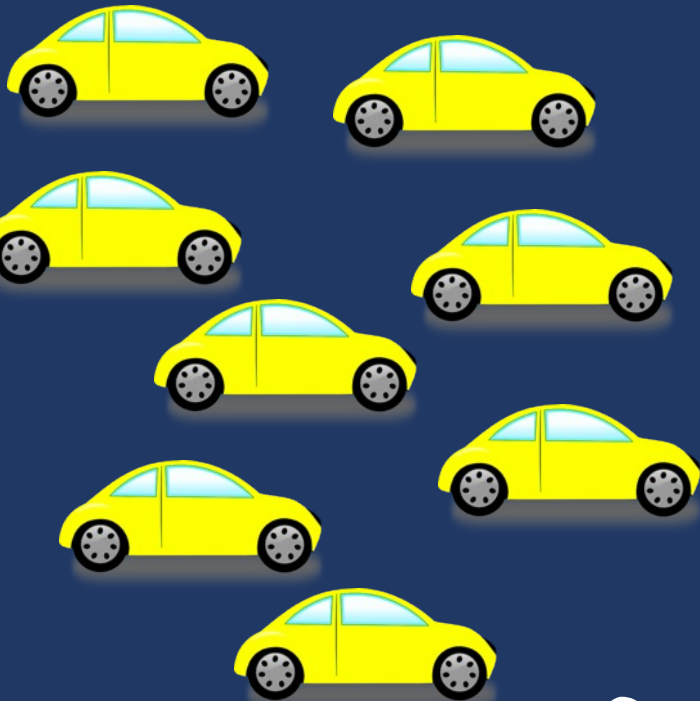


Equal Groups

(Partitive Division)

Division

Show the dividend, divide equally among divisor, count quotient



$$8 \div 2 = 4$$

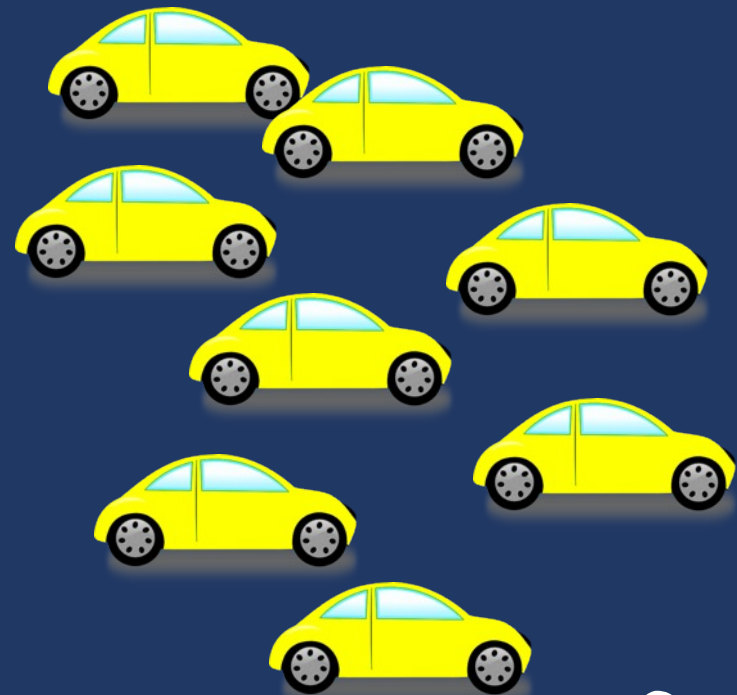


Equal Groups

(Quotative Division)

Division

Show the dividend, make groups of the divisor, count groups



$$8 \div 2 = 4$$



Equal Groups

Division

Groups multiplied by **number in each group** for a **product**

Stefanie has **12** pencils. She wants to share them equally among her **2** friends. How many pencils will each friend receive?

Nicole has **12** pencils. She put them into pencil pockets with **6** pencils each. How many pencil pockets did Nicole use?



Equal Groups

(Partitive Division)
(Quotative Division)

$$10 \div 5 = \underline{\quad}$$

$$12 \div 4 = \underline{\quad}$$

$$8 \div 2 = \underline{\quad}$$



Which representations would use to help students understand division?

Addition	Subtraction
Multiplication	Division

Build fluency with math facts.

- Addition: single-digit addends
- Subtraction: single-digit subtrahend
- Multiplication: single-digit factors
- Division: single-digit divisor

$$\begin{array}{r} 5 \\ + 8 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ - 4 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times 7 \\ \hline \end{array}$$

$$\begin{array}{r} 56 \\ \div 8 \\ \hline \end{array}$$



DAILY

BRIEF

Work on small sets of facts

Work on unknown facts
(in combination with known facts)



Dice



Roll the Dice



$$\underline{8} + \underline{7} = \underline{15}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} - \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} - \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} - \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} - \underline{\quad} = \underline{\quad}$$

Beach Ball

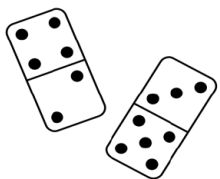


4 plus 6 equals 10.

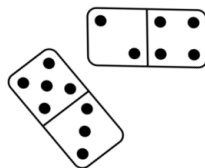
7 plus 6 equals 13.

2 plus 2 equals 4.

Dominoes



Dominoes



$$\underline{4} + \underline{6} = \underline{10}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} - \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} - \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} \times \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} \times \underline{\quad} = \underline{\quad}$$

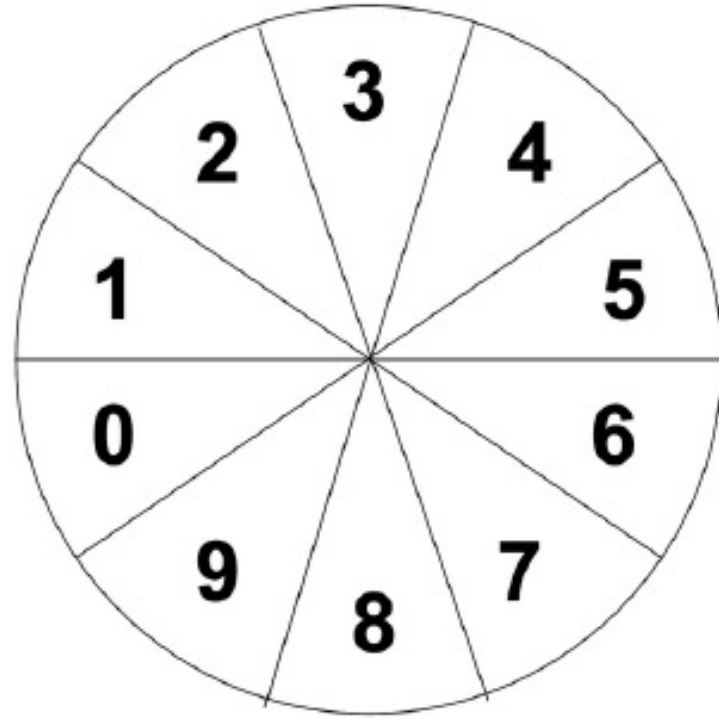


Spinner

2 times 4 equals 8.

6 times 9 equals 54.

7 times 1 equals 7.



Playing Cards

Cards



$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{6} - \underline{2} = \underline{4}$$

$$\underline{\quad} - \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

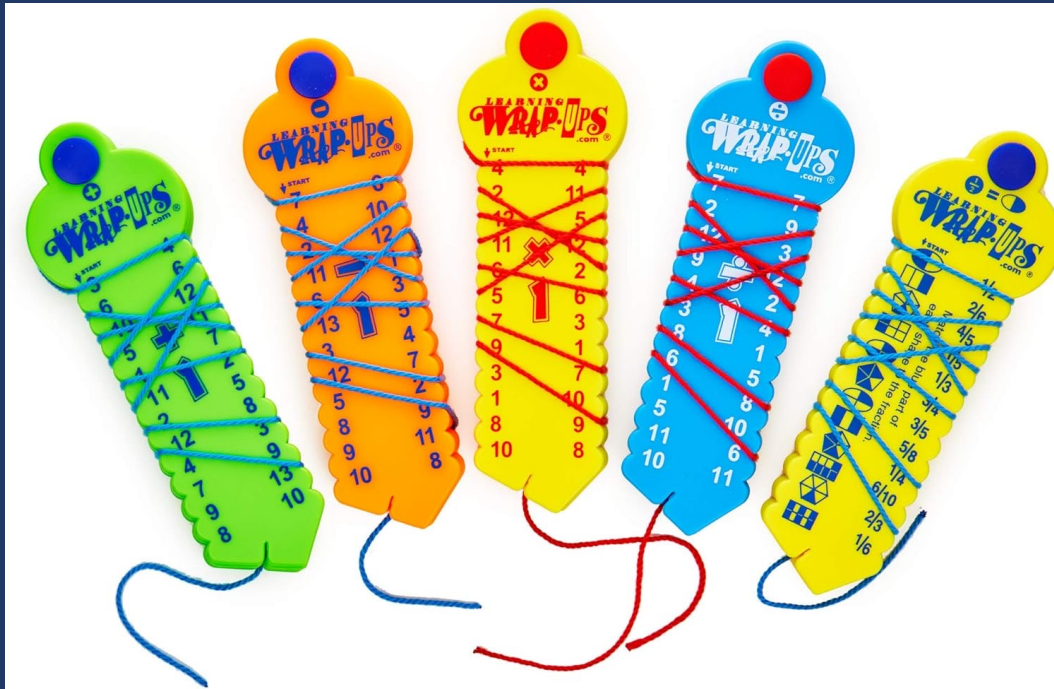
$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} - \underline{\quad} = \underline{\quad}$$

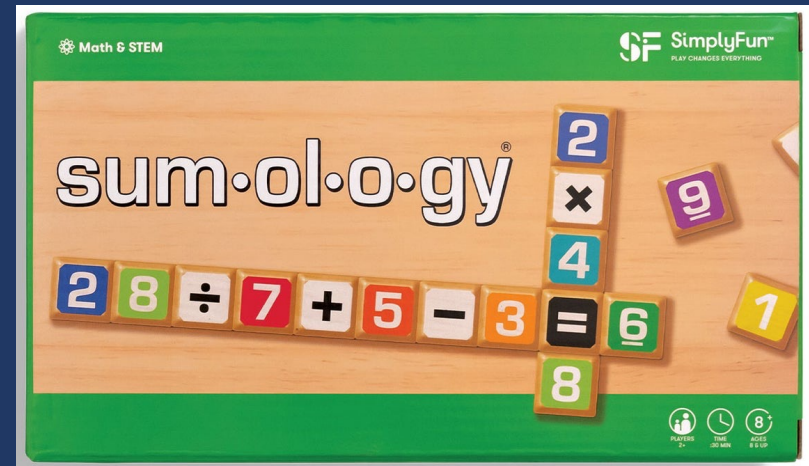
$$\underline{\quad} - \underline{\quad} = \underline{\quad}$$



Wrap-Ups



Mobi Math

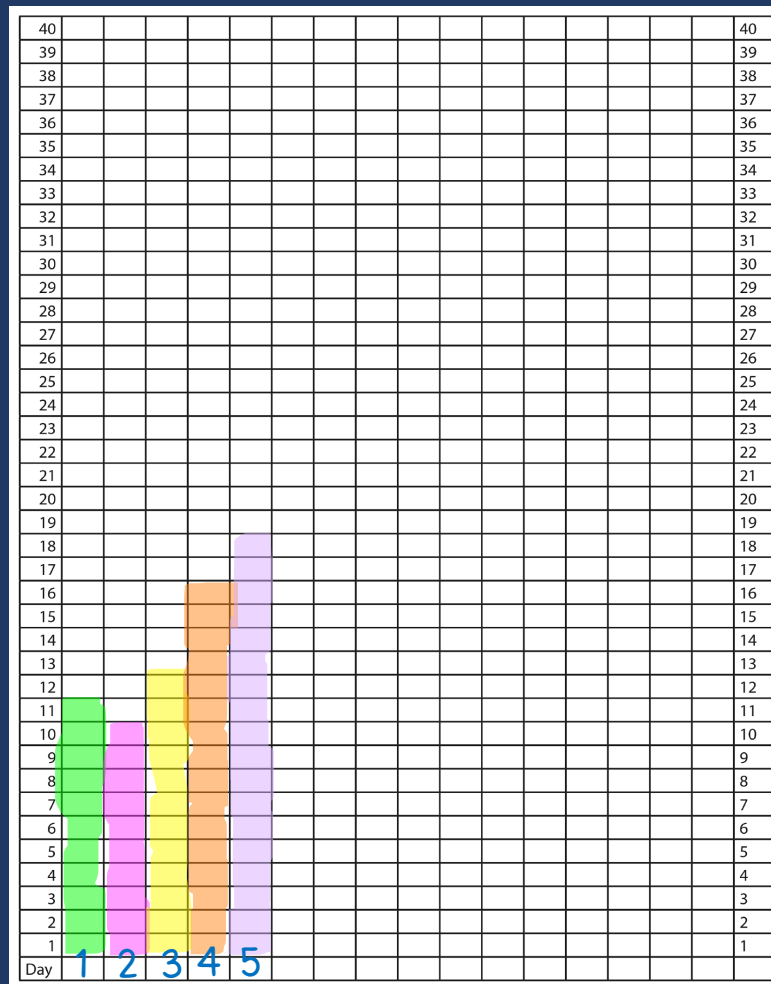


Flash Cards

$$\begin{array}{r} 7 \\ - 6 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ - 4 \\ \hline \end{array}$$

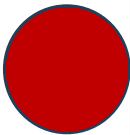
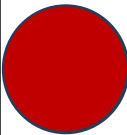
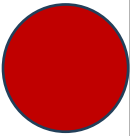
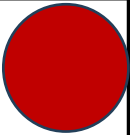
$$\begin{array}{r} 11 \\ - 3 \\ \hline \end{array}$$



Bingo

Math Bingo

Your teacher will call out a math problem. Quickly solve the problem. If you have the answer on your bingo card, cover it with a chip. The middle space is free!
The first person to finish the pattern your teacher decides wins!
(5 in a row, 4 corners, etc.)

12		24	100	
15	0	42	16	20
8	35		6	4
	2	40	27	7
50	10	30	48	14

8 times 10 equals...

3 times 1 equals...

2 plus 3 equals...

Magic Squares

Magic Squares Board

- Place the sum or product in the bottom right corner.
- In the bottom row, create a fact with a sum or product of the bottom right corner.
- In the right column, create a fact with a sum or product of the bottom right corner.
- Create two columns with a sum or product of the bottom number.
- Create two rows with a sum or product of the right column number.
- Write the created facts below.

0	2	2
5	4	9
5	6	11

$$0 + 2 = 2$$

$$2 - 0 = 2$$

$$5 + 4 = 9$$

$$9 - 5 = 4$$

4	5	9
2	0	2
6	5	11

7	3	10
1	0	1
8	3	11

6	1	7
3	2	5
9	3	12

4	4	8
2	2	4
6	6	12

5	1	6
4	3	7
9	4	13

5	1	6
3	4	7
8	5	13

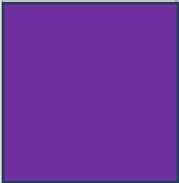
6	3	9
2	3	5
8	6	14

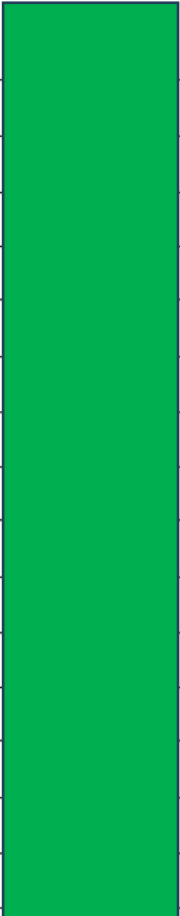
1	5	6
6	2	8
7	7	14

6	2	8
3	4	7
9	6	15



Cover, Copy, Compare

Cover, Copy, Compare			
	$\begin{array}{r} 9 \\ \times 6 \\ \hline 54 \end{array}$	$\begin{array}{r} 8 \\ \times 6 \\ \hline 48 \end{array}$	
$\begin{array}{r} 7 \\ \times 8 \\ \hline 56 \end{array}$		$\begin{array}{r} 6 \\ \times 5 \\ \hline 30 \end{array}$	
$\begin{array}{r} 9 \\ \times 9 \\ \hline 81 \end{array}$		$\begin{array}{r} 7 \\ \times 9 \\ \hline 63 \end{array}$	
$\begin{array}{r} 6 \\ \times 7 \\ \hline 42 \end{array}$		$\begin{array}{r} 8 \\ \times 5 \\ \hline 40 \end{array}$	
$\begin{array}{r} 8 \\ \times 8 \\ \hline 64 \end{array}$		$\begin{array}{r} 7 \\ \times 7 \\ \hline 49 \end{array}$	

File folder	
$6 + 3 = 9$	
$1 + 7 = 8$	
$6 + 4 = 10$	
$7 + 3 =$	
$2 + 7 =$	
$5 + 6 =$	
$4 + 7 =$	
$7 + 8 =$	
$6 + 7 =$	
$7 + 9 =$	
$7 + 6 =$	
$8 + 7 =$	
$7 + 0 =$	
$9 + 6 =$	
$6 + 0 =$	
$6 + 8 =$	



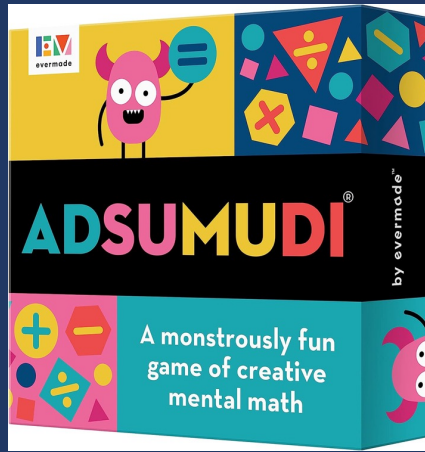
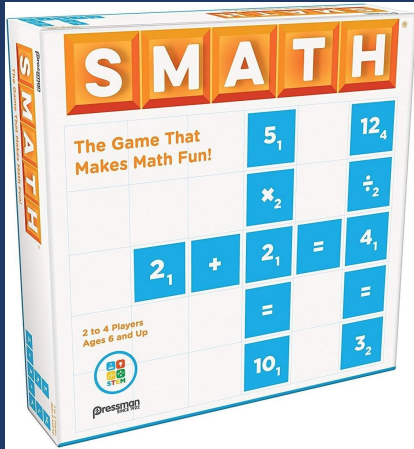
Taped Problems

Taped Problems

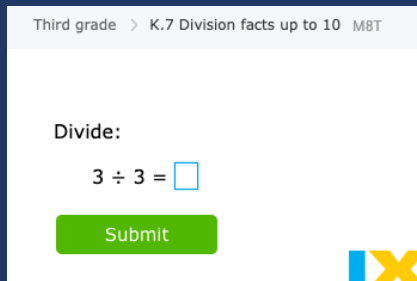
$\begin{array}{r} 8 \\ \times 8 \\ \hline 64 \end{array}$	$\begin{array}{r} 7 \\ \times 7 \\ \hline 49 \end{array}$	$\begin{array}{r} 8 \\ \times 7 \\ \hline \end{array}$
$\begin{array}{r} 6 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ \times 7 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ \times 8 \\ \hline \end{array}$
$\begin{array}{r} 5 \\ \times 7 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ \times 6 \\ \hline \end{array}$
$\begin{array}{r} 6 \\ \times 6 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 6 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ \times 6 \\ \hline \end{array}$
$\begin{array}{r} 7 \\ \times 8 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ \times 5 \\ \hline \end{array}$



Games



Technology



Reflex

Get your free 30-day trial

Help your students attain math fact fluency success whether in-person, remote, or through hybrid learning

Game-based system to improve math fact fluency for grades 2-6 in less than 30 days!

A video player showing a scene from "Crabby's Fact Fair". The scene depicts a fairground with a sign that says "CRABBY'S FACT FAIR". There are colorful lights and structures in the background.

FACT MONSTER

Games / Flashcard

Flashcard

subtraction Level 3 1:51

13
- 6



DAILY

BRIEF

Work on small sets of facts

Work on unknown facts
(in combination with known facts)



Multiplication Computation

$$13 \times 47 =$$

$$123 \times 24 =$$



Standard

A.

$$\begin{array}{r} \\ \cancel{X} \\ 24 \\ \times 43 \\ \hline 172 \\ +960 \\ \hline 1,032 \end{array}$$

B.

$$\begin{array}{r} \\ 132 \\ \times 53 \\ \hline 396 \\ +6600 \\ \hline 6,996 \end{array}$$

Partial Products

A.

$$\begin{array}{r} 24 \\ \times 43 \\ \hline 800 \\ 160 \\ 60 \\ + 12 \\ \hline 1,032 \end{array}$$

B.

$$\begin{array}{r} 132 \\ \times 53 \\ \hline 5000 \\ 15000 \\ 1000 \\ 3000 \\ 90 \\ + 6 \\ \hline 6996 \end{array}$$

Area (Array)

A.

$$\begin{array}{r} 24 \\ \times 43 \\ \hline 800 \\ 160 \\ 60 \\ + 12 \\ \hline 1,032 \end{array}$$

20	4	
800	160	40
60	12	3

B.

$$\begin{array}{r} 132 \\ \times 53 \\ \hline 5000 \\ 1500 \\ 100 \\ 300 \\ 90 \\ + 6 \\ \hline 6996 \end{array}$$

100	30	2	
5000	1500	100	50
300	90	6	3

Lattice

A.

$$\begin{array}{r} 24 \\ \times 43 \\ \hline \end{array}$$

2 4

0	8	4
0	6	3

3 2

1,032

B.

$$\begin{array}{r} 132 \\ \times 53 \\ \hline \end{array}$$

1 3 2

0	5	1	0	5
0	3	0	9	6

9 9 6

6,996

Show:

$$13 \times 47$$

$$123 \times 24$$



Which representations would you use for multiplication computation?

Division Computation

$$804 \div 12 =$$

$$1,746 \div 18 =$$



Standard

$$\begin{array}{r} 13 \text{ R}2 \\ 12 \overline{) 158} \\ \underline{-12} \downarrow \\ 38 \\ \underline{-36} \\ 2 \end{array}$$

$$\begin{array}{r} 28 \text{ R}18 \\ 34 \overline{) 89170} \\ \underline{-68} \downarrow \\ 2910 \\ \underline{-272} \\ 18 \end{array}$$

Partial Quotients

$$\begin{array}{r} \text{A. } 12 \overline{) 158} \\ \underline{- 120} \\ 38 \\ \underline{- 36} \\ 2 \end{array} \quad \begin{array}{r} 10 \\ + 3 \\ \hline 13 \text{ R} 2 \end{array}$$

$$\begin{array}{r} \text{B. } 34 \overline{) 170} \\ \underline{- 680} \\ 290 \\ \underline{- 170} \\ 120 \\ \underline{- 102} \\ 18 \end{array} \quad \begin{array}{r} 20 \\ 5 \\ + 3 \\ \hline 28 \text{ R} 18 \end{array}$$

Lattice

A. $12 \overline{)158} \quad 13 \text{ R}2$

12 $\begin{array}{|c|} \hline 0 \\ \hline 1 \\ \hline \end{array}$ $\begin{array}{|c|} \hline 1 \\ \hline 3 \\ \hline \end{array}$ $\begin{array}{|c|} \hline 3 \\ \hline 2 \\ \hline \end{array}$

B. $34 \overline{)970} \quad 28 \text{ R}18$

34 $\begin{array}{|c|} \hline 0 \\ \hline 9 \\ \hline \end{array}$ $\begin{array}{|c|} \hline 2 \\ \hline 29 \\ \hline \end{array}$ $\begin{array}{|c|} \hline 8 \\ \hline 18 \\ \hline \end{array}$

Show:

$$804 / 12$$

$$1,746 / 18$$



Which representations would you use for division computation?

Operations and Place Value

Fractions and Decimals

Integers and Algebra

Geometry



Fractions

Fraction	Length	Area	Set
$\frac{2}{3}$			
$\frac{1}{4}$			
$1\frac{1}{2}$			
$\frac{3}{7}$			



LENGTH

AREA

SET

LENGTH

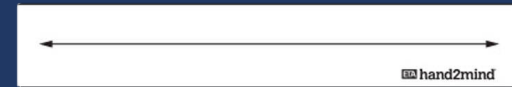
Fractions are appropriated by length



Fraction tiles



Cuisenaire rods



Number lines

LENGTH

Fractions are appropriated by length

$$\frac{2}{3}$$

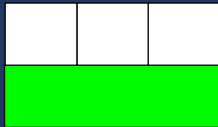


Fraction tiles/bars

LENGTH

Fractions are appropriated by length

$$\frac{2}{3}$$

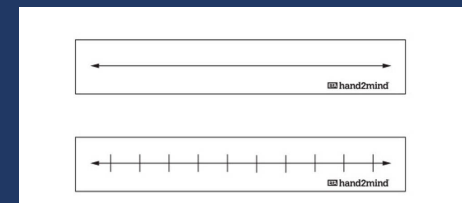


Cuisenaire Rods

LENGTH

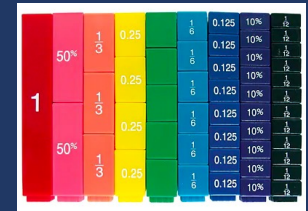
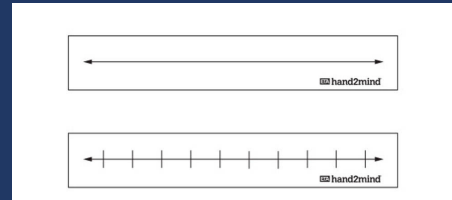
Fractions are appropriated by length

$$\frac{2}{3}$$



Number Lines

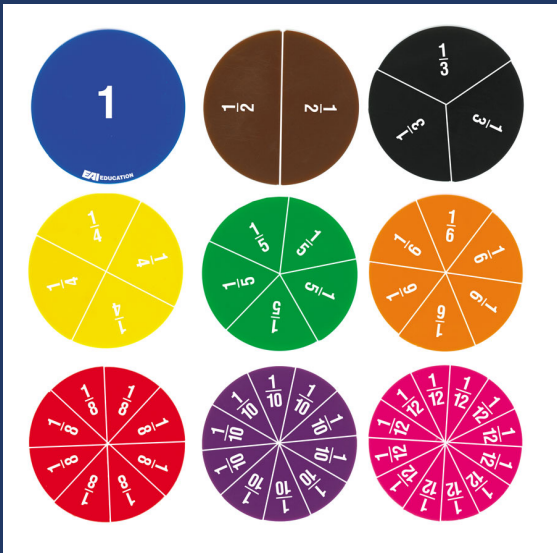
LENGTH



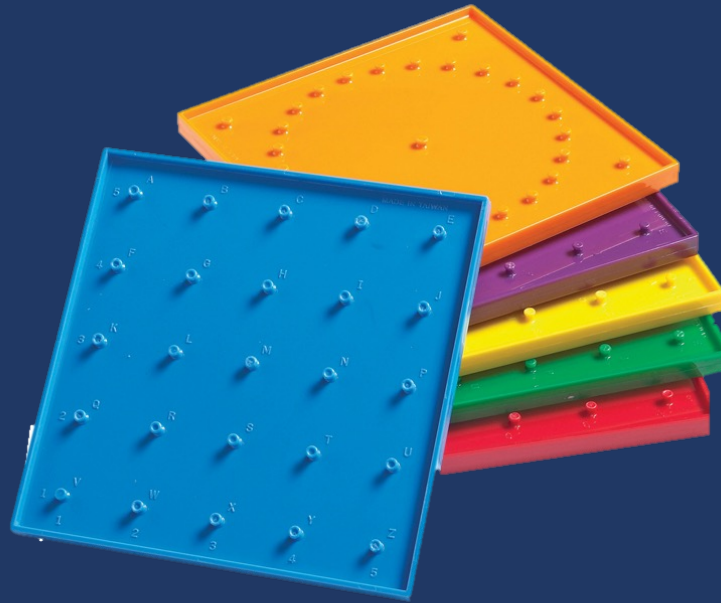
Use representations to show fractions according to length.

AREA

Areas divided into equal sections



Fraction circles



Geoboards

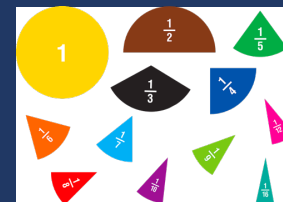


Pattern blocks

AREA

Areas divided into equal sections

$$\frac{2}{3}$$

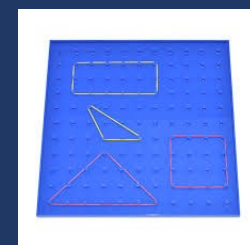
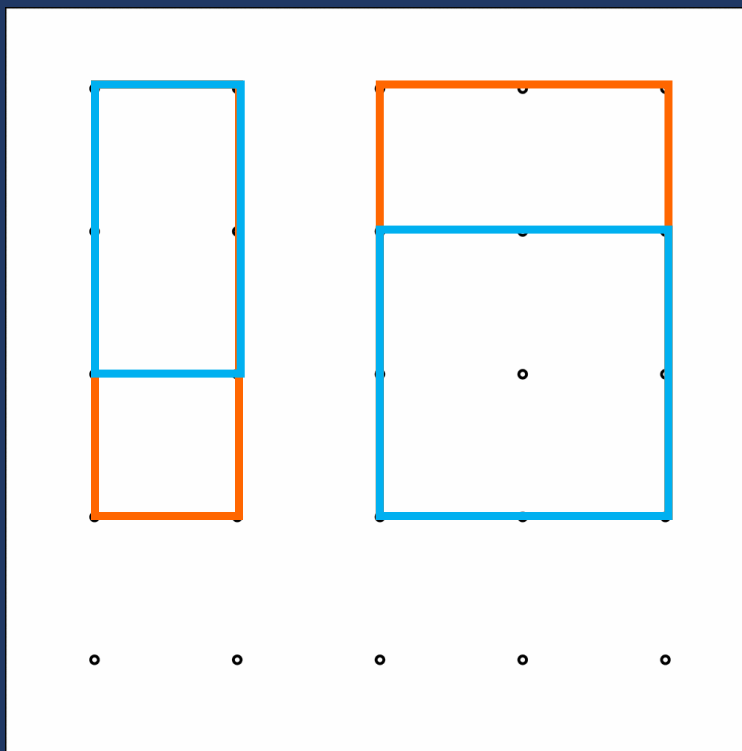


Fraction Circles

AREA

Areas divided into equal sections

$$\frac{2}{3}$$

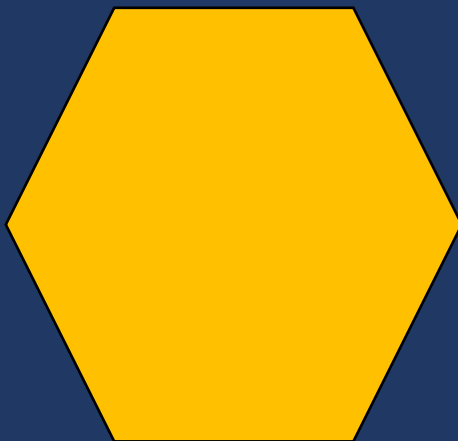


Geoboards

AREA

Areas divided into equal sections

$$\frac{2}{3}$$

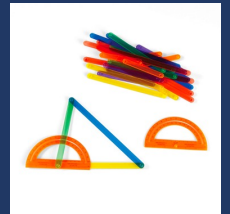


Pattern Blocks

AREA

Areas divided into equal sections

$$\frac{2}{3}$$



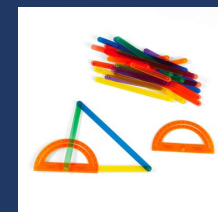
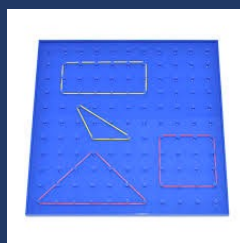
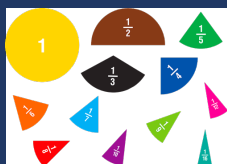
Anglegs



Legos



AREA



Use representations to show fractions according to area.

SET

Individual items show a collection



Two-color counters



Unifix cubes
Snap cubes

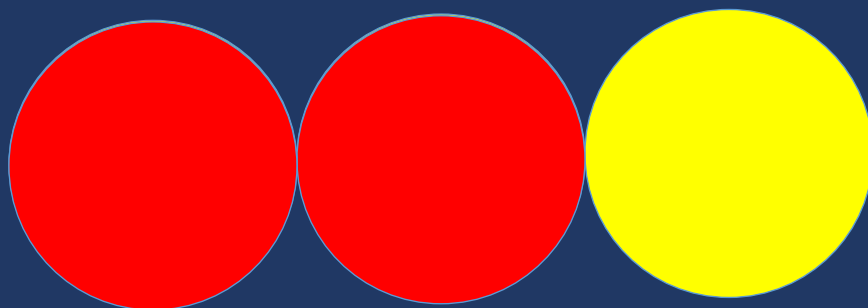


Counters

SET

Individual items show a collection

$$\frac{2}{3}$$



Two-Color Counters

SET

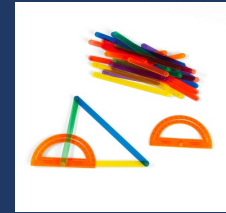
Individual items show a collection

$$\frac{2}{3}$$



Color Tiles

SET



Use representations to show fractions according to a set.

Fraction Addition and Subtraction

Problem	Representation
$\frac{1}{5} + \frac{3}{5}$	
$\frac{2}{8} + \frac{5}{8}$	
$\frac{1}{2} + \frac{1}{4}$	
$\frac{4}{6} + \frac{1}{3}$	
$\frac{4}{5} - \frac{1}{5}$	
$\frac{6}{8} - \frac{3}{8}$	
$\frac{7}{8} - \frac{2}{4}$	
$\frac{8}{9} - \frac{1}{3}$	



Fraction Multiplication and Division

Problem	Representation
2×3	
$\frac{1}{2} \times 2$	
$\frac{1}{2} \times \frac{4}{4}$	
$\frac{1}{2} \times \frac{2}{4}$	
$\frac{1}{2} \times \frac{3}{4}$	
$\frac{2}{3} \times \frac{3}{4}$	
$\frac{4}{4} \div \frac{1}{2}$	
$\frac{2}{4} \div \frac{1}{2}$	
$\frac{3}{4} \div \frac{1}{2}$	
$\frac{5}{6} \div \frac{2}{3}$	



1.2	0.88	1.034
2.8	1.04	0.829

Decimals



Ones



Tenths

Hundredths

Thousandths



Decimal Computation

Problem	Representation
$\begin{array}{r} 2.34 \\ + 1.61 \\ \hline \end{array}$	
$\begin{array}{r} 1.98 \\ + 0.34 \\ \hline \end{array}$	
$\begin{array}{r} 2.34 \\ - 1.61 \\ \hline \end{array}$	
$\begin{array}{r} 3.09 \\ - 1.88 \\ \hline \end{array}$	



Operations and Place Value

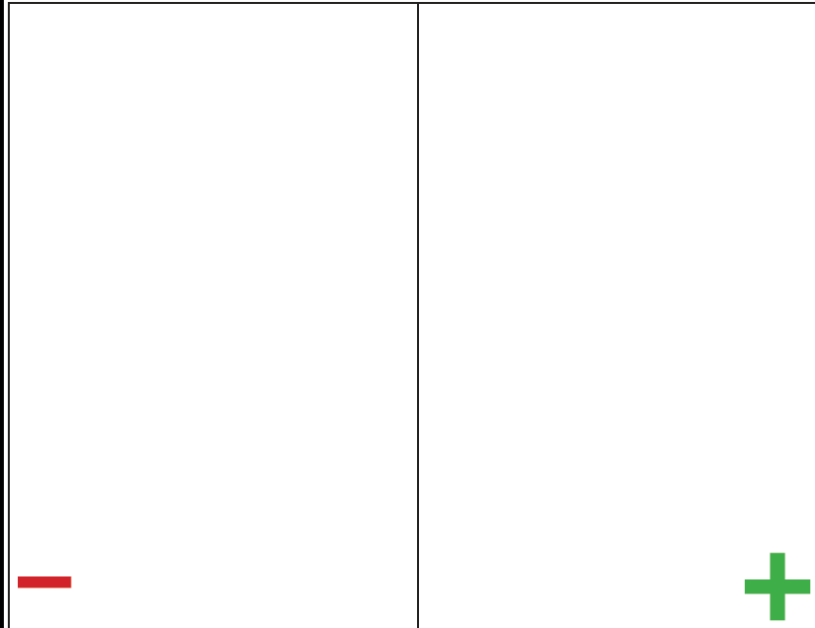
Fractions and Decimals

Integers and Algebra

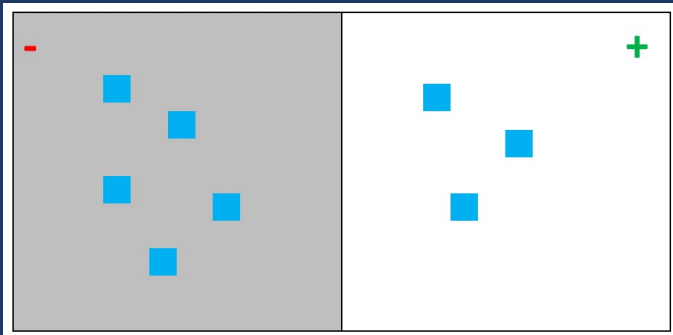
Geometry



Integers



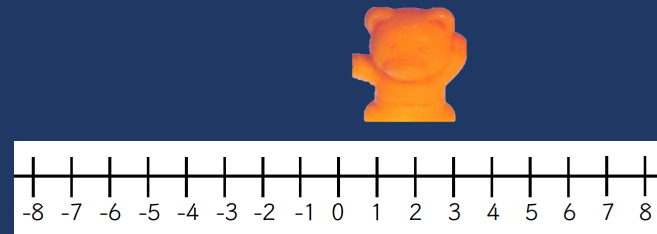
MATH



Mat and counters



Two-color counters



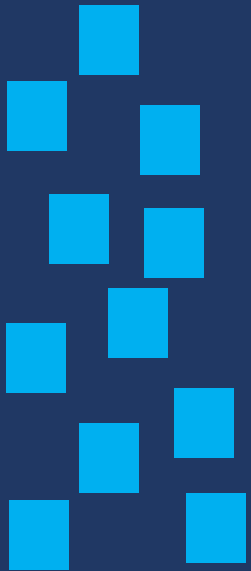
Number line

Integer Addition and Subtraction

Problem	Representation
$3 + 5$	
$3 + (-5)$	
$-2 + 6$	
$-6 + (-3)$	
$5 - 3$	
$-3 - 4$	
$-2 - (-6)$	

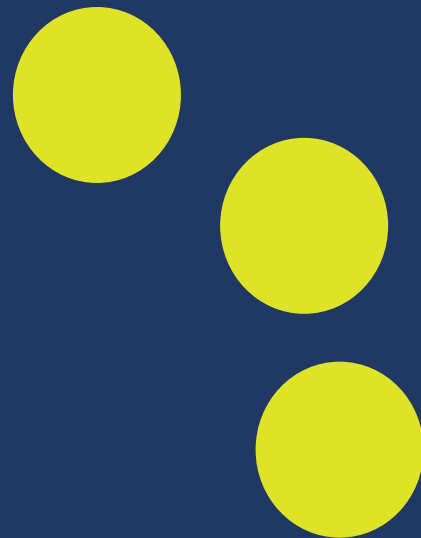


$$3 + (-5)$$

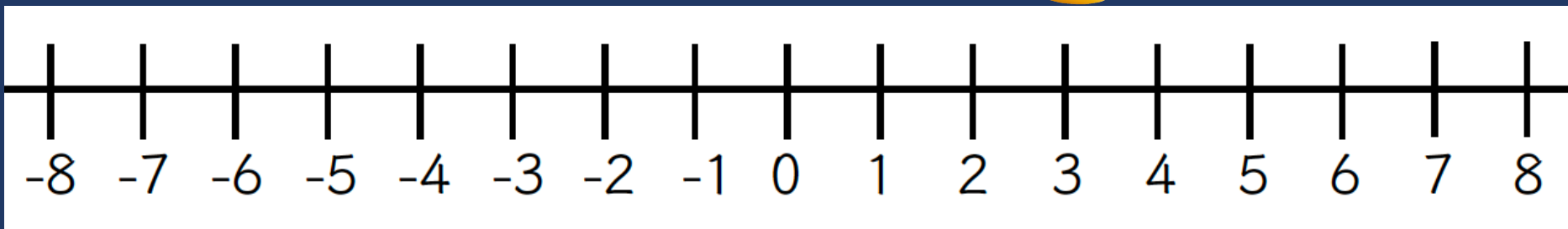


A large rectangular area divided into two vertical panels. The left panel is light gray and contains a small red minus sign (-) in the top-left corner. The right panel is dark blue and contains a small green plus sign (+) in the top-right corner.

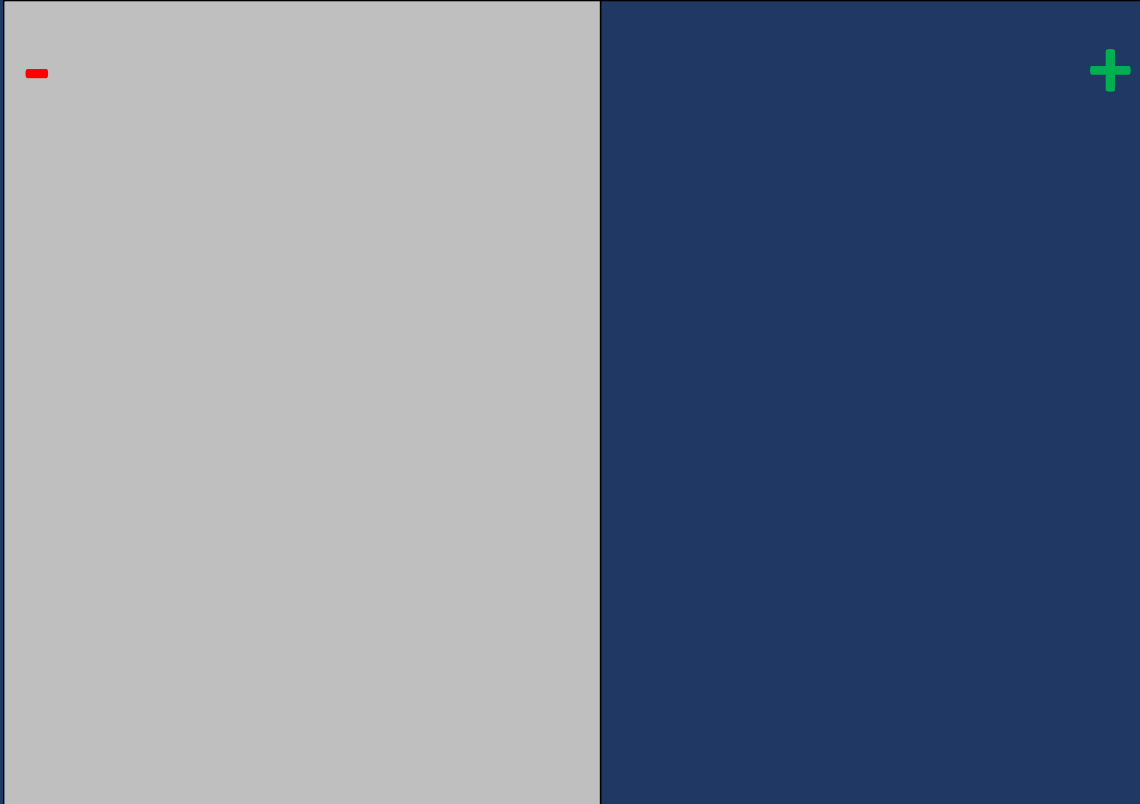
$$3 + (-5)$$



$$3 + (-5)$$



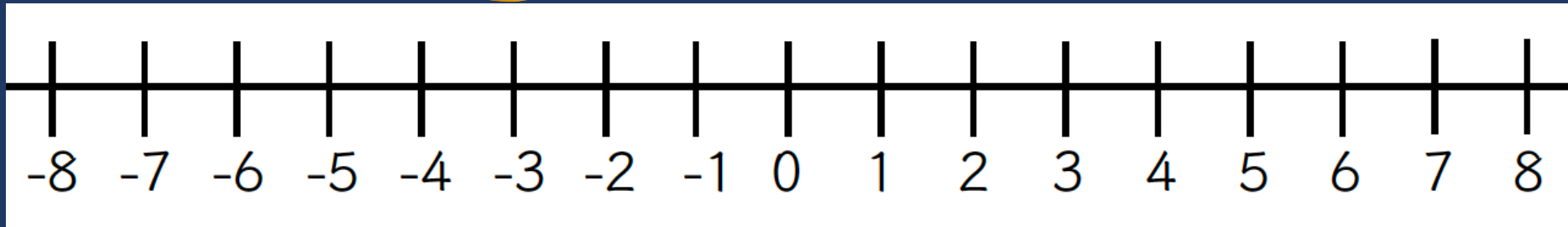
$$-3 - 4$$



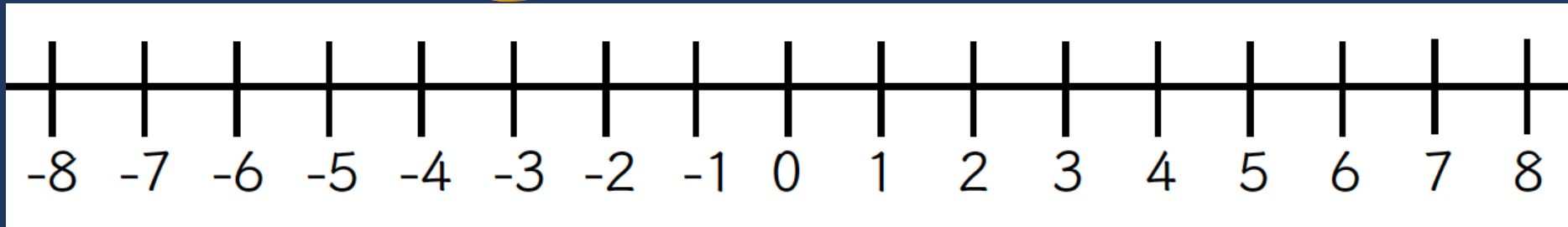
$$-3 - 4$$



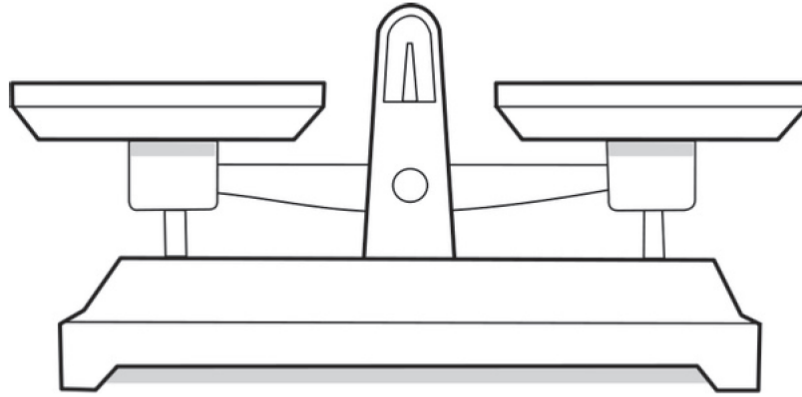
$$-3 - 4$$



$$-3 - 4$$



Equation Solving



$$\underline{\quad} + 3 = 7$$

$$9 - \underline{\quad} = 6$$

$$6 = 2 + \underline{\quad}$$

$$6 = \underline{\quad} - 2$$

$$3 + 5 = 4 + \underline{\quad}$$

$$9 - 6 = 7 - \underline{\quad}$$

$$5 + 4 = \underline{\quad} + 2$$

$$6 - \underline{\quad} = 7 - 3$$

$$7 = \underline{\quad}$$



zero pairs

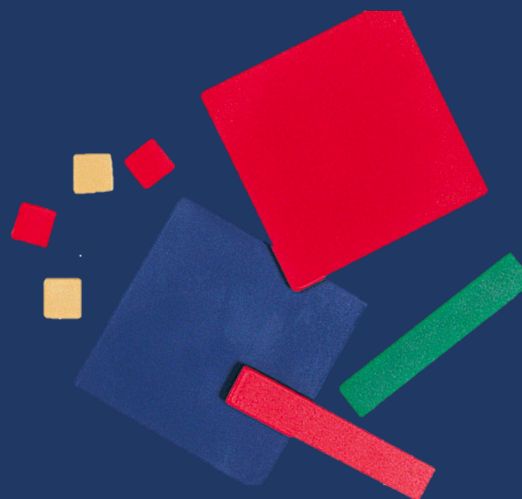
-1 and 1 equal 0

properties
of
equality

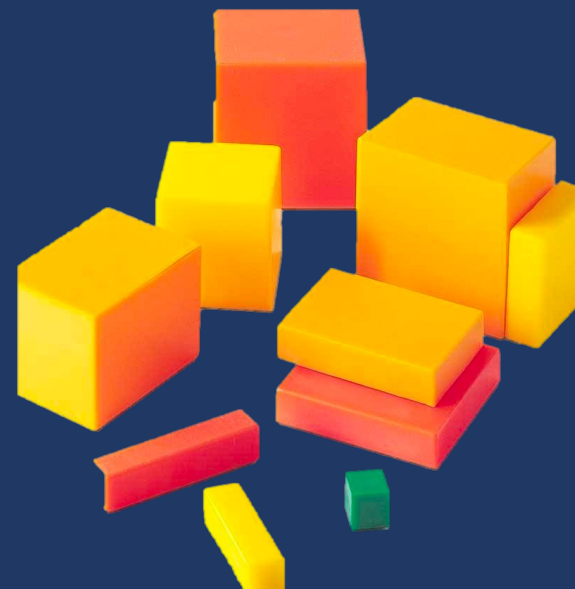
If you do something to one side of the equal sign, you do the same thing to the other side.



Cups and counters



Algebra tiles



Algeblocks

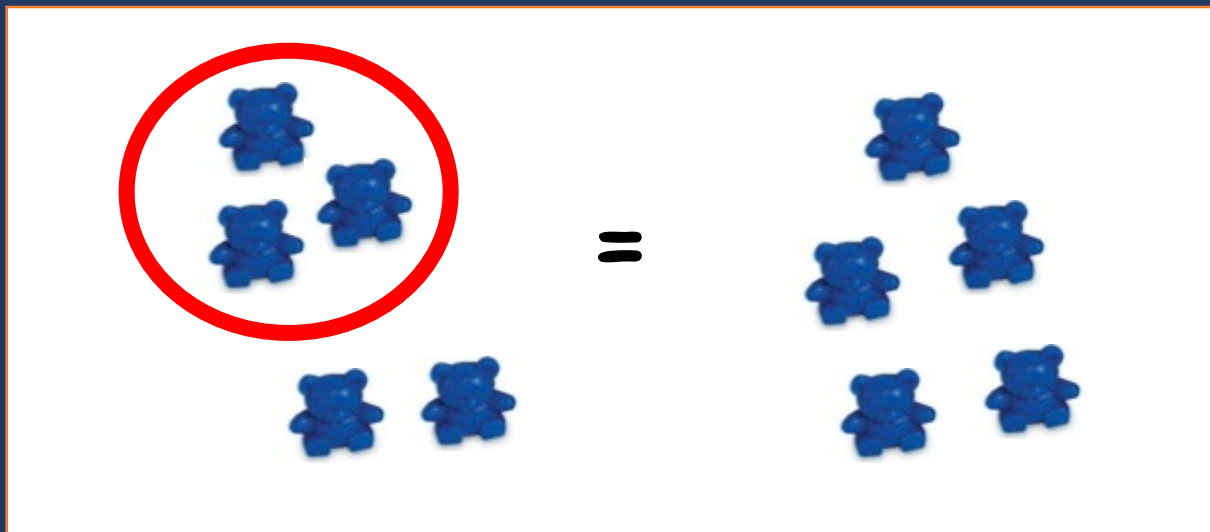
Equation Solving with Cups/Plates and Counters

Problem	Representations
$x + 2 = 5$	
$x + 2 = 5$	
$4 + x = 6$	
$4 + x = 6$	
$5 = x - 3$	
$-2 = x + 3$	



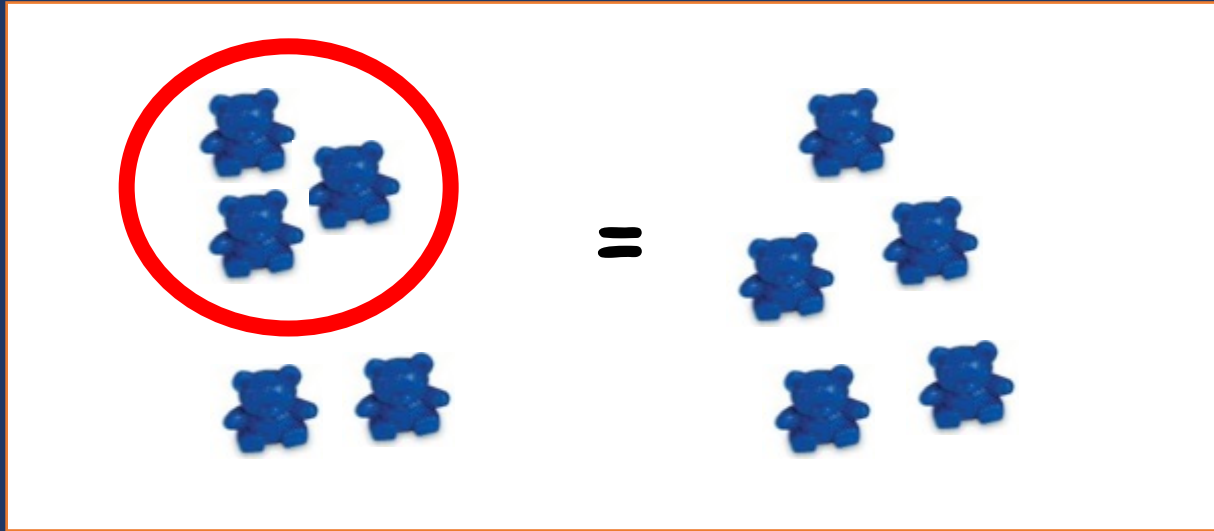
cups/plates
and counters

$$x + 2 = 5$$



cups/plates
and counters

$$x + 2 = 5$$



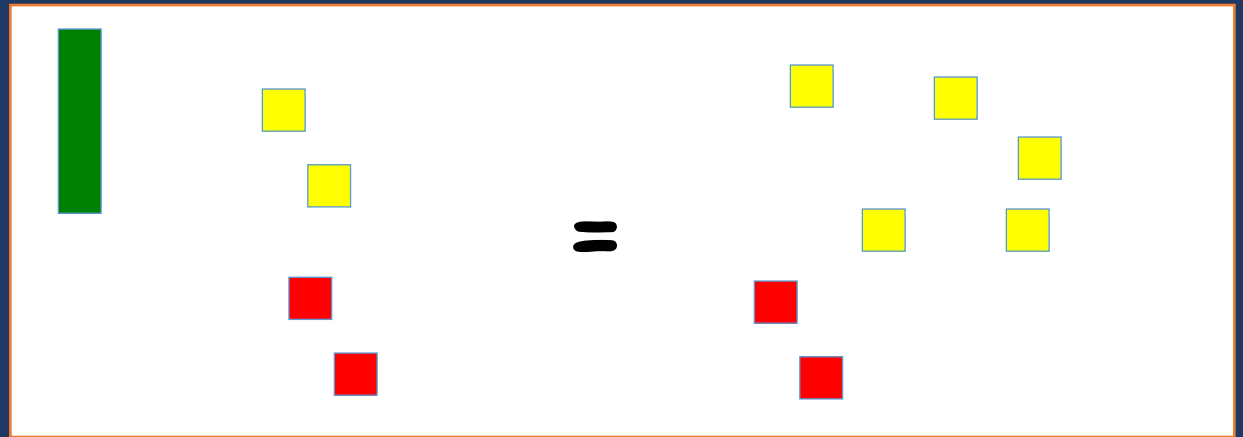
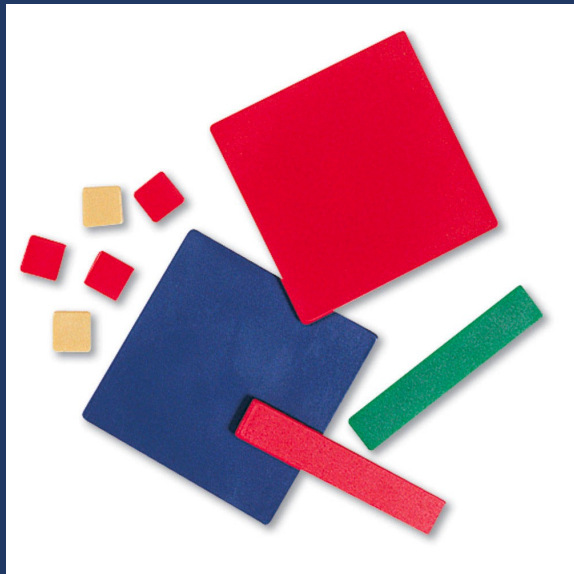
Equation Solving with Algebra Tiles

Problem	Representations
$x + 2 = 5$	
$x + 2 = 5$	
$4 + x = 6$	
$5 = x - 3$	
$-2 = x + 3$	
$x + 3 = -7$	
$y - 4 = 2$	



algebra tiles

$$x + 2 = 5$$



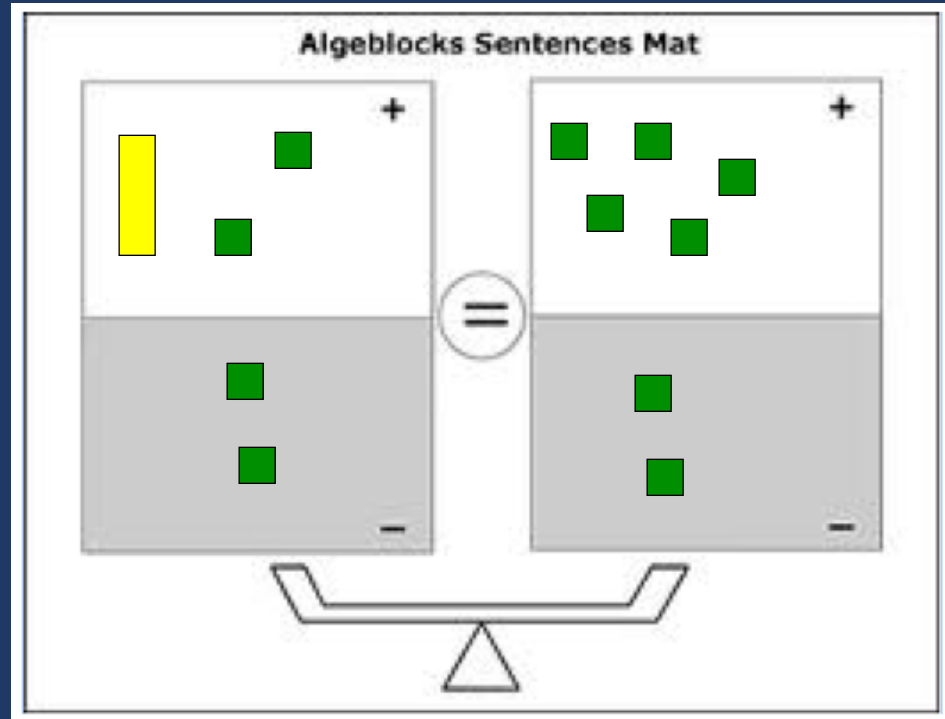
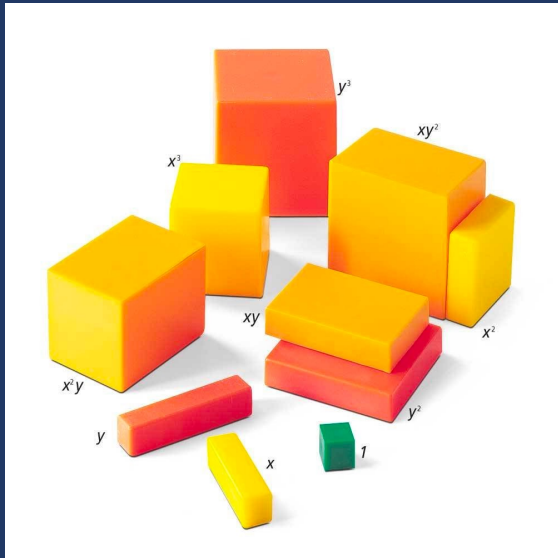
Equation Solving with Algeblocks

Problem	Representations
$x + 2 = 5$	
$4 + x = 6$	
$-2 = x + 3$	
$-1 = y - 4$	
$2x + 2 = 6$	
$x + 4 = 3x$	
$2(x + 3) = x + 4$	
$2x - 4 = 1 + 3x$	
$3y - 5 = -y - 1$	
$1 - x = x + 1$	



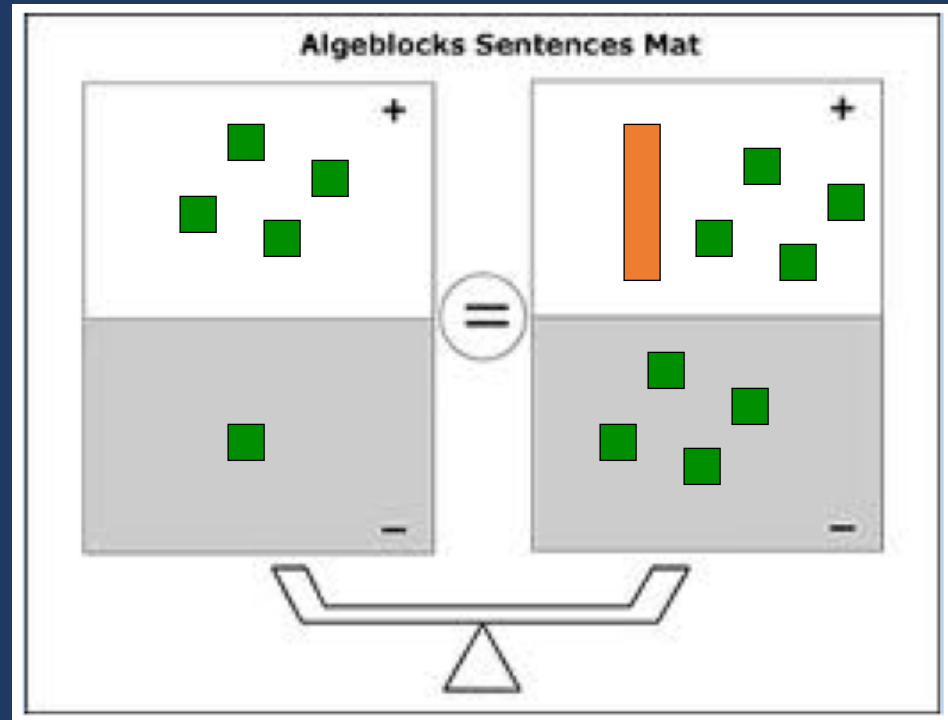
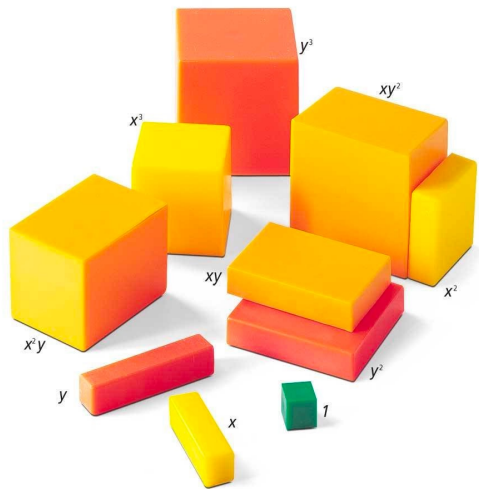
Algeblocks

$$x + 2 = 5$$



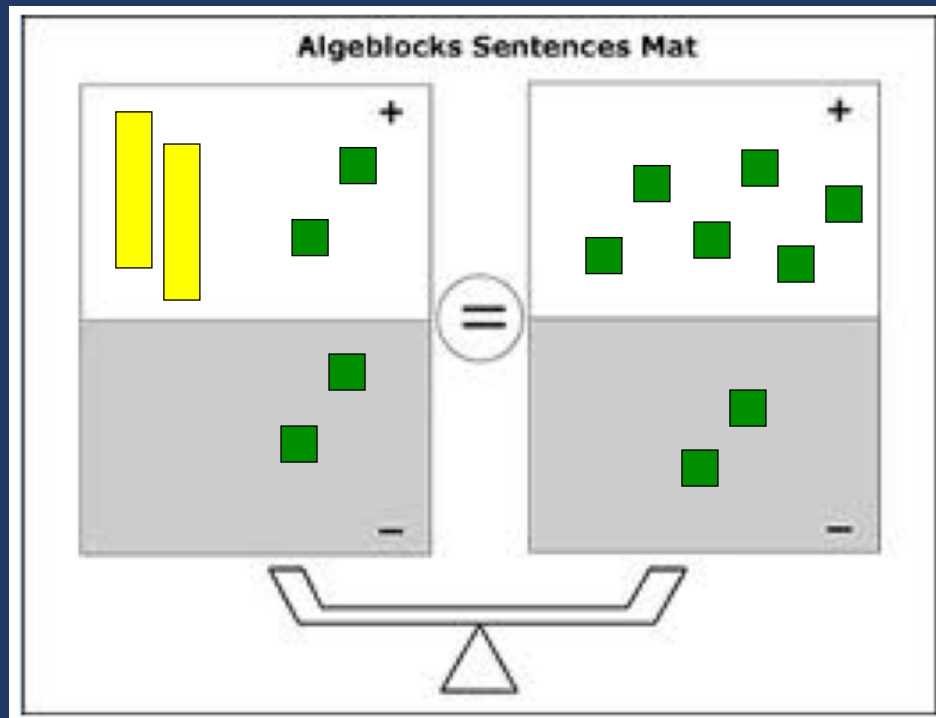
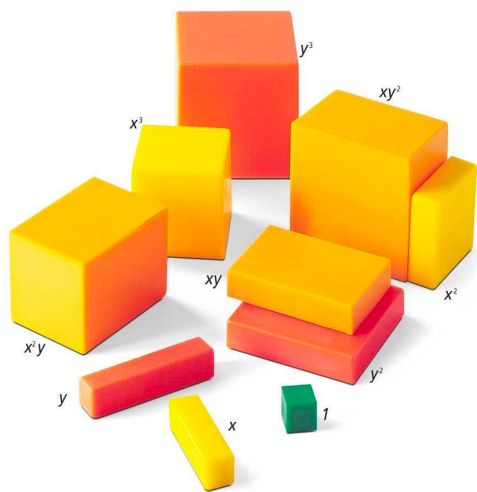
Algeblocks

$$-1 = y - 4$$



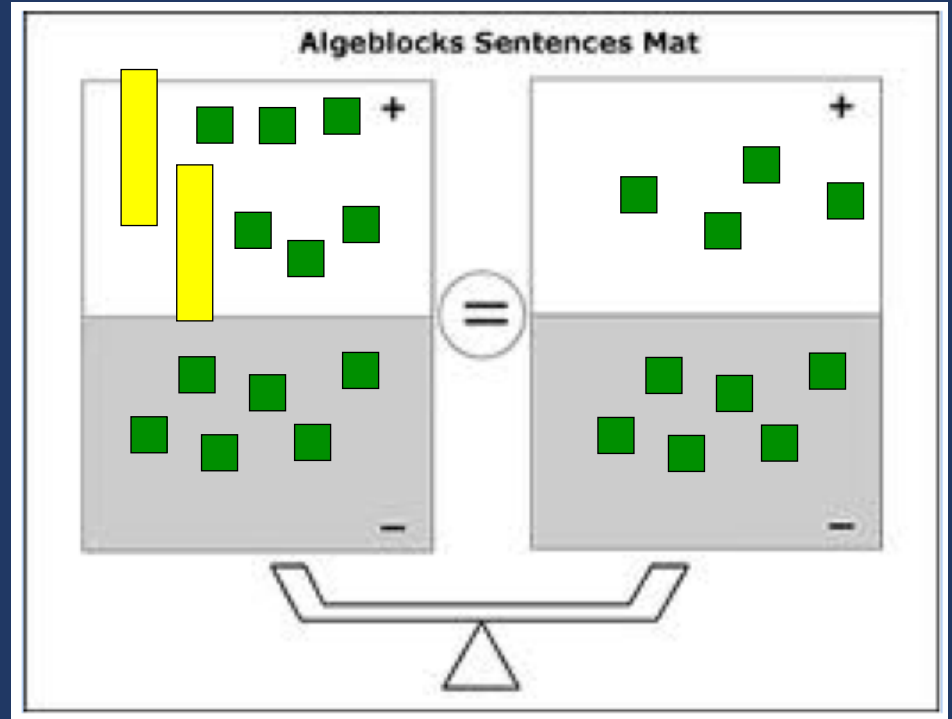
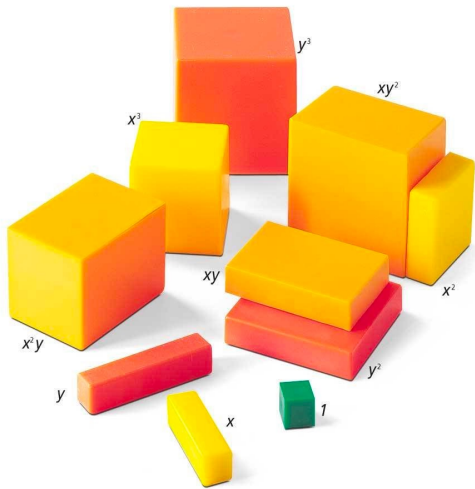
Algeblocks

$$2x + 2 = 6$$



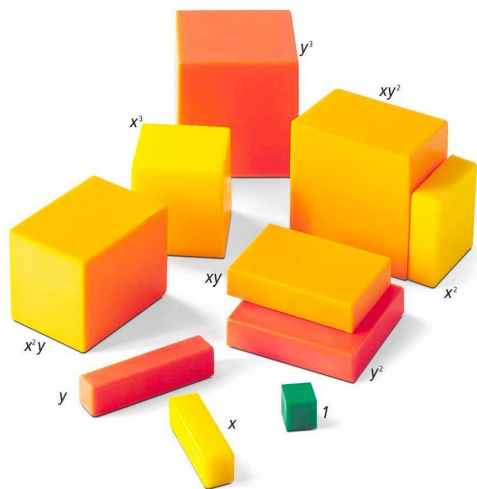
Algeblocks

$$2(x + 3) = 4$$

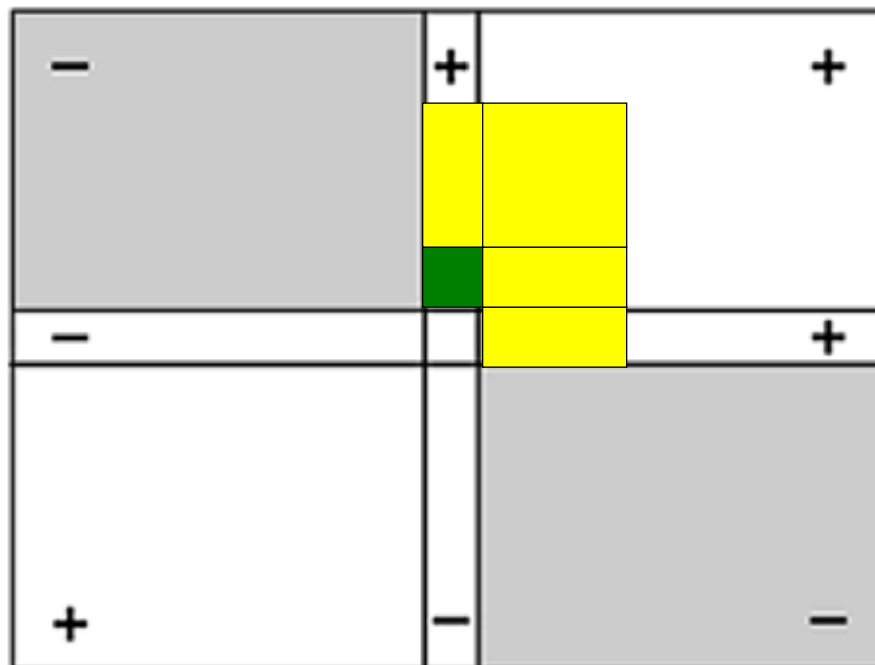


Algeblocks

$$x(1 + x)$$

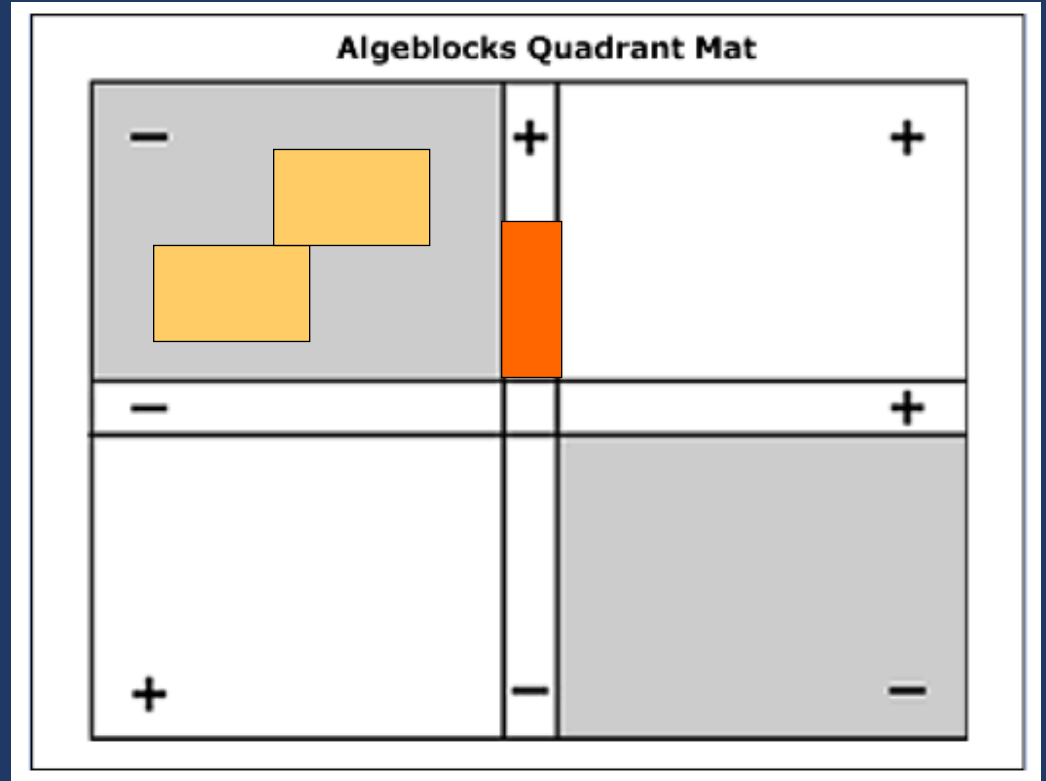
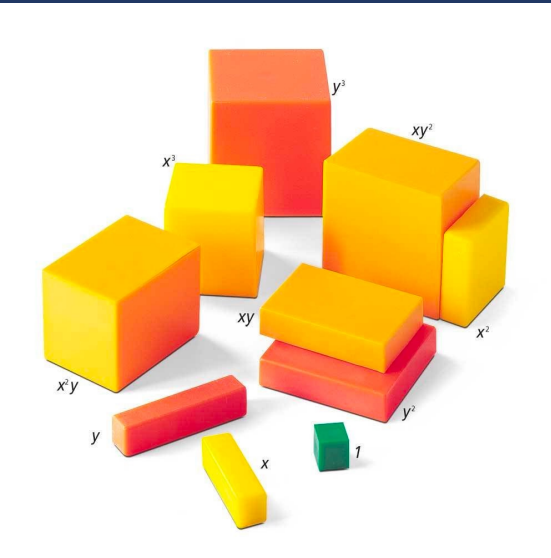


Algeblocks Quadrant Mat



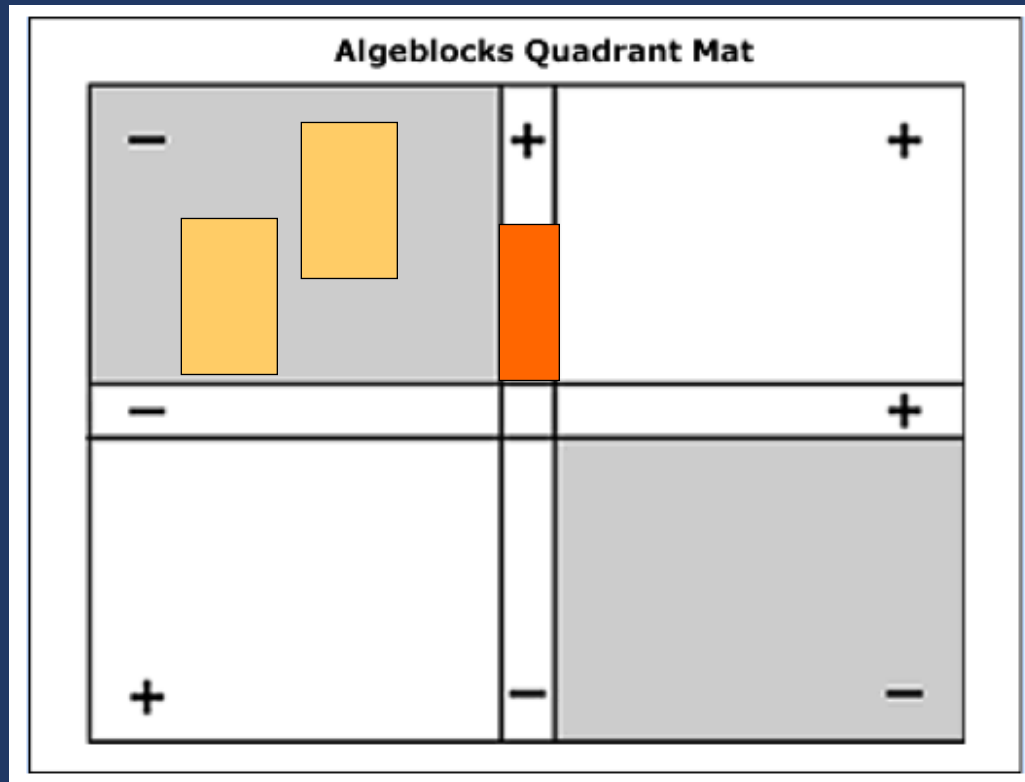
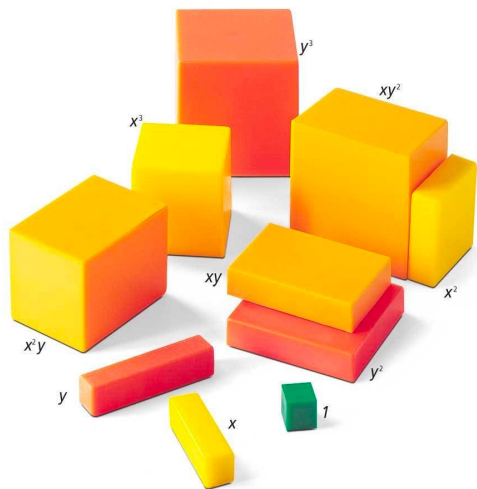
Algeblocks

$$-2xy \div y$$



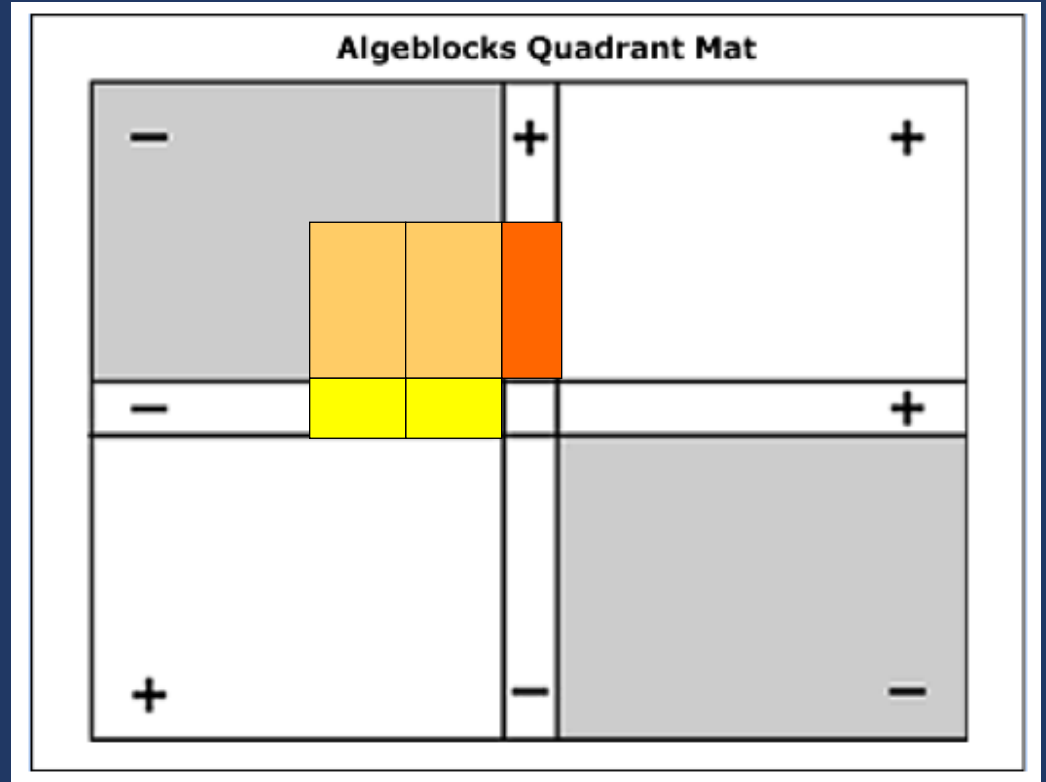
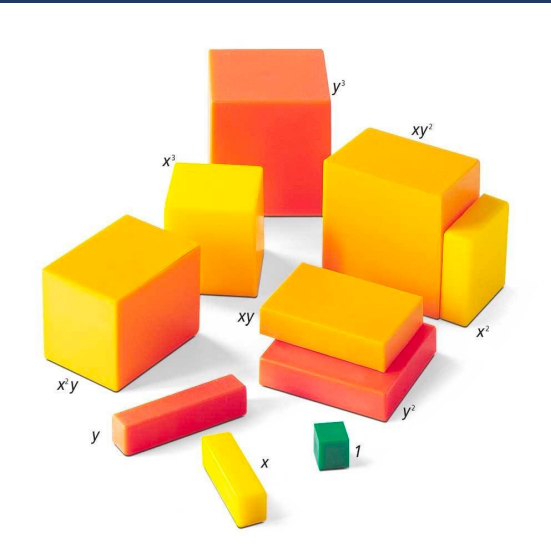
Algeblocks

$$-2xy \div y$$



Algeblocks

$$-2xy \div y$$



Operations and Place Value

Fractions and Decimals

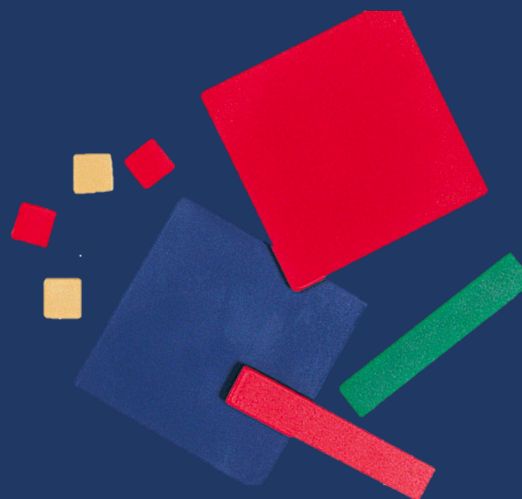
Integers and Algebra

Geometry

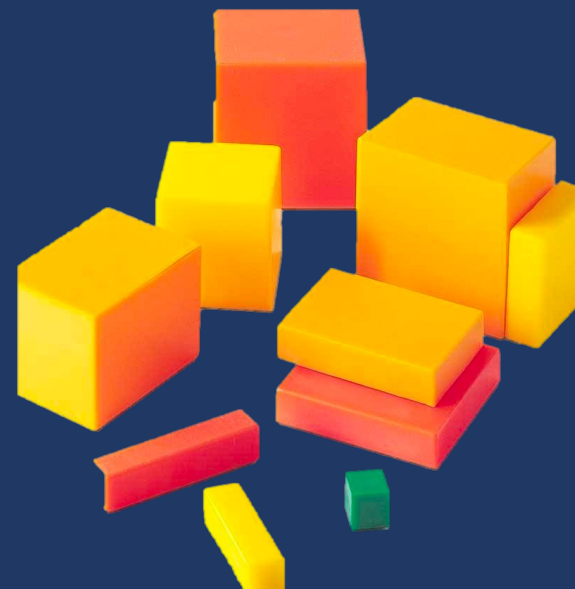




Cups and counters



Algebra tiles



Algeblocks

Triangles

Name	Properties	Examples
Equilateral		
Isosceles		
Scalene		
Acute		
Obtuse		
Right		

Quadrilaterals

Name	Properties	Examples
Parallelogram		
Rectangle		
Rhombus		
Square		
Kite		
Trapezoid		



Spatial Reasoning

1. Tangrams

2. Pentominoes

3. Tessellations



Spatial Reasoning

1. Tangrams

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Three-Dimensional Figures

Name	Properties (Faces, Edges, Vertices)	Examples
Rectangular Prism		
Cube		
Triangular Prism		
Hexagonal Prism		
Rectangular Pyramid		
Triangular Pyramid		
Hexagonal Pyramid		
Cylinder		
Cone		
Sphere		



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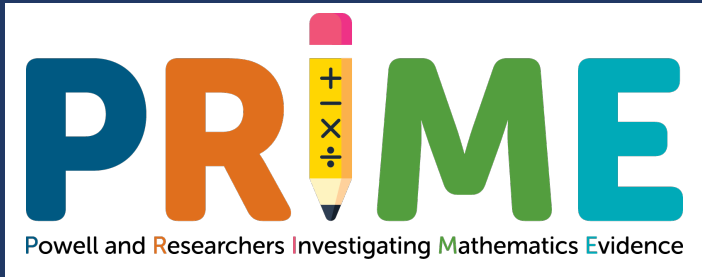
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