Mathematics Manipulatives Mania!

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PRIME
Powell and Researchers Investigating Mathematics Evidence

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Introduce yourself.

Describe your strengths in supporting mathematics.

Describe an opportunity for growth.

Operations and Place Value

Fractions and Decimals

Integers and Algebra

Geometry

Math Manipulatives Mania!

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Instructional Platform

Instructional Delivery

Instructional Strategies

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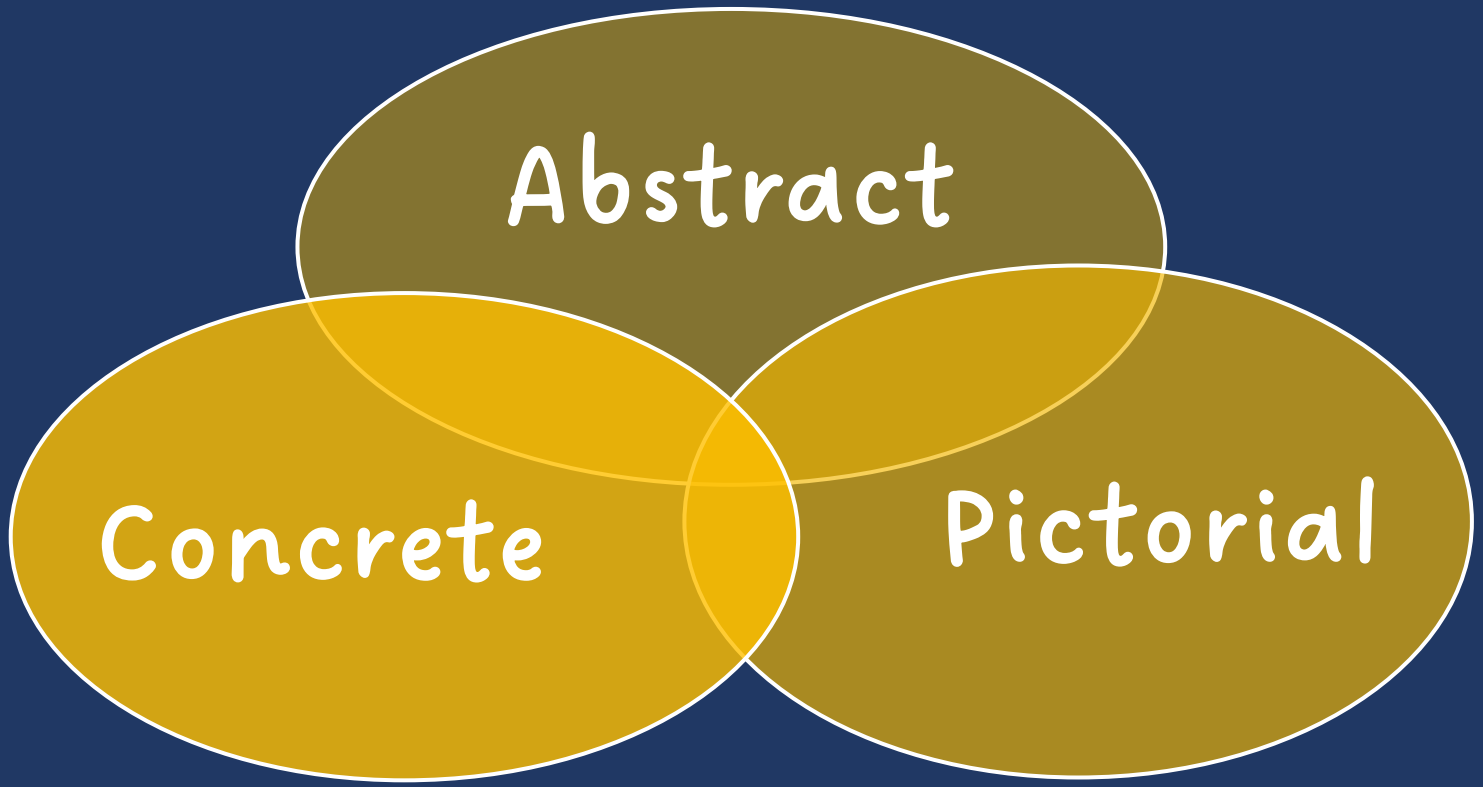
Instructional Platform

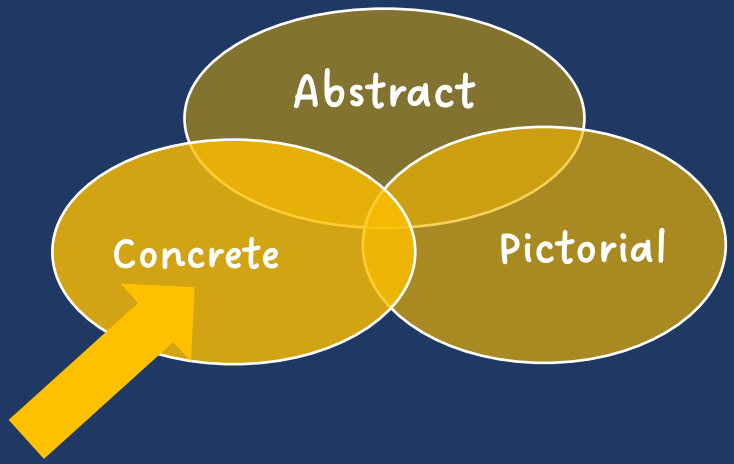
INSTRUCTIONAL DELIVERY

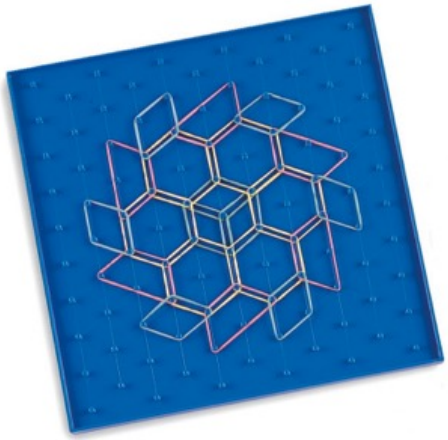
- Vocabulary
- Representations
- Model and Practice

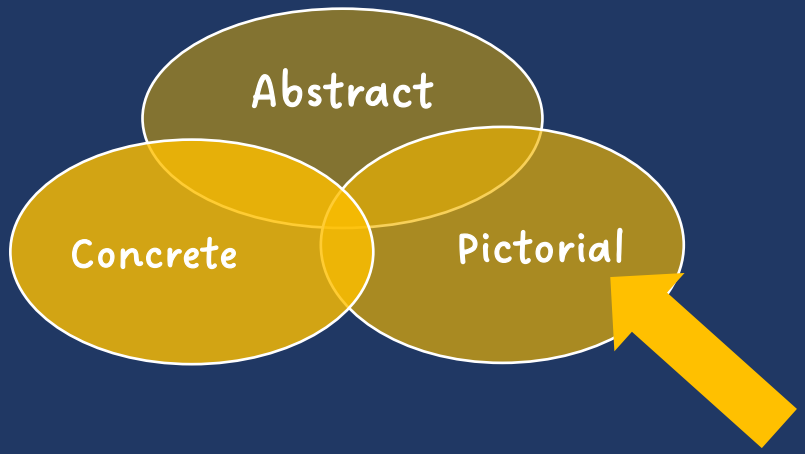
INSTRUCTIONAL STRATEGIES

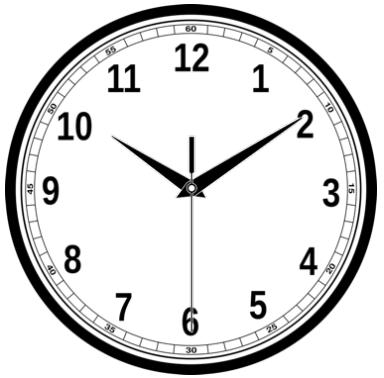
- Fluency
- Word Problems

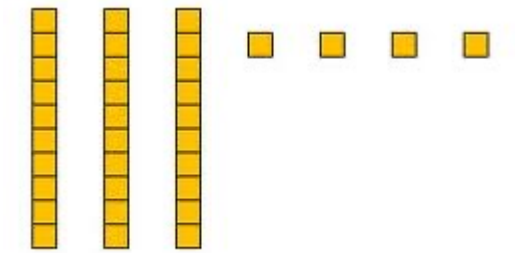


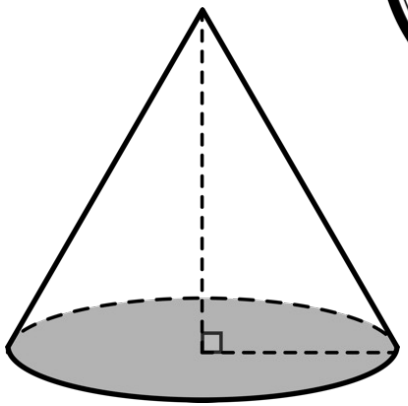


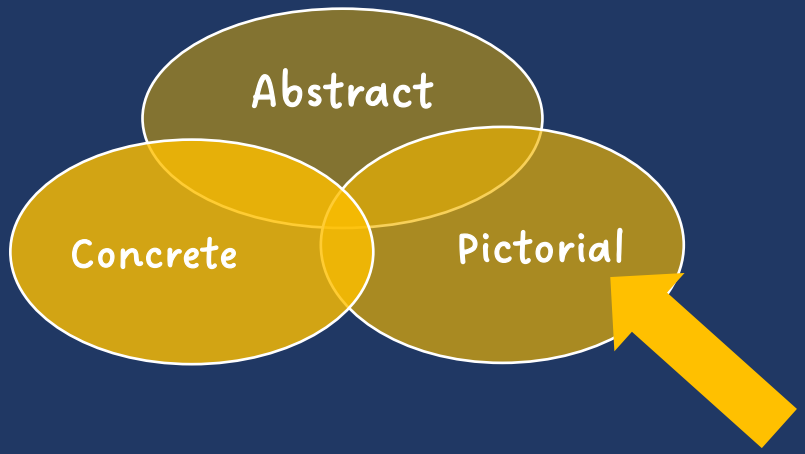


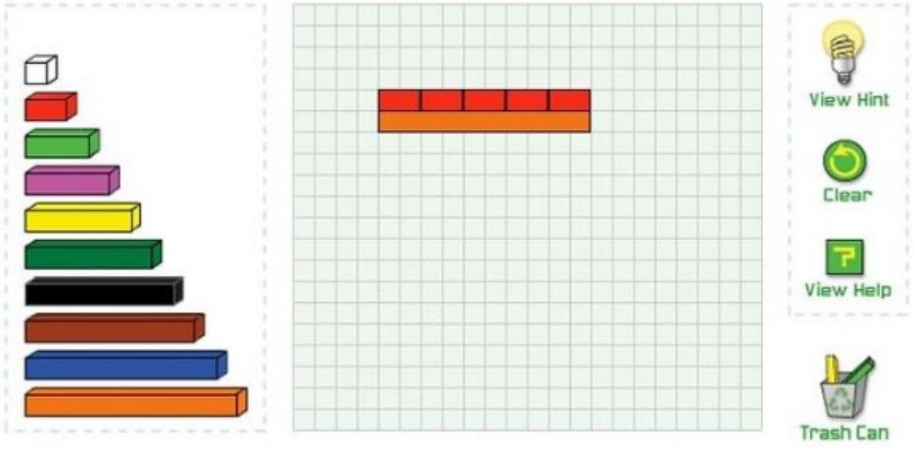


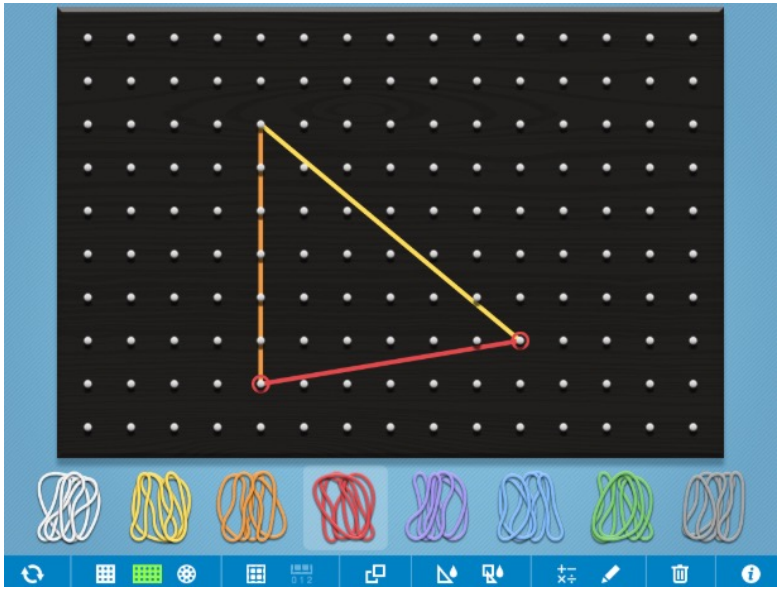


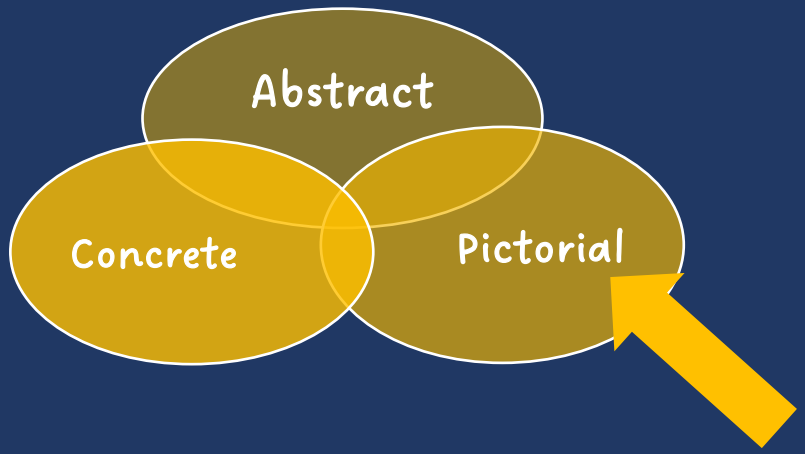


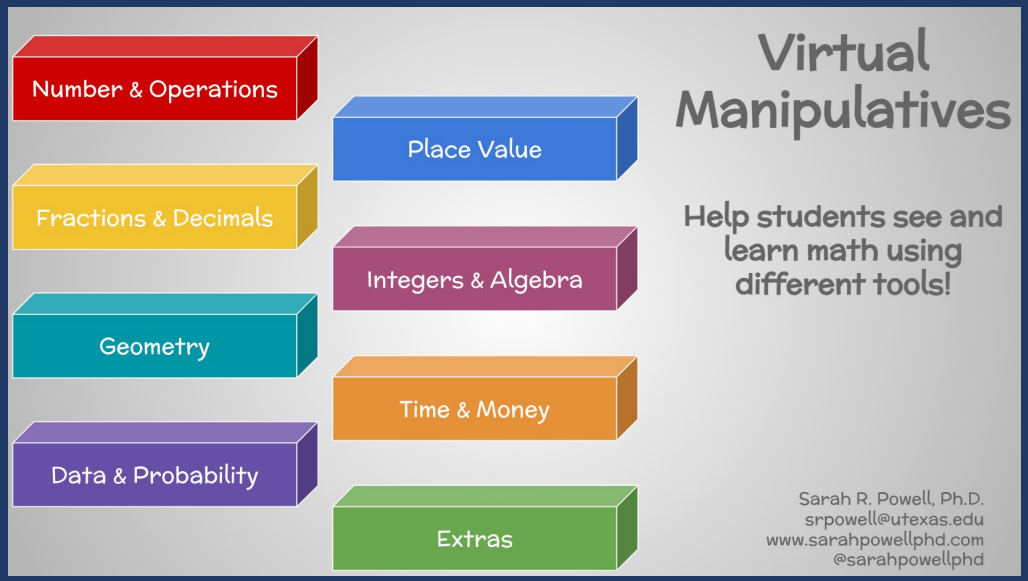












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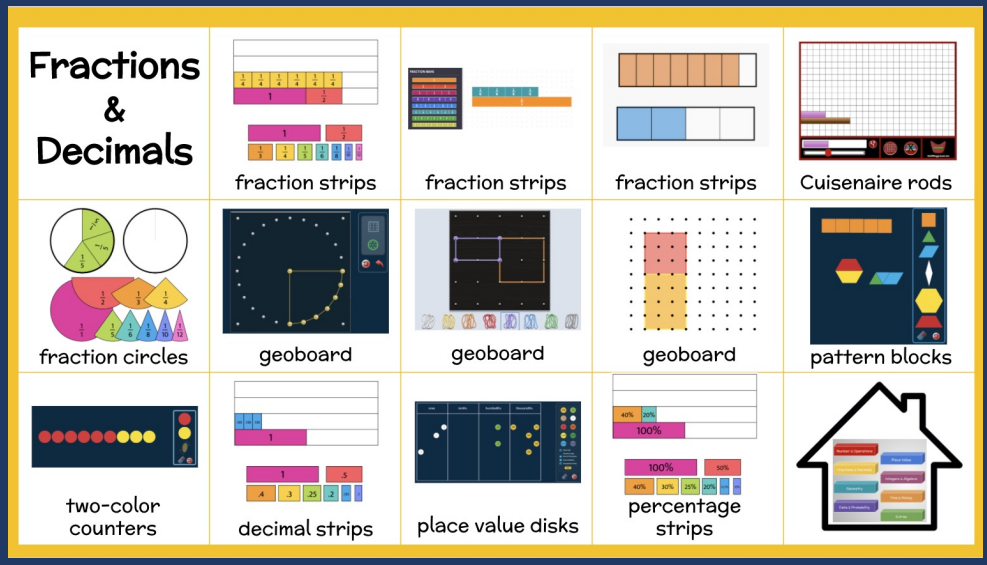


Operations and Place Value

Fractions and Decimals

Integers and Algebra

Geometry

Unifix cubes
Snap cubes

Math links

Mini motors

Two-color counters

Dice

Dominoes

Addition Concepts

Subtraction Concepts

Addition

100 addition facts

Single-digit addends sum to a single- or double-digit number

$$\begin{array}{rl} 5 & \text{(addend)} \\ +\ 4 & \text{(addend)} \\ \hline 9 & \text{(sum)} \end{array}$$

Total

Addition

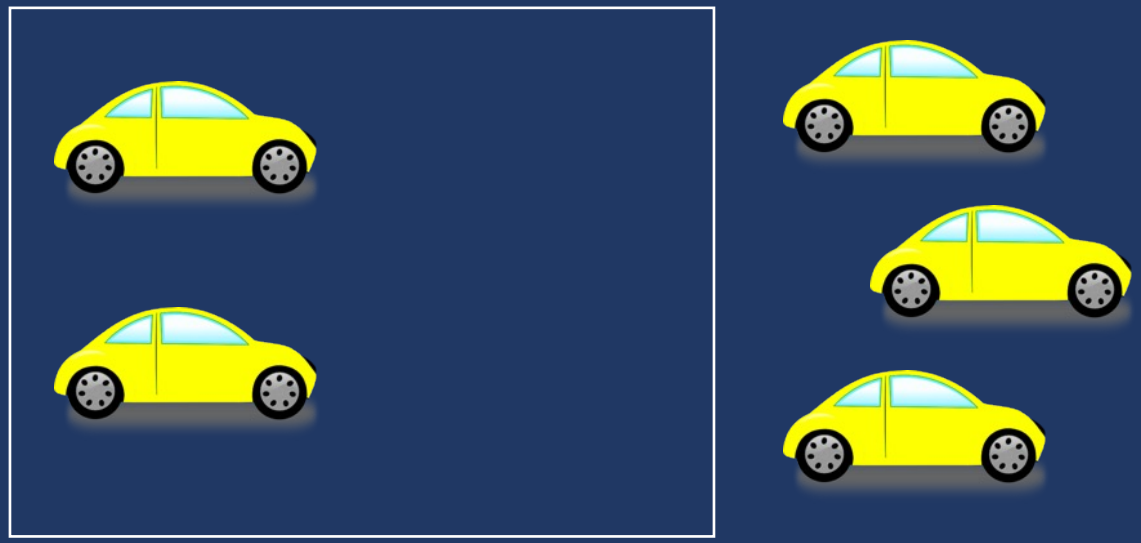
Count one set, count another set, put sets together, count sum

$$2 + 3 = 5$$

Change

Addition

Start with a set, add the other set, count sum



$$2 + 3 = 5$$

Total

Addition

Parts put together into a **total**

Karly saw **4** cardinals and **5** blue jays. How many birds did Karly see?

Change

Addition

An amount that **increases** or decreases

Premila had $4. Then they earned $5 for cleaning their room. How much money does Premila have now?

Total

Change

7 + 4 = __

5 + 8 = __

9 + 2 = __

Which representations would you use to help students understand addition?

Subtraction

100 subtraction facts

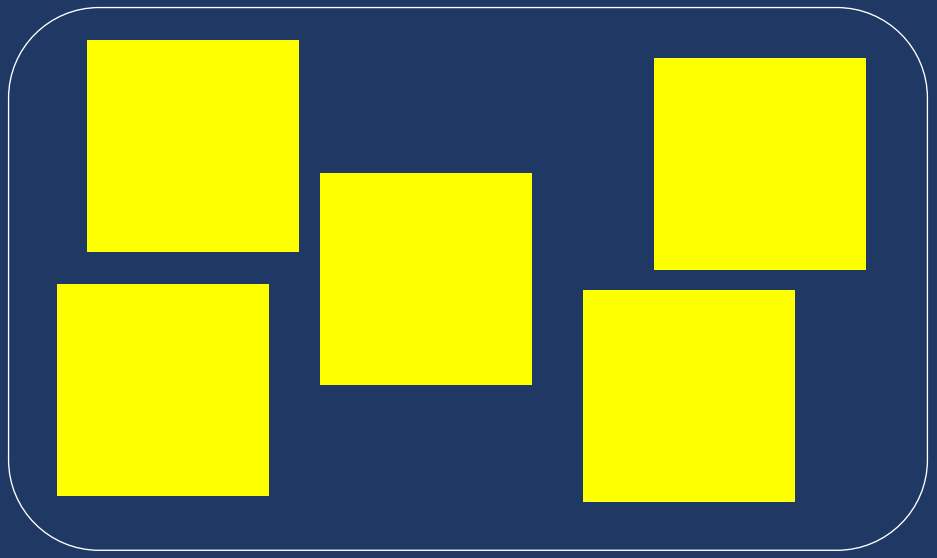
Subtrahend and difference are single-digit numbers and minuend is single- or double-digit number

$$\begin{array}{rl} 16 & \text{(minuend)} \\ -\ \ 8 & \text{(subtrahend)} \\ \hline 8 & \text{(difference)} \end{array}$$

Change

Subtraction

Start with a set, take away from that set, count difference

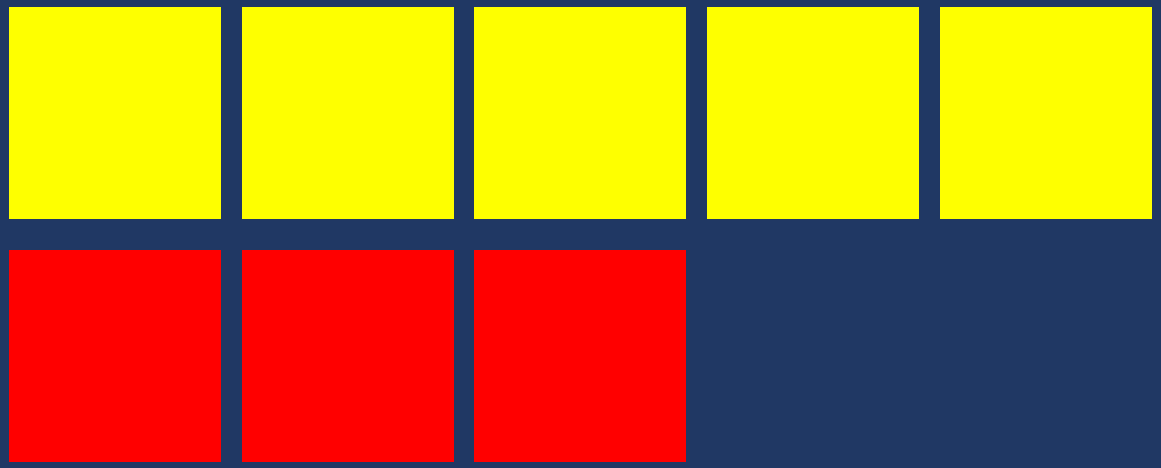


$$5 - 3 = 2$$

Difference

Subtraction

Compare two sets, count difference



$$5 - 3 = 2$$

Change

Subtraction

An amount that increases or **decreases**

Bronwyn had 9 cookies. Then they ate 2 of the cookies. How many cookies does Bronwyn have now?

Difference

Subtraction

Greater and lesser amounts compared for a difference

Rachel has 9 apples. Jodie has 2 apples. How many more apples does Rachel have? (How many fewer does Jodie have?)

Change

Difference

13 - 5 = __

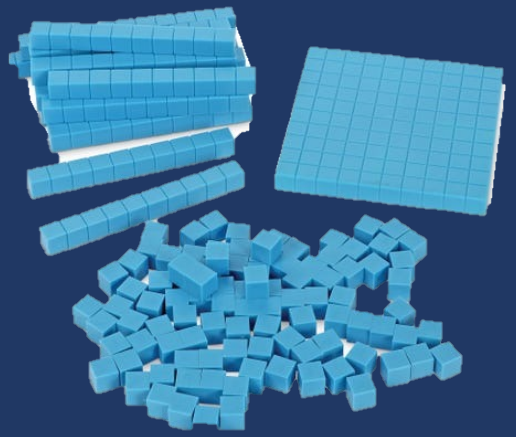
12 - 9 = __

15 - 8 = __

Which representations would you use for subtraction?

Unifix cubes
Snap cubes

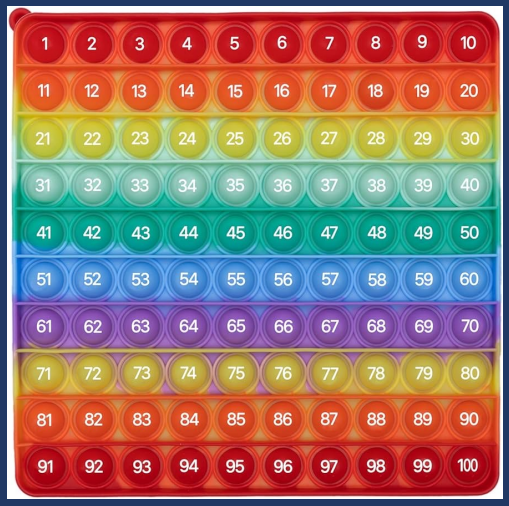
Math links



Base-10 Blocks

Place Value Disks

Hundred Chart



Hundred Pop It

Ten Frame

Tens and Ones

Count sets with 0-19 items in the set using a ten frame.

Determine how many sets of ten. Determine how many ones.

Read as:
14 is 1 ten and 4 ones.

Tens and Ones

Count sets with 0-19 items in the set using items that can be linked or connected.

Determine how many sets of ten. Determine how many ones.

Read as:
14 is 1 ten and 4 ones.

Tens and Ones

Use a hundred chart to identify patterns with tens and ones.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

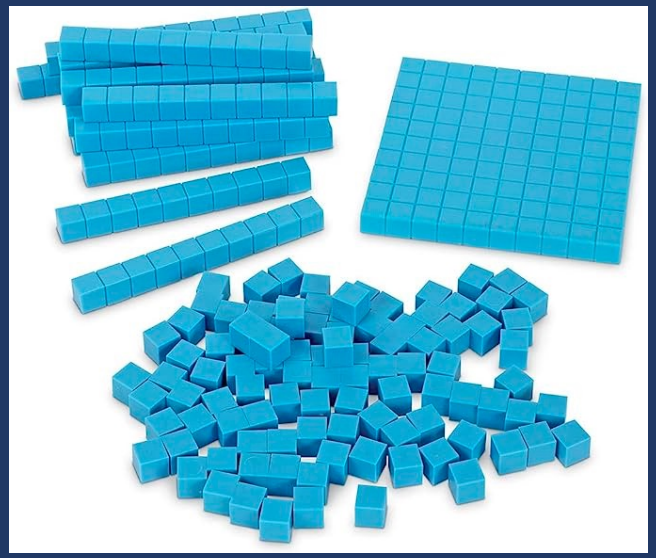
Standard Form and Expanded Notation

Roll dice to create numbers with tens and ones.

Write in standard form and expanded form.

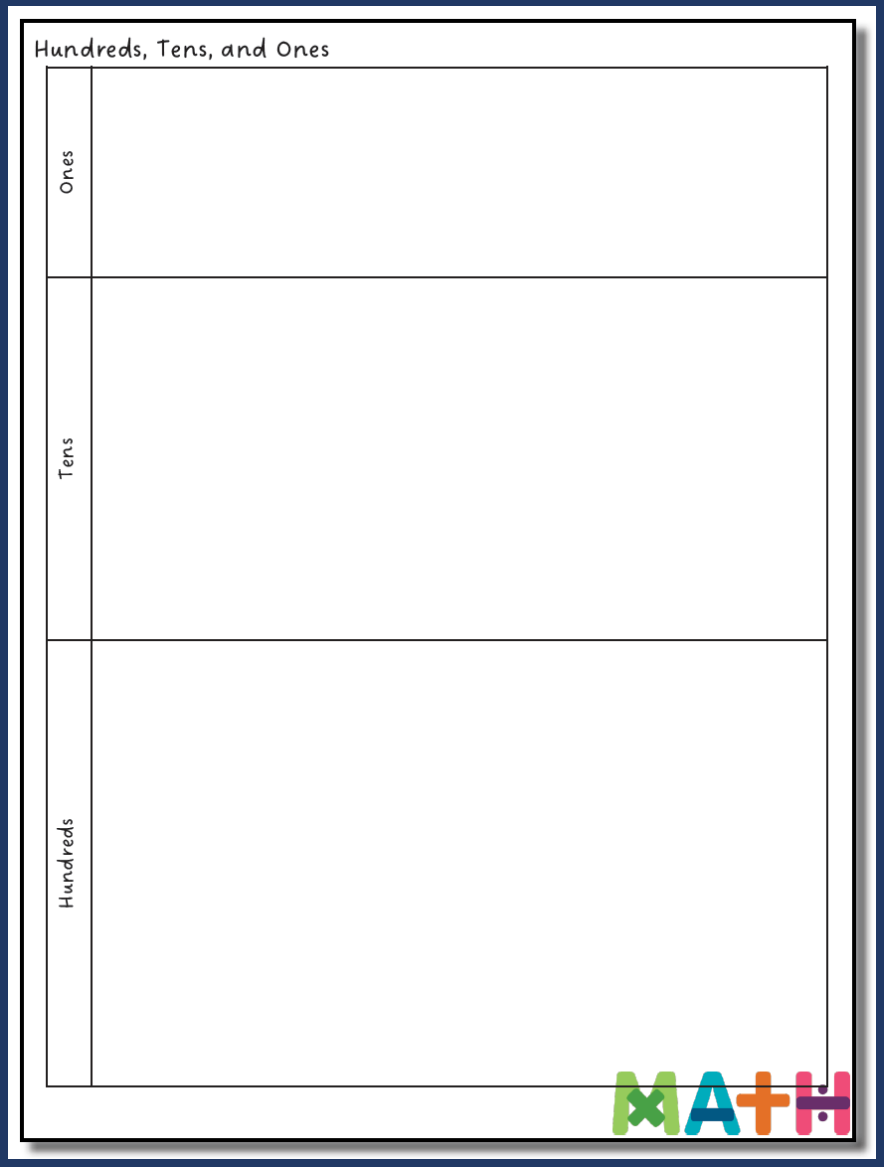
1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____
11. _____
12. _____
13. _____
14. _____
15. _____
16. _____
17. _____
18. _____
19. _____
20. _____

Hundreds, Tens, and Ones



Proportional materials

Non-proportional materials

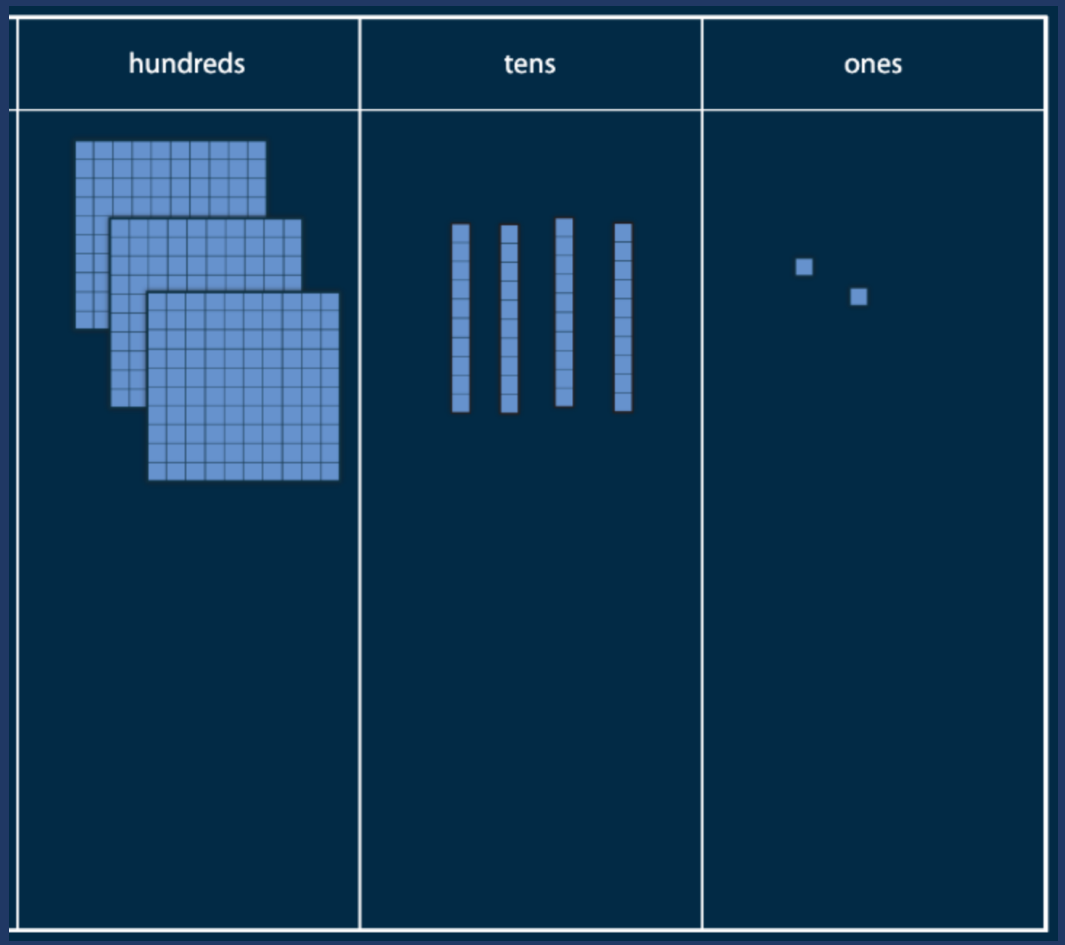


Hundreds, Tens, and Ones

Show numbers 0-999.

Determine how many hundreds, tens, and ones.

Read as:
342 is 3 hundreds, 4 tens, and 2 ones.

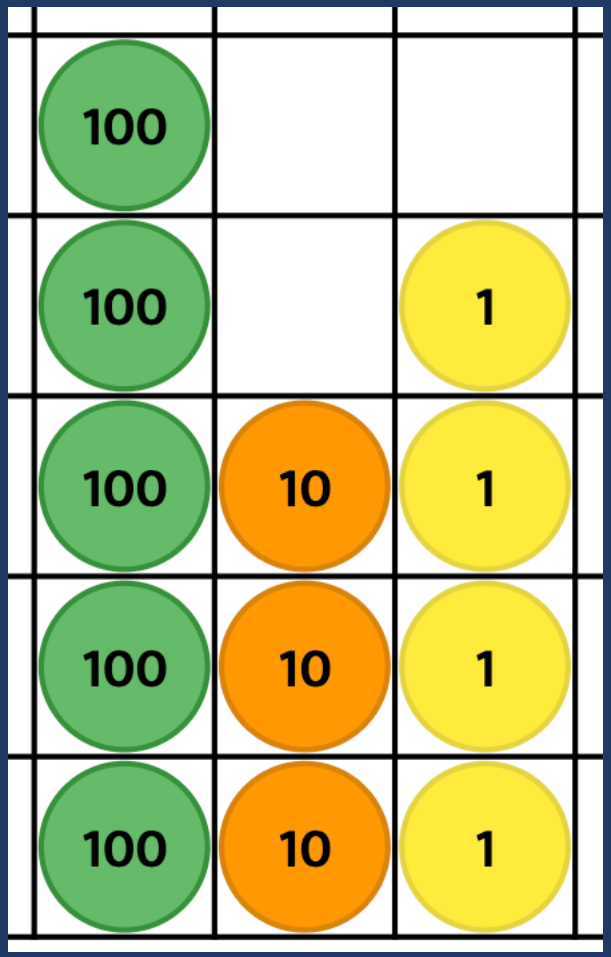


Hundreds, Tens, and Ones

Show numbers 0-999.

Determine how many hundreds, tens, and ones.

Read as:
534 is 5 hundreds, 3 tens, and 4 ones.



Show:

14

41

163

596

Which representations would you use for place value?

Addition Computation

24 + 35 =

64 + 29 =

Standard

A.

$$\begin{array}{r} {\scriptstyle 1} \\ 74 \\ +\ 18 \\ \hline 92 \end{array}$$

B.

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 725 \\ +\ 365 \\ \hline 1{,}090 \end{array}$$

Partial Sums

A.

$$\begin{array}{r} 74 \\ +\ 18 \\ \hline 80 \\ +12 \\ \hline 92 \end{array}$$

B.

$$\begin{array}{r} 725 \\ +\ 365 \\ \hline 1{,}000 \\ 80 \\ +\ 10 \\ \hline 1{,}090 \end{array}$$

Opposite Change

A.

$$\begin{array}{r} 74 \xrightarrow{-4} 70 \\ +\ 18 \xrightarrow{+4} +22 \\ \hline 92 \end{array}$$

B.

$$\begin{array}{r} 725 \xrightarrow{+5} 730 \\ +\ 365 \xrightarrow{-5} +360 \\ \hline 1{,}090 \end{array}$$

Show:

24 + 35

64 + 29

Which representations would you use for addition computation?

Subtraction Computation

75 - 42 =

61 - 38 =

Standard

A.

$$\begin{array}{r} 5 \\ \not{6}\,{}^{1}2 \\ -\ 17 \\ \hline 45 \end{array}$$

B.

$$\begin{array}{r} 2\ 9 \\ \not{3}\,{}^{1}\not{0}\,{}^{1}5 \\ -\ 9\ 6 \\ \hline 209 \end{array}$$

Partial Differences

A.

$$\begin{array}{r} 62 \\ -\ 17 \\ \hline +50 \\ -\ 5 \\ \hline 45 \end{array}$$

B.

$$\begin{array}{r} 305 \\ -\ 96 \\ \hline +300 \\ -90 \\ -1 \\ \hline 209 \end{array}$$

Same Change

A.

$$\begin{array}{r} 62 \xrightarrow{+3} 65 \\ -\ 17 \xrightarrow{+3} -20 \\ \hline 45 \end{array}$$

B.

$$\begin{array}{r} 305 \xrightarrow{+4} 309 \\ -\ 96 \xrightarrow{+4} -100 \\ \hline 209 \end{array}$$

Add Up

A.

$$\begin{array}{r} 62 \\ -\ 17 \\ \hline \end{array}$$

$$\begin{array}{r} 17 \\ 20 \quad 3 \\ 60 \quad 40 \\ 62 \quad +\ 2 \\ \hline 45 \end{array}$$

B.

$$\begin{array}{r} 305 \\ -\ 96 \\ \hline \end{array}$$

$$\begin{array}{r} 96 \\ 100 \quad 4 \\ 300 \quad 200 \\ 305 \quad +\ 5 \\ \hline 209 \end{array}$$

Show:

75 - 42

61 - 38

Which representations would you use for subtraction computation?

Multiplication Concepts

Division Concepts

Multiplication

100 multiplication facts

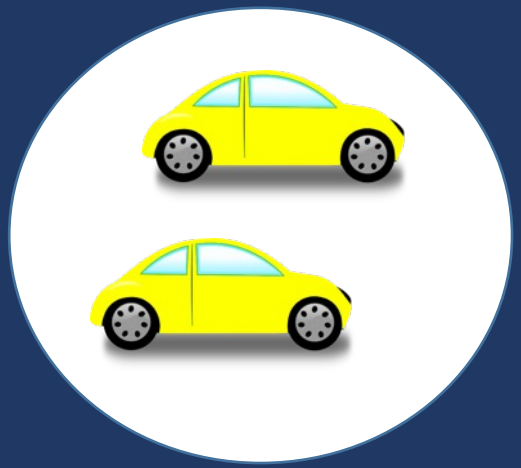
Multiplication of single-digit factors results in a single- or double-digit product

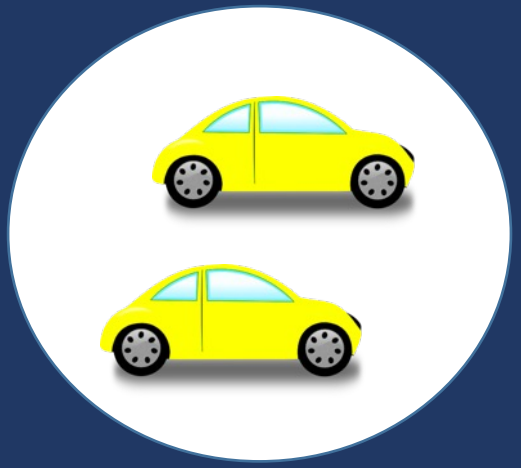
$$\begin{array}{rl} 2 & \text{(factor)} \\ \times\ 3 & \text{(factor)} \\ \hline 6 & \text{(product)} \end{array}$$

Equal Groups

Multiplication

Show the groups, show the amount for each group, count product



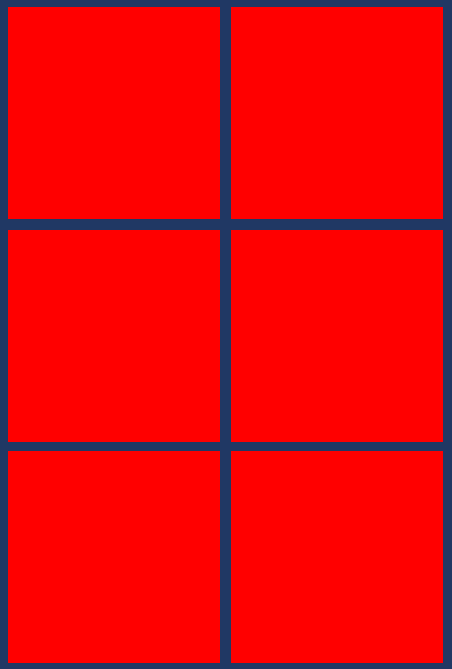


$3 \times 2 = 6$

Equal Groups

Multiplication

Show the groups, show the amount for each group, count product

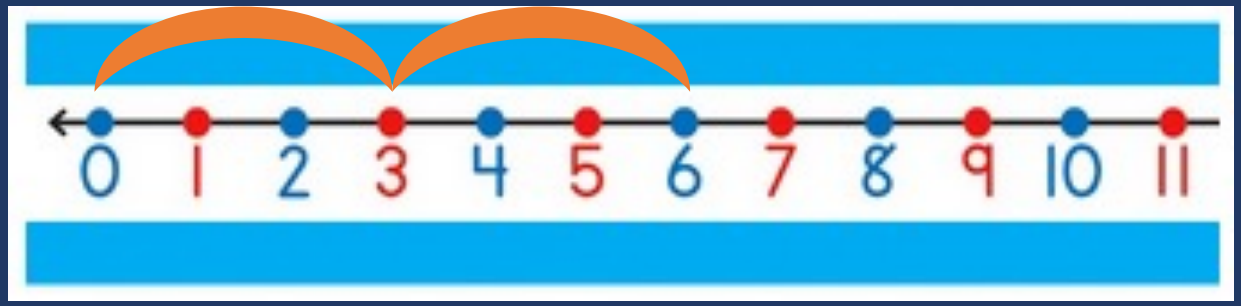


$3 \times 2 = 6$

Comparison

Multiplication

Show a set, then multiply the set



$$3 \times 2 = 6$$

Equal Groups

Multiplication

Groups multiplied by number in each group for a product

Rhiannon has 2 boxes of crayons. There are 12 crayons in each box. How many crayons does Rhiannon have altogether?

Comparison

Multiplication

Set multiplied by a number of times for a product

Vivienne had 12 stickers. Jessica had 2 times as many stickers as Vivienne. How many stickers did Jessica have?

Equal Groups

Comparison

$2 \times 5 = \_\_$

$3 \times 4 = \_\_$

$6 \times 2 = \_\_$

Which representations would you use to help students understand multiplication?

Division

90 division facts

Divisor and quotient are single-digit numbers and dividend is single- or double-digit number

8 ÷ 4 = 2

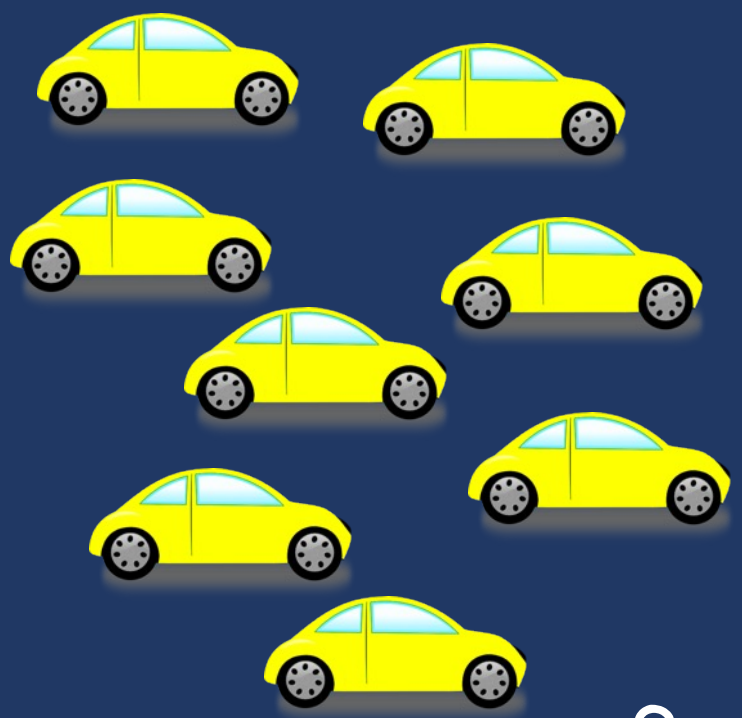
(dividend) (divisor) (quotient)

Equal Groups

(Partitive Division)

Division

Show the dividend, divide equally among divisor, count quotient



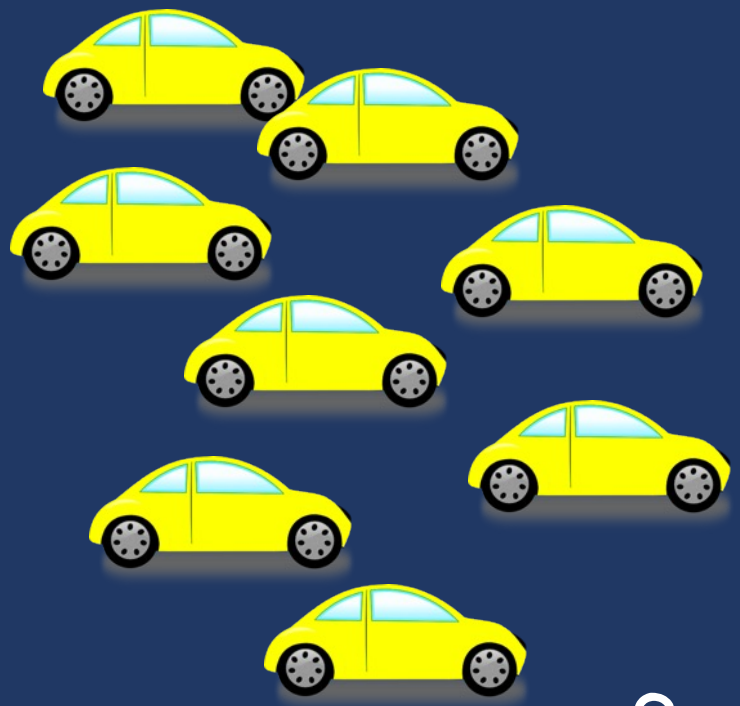
$8 \div 2 = 4$

Equal Groups

(Quotative Division)

Division

Show the dividend, make groups of the divisor, count groups



$$8 \div 2 = 4$$

Equal Groups

Division

Groups multiplied by number in each group for a product

Stefanie has 12 pencils. She wants to share them equally among her 2 friends. How many pencils will each friend receive?

Nicole has 12 pencils. She put them into pencil pockets with 6 pencils each. How many pencil pockets did Nicole use?

Equal Groups

(Partitive Division)
(Quotative Division)

10 ÷ 5 = __

12 ÷ 4 = __

8 ÷ 2 = __

Which representations would use to help students understand division?

Addition	Subtraction
Multiplication	Division

Build fluency with math facts.

- Addition: single-digit addends
- Subtraction: single-digit subtrahend
- Multiplication: single-digit factors
- Division: single-digit divisor

$$\begin{array}{r} 5 \\ +\ 8 \\ \hline \end{array} \qquad \begin{array}{r} 9 \\ -\ 4 \\ \hline \end{array} \qquad \begin{array}{r} 6 \\ \times\ 7 \\ \hline \end{array} \qquad \begin{array}{r} 56 \\ \div\ 8 \\ \hline \end{array}$$

DAILY

BRIEF

Work on small sets of facts

Work on unknown facts
(in combination with known facts)

Dice

Roll the Dice

8 + 7 = 15

____ + ____ = ____

____ + ____ = ____

____ + ____ = ____

____ − ____ = ____

____ − ____ = ____

____ − ____ = ____

____ − ____ = ____

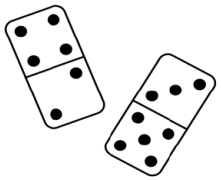
Beach Ball

4 plus 6 equals 10.

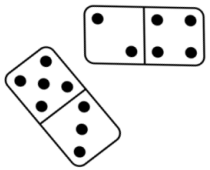
7 plus 6 equals 13.

2 plus 2 equals 4.

Dominoes



Dominoes



4 + 6 = 10

____ + ____ = ____

____ − ____ = ____

____ − ____ = ____

____ × ____ = ____

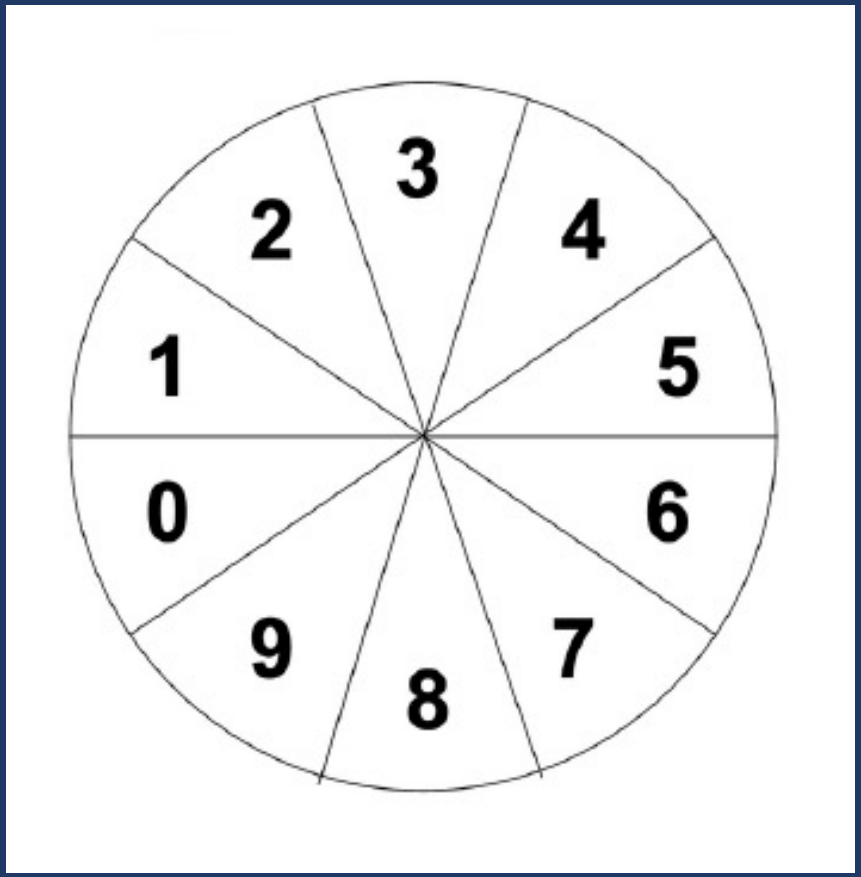
____ × ____ = ____

Spinner

2 times 4 equals 8.

6 times 9 equals 54.

7 times 1 equals 7.



Playing Cards

Cards 

____ + ____ = ____

____ + ____ = ____

6 − 2 = 4

____ − ____ = ____

____ + ____ = ____

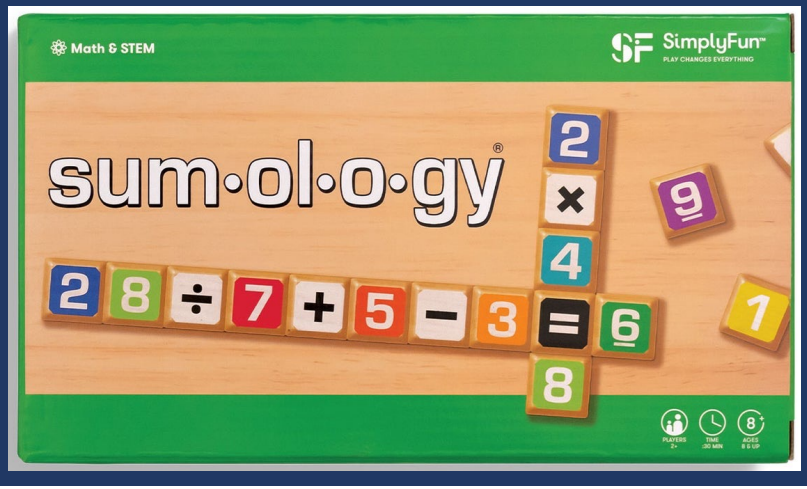
____ + ____ = ____

____ − ____ = ____

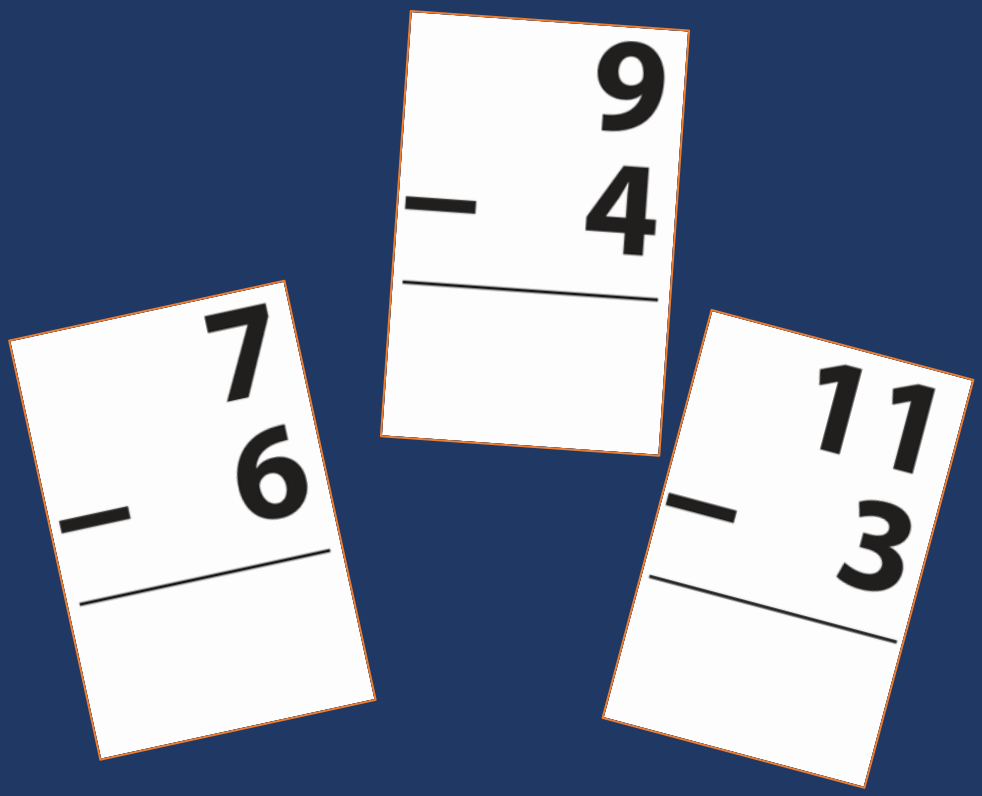
____ − ____ = ____

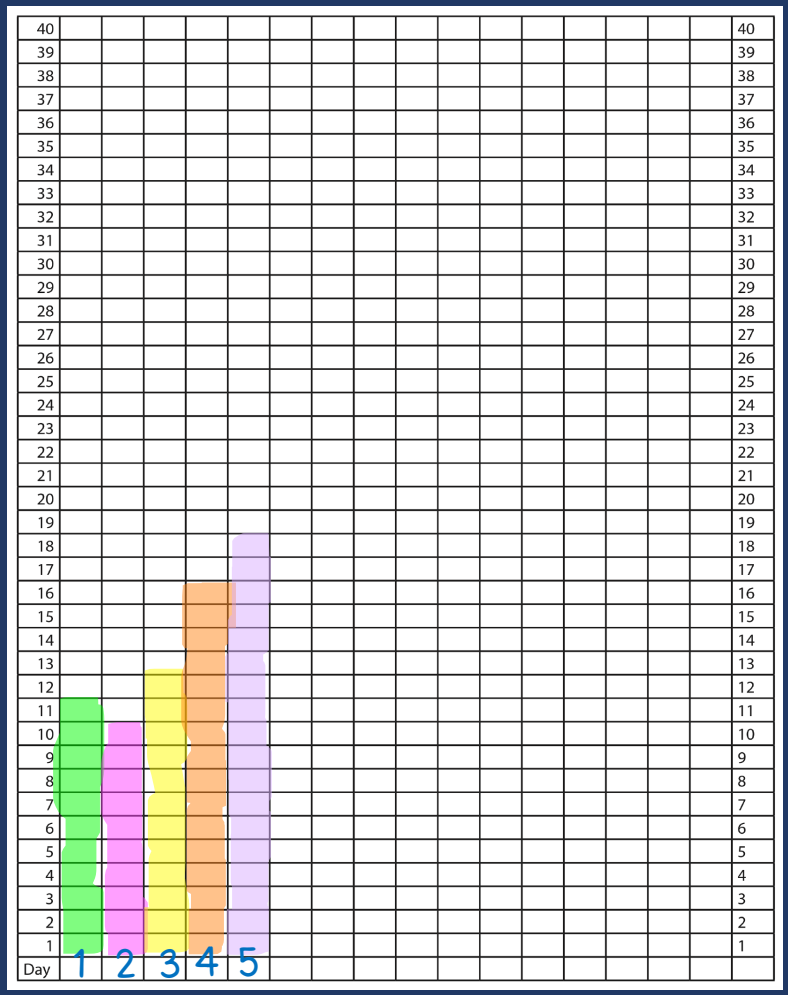
Wrap-Ups

Mobi Math



Flash Cards

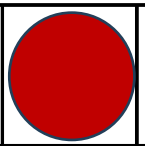
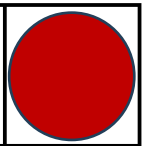
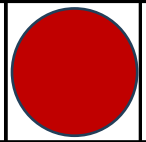
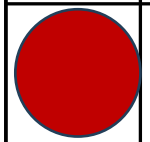




Bingo

Math Bingo

Your teacher will call out a math problem. Quickly solve the problem. If you have the answer on your bingo card, cover it with a chip. The middle space is free!
The first person to finish the pattern your teacher decides wins!
(5 in a row, 4 corners, etc.)

12		24	100	
15	0	42	16	20
8	35		6	4
	2	40	27	7
50	10	30	48	14

8 times 10 equals...

3 times 1 equals...

2 plus 3 equals...

Magic Squares

Magic Squares Board

1. Place the sum or product in the bottom right corner.
2. In the bottom row, create a fact with a sum or product of the bottom right corner.
3. In the right column, create a fact with a sum or product of the bottom right corner.
4. Create two columns with a sum or product of the bottom number.
5. Create two rows with a sum or product of the right column number.
6. Write the created facts below.

0	2	2
5	4	9
5	6	11

$0 + 2 = 2$

$5 + 4 = 9$

$2 - 0 = 2$

$9 - 5 = 4$

4	5	9
2	0	2
6	5	11

7	3	10
1	0	1
8	3	11

6	1	7
3	2	5
9	3	12

4	4	8
2	2	4
6	6	12

5	1	6
4	3	7
9	4	13

5	1	6
3	4	7
8	5	13

6	3	9
2	3	5
8	6	14

1	5	6
6	2	8
7	7	14

6	2	8
3	4	7
9	6	15

4 5 2 0

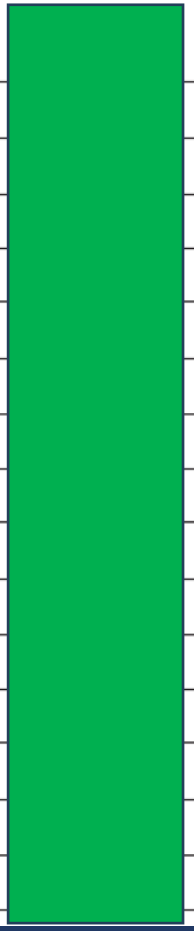
5 6 9 11 2

Cover, Copy, Compare

Cover, Copy, Compare

	9 x 6 54	8 × 6 48	
7 × 8 56		6 × 5 30	
9 × 9 81		7 × 9 63	
6 × 7 42		8 × 5 40	
8 × 8 64		7 × 7 49	

File Folder



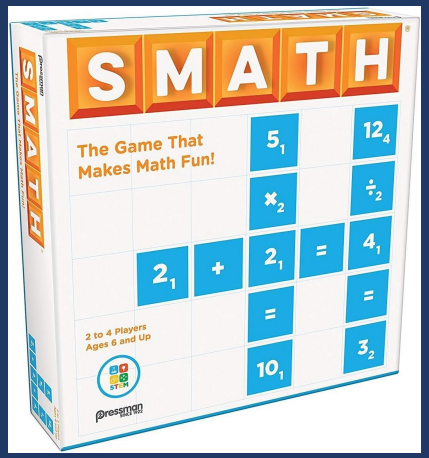
6 + 3 = 9
1 + 7 = 8
6 + 4 = 10
7 + 3 =
2 + 7 =
5 + 6 =
4 + 7 =
7 + 8 =
6 + 7 =
7 + 9 =
7 + 6 =
8 + 7 =
7 + 0 =
9 + 6 =
6 + 0 =
6 + 8 =

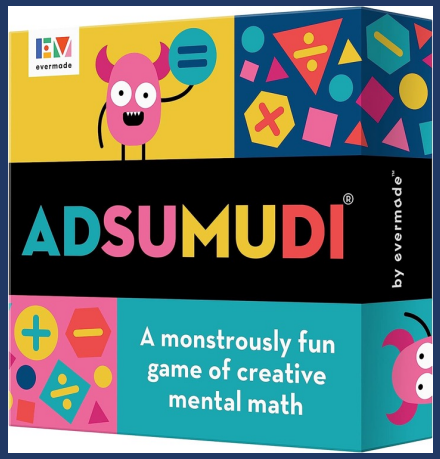
Taped Problems

Taped Problems

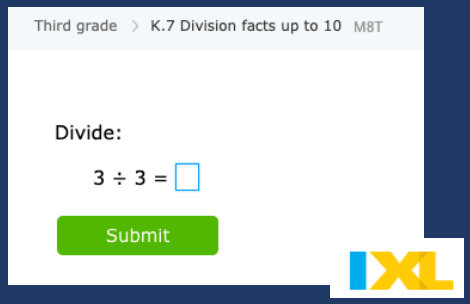
$8 \times 8 = 64$	$7 \times 7 = 49$	$8 \times 7 =$
$6 \times 5 =$	$6 \times 7 =$	$6 \times 8 =$
$5 \times 7 =$	$5 \times 5 =$	$5 \times 6 =$
$6 \times 6 =$	$8 \times 6 =$	$7 \times 6 =$
$7 \times 8 =$	$8 \times 5 =$	$7 \times 5 =$

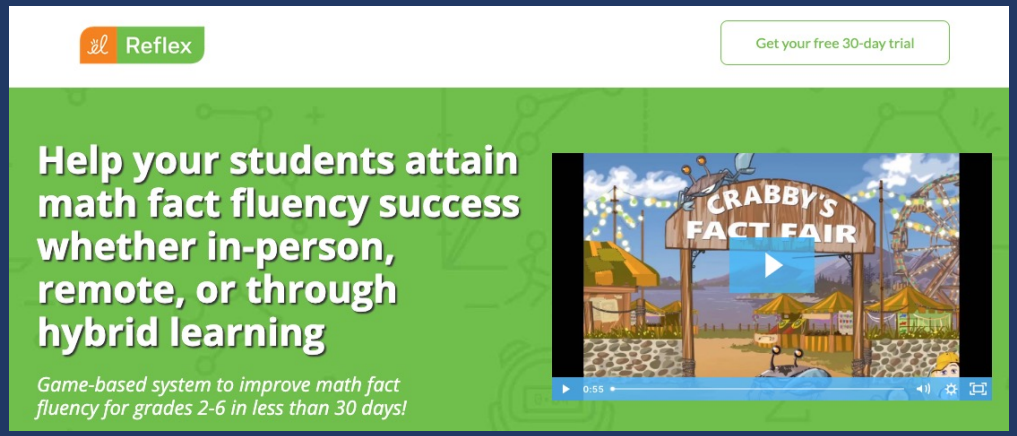
Games

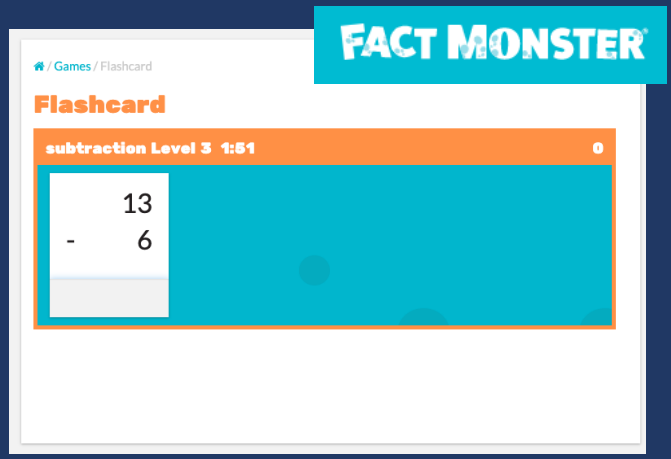




Technology







DAILY

BRIEF

Work on small sets of facts

Work on unknown facts
(in combination with known facts)

Multiplication Computation

13 × 47 =

123 × 24 =

Standard

A.

$$\begin{array}{r} {\scriptstyle 1} \\ 24 \\ \times\ 43 \\ \hline {\scriptstyle 1\,1}72 \\ +960 \\ \hline 1{,}032 \end{array}$$

B.

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 132 \\ \times\ 53 \\ \hline 396 \\ +6600 \\ \hline 6{,}996 \end{array}$$

Partial Products

A.

$$
\begin{array}{r}
24 \\
\times\ 43 \\
\hline
800 \\
160 \\
60 \\
+\ 12 \\
\hline
1{,}032
\end{array}
$$

B.

$$
\begin{array}{r}
132 \\
\times\ 53 \\
\hline
5000 \\
1500 \\
100 \\
300 \\
90 \\
+\ 6 \\
\hline
6996
\end{array}
$$

Area (Array)

A.

$$\begin{array}{r} 24 \\ \times\ 43 \\ \hline \end{array}$$

	20	4	
	800	160	40
	60	12	3

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 800 \\ 160 \\ 60 \\ +\ 12 \\ \hline 1{,}032 \end{array}$$

B.

$$\begin{array}{r} 132 \\ \times\ 53 \\ \hline \end{array}$$

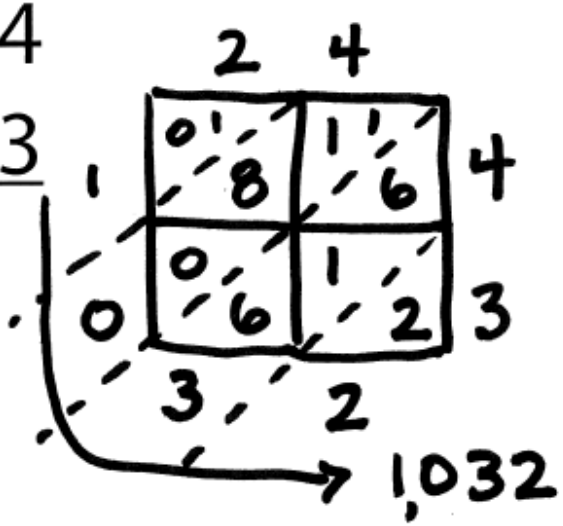
	100	30	2	
	5000	1500	100	50
	300	90	6	3

$$\begin{array}{r} 5000 \\ 1500 \\ 100 \\ 300 \\ 90 \\ +\ 6 \\ \hline 6996 \end{array}$$

Lattice

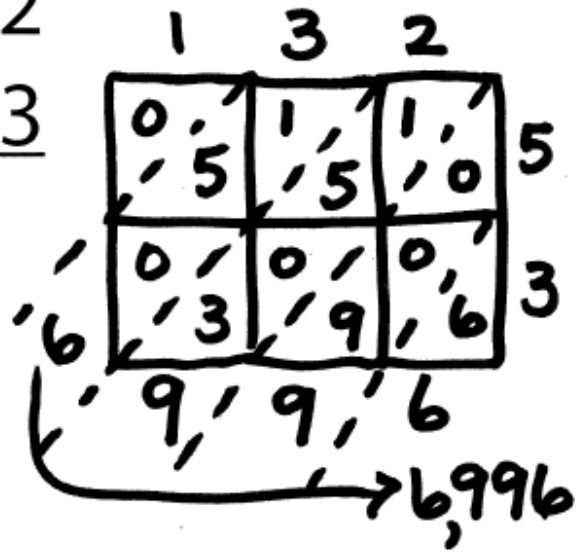
A.

$$\begin{array}{r} 24 \\ \times\ 43 \\ \hline \end{array}$$



B.

$$\begin{array}{r} 132 \\ \times\ 53 \\ \hline \end{array}$$



Show:

13 x 47

123 x 24

Which representations would you use for multiplication computation?

Division Computation

804 ÷ 12 =

1,746 ÷ 18 =

Standard

A.

$$\begin{array}{r} 13\ R2 \\ 12\overline{)158} \\ -12\downarrow \\ \hline 38 \\ -36 \\ \hline 2 \end{array}$$

B.

$$\begin{array}{r} 28\ R18 \\ 34\overline{)\cancel{89}\,170} \\ -68\downarrow \\ \hline 2\cancel{8}\,10 \\ -272 \\ \hline 18 \end{array}$$

Partial Quotients

A.

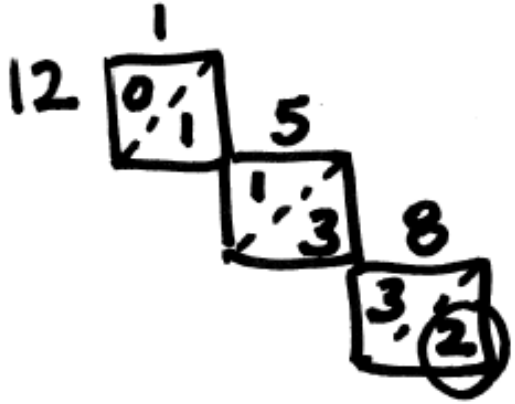
12)158	
−120	10
38	
−36	+3
2	13 R2

B.

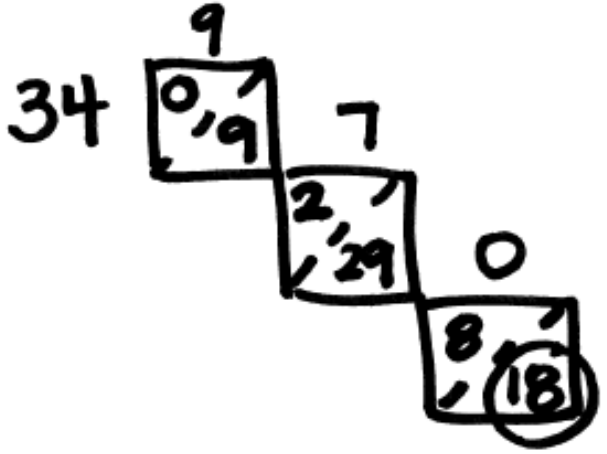
34)~~88~~170	
−680	20
290	
−170	5
~~11~~2~~1~~0	
102	+3
18	28 R 18

Lattice

A. $12\overline{)158}$ R2 = 13



B. $34\overline{)970}$ R 18 = 28



Show:

804 / 12

1,746 / 18

Which representations would you use for division computation?

Operations and Place Value

Fractions and Decimals

Integers and Algebra

Geometry

Fractions

Fraction	Length	Area	Set
$\frac{2}{3}$			
$\frac{1}{4}$			
$1\frac{1}{2}$			
$\frac{3}{7}$			

LENGTH

AREA

SET

LENGTH

Fractions are appropriated by length

Fraction tiles

Cuisenaire rods

Number lines

LENGTH

Fractions are appropriated by length

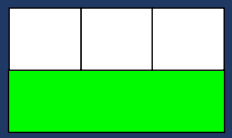
$\frac{2}{3}$

Fraction tiles/bars

LENGTH

Fractions are appropriated by length

$\frac{2}{3}$

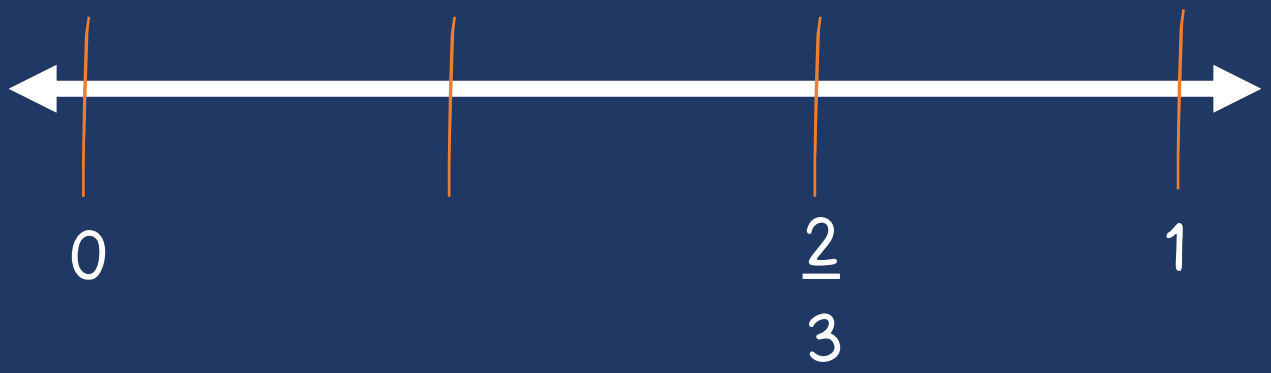


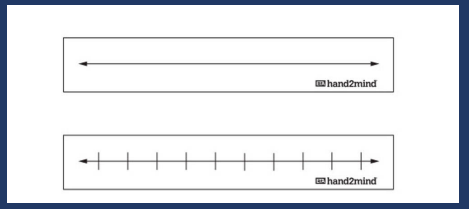
Cuisenaire Rods

LENGTH

Fractions are appropriated by length

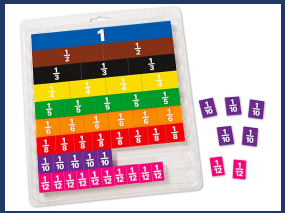
$\frac{2}{3}$

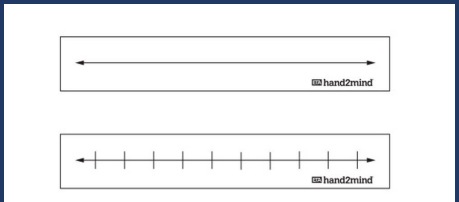


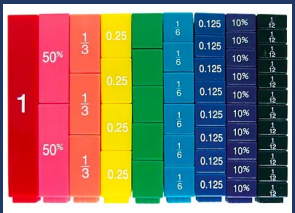


Number Lines

LENGTH



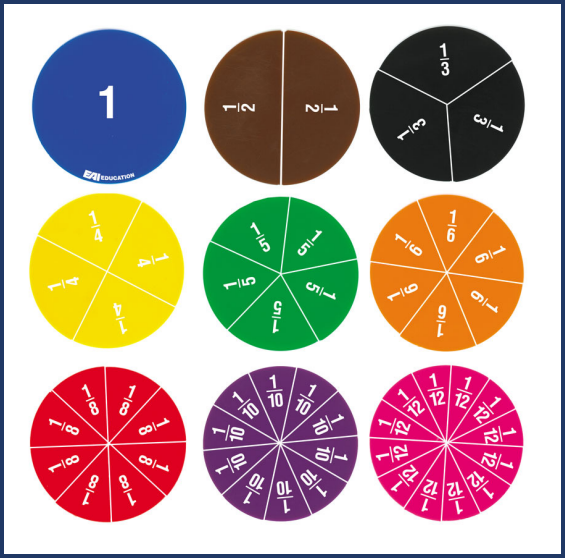




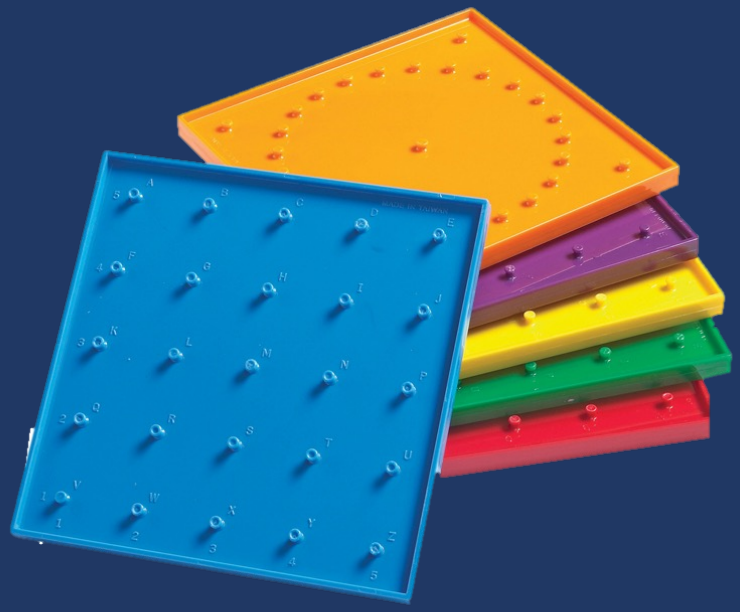
Use representations to show fractions according to length.

AREA

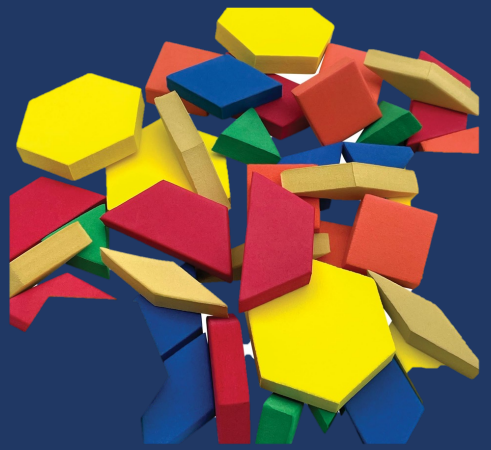
Areas divided into equal sections



Fraction circles



Geoboards

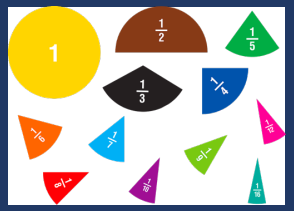


Pattern blocks

AREA

Areas divided into equal sections

$\frac{2}{3}$

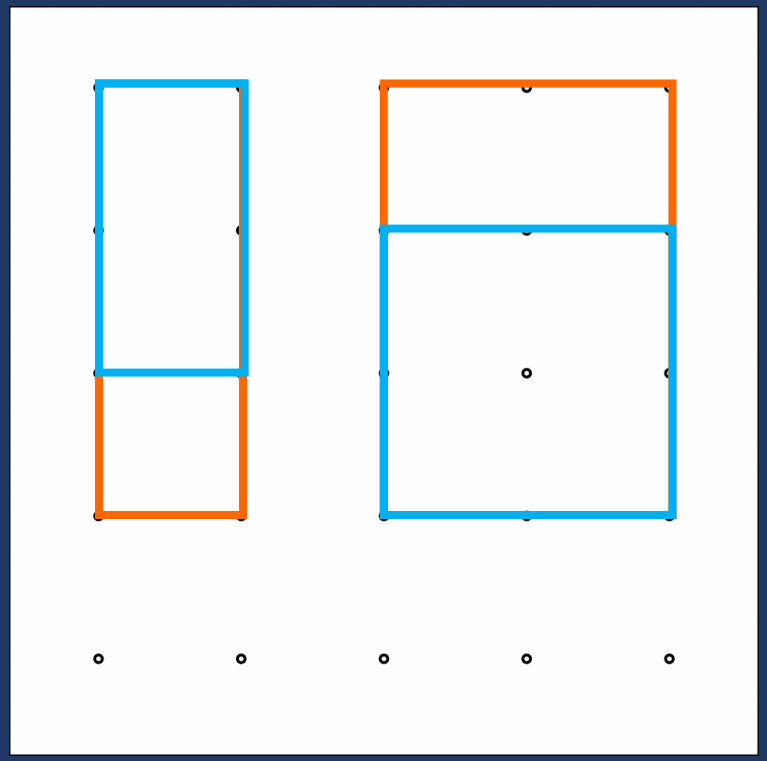


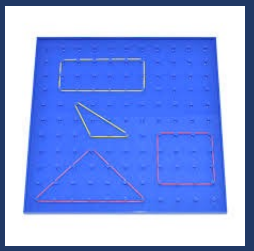
Fraction Circles

AREA

Areas divided into equal sections

$\frac{2}{3}$



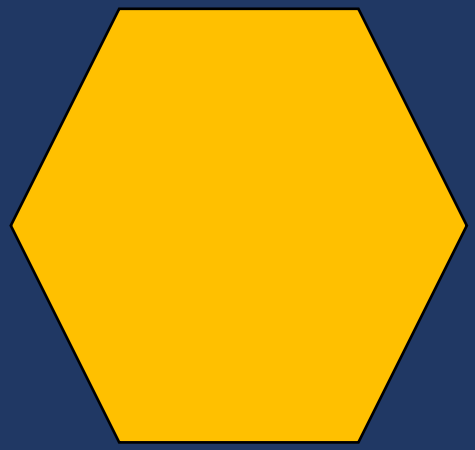


Geoboards

AREA

Areas divided into equal sections

$\frac{2}{3}$

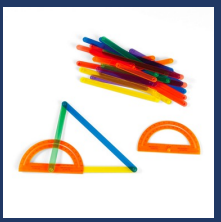


Pattern Blocks

AREA

Areas divided into equal sections

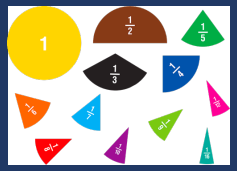
$\frac{2}{3}$

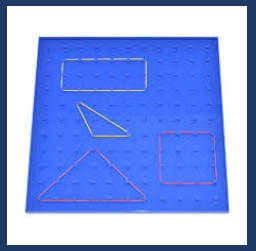


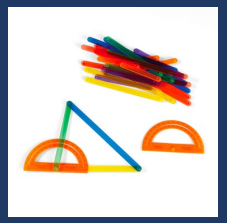
Anglegs

Legos

AREA







Use representations to show fractions according to area.

SET

Individual items show a collection

Two-color counters

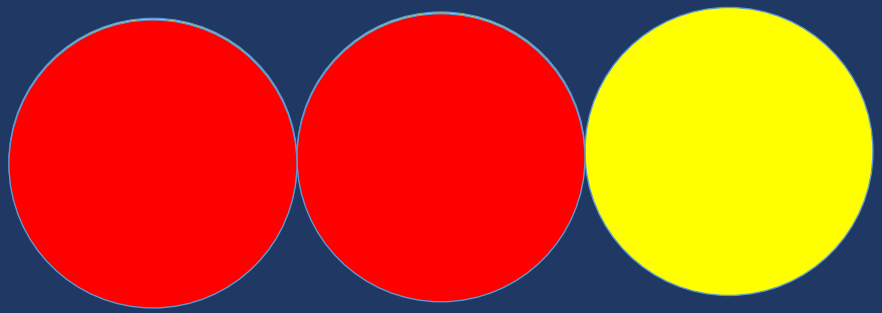
Unifix cubes
Snap cubes

Counters

SET

Individual items show a collection

$\frac{2}{3}$



Two-Color Counters

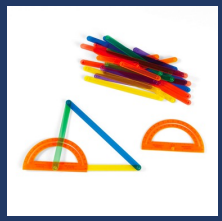
SET

Individual items show a collection

$\frac{2}{3}$

Color Tiles

SET



Use representations to show fractions according to a set.

Fraction Addition and Subtraction

Problem	Representation
$\frac{1}{5} + \frac{3}{5}$	
$\frac{2}{8} + \frac{5}{8}$	
$\frac{1}{2} + \frac{1}{4}$	
$\frac{4}{6} + \frac{1}{3}$	
$\frac{4}{5} - \frac{1}{5}$	
$\frac{6}{8} - \frac{3}{8}$	
$\frac{7}{8} - \frac{2}{4}$	
$\frac{8}{9} - \frac{1}{3}$	

Fraction Multiplication and Division

Problem	Representation
2×3	
$\frac{1}{2} \times 2$	
$\frac{1}{2} \times \frac{4}{4}$	
$\frac{1}{2} \times \frac{2}{4}$	
$\frac{1}{2} \times \frac{3}{4}$	
$\frac{2}{3} \times \frac{3}{4}$	
$\frac{4}{4} \div \frac{1}{2}$	
$\frac{2}{4} \div \frac{1}{2}$	
$\frac{3}{4} \div \frac{1}{2}$	
$\frac{5}{6} \div \frac{2}{3}$	

Decimals

1.2	0.88	1.034
2.8	1.04	0.829

Ones

Tenths

Hundredths

Thousandths

Decimal Computation

Problem	Representation
2.34 + 1.61	
1.98 + 0.34	
2.34 − 1.61	
3.09 − 1.88	

Operations and Place Value

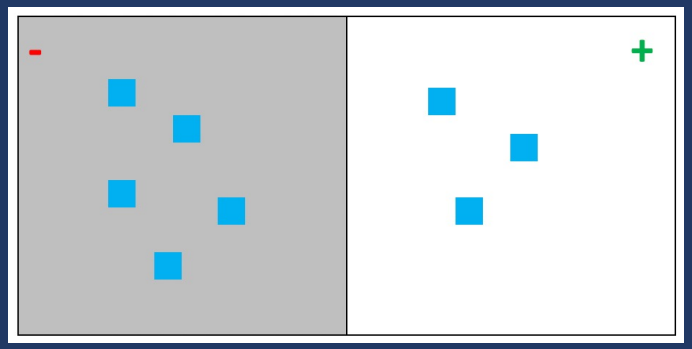
Fractions and Decimals

Integers and Algebra

Geometry

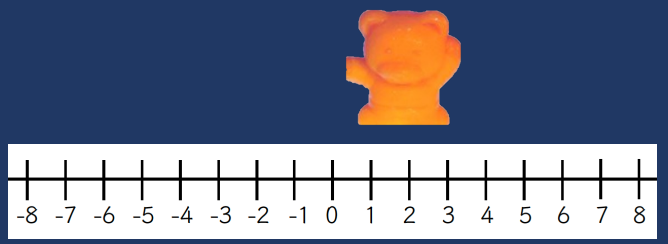
Integers

−	+



Mat and counters

Two-color counters

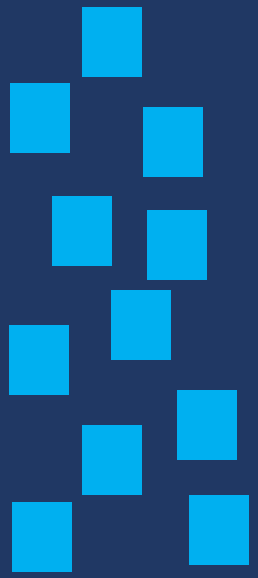


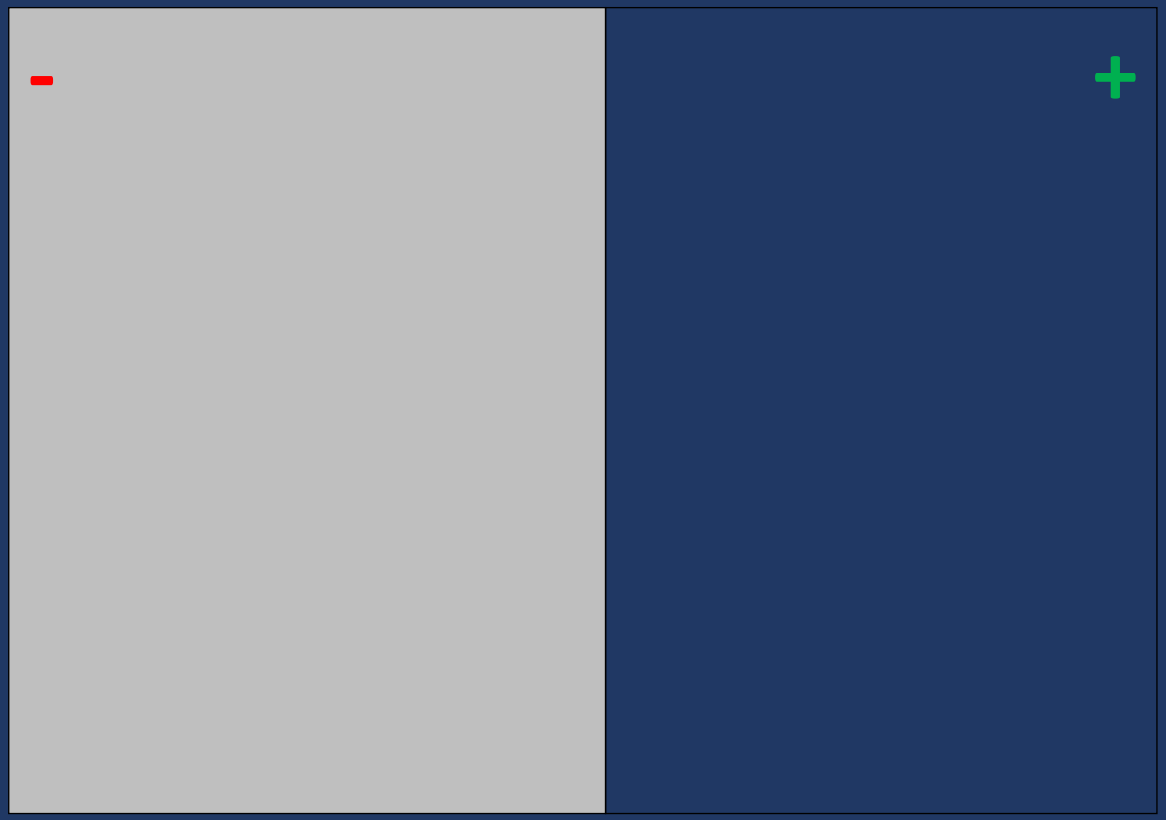
Number line

Integer Addition and Subtraction

Problem	Representation
3 + 5	
3 + (-5)	
-2 + 6	
-6 + (-3)	
5 – 3	
-3 – 4	
-2 – (-6)	

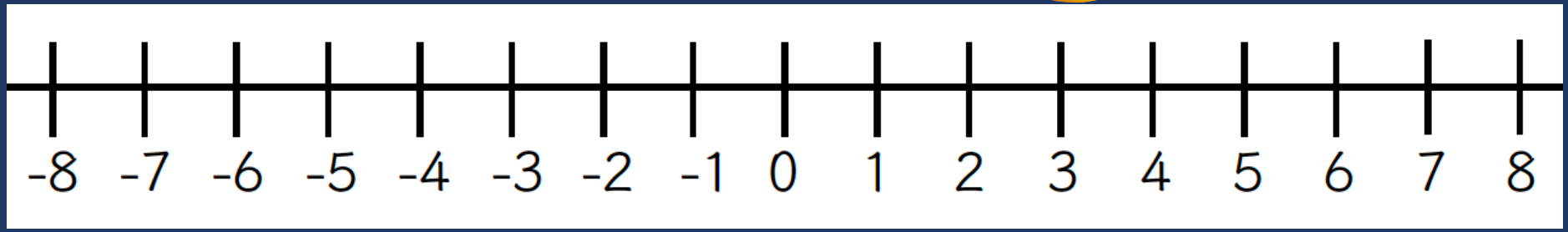
3 + (-5)



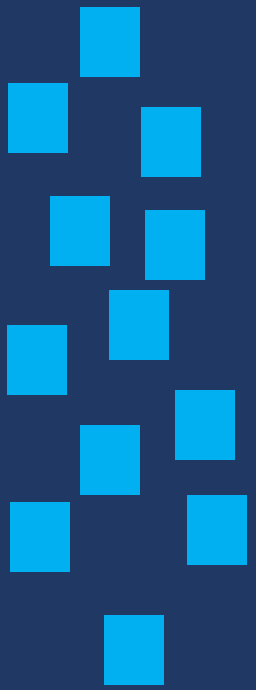


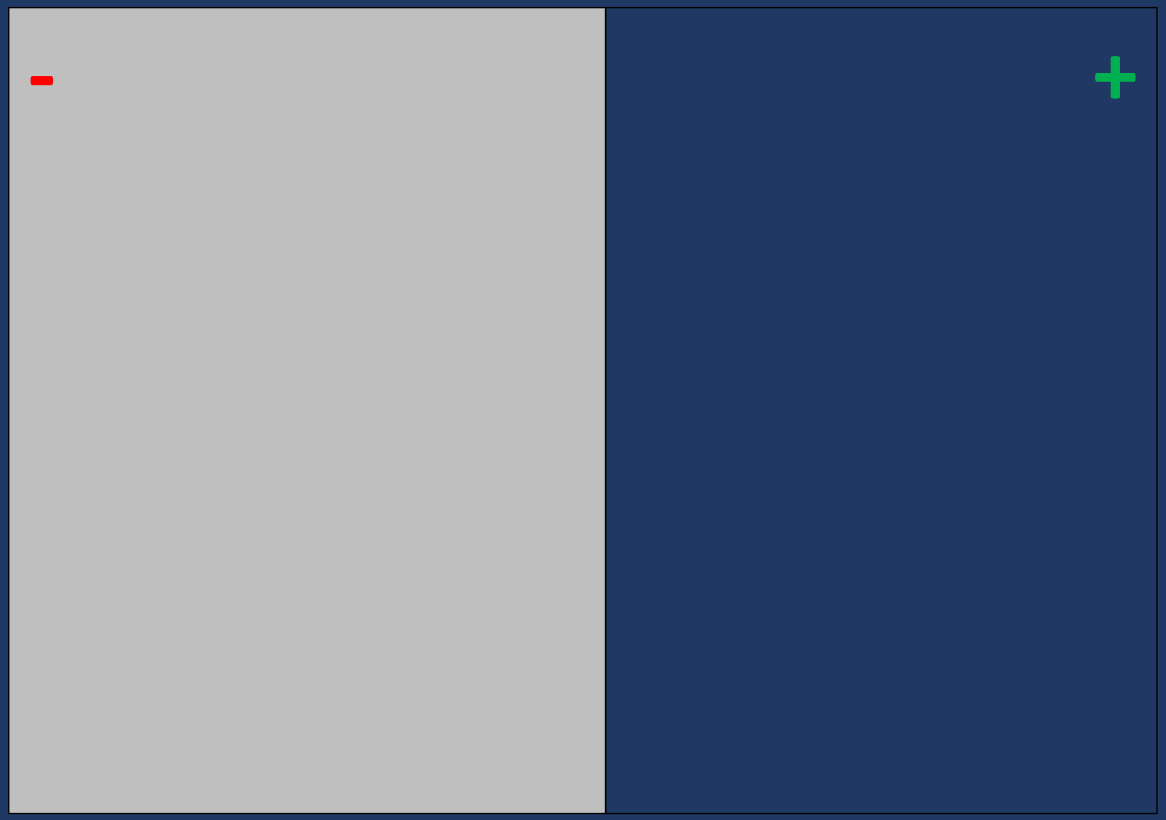
$3 + (-5)$

$3 + (-5)$

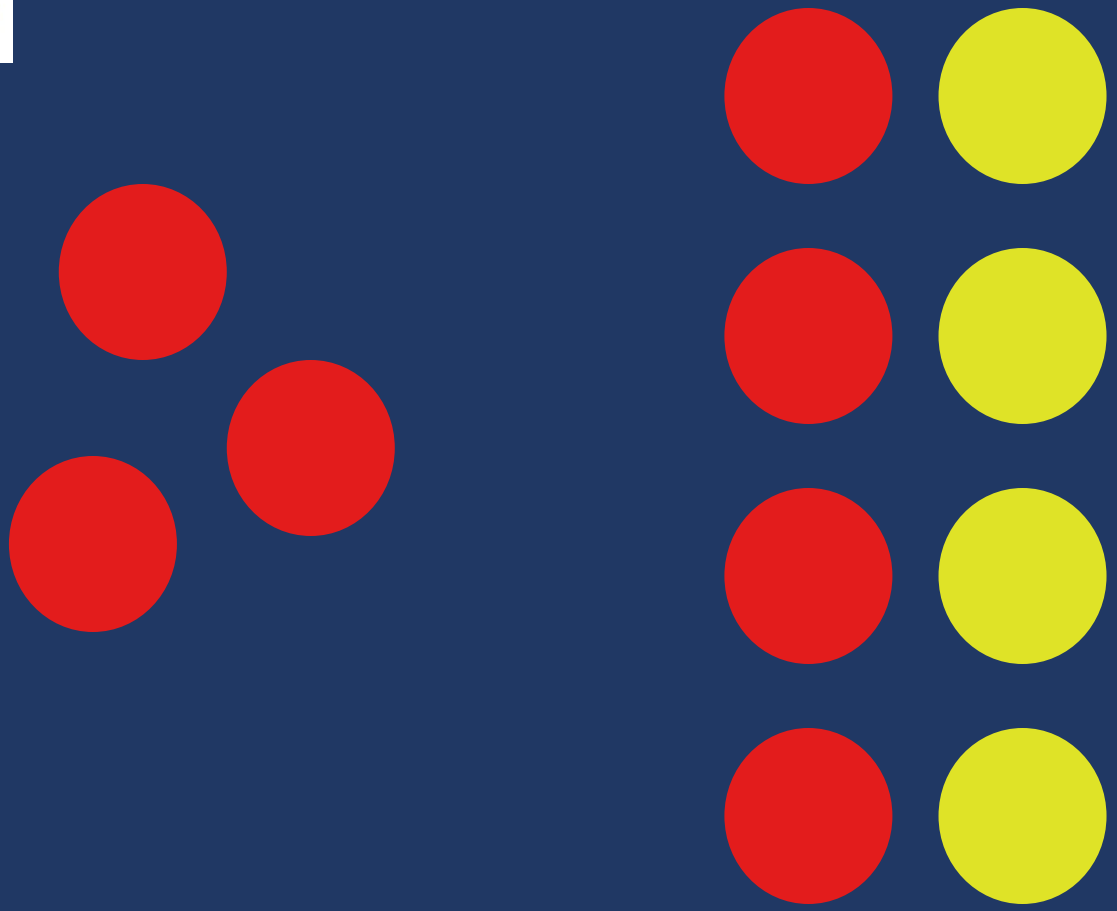


-3 – 4

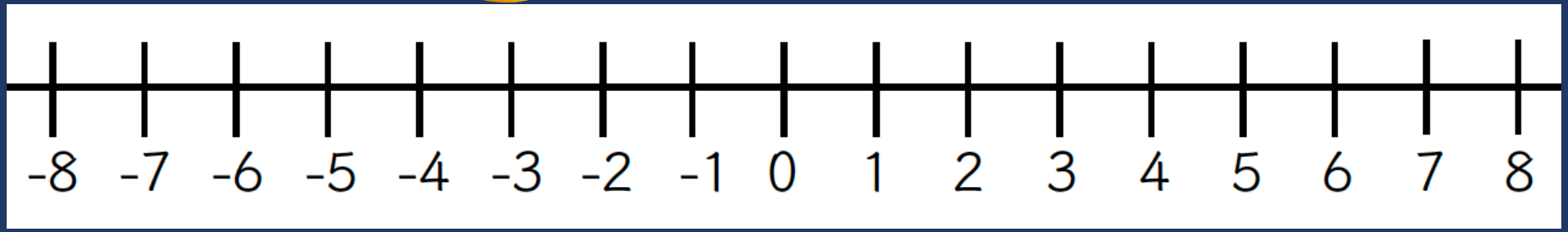




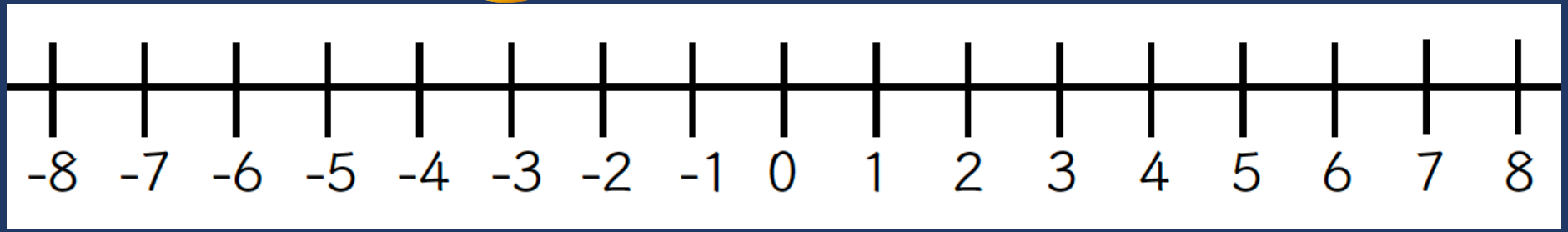
-3 – 4



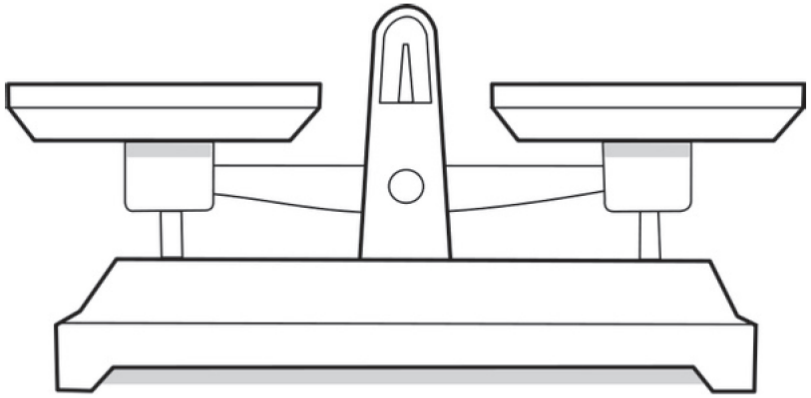
-3 – 4



-3 – 4



Equation Solving



___ + 3 = 7

6 = 2 + ___

3 + 5 = 4 + ___

5 + 4 = ___ + 2

7 = ___

9 − ___ = 6

6 = ___ − 2

9 − 6 = 7 − ___

6 − ___ = 7 − 3

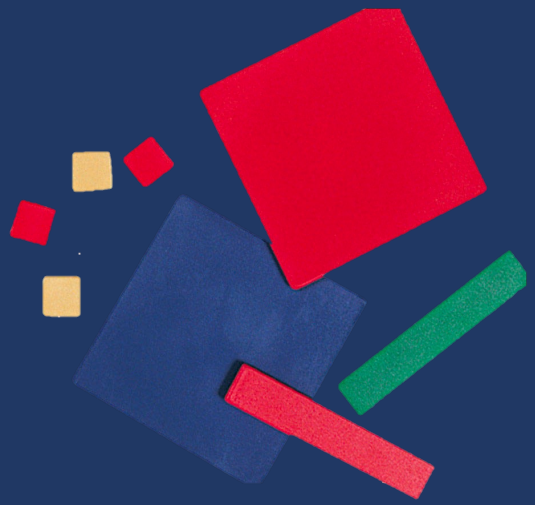
zero pairs

-1 and 1 equal 0

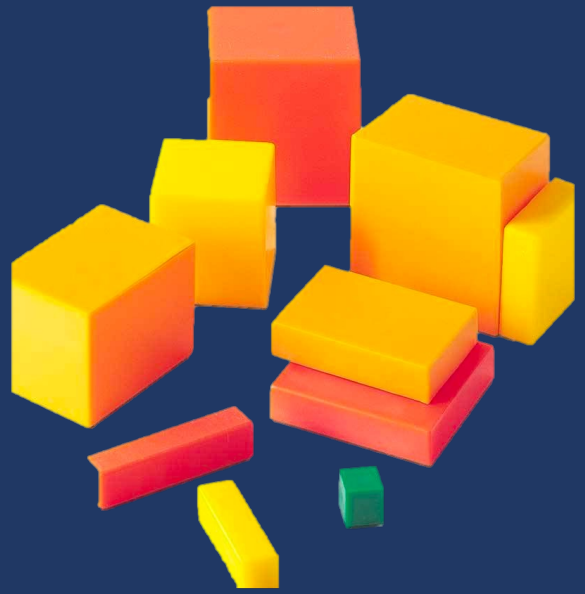
properties of equality

If you do something to one side of the equal sign, you do the same thing to the other side.

Cups and counters



Algebra tiles



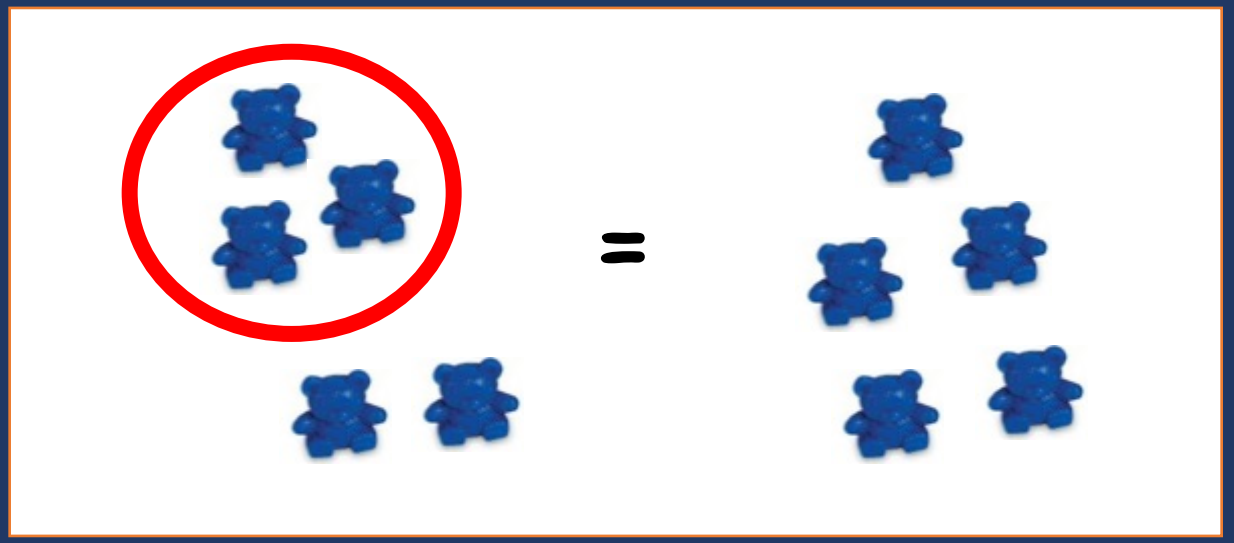
Algeblocks

Equation Solving with Cups/Plates and Counters

Problem	Representations
$x + 2 = 5$	
$x + 2 = 5$	
$4 + x = 6$	
$4 + x = 6$	
$5 = x - 3$	
$-2 = x + 3$	

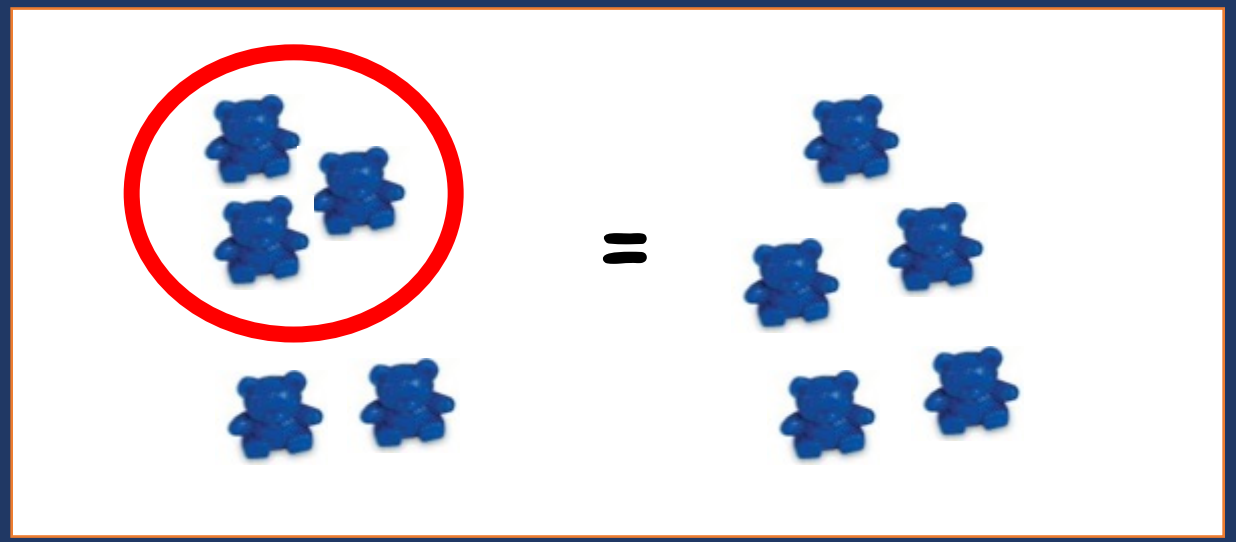
cups/plates and counters

$x + 2 = 5$



cups/plates and counters

$x + 2 = 5$

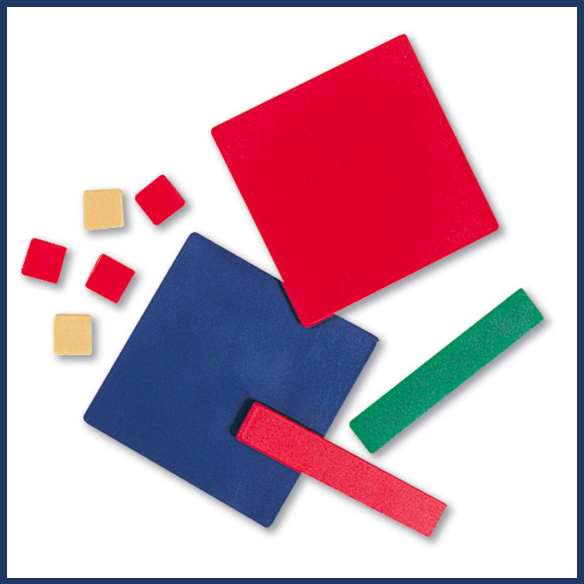


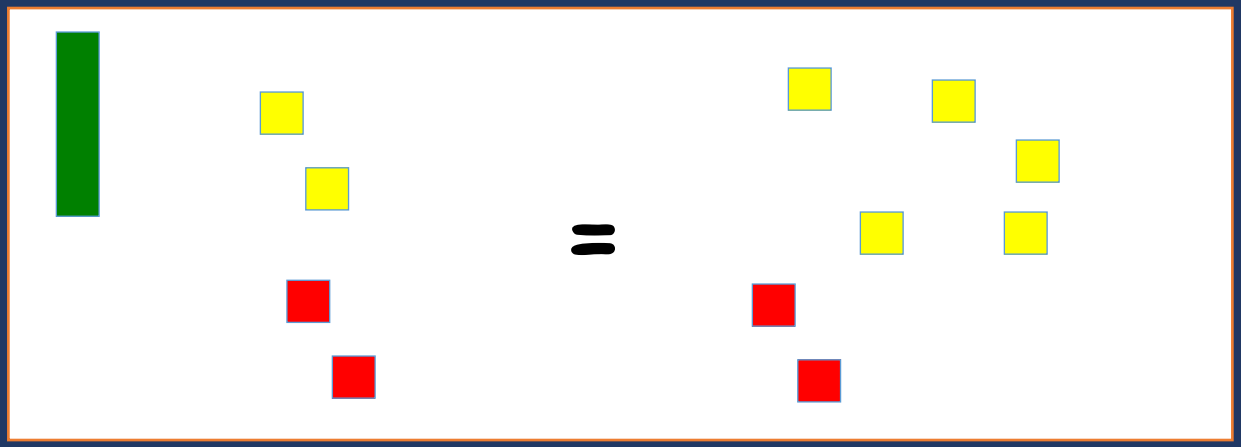
Equation Solving with Algebra Tiles

Problem	Representations
$x + 2 = 5$	
$x + 2 = 5$	
$4 + x = 6$	
$5 = x - 3$	
$-2 = x + 3$	
$x + 3 = -7$	
$y - 4 = 2$	

algebra tiles

$x + 2 = 5$



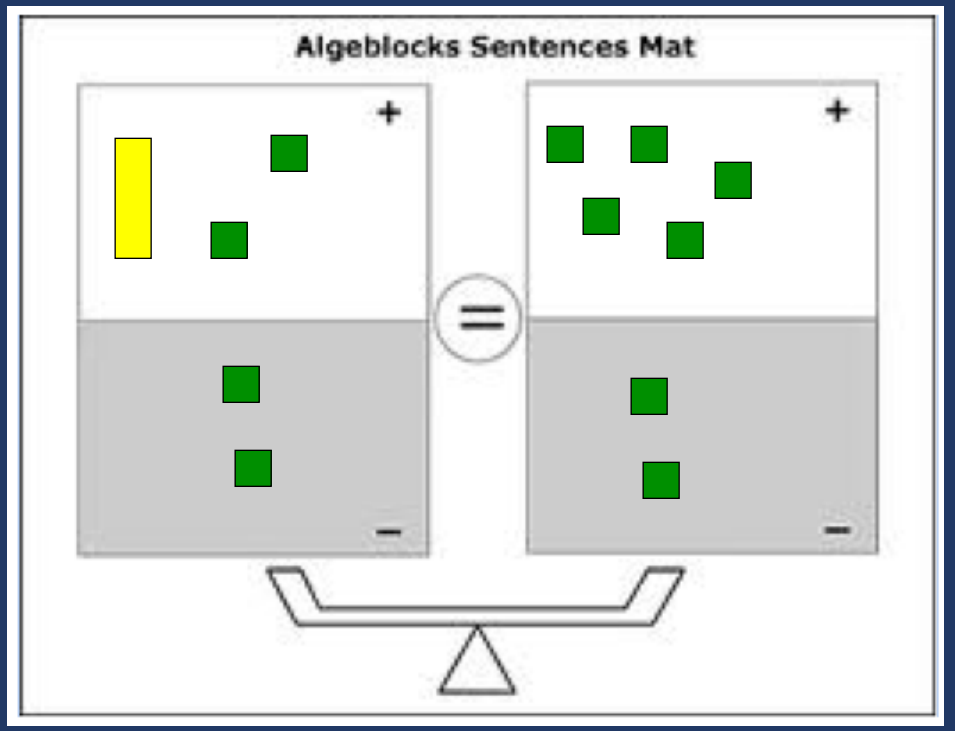


Equation Solving with Algeblocks

Problem	Representations
$x + 2 = 5$	
$4 + x = 6$	
$-2 = x + 3$	
$-1 = y - 4$	
$2x + 2 = 6$	
$x + 4 = 3x$	
$2(x + 3) = x + 4$	
$2x - 4 = 1 + 3x$	
$3y - 5 = -y - 1$	
$1 - x = x + 1$	

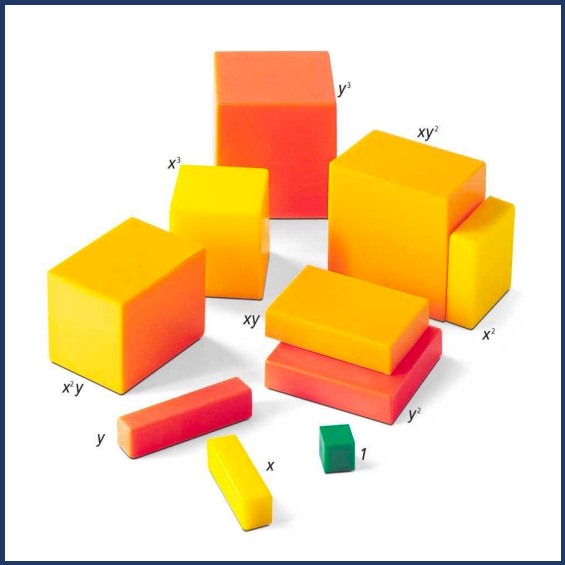
Algeblocks

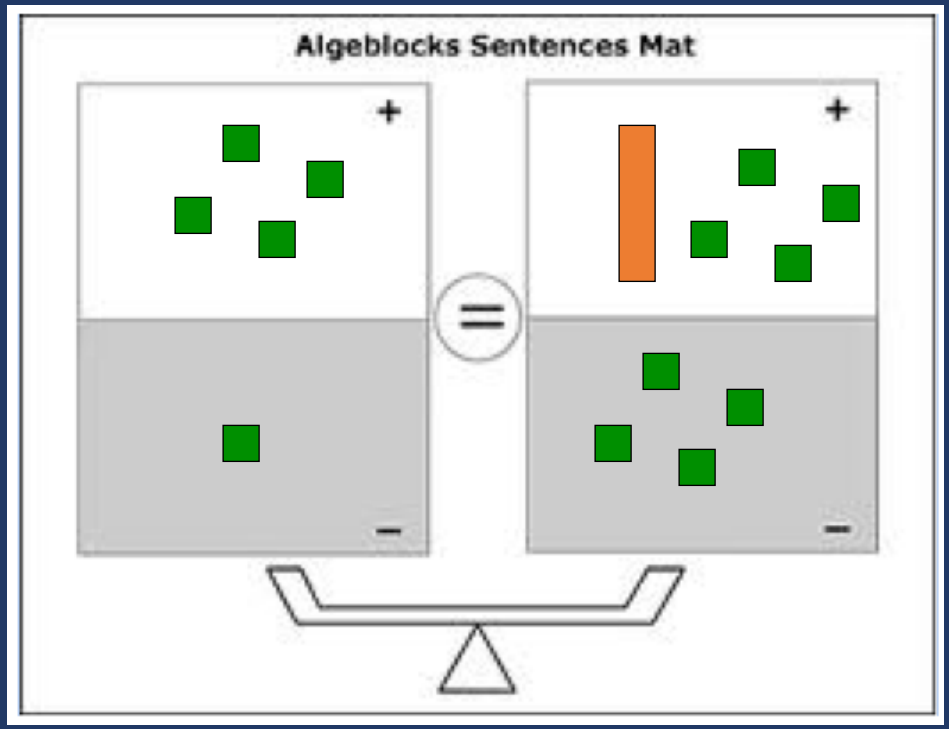
$x + 2 = 5$



Algeblocks

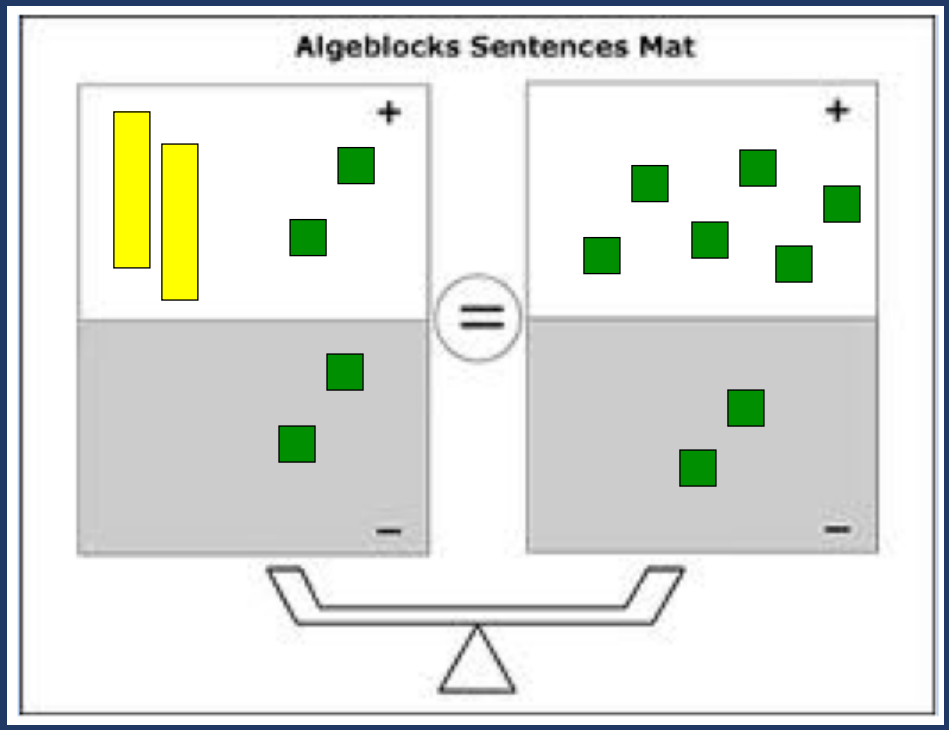
$-1 = y - 4$





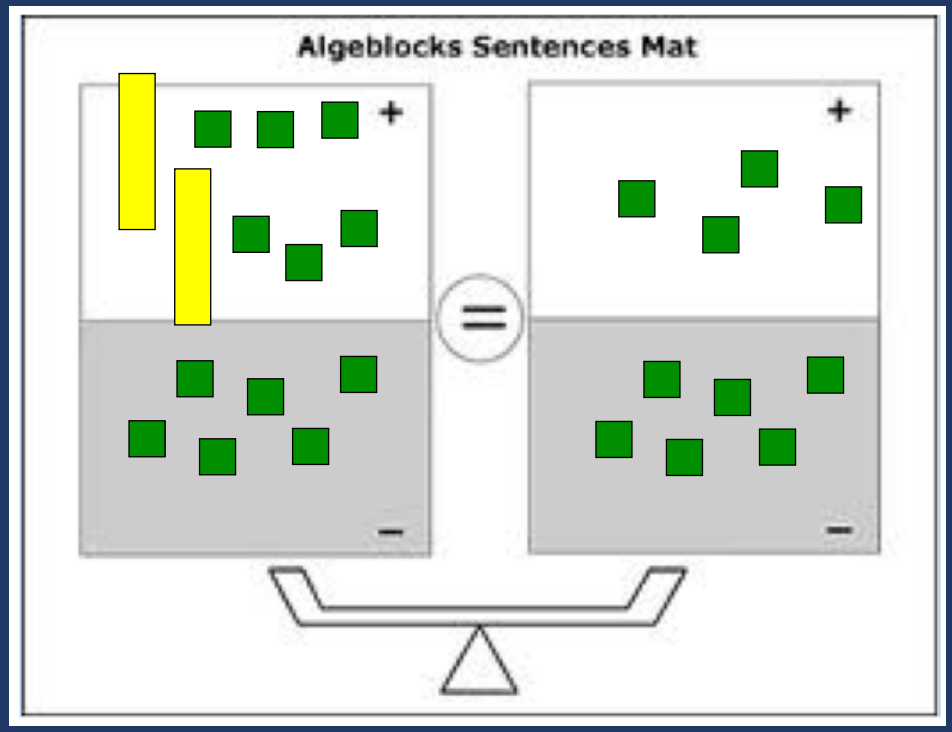
Algeblocks

$2x + 2 = 6$



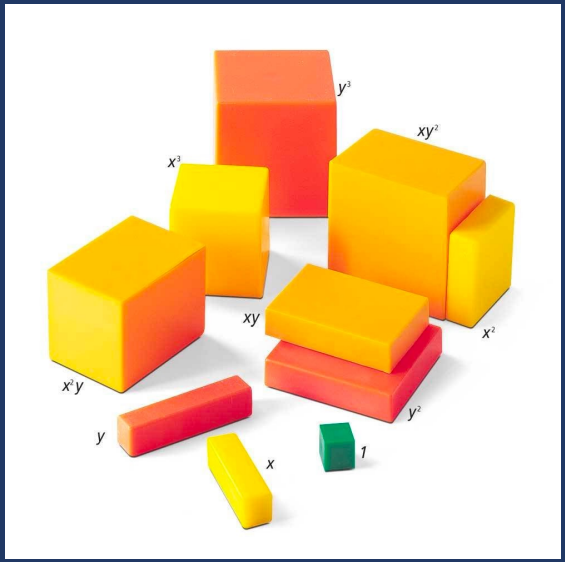
Algeblocks

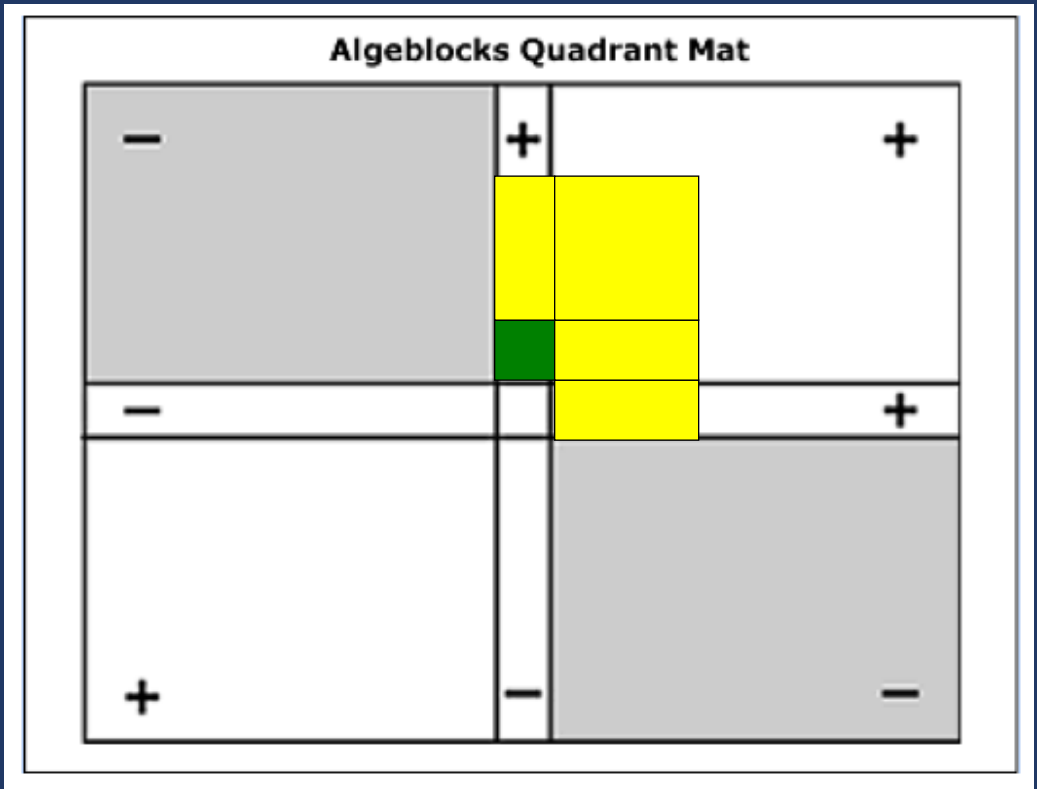
$$2(x + 3) = 4$$



Algeblocks

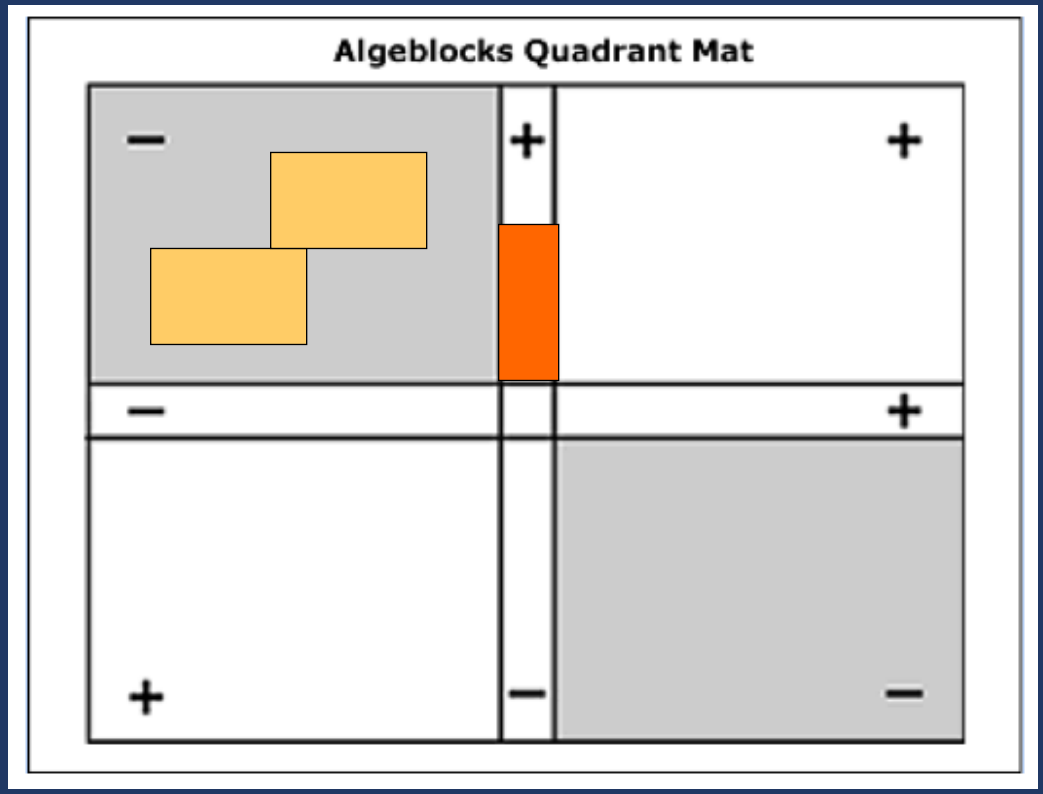
$x(1 + x)$





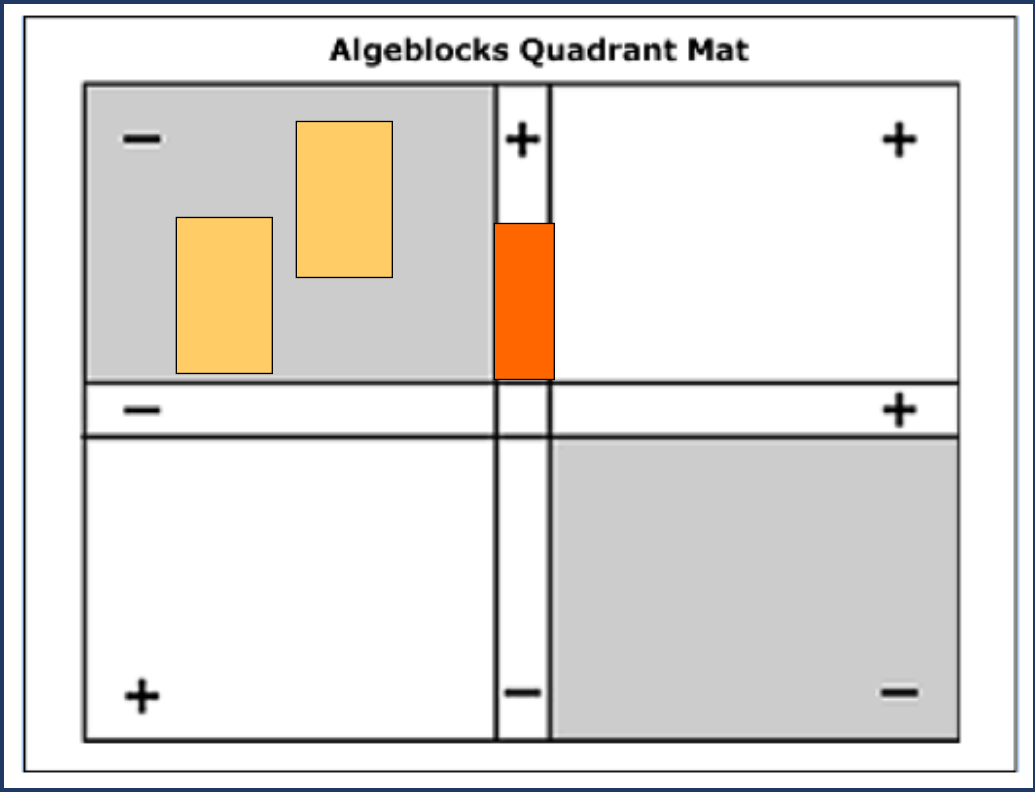
Algeblocks

$-2xy \div y$



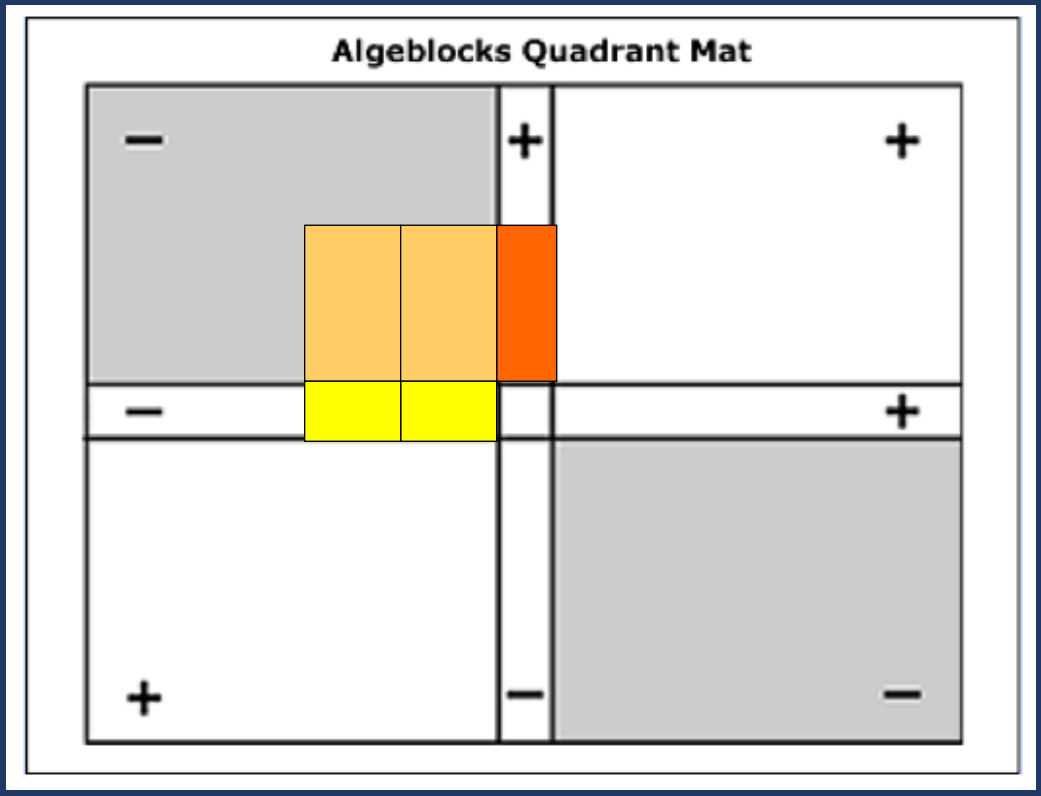
Algeblocks

$-2xy \div y$



Algeblocks

$-2xy \div y$



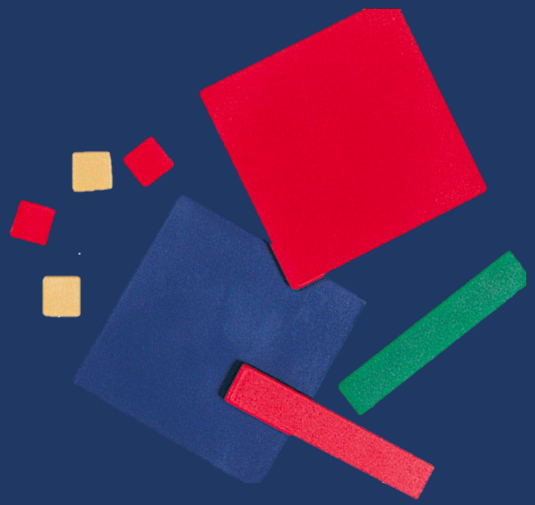
Operations and Place Value

Fractions and Decimals

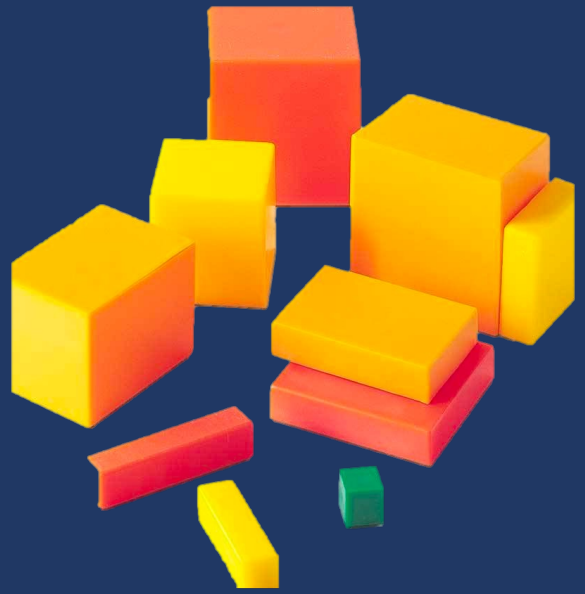
Integers and Algebra

Geometry

Cups and counters



Algebra tiles



Algeblocks

Triangles

Name	Properties	Examples
Equilateral		
Isosceles		
Scalene		
Acute		
Obtuse		
Right		

Quadrilaterals

Name	Properties	Examples
Parallelogram		
Rectangle		
Rhombus		
Square		
Kite		
Trapezoid		

Spatial Reasoning

1. Tangrams

2. Pentominoes

3. Tessellations

Spatial Reasoning

1. Tangrams

2. Pentominoes

3. Tessellations

Three-Dimensional Figures

Name	Properties (Faces, Edges, Vertices)	Examples
Rectangular Prism		
Cube		
Triangular Prism		
Hexagonal Prism		
Rectangular Pyramid		
Triangular Pyramid		
Hexagonal Pyramid		
Cylinder		
Cone		
Sphere		

Operations and Place Value

Fractions and Decimals

Integers and Algebra

Geometry

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PRIME
Powell and Researchers Investigating Mathematics Evidence

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