The Loyalty-Efficiency Tradeoff in Authoritarian Repression

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Abstract

Many argue that dictators face a loyalty-efficiency tradeoff between (1) personally loyal ethnic militaries that pose a low insider coup threat and (2) nationally recruited militaries that more efficiently repress outsider threats. However, empirically, ethnic militaries often perform better than national militaries against outsider threats, especially non-radical social movements. This paper formally studies repression agency problems alongside heterogeneous outsider threats. A dictator chooses either an ethnic or national military, which can repress for the dictator, stage a coup, or transition to outsider rule. Non-radical outsider threats eliminate the loyalty-efficiency tradeoff. The ethnic military's lower reservation value under outsider rule yields considerably stronger repression incentives than the national military, engendering greater equilibrium efficiency for the ethnic military. The dictator's strict preference for the ethnic military also implies that stronger outsider threats do not raise equilibrium coup propensity. However, a strong, radical threat encourages choosing a national military despite raising coup likelihood.

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1 INTRODUCTION

Non-democratic rulers pursue diverse strategies to survive in power. Many contemporary dictators oversee a formal party, compete in elections, and make concessions to the opposition by accepting constraints imposed by legislatures and written constitutions, which considerable recent research has analyzed (Gandhi 2008; Geddes et al. 2018; Meng 2019). However, the military provides the survival tool of last resort for any dictator. Although authoritarian rulers engage in low-intensity repressive techniques to prevent mass citizen mobilization from occurring in the first place as well as attempt to deter rebel organization and foreign invasions, when facing a perilous outsider threat, the regime will survive only if the military successfully disbands protesters and fights effectively to defeat domestic or foreign armed groups. Mass uprisings, rebel victory in civil war, and foreign invasions collectively accounted for 29% percent of authoritarian regime collapses between 1945 and 2010 (Geddes et al. 2018, 179), indicating the prevalence of these threats. Yet the military itself can also threaten the regime from the inside. Empirically, successful coups accounted for 35% of authoritarian regime collapses between 1945 and 2010 (Geddes et al. 2018, 179). Given these dual threat sources, how do dictators construct a military apparatus to facilitate their survival?

Many argue that simultaneous insider and outsider threats create a ***loyalty-efficiency*** tradeoff that constrains any dictator's choice over the strength and organization of its military. For example, Powell (2014, 2) argues that leaders "find themselves mired in a paradox in which a weak military can leave them vulnerable to invasion or civil war, while a strong military could expedite their exit through a coup d'etat." Similarly, Greitens (2016, 4) proclaims: "Because coup-proofing calls for fragmented and socially exclusive organizations, while protecting against popular unrest demands unitary and inclusive ones, autocrats cannot simultaneously maximize their defenses against both threats." Throughout, I distinguish between (1) strong, unitary, and inclusive "national" militaries that recruit officers and soldiers from broad strata of society into a professional apparatus distinguished by meritocratic promotion and a disciplined hierarchical command, and (2) weak, fragmented, and socially exclusive "ethnic" militaries that stack the officer corps with unqualified family members and co-ethnics. According to the existing loyalty-efficiency logic, national militaries should more effectively defeat outsider threats such as rebel groups and mass citizen movements—which I refer to as repressive efficiency—whereas ethnic militaries should present a lesser insider threat, which I refer to as coup loyalty.

Given this loyalty-efficiency tradeoff, many argue that the strength of outsiders that threaten the regime determines dictators' military choices. The more immediate threat of insider overthrow via a coup causes many dictators to create "coup-proofed" ethnic militaries despite considerable evidence that protecting against insider disloyalty diminishes military efficiency (Quinlivan 1999; Pilster and Böhmelt 2011; Talmadge 2015). Roessler (2016) characterizes a similar tradeoff whereby fear of a coup may cause a ruler to exclude rival ethnic groups from power, which hinders the state's counterinsurgency capacity by disrupting the government's intelligence network in the excluded group's regional base. Yet a dictator that faces a particularly strong outsider threat willingly sacrifices coup loyalty for the increased repressive efficiency that a national military should deliver (Acemoglu et al. 2010; Besley and Robinson 2010; Svolik 2013). Therefore, the loyalty-efficiency tradeoff also creates a ***guardianship dilemma*** for dictators: the stronger guards needed to defeat a severe threat can more easily overthrow the dictator via a coup.[1]

Applying the loyalty-efficiency logic, however, leads to strange interpretations of many empirical cases. Although the main virtue of strong national militaries is supposedly to more efficiently defeat outsider threats, empirically, ethnic militaries often save the regime whereas national militaries refuse to shoot. For example, nationally representative militaries in Tunisia and Egypt were ultimately unwilling to repress pro-democracy protesters in early 2011 amid the Arab Spring uprisings, but ethnically stacked militaries in Bahrain, Syria, and Libya reacted with harsh crackdowns. In Latin America in the 1980s, many democratic transitions occurred when national militaries negotiated deals with broad societal groups (e.g., Uruguay) or with moderate rebel groups (e.g., El Salvador), whereas African dictators such as José Eduardo dos Santos in Angola and Robert Mugabe in Zimbabwe relied on ethnically exclusive militaries to defeat rebel groups and other anti-regime movements that arose between the 1980s and 2000s. These cases exemplify a central shortcoming of the efficiency component of the posited loyalty-efficiency tradeoff, which overlooks that exercising repression is a strategic choice: even a military endowed with high coercive capacity will exhibit low efficiency in equilibrium—that is, defeat an outsider threat with low probability—if it does not fight hard against rebel groups and repress protesters when given orders.

Although other recent contributions (not specifically focused on the loyalty-efficiency tradeoff or guardianship dilemma) address the agency problem posed by militaries,[2] existing theories largely overlook that

[1] Below I discuss McMahon and Slantchev's (2015) insightful critique of the guardianship dilemma.

[2] Tyson (2018), Dragu and Lupu (2018), and Dragu and Przeworski (2019) formally analyze various as-

outsider threats vary in their goals—which in turn affects the military's incentives to exercise repression. In the aforementioned examples of Egypt, Tunisia, Uruguay, and El Salvador, the opposition did not seek to fundamentally reconstruct the social order, and these *non-radical* goals likely influenced the national militaries' decisions to cut a deal. However, generals in these militaries may have chosen differently if they faced a more *radical* threat (e.g., communist rebels) that likely would have engendered a worse post-exit fate for military leaders—and narrow ethnically stacked militaries such as those in Bahrain, Syria, Angola, and Zimbabwe usually fear rule by any other societal group. To better understand cases such as these, as well as the deeper theoretical logic underpinning how dictators construct their militaries, we need to understand how attributes of outsider threats interact with attributes of the military to affect repression decisions.

This paper analyzes a formal model that studies the core strategic tradeoffs that dictators face when constructing their military apparatus, and consequences for regime survival. The dictator—who faces an exogenous outsider threat that can overthrow the regime—chooses how to organize its coercive apparatus by delegating authority to either an ethnic or a national military. The dictator faces a dual agency problem. To survive, it needs the military to defend the regime by exercising repression, but the military can alternatively decide to either negotiate a transition with the outsider or attempt a coup. Compared to an ethnic military, a national military is more likely to be *able* to successfully repress the opposition. However, because national militaries recruit from broad segments of society, and merit rather than personal fealty to the incumbent dictator determines promotion decisions, national militaries fare better than ethnic militaries under outsider rule. National militaries are also more likely to have an opportunity to successfully stage a coup.

The analysis rethinks the logic of the loyalty-efficiency tradeoff and the guardianship dilemma. To yield the first main result, I open up key implicit assumptions undergirding the loyalty-efficiency tradeoff by modeling endogenous repression compliance and by allowing the outsider threat to vary in its radicalism. Facing a non-radical threat, stacking the military with sycophants can reduce the probability of outsider overthrow—that is, *raise* repressive efficiency—relative to choosing a national military. Whereas national militaries fare

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pects of the dictator's agency problem (focusing on coordination problems), and other recent formal models study additional strategic aspects of authoritarian repression (Pierskalla 2010; Ritter 2014; Gibilisco 2017). McLauchlin (2010), Bellin (2012), and Barany (2016) present case studies of militaries' repression decisions, and an older literature addresses the frequency with which states experienced military intervention in politics following independence from European colonial rule, such as Huntington (1957) and Finer (2002).

relatively well under rule by a non-radical outsider, ethnic militaries do not because of their patrimonial ties to the incumbent. This discrepancy causes ethnic militaries to exercise repression on behalf of the incumbent with considerably greater likelihood, yielding a lower probability of outsider takeover despite the ethnic military's weaker coercive endowment. Therefore, non-radical threats eliminate the loyalty-efficiency tradeoff: the ethnic military exhibits higher repressive efficiency and higher loyalty (i.e., lower coup propensity, which is true for all parameter values). This revised loyalty-efficiency logic also explains why, contrary to arguments that dictators choose broad-based militaries if the dominant perceived threat when coming to power is an outsider rebellion, dictators facing a low coup threat may still choose an ethnic military.

Instead, dictators only face a loyalty-efficiency tradeoff when encountering strong *radical* threats, such as communist guerrillas in many East and Southeast Asian countries following World War II, and anti-monarchical rebellions in the Middle East in the 1950s and 1960s. Radical threats pose an existential crisis for ethnic *and* national militaries—because the outsider seeks to upend the existing social structure and elites—which incentivizes either type of military to fight to defend the regime. The national military's greater coercive endowment causes it to defeat the threat with higher probability than the ethnic military, and stronger outsider threats widen this discrepancy. Therefore, if the threat is strong and radical, then the dictator chooses a national military to maximize repressive efficiency—despite sacrificing coup loyalty.

The second main result shows that the dictator faces a guardianship dilemma if and only if it faces a loyalty-efficiency tradeoff. If the threat is radical, then the dictator's willingness to sacrifice coup loyalty for higher repressive efficiency as the outsider threat grows in strength creates a non-monotonic relationship between threat size and equilibrium coup probability. The coup probability exhibits a discrete increase at an intermediate threat level in which the dictator switches from an ethnic to a national military—recovering the traditional guardianship dilemma logic. However, at all other threat levels, the equilibrium probability of a coup *decreases* in outsider threat strength because the dictator does not change its choice of military and a stronger outsider threat increases the difficulty of installing a military dictatorship.

By contrast, a non-radical threat eliminates the guardianship dilemma for the same reason that it obviates the loyalty-efficiency tradeoff. In this case, the ethnic military is more repressively efficient regardless of threat strength. This implies that increasing the severity of a non-radical threat does not cause the dictator to switch to the less loyal national military, and equilibrium coup likelihood strictly decreases in the size of the threat. Table 1 summarizes the main theoretical findings.

Table 1: Summary of Main Findings

Non-radical threat	Radical threat
(National military has high value to outsider rule)	(National military has low value to outsider rule)
⇓	⇓
Ethnic military more repressively efficient regardless of threat size	Ethnic more efficient if weak threat National more efficient if strong threat
⇓	⇓
No loyalty-efficiency tradeoff	Loyalty-efficiency tradeoff
⇓	⇓
No guardianship dilemma	Guardianship dilemma

2 Related Research

My analysis departs from existing studies of the loyalty-efficiency tradeoff (cited above) by establishing the *conditions* under which a loyalty-efficiency tradeoff exists for authoritarian militaries, rather than assuming that dictators necessarily trade off between loyal and efficient militaries. This also distinguishes the model from two existing formal theoretic models that analyze how dictators choose between competent and incompetent agents. Zakharov (2016) characterizes a dynamic loyalty-efficiency tradeoff between high-quality advisers that generate a high fixed payoff for the dictator, and low-quality advisers that endogenously demonstrate higher loyalty to the incumbent dictator because they have a lower outside option to betraying the incumbent than high-quality advisers. This resembles the present idea that national militaries have a higher reservation value to negotiating a transition with society. However, in my model, the dictator's utility depends on whether the military *chooses* to exert repressive effort, contrary to Zakharov's (2016) assumption that dictators accrue a *fixed* rent from particular types of agents. Therefore, whereas in his model rulers always face a loyalty-efficiency tradeoff, here, an ethnic military—despite a weaker coercive endowment—may exhibit greater repressive efficiency than a national military. This discrepancy is crucial for explaining the conditions under which a dictator faces a loyalty-efficiency tradeoff and, consequently, a guardianship dilemma. My model also departs from Egorov and Sonin (2011), in which rulers always face a loyalty-efficiency tradeoff because of different informational endowments. In their model, agents do not differ in their coercive ability to defend the regime.

The findings also depart from existing theories of the guardianship dilemma by showing the intimate relationship with the loyalty-efficiency tradeoff. I follow McMahon and Slantchev (2015) by critiquing the guardianship logic, although my critique is more fundamental. I adopt their core assumption that the outsider threat endogenously affects the military's incentives to stage a coup,[3] but this is insufficient to eliminate the guardianship mechanism. Despite this assumption, if the outsider threat is radical, then a large-enough increase in the magnitude of the threat causes the dictator to switch from an ethnic to a national military. This raises the equilibrium probability of a coup attempt—which is the guardianship mechanism. Instead, I show that only a *non*-radical threat eliminates the guardianship dilemma by obviating the loyalty-efficiency tradeoff. This insight follows from the novel aspects of my model: assuming the military faces a strategic choice to exercise repression, and parameterizing the outsider threat's radicalism.[4]

Analyzing how militaries expect to fare under outsider rule instead relates to broader considerations in the political regimes literature. Geddes (1999) argues that military regimes often acquiesce to democratization because the military will survive as an intact organization. Most of her examples are from Latin America in the 1980s which, consistent with my analysis, tended to feature national militaries facing non-radical threats. Debs (2016) proposes a different mechanism based on expected post-transition fate: military dictators are more willing than other types of dictators to democratize because they are less likely to face punishment for their comparative advantage in coercion under a democratic than an authoritarian regime. Albertus and Menaldo (2018) argue that dictators more willingly democratize after enacting a constitution that affords elite protection against political participation by the masses.

This paper also offers an important conceptual consideration. Following most of the literature, my conceptualization of loyalty and efficiency distinguishes the dictator's tradeoff between insider threats (concerns about loyalty) and outsider threats (concerns about efficiency). However, it is also possible to conceptualize loyalty more broadly to encompass not only the military's coup decision, but also the repression decision. For example, Levitsky and Way (2010) refer to militaries' "cohesion" in the face of outsider threats and Bellin (2012) to their "will" to crush enemies. This alternative conceptualization does not alter the core logic, but instead simply requires some relabeling. In this interpretation, my model highlights two distinct mechanisms that undermine the loyalty of national militaries. The analysis of the repressive dimension of loyalty explains why, when facing non-radical threats, broader-based militaries are not better at defeating

[3]By contrast, in earlier models, the outsider threat disappears if the military takes over.

[4]McMahon and Slantchev (2015) instead assume that the military consumes 0 if the outsider takes over, which corresponds to the ideal-type radical threat in my model. Consequently, even if given the choice, in their model the military would never transition to outsider rule.

outsider threats (contrary to core ideas from research such as Acemoglu et al. 2010, Besley and Robinson 2010, Svolik 2013, Powell 2014, Greitens 2016, and Roessler 2016)—even if the decision to exercise repression against outsiders is not considered as a component of the military's "efficiency." Given different possible conceptualization schemes, another contribution here is simply to distinguish different aspects of the military's calculus to provide a common vocabulary for the literature. Section 6 provides a more extensive analysis of loyalty mechanisms.

A related conceptual consideration, discussed in the conclusion, is that the present analysis brings together research on civil conflict, contentious politics, and authoritarian regimes. Rather than treat urban protesters and guerrilla groups as qualitatively different phenomena, it parameterizes outsider threats by their strength and the radicalism of their goals.

3 Setup of Baseline Model

Section 3.1 presents the sequence of moves in the baseline game, Section 3.2 motivates key assumptions, and Section 3.3 summarizes the extensions.

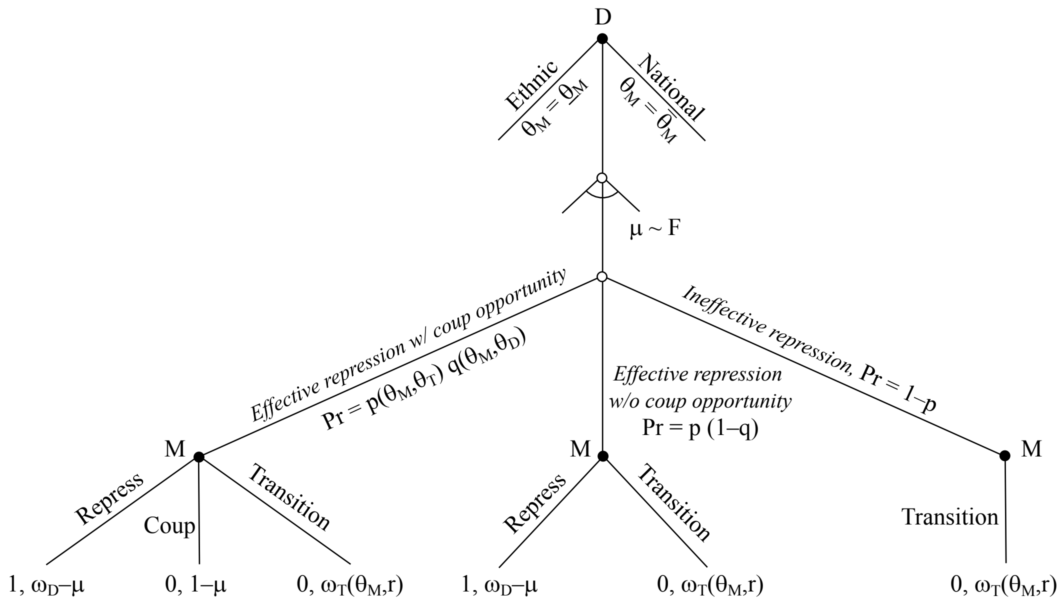
3.1 Players, Choices, and Payoffs

Two strategic players that face an exogenous outsider threat make sequential choices, as Figure 1 shows. The dictator's coercive endowment is $\theta_D \in \left(0, \overline{\theta}_D\right)$, for $\overline{\theta}_D > 0$. The exogenous outsider threat's coercive capacity is $\theta_T \in \left(0, \overline{\theta}_T\right)$, for $\overline{\theta}_T > 0$. The radicalism of the outsider threat is parameterized by $r \in (0, 1)$, and higher r corresponds to a more radical threat. In the first strategic choice, the dictator chooses between an ethnic military with low coercive capacity $\theta_M = \underline{\theta}_M > 0$ and a high-capacity national military with $\theta_M = \overline{\theta}_M > \underline{\theta}_M$.

In between the two strategic moves, three Nature moves ensure that the dictator is uncertain about the military's actions, but it knows the underlying distribution of the Nature draws (including the input parameters θ_T and θ_D). The Nature moves reflect inherent variability in (1) outsiders' ability to threaten the regime, (2) costs of exercising repression, and (3) chances for the military to successfully overthrow the ruler via a coup. The military knows the realization of these draws, and all other parameters are common knowledge.

The military chooses from up to three possibilities. The first choice, which is available at every information

Figure 1: Game Tree for Baseline Model



set for the military, is to transition to outsider rule. This yields consumption of $\omega_T\left(\theta_M, r\right)$ for the military. Below I discuss the key assumptions about how each input affects this and the following functions.

The second potential choice for the military is to exercise repression on behalf of the dictator, although Nature may deny this possibility. With probability $p(\theta_M, \theta_T) \in (0, 1)$, the military can effectively repress, that is, succeed at repression with probability 1. By contrast, with probability $1-p(\cdot)$, Nature does not allow the military to exercise repression.[5] For either type of military, consumption under the status quo regime equals $\omega_D \in (0, 1)$. The cost of exercising repression equals μ, which Nature draws from a smooth cdf $F(\cdot)$ with full support over $\left[0, \overline{\mu}\right]$, for $\overline{\mu}$ defined later. The associated pdf is denoted as $f(\cdot)$.[6]

The third potential choice for the military is to stage a coup. Conditional on Nature drawing effective repression, with probability $q(\theta_M, \theta_D) \in (0, 1)$, the military has a coup opportunity, that is, Nature enables the military to succeed at a coup attempt with probability 1. By contrast, with probability $1 - p(\cdot) \cdot q(\cdot)$, Nature does not allow the military to attempt a coup. The military needs repression to be effective in order to repress society to install a military dictatorship, and therefore the military pays the same repression cost μ if it stages a coup. Excluding the cost of repression, the military consumes 1 under military dictatorship,

[5]Assuming that the military knows whether or not repression will succeed prior to making its choice simplifies the exposition without altering the main intuitions, as Appendix Section B.4 shows.

[6]Several proofs require assuming $f'(\cdot) \leq 0$ which, for example, the uniform distribution satisfies.

which exceeds its payoff in the status quo regime.

The dictator's only goal is political survival: it consumes 1 if it survives in power, and 0 otherwise. The dictator survives if and only if the military exercises repression without staging a coup. To focus only on elements needed to generate the core tradeoffs, the dictator does not pay costs of repression or of military-building. Appendix Table A.1 summarizes the formal notation.

3.2 KEY ASSUMPTIONS

Rulers throughout history have organized their militaries in various manners. The present distinction between ethnic and national militaries captures several key differences across militaries in a parsimonious manner: fighting and repression capacity, opportunities for staging coups, and prospects under outsider rule. Of course, in some empirical situations, a certain type of military is both more coercively effective and less able to stage a coup than alternatives, such as janissaries (slave soldiers) in the Ottoman empire and militaries created during the Russian and Chinese communist revolutions. I instead focus on the more strategically interesting scenario in which the dictator faces a tradeoff.

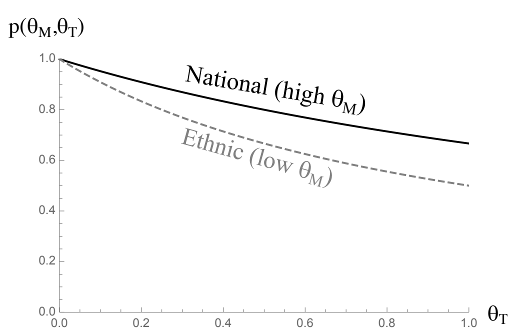
3.2.1 Probability of Effective Repression

The first distinction is that national militaries exhibit greater ability to fight and repress than ethnic militaries. Whereas national militaries recruit from broad segments of society, ethnic militaries' narrow recruitment can create manpower deficits (Quinlivan 1999). Ethnically biased recruitment can also undermine intelligence networks in areas populated by excluded ethnic groups, which hinders counterinsurgency capabilities (Roessler 2016). Broad recruitment strategies often correspond with military professionalization. A hierarchical chain of command minimizes the probability of splits and defections, which increases national militaries' ability to act collectively to defeat insurgents or to disperse protesters. By contrast, ethnic militaries feature more extensive coup-proofing measures—such as preventing officers from communicating with each other and diverting resources to paramilitary organizations such as presidential guards—which undermines their fighting capacity (Quinlivan 1999; Pilster and Böhmelt 2011; Talmadge 2015). Therefore, θ_M should be lower for ethnic militaries not only because of manpower and intelligence deficits. Allocating resources to a parallel coercive organization reduces available resources to supply for the conventional military—the organization usually called upon if facing a guerrilla group or as the measure of last resort when confronting a major urban movement.

The variable θ_M is the systematic component in the probability that the military can effectively repress. The stochastic component captures inherent variability in outsiders' ability to threaten the regime. Lacking a repressive opportunity corresponds to various possible circumstances: killing enough people to end a rebellion would be prohibitively costly, as in Indonesia in 1999 prior to democratizing; lower-level officers refuse to follow orders or mutiny, as in Iran in 1978-9; or prior battles cause the military to disintegrate, as in Ethiopia in 1991 at the conclusion of the civil war that overthrew Mengistu. Similarly, even if the military can effectively repress, the exact costs of repressing also depend on factors related to the extent of societal mobilization that are outside the regime's direct control, justifying the assumed variability in μ.

Formally, the function that determines the probability that repression will be effective satisfies several intuitive properties, which Figure 2 depicts by plotting $p(\theta_M, \theta_T)$ as a function of θ_T while fixing other parameter values and assuming congruous functional forms. Either military surely defeats the weakest possible threat: $p(\theta_M, 0) = 1$ for all $\theta_M > 0$. Assuming $\frac{\partial p}{\partial \theta_M} > 0$ implies that for all $\theta_T > 0$, the national military has a higher probability of effective repression. This probability decreases in the magnitude of the outsider threat, $\frac{\partial p}{\partial \theta_T} < 0$,[7] with increasing differences, $\frac{\partial^2 p}{\partial \theta_M \partial \theta_T} > 0$. The cross-partial captures the intuitive idea that the coercive advantages of national militaries are more pronounced when facing stronger threats.[8]

Figure 2: Assumptions about Repression Effectiveness



Probability of effective repression

	Low θ_T	High θ_T
National military	High	Medium
Ethnic military	High	Low

Notes: Figure 2 uses the parameter values $\overline{\theta}_T = 1$, $\underline{\theta}_M = 1$, and $\overline{\theta}_M = 2$, and assumes $p(\theta_M, \theta_T) = \frac{\theta_M}{\theta_M + \theta_T}$.

[7] This implies $p(\theta_M, \theta_T) \in (0, 1)$ for all $\theta_T > 0$.

[8] For example, the ratio-form contest function $p(\theta_M, \theta_T) = \frac{\theta_M}{\theta_M + \theta_T}$ satisfies these assumptions, including the complementarity assumption for any $\theta_M > \theta_T > 0$. Assuming $\overline{\theta}_T < \underline{\theta}_M$ incorporates the reasonable premise that the government's military has a greater coercive endowment than the outsider threat. The linear contest function $p(\theta_M, \theta_T) = 1 - \theta_T \cdot (1 - \theta_M)$ satisfies these assumptions for all $(\theta_M, \theta_T) \in [0, 1]^2$.

3.2.2 Probability of a Coup Opportunity

Even an aggrieved military may lack the opportunity to seize power because of the inherent secrecy and stealth involved with planning and executing a coup (Finer 2002; Luttwak 2016). Although militaries possess the guns, dictators retain non-coercive sources for thwarting coups such as mobilizing citizens to protest a coup attempt (i.e., popularity) and non-domestic sources of support from allied states, which correspond with higher θ_D. The type of military also affects the opportunity to stage a coup, providing another distinction in the model between national and ethnic militaries. Measures that hinder fighting capacity that are prevalent in ethnic militaries, such as inhibiting communication among officers and building a presidential guard, specifically aim to guard against coups. This consideration also follows the standard assumption in the guardianship dilemma literature that more capable militaries are better-situated to stage a coup (Acemoglu et al. 2010; Besley and Robinson 2010; Svolik 2013; McMahon and Slantchev 2015).

The formal assumptions resemble those for $p(\cdot)$, which Figure 3 illustrates by plotting $q(\theta_M, \theta_D)$ as a function of θ_D. If the dictator has the highest possible popularity endowment, then neither type of military can launch a coup: $q\big(\theta_M, \overline{\theta}_D\big) = 0$ for all $\theta_M > 0$. Assuming $\frac{\partial q}{\partial \theta_M} > 0$ implies that for all $\theta_D < \overline{\theta}_D$, the national military has a higher probability of a coup opportunity. The probability of a coup opportunity increases as the dictator's coup-proofing ability decreases, $-\frac{\partial q}{\partial \theta_D} > 0$, with increasing differences, $-\frac{\partial^2 q}{\partial \theta_D \partial \theta_M} > 0$,[9] capturing the intuitive idea that less popular dictators enhance the coup advantages of national militaries. I also assume that if the dictator has the lowest popularity endowment, then it cannot prevent a coup attempt by the national military: $q\big(\overline{\theta}_M, 0\big) = 1$.[10] Whereas low societal support ($\theta_D = 0$) should completely incapacitate an ideal-type unpopular dictator from preventing a coup by a national military, built-in coup-proofing measures facilitate possibly preventing a coup by an ethnic military.

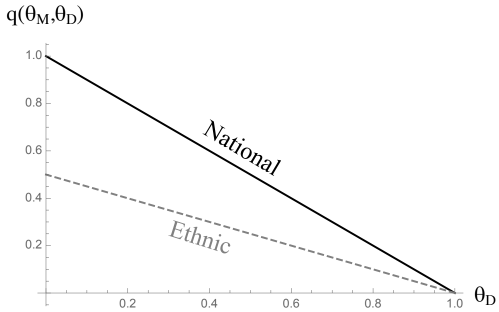
3.2.3 Military's Payoff Under Outsider Rule

The military's fate under outsider rule depends on (1) the outsider's radicalism and (2) military type. First, a radical outsider seeks to transform the composition of the elite class, and perhaps the entire social structure. Many takeovers by radical, or revolutionary, groups involve arbitrary arrests and executions, as with

[9] I assume that the cross-partial is large in magnitude (see Appendix Assumption A.1).

[10] The negative cross-partial additionally implies $q\big(\underline{\theta}_M, 0\big) < 1$.

Figure 3: Assumptions about Coup Opportunity



Probability of coup opportunity

	Low θ_D	High θ_D
National military	High	Low
Ethnic military	Medium	Low

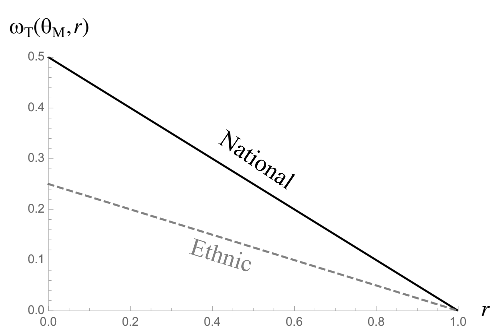
Notes: Figure 3 uses the parameter values $\underline{\theta}_M = 1$ and $\overline{\theta}_M = 2$, and assumes $q(\theta_M, \theta_D) = \left(\theta_M / \overline{\theta}_M\right) \cdot (1 - \theta_D)$.

the formation of the Soviet Union, or general chaos and destruction even in cases that lack a distinctive revolutionary ideology, as with Genghis Khan in the thirteenth century. Many twentieth-century communist movements fit the radical revolutionary characterization. For example, the Chinese Communist party implemented a massive land reform during and after its struggle to capture power in 1949. This was necessary to "destroy the gentry-landlord class (and thus eliminate a potential counterrevolutionary threat), establish Communist political power within the villages, and thus promote the building of a centralized state with firm administrative control over the countryside" (Meisner 1999, 92). Levitsky and Way (2013, 7) discuss the broad goals of revolutionaries to destroy traditional ruling and religious institutions, political parties, and the old army: "In most revolutions, preexisting armies either dissolved with the fall of the dictator (Cuba and Nicaragua) or were destroyed by civil war (China, Mexico, and Russia)."

The second consideration about the military's fate under outsider rule highlights another important difference between militaries: national militaries' broader societal recruitment yields better fates than ethnic militaries under outsider rule. Although this discrepancy in exit options is muted when facing a radical threat—because both types of militaries face existential crises—non-radical outsiders create divergent payoffs. A national military expects minimal restructuring because it will remain largely intact in a non-radical regime, but an ethnic military composed largely of soldiers tied to the previous regime expects extensive purging. For example, amid pro-democracy protests that emerged across Arab countries in early 2011, the nationally recruited Egyptian army eventually acquiesced to regime transition but the ethnically stacked army in Syria feared takeover by non-Alawites. The civil war that resulted from the Syrian military's willingness to fight outsiders is still ongoing as of 2019. Although members of the al-Asad regime may consider the Sunni opposition as radical, in cases such as this, a national military would likely consider the protesters' and rebels' espoused democratization goals as non-radical—implying that disparities in repression incentives arise from differences between narrow- and broad-based militaries rather than from the outsider's radicalism.

These considerations motivate the following assumptions about the military's payoff under outsider rule, which Figure 4 depicts by plotting $\omega_T\left(\theta_M, r\right)$ as a function of r. Both military types expect dire fates under an ideal-type radical threat ($r = 1$) because both expect executions, disbandment, and other punishments: $\omega_T\left(\theta_M, 1\right) = 0$ for all $\theta_M > 0$. For any $r < 1$, the national military fares better than the ethnic military under outsider rule, which follows from assuming $\frac{\partial \omega_T}{\partial \theta_M} > 0$. A decrease in the outsider's radicalism increases the payoff for either type of military under outsider rule, $-\frac{\partial \omega_T}{\partial r} > 0$, with increasing differences, $-\frac{\partial^2 \omega_T}{\partial \theta_M \partial r} > 0$.[11] The cross-partial captures the intuitive idea, discussed above, that national militaries fare considerably better than ethnic militaries under rule by a non-radical outsider. I also assume for the least radical type of threat, $r = 0$, that the national military consumes the same as under the status quo authoritarian regime, $\omega_T\left(\overline{\theta}_M, 0\right) = \omega_D$.[12] These assumptions imply that $\omega_T < \omega_D$ for all $r \in (0, 1)$, which focuses the analysis on the strategically non-trivial case in which any military receives certain perks under the incumbent regime that it would lose following a transition.

Figure 4: Assumptions about Military's Payoff Under Outsider Rule



Payoff under outsider rule

	Low r	High r
National military	High	Low
Ethnic military	Medium	Low

Notes: Figure 4 uses the parameter values $\underline{\theta}_M = 1$, $\overline{\theta}_M = 2$, $\omega_D = 0.5$, and assumes $\omega_T(\theta_M, r) = \left(\theta_M / \overline{\theta}_M\right) \cdot (1 - r) \cdot \omega_D$.

3.3 Simplifying Assumptions and Extensions

To focus on the main tradeoffs of interest, the model contains many simplifying assumptions. One is that the dictator can freely choose how to construct its military, whereas in reality rulers face constraints. For example, Ahram (2011) discusses the difficulties that Saddam Hussein and earlier Iraqi leaders faced when

[11] I assume that the cross-partial is large in magnitude (see Appendix Assumption A.1).

[12] The negative cross-partial additionally implies $\omega_T\left(\underline{\theta}_M, 0\right) < \omega_D$.

trying to devolve military control to state-sponsored militias given Iraq's prior history with a conventional military. However, in many cases, dictators enjoy considerable leeway in their ability to personalize (or refrain from personalizing) the officer corps early in their tenure. Greitens (2016) demonstrates this point with case studies from East Asia,[13] and Geddes et al. (2018, 85-89) provide quantitative data on the large extent of personalization that occurs earlier in rulers' tenures. Even in Iraq, Saddam succeeding at stacking the officer corps with Tikriti despite the constraints imposed by an established military apparatus. Additionally, although altering the model to constrain the dictator's choice may better account for certain empirical cases, this would distract from my focus on understanding the strategic incentives that dictators face in scenarios where they *choose* military type.

Dictators may also face multiple outsider threats that can change over time. We can conceive θ_T and r as a weighted average of the outsider threats that the dictator anticipates when shaping its military, for which, as noted, the most consequential choices often occur at the beginning of the ruler's tenure. This resembles Greitens's (2016) focus on the main threat a dictator faces when achieving power.

Even with these and other simplifications, the model still contains four key elements that existing models treat separately: the dictator chooses its type of military, and the military chooses among repressing, staging a coup, and transitioning to outsider rule. The military's coercive endowment θ_M affects all three of its choices. Although including all these elements in the model is necessary to address the motivating questions regarding the loyalty-efficiency tradeoff, they impose tractability constraints. Despite the otherwise parsimonious setup, θ_M enters the dictator's objective function in three places (see Equation 4), which causes the number of indirect effects to proliferate in higher-order derivatives of the objective function. Table 2 summarizes numerous extensions presented later in the paper and in Appendix B that relax certain simplifying assumptions in the baseline model.

[13] See, for example, pp. 37-39.

Table 2: Summary of Extensions

Assumption in baseline model	Alteration	Section
Both militaries consume ω_D in incumbent regime, which is less-preferred than military rule	Heterogeneous military valuation of incumbent regime yields differential "inherent" loyalty	6.1, B.1
Military's consumption under outsider rule does not affect its coup calculus	Possibility that a coup might yield outsider rule yields differential "strategic" loyalty	6.2, B.2
Both types are standing militaries that cannot flee	Flee option lowers mercenary armies' reliability relative to militias	6.3, B.3
Repression success or not known at military's information set	Probabilistic repression success	B.4
Dictator's choice over military type is binary	Continuous choice over military type	B.5
Dictator cares only about political survival	Dictator's consumption under outsider rule varies in θ_D and r	B.6
Radicalism of outsider threat is exogenous	Dictator chooses outsider's radicalism	B.6

4 Analysis of Military's Decision

4.1 Repression, Coup, or Transition?

Solving backwards, Table 3 shows the military's optimal choices. If repression is ineffective, then the military's only option is to transition to outsider rule. If instead repression is effective, then its optimal choice depends on the other Nature draws. If the military has a coup opportunity, then it will stage a coup. It prefers creating a military dictatorship to defending the regime because $\omega_D < 1$, and necessarily prefers a coup to a transition because I assume that the maximum possible repression cost is sufficiently low, $\overline{\mu} = 1 - \omega_D$. But if the military is effective at repression while lacking a coup opportunity, then its optimal choice depends on repression costs. If $\mu < \hat{\mu}(\theta_M) \equiv \omega_D - \omega_T(\theta_M, r)$, then the military represses for the regime, whereas the military transitions if μ is higher. The critical repression-cost threshold $\hat{\mu}(\theta_M)$ depends on θ_M because this parameter affects the military's payoff under outsider rule. The assumed distributions for the Nature variables enables writing the probability of each outcome conditional on the dictator's military choice, which Lemma 1 states.

Table 3: Military's Optimal Choice

Effective repression with coup opportunity Pr=$p(\theta_M, \theta_T) \cdot q(\theta_M, \theta_D)$	**Effective repression without coup opportunity** Pr=$p \cdot (1 - q)$	**Ineffective repression** Pr=$1 - p$
Coup	Repress if μ is low Transition if μ is high	Transition

Lemma 1 (Outcome probabilities conditional on military type)**.** *Given θ_M, the equilibrium probability of each outcome is:*

$$Pr(repress) = \underbrace{\Big[1-q\big(\theta_M,\theta_D\big)\Big]}_{\textit{No coup opportunity}} \cdot \underbrace{p\big(\theta_M,\theta_T\big)}_{\textit{Effective repression}} \cdot \underbrace{F\Big(\omega_D-\omega_T\big(\theta_M,r\big)\Big)}_{\textit{Low repression costs}}$$

$$Pr(coup) = \underbrace{q\big(\theta_M,\theta_D\big)\cdot p\big(\theta_M,\theta_T\big)}_{\textit{Effective repression and coup opportunity}}$$

$$Pr(transition) = \underbrace{1-p\big(\theta_M,\theta_T\big)}_{\textit{Ineffective repression}} + \Big[1-q\big(\theta_M,\theta_D\big)\Big]\cdot p\big(\theta_M,\theta_T\big)\cdot \underbrace{\Big[1-F\Big(\omega_D-\omega_T\big(\theta_M,r\big)\Big)\Big]}_{\textit{High repression costs}}$$

Lemma 1 yields two immediate implications. First, the ethnic military is less likely to attempt a coup—indicating higher loyalty—because the national military is more likely to have a coup opportunity. Second, conditional on effective repression and the military lacking a coup opportunity, the ethnic military defends the regime with higher probability: its lower consumption under outsider rule increases the range of μ values small enough that it optimally represses.

Lemma 2 (Coup loyalty)**.**

$$\underbrace{1-p\big(\overline{\theta}_M,\theta_T\big)\cdot q\big(\overline{\theta}_M,\theta_D\big)}_{\textit{National}} < \underbrace{1-p\big(\underline{\theta}_M,\theta_T\big)\cdot q\big(\underline{\theta}_M,\theta_D\big)}_{\textit{Ethnic}}$$

Lemma 3 (Conditional probability of repression)**.**

$$\underbrace{F\Big(\omega_D-\omega_T\big(\overline{\theta}_M,r\big)\Big)}_{\textit{National}} < \underbrace{F\Big(\omega_D-\omega_T\big(\underline{\theta}_M,r\big)\Big)}_{\textit{Ethnic}}$$

4.2 Loyalty and Efficiency Mechanisms

These lemmas enable characterizing the relative advantages of each military type from the dictator's perspective, which Table 4 summarizes. Recovering conventional wisdom about the loyalty-efficiency tradeoff, ethnic militaries exhibit higher ***loyalty*** through their lower probability of a coup (Lemma 2). A necessary and sufficient for a coup is for Nature to draw both a coup opportunity and effective repression—both of which advantage the national military. Notably, the coup loyalty result follows solely from differential *opportunities* to stage a coup rather than from differences in the militaries' *preferences* for the incumbent.

In other words, conventional ideas such as officers favoring co-ethnic rule are not necessary to generate the loyalty side of the loyalty-efficiency tradeoff. Section 6 discusses this consideration in more detail and demonstrates alternative ways to model loyalty that yield a similar logic.

With regard to repressive ***efficiency***, national and ethnic militaries exhibit mixed considerations. On the one hand, the higher probability with which the national military can repress effectively—which arises from assuming that $p\big(\theta_M, \theta_T\big)$ strictly increases in θ_M—creates an efficiency advantage. However, the national military's higher reservation value to outsider rule creates a countervailing effect. Conditional on the military being able to repress effectively, the national military is less likely to defend the regime (Lemma 3). This countervailing efficiency mechanism—largely overlooked in existing studies positing that rulers face a loyalty-efficiency tradeoff—creates the possibility that an ethnic military can exhibit higher repressive efficiency despite its weaker coercive endowment.

Table 4: Dictator's Tradeoff Between Military Types

Mechanism	***Probability term***	***National military***	***Ethnic military***
Loyalty	Pr(coup)		✓
Efficiency #1	Pr(repression is effective)	✓	
Efficiency #2	Pr(repress | repression is effective)		✓

4.3 A Loyalty-Efficiency Tradeoff?

Does the dictator trade off between coup loyalty and repressive efficiency? Repressive efficiency equals the probability of no outsider overthrow conditional on no coup:

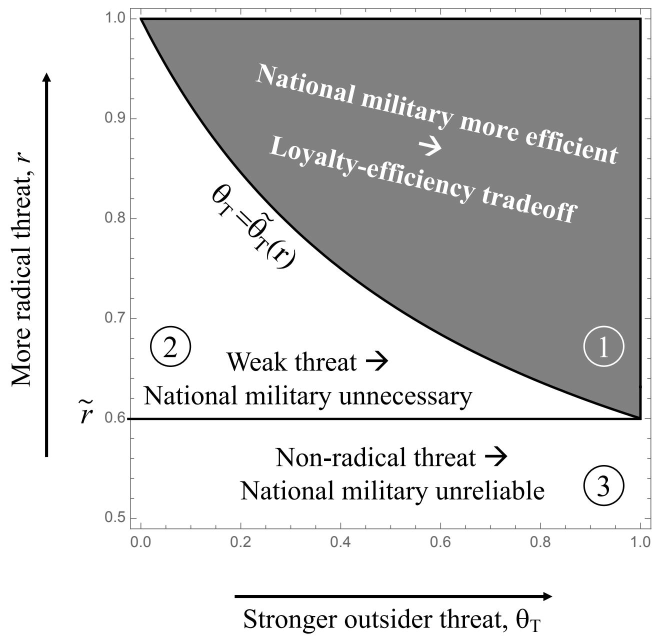
$$E^*\big(\theta_M, \theta_T, r\big) \equiv \underbrace{p\big(\theta_M, \theta_T\big)}_{\text{Pr(repression is effective)}} \cdot \underbrace{F\Big(\omega_D - \omega_T\big(\theta_M, r\big)\Big)}_{\text{Pr(repress | repression is effective)}} \tag{1}$$

Figure 5 presents a region plot as a function of outsider threat strength, θ_T (horizontal axis), and the outsider's radicalism, r (vertical axis). It highlights two regions in white in which the dictator does not face a loyalty-efficiency tradeoff because the ethnic military exhibits higher equilibrium efficiency than the national military. First, the following threshold value of r defines region 3:

$$E^*\big(\underline{\theta}_M, \overline{\theta}_T, \tilde{r}\big) = E^*\big(\overline{\theta}_M, \overline{\theta}_T, \tilde{r}\big) \tag{2}$$

Showing the importance of modeling the endogenous choice to exercise repression, $r < \tilde{r}$ makes the national military unreliable. When facing a non-radical threat, the national military fares considerably better under outsider rule than the ethnic military, that is, $\omega_T\left(\overline{\theta}_M, r\right)$ is considerably larger than $\omega_T\left(\underline{\theta}_M, r\right)$, which Figure 4 depicts. This creates a large gap in the two militaries' probability of exercising repression conditional on repression being effective, that is, the second efficiency mechanism in Table 4 is large in magnitude. If r is low, then this mechanism swamps the repressive advantages of the national military even if θ_T is large.

Figure 5: Repressive Efficiency



Notes: Figure 5 uses the same parameter values and functional form assumptions as Figures 2 through 4, and $\omega_D = 0.8$ and $\mu \sim U(0, 1 - \omega_D)$.

Second, a threshold value of θ_T defines region 2:

$$E^*\left(\underline{\theta}_M, \tilde{\theta}_T, r\right) = E^*\left(\overline{\theta}_M, \tilde{\theta}_T, r\right) \tag{3}$$

If $\theta_T < \tilde{\theta}_T(r)$, then the national military is unnecessary. Facing a weak threat, the gap between $p\left(\overline{\theta}_M, \theta_T\right)$ and $p\left(\underline{\theta}_M, \theta_T\right)$ is small because either type of military likely can effectively repress a weak threat, which Figure 2 depicts. That is, the first efficiency mechanism in Table 4 is small in magnitude.

Only if both r and θ_T are large (region 1 in gray) is the national military more repressively efficient. Combining this result with the ethnic military exhibiting higher coup loyalty for all parameter values (Lemma

2) implies that the dictator faces a loyalty-efficiency tradeoff if and only if the threat is strong and radical. High θ_T widens the national military's coercive advantage relative to the ethnic military, and high r raises the relative likelihood with which the national military exercises repression—which also explains why the minimum value of θ_T at which the national military is more efficient strictly decreases in r. Appendix A proves Lemma 4 and all other results in the baseline model that do not follow directly from the text.

Lemma 4 (Loyalty-efficiency tradeoff)**.**

Part a. Non-radical threat. *If $r < \tilde{r}$, then the dictator does not face a loyalty-efficiency tradeoff because the ethnic military exhibits higher repressive efficiency for all $\theta_T \in \big(0, \overline{\theta}_T\big)$: $E^*\big(\underline{\theta}_M, \theta_T, r\big) > E^*\big(\overline{\theta}_M, \theta_T, r\big)$. Region 3 of Figure 5 depicts these parameter values.*

Part b. Radical threat. *If $r > \tilde{r}$, then:*

- *If $\theta_T < \tilde{\theta}_T$, then the dictator does not face a loyalty-efficiency tradeoff because the ethnic military exhibits higher repressive efficiency: $E^*\big(\underline{\theta}_M, \theta_T, r\big) > E^*\big(\overline{\theta}_M, \theta_T, r\big)$. Region 2 of Figure 5 depicts these parameter values.*
- *If $\theta_T > \tilde{\theta}_T$, then the dictator faces a loyalty-efficiency tradeoff because the national military exhibits higher repressive efficiency: $E^*\big(\underline{\theta}_M, \theta_T, r\big) < E^*\big(\overline{\theta}_M, \theta_T, r\big)$. Region 1 of Figure 5 depicts these parameter values.*

5 Analysis of Dictator's Decision

5.1 Optimal Military Choice

The dictator weighs the military's coup propensity and repressive efficiency. It maximizes its probability of survival, which equals the probability that the military exercises repression to defend the regime:

$$S^*\big(\theta_M, \theta_T, \theta_D, r\big) \equiv \underbrace{\Big[1 - q\big(\theta_M, \theta_D\big)\Big]}_{\text{No coup opportunity}} \cdot \underbrace{p\big(\theta_M, \theta_T\big) \cdot F\Big(\omega_D - \omega_T\big(\theta_M, r\big)\Big)}_{\text{Repressive efficiency}} \tag{4}$$

There are two cases to consider. First, if the national military's coup likelihood is sufficiently high, which is true for low θ_D, then the dictator will choose the ethnic military regardless of repressive efficiency considerations. Extremely unpopular dictators cannot harness the (possible) repressive advantages of a national military. This threshold is defined as:

$$S^*\big(\underline{\theta}_M, \overline{\theta}_T, 1, \tilde{\theta}'_D\big) = S^*\big(\overline{\theta}_M, \overline{\theta}_T, 1, \tilde{\theta}'_D\big) \tag{5}$$

Given the logic described above for the loyalty-efficiency tradeoff, if the dictator's survival probability under the national military is lower than that for the ethnic military even if the threat is strong and radical, then the dictator prefers the ethnic military for all values of θ_T and r.

The second and more strategically interesting case occurs for $\theta_D > \tilde{\theta}'_D$ because the dictator's military choice of military depends on repressive efficiency. The national military is more likely to attempt a coup, which implies that the dictator chooses the ethnic military under all parameter values in Lemma 4 in which the ethnic military exhibits higher repressive efficiency—if the outsider threat is non-radical and/or weak in magnitude. These are regions 2 and 3 in Figures 5 and 6. However, even for parameter values in which the national military is more repressively efficient, the loyalty-efficiency tradeoff implies that the dictator does not necessarily choose the national military. Although the dictator follows a similar threshold strategy as characterized in Lemma 4, it optimally chooses the national military for a smaller range of parameter values than those for which the national military exhibits higher repressive efficiency. Figure 6 shows this by distinguishing region 1a in black, in which the dictator chooses the national military, from the gray regions 1b and 1c in which the dictator chooses the ethnic military. These areas collectively compose region 1 in Figure 5.[14] The respective analogies to the r and θ_T thresholds defined for repressive efficiency are:

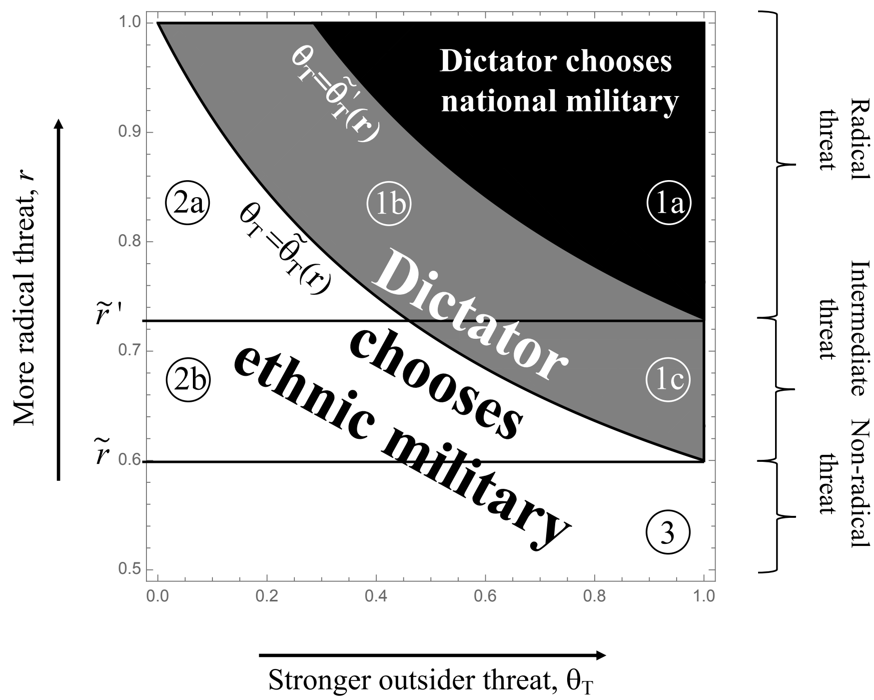
$$S^*\left(\underline{\theta}_M, \overline{\theta}_T, \tilde{r}', \theta_D\right) = S^*\left(\overline{\theta}_M, \overline{\theta}_T, \tilde{r}', \theta_D\right) \tag{6}$$

$$S^*\left(\underline{\theta}_M, \tilde{\theta}'_T, r, \theta_D\right) = S^*\left(\overline{\theta}_M, \tilde{\theta}'_T, r, \theta_D\right) \tag{7}$$

Proposition 1 and the actions from Table 3 characterize the unique subgame perfect Nash equilibrium.

[14]Note that there is an intermediate threat range, $r \in \left(\tilde{r}, \tilde{r}'\right)$, in which the national military is more repressively efficient for large enough θ_T (region 1c in Figure 6), but the dictator chooses the ethnic military for all θ_T. This is because the repressive efficiency advantage of the national military is low enough in magnitude that the dictator is unwilling to sacrifice loyalty for efficiency.

Figure 6: Optimal Military Choice



Notes: Figure 6 uses the same parameter values and functional form assumptions as the previous figures. In the white regions, the dictator does not face a loyalty-efficiency tradeoff and optimally chooses the ethnic military. In the gray region, the dictator faces a loyalty-efficiency tradeoff and chooses the ethnic military. In the black region, the dictator faces a loyalty-efficiency tradeoff and chooses the national military.

Proposition 1 (Optimal military choice)**.**

> ***Part a. Unpopular dictators.*** *If $\theta_D < \tilde{\theta}_D'$, then the dictator chooses the ethnic military: $S^*\big(\underline{\theta}_M, \theta_T, \theta_D, r\big) > S^*\big(\overline{\theta}_M, \theta_T, \theta_D, r\big)$. Figure 6 does not depict these parameter values.*
>
> ***Part b. Non-radical (and intermediate) threat.*** *If $\theta_D > \tilde{\theta}_D'$ and $r < \tilde{r}'$, then the dictator chooses the ethnic military: $S^*\big(\underline{\theta}_M, \theta_T, \theta_D, r\big) > S^*\big(\overline{\theta}_M, \theta_T, \theta_D, r\big)$. Regions 1c, 2b, and 3 in Figure 6 depict these parameter values.*
>
> ***Part c. Radical threat.*** *If $\theta_D > \tilde{\theta}_D'$ and $r > \tilde{r}'$, then:*
>
> - *If $\theta_T < \tilde{\theta}_T'$, then the dictator chooses the ethnic military: $S^*\big(\underline{\theta}_M, \theta_T, \theta_D, r\big) > S^*\big(\overline{\theta}_M, \theta_T, \theta_D, r\big)$. Regions 1b and 2a in Figure 6 depict these parameter values.*
> - *If $\theta_T > \tilde{\theta}_T'$, then the dictator chooses the national military: $S^*\big(\underline{\theta}_M, \theta_T, \theta_D, r\big) < S^*\big(\overline{\theta}_M, \theta_T, \theta_D, r\big)$. Region 1a in Figure 6 depicts these parameter values.*

5.2 A GUARDIANSHIP DILEMMA?

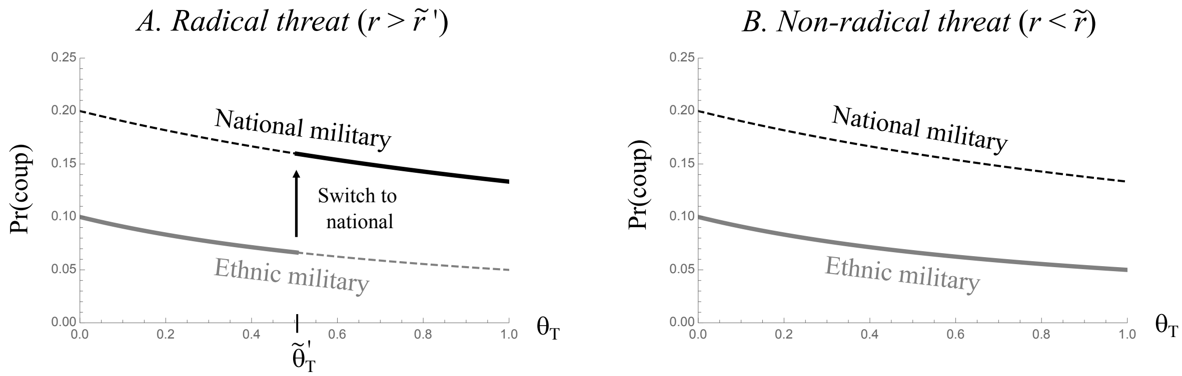
Do stronger outsider threats trigger a guardianship dilemma, that is, raise the equilibrium probability of a coup attempt by causing the dictator to shift to a less loyal type of military? Existing arguments are incomplete because they do not consider either the military agency problem or heterogeneity in the outsider threats that regimes face. Figure 7 depicts the relationship between θ_T and the equilibrium probability of a coup, distinguishing between radical (Panel A) and non-radical (Panel B) threats. An increase in θ_T generates both a direct and an indirect effect. The direct effect is that higher θ_T decreases the probability that the military can effectively exercise repression, thereby decreasing the probability with which the military can install a military dictatorship. Contrary to the guardianship logic, this mechanism yields a *negative* relationship between outsider threat strength and equilibrium coup probability. This logic is independent of military type or the radicalism of the outsider threat, as shown by the downward slope of all four curves in Figure 7 that fix the dictator's choice at either the national or ethnic military. This resembles McMahon and Slantchev's (2015) finding that stronger outsider threats diminish the equilibrium probability of a coup attempt by decreasing the value of holding office.

However, the indirect effect of increasing θ_T recovers the traditional guardianship dilemma mechanism, contrary to McMahon and Slantchev's (2015) critique. If the outsider threat is radical $\left(r > \tilde{r}'\right)$, then a strong outsider threat $\left(\theta_T > \tilde{\theta}_T\right)$ creates a loyalty-efficiency tradeoff for the dictator, which regions 1a and 1b in Figure 6 depict. For any increase in θ_T large enough to cross the $\theta_T = \tilde{\theta}'_T$ threshold that separates regions 1a and 1b, the dictator switches from the ethnic to the national military. This effect discretely *increases* the equilibrium probability of a coup attempt because the national military exhibits higher coup propensity than the ethnic military (see Table 4). Therefore, a radical threat generates both a loyalty-efficiency tradeoff and a guardianship dilemma, which Panel A of Figure 7 shows.

By contrast, a non-radical threat ($r < \tilde{r}$) eliminates the guardianship mechanism by eliminating the dictator's loyalty-efficiency tradeoff (see Lemma 4). The dictator never chooses the national military, which implies that at no θ_T values does the equilibrium probability of a coup attempt discretely increase. Panel B of Figure 7 depicts this result.[15]

[15] A loyalty-efficiency tradeoff is necessary but not sufficient for a guardianship dilemma (see regions 1c and 2b in Figure 6). For intermediate values $r \in \left(\tilde{r}, \tilde{r}'\right)$, although the national military is more efficient than the ethnic military for high enough θ_T, the dictator prefers the ethnic military even at $\theta_T = \overline{\theta}_T$ because the

Figure 7: Equilibrium Probability of a Coup



Notes: Solid segments of curves correspond with parameter values in which the dictator optimally chooses the specified type of military, and dashed segments correspond with off-the-equilibrium path outcomes. Therefore, the equilibrium coup probability equals the piecewise function created by the solid segments of curves. Both panels use the same parameter values and functional form assumptions as previous figures. In Panel A, $r = 0.88$. In Panel B, $r = 0.7$.

Proposition 2 (Threat strength and equilibrium coup probability)**.** *Given the thresholds stated in Proposition 1:*

Part a. Radical threat. *If $\theta_D > \tilde{\theta}_D'$ and $r > \tilde{r}'$, then the equilibrium probability of a coup strictly decreases in θ_T for $\theta_T \in \left(0, \tilde{\theta}_T'\right) \cup \left(\tilde{\theta}_T', \overline{\theta}_T\right)$, and exhibits a discrete increase at $\theta_T = \tilde{\theta}_T'$.*

Part b. Non-radical/intermediate threat. *If $\theta_D < \tilde{\theta}_D'$ or $r < \tilde{r}'$, then the equilibrium probability of a coup strictly decreases in θ_T for all $\theta_T \in \left(0, \overline{\theta}_T\right)$.*

Proposition 3 (Threat radicalism, loyalty-efficiency tradeoff, and guardianship dilemma)**.** *Given the thresholds stated in Lemma 4 and Proposition 1, if $\theta_D > \tilde{\theta}_D'$, then:*

Part a. Radical threat. *If $r > \tilde{r}'$, then the dictator faces both a loyalty-efficiency tradeoff and a guardianship dilemma.*

Part b. Non-radical threat. *If $r < \tilde{r}$, then the dictator faces neither a loyalty-efficiency tradeoff nor a guardianship dilemma.*

Part c. Intermediate threat. *If $r \in \left(\tilde{r}, \tilde{r}'\right)$, then the dictator faces a loyalty-efficiency tradeoff but not a guardianship dilemma.*

5.3 Effects of Dictator Popularity

Although the analysis focuses primarily on how characteristics of the outsider threat affect the dictator's optimal military choice, the dictator's endowed strength θ_D—which encompasses broader political institutions

difference in repressive efficiency is not large enough to compensate for the difference in coup likelihood.

and popular support—also affects its choice. A dictator with high θ_D faces low coup vulnerability. Existing arguments posit that dictators should favor more broadly based national militaries when facing a low coup threat. For example, Greitens (2016, 18) argues that dictators resolve their dual coup and outsider rebellion threats by "configuring their internal security apparatus to address the *dominant perceived threat* at the time they come to power. Prioritizing the threat of a coup leads to higher fragmentation and exclusivity, whereas focusing on the threat of popular uprising leads to a more unitary and socially inclusive apparatus."

The model produces two findings about the effects of increasing θ_D. The first supports Greitens' argument but the second does not. First, higher θ_D causes the dictator to weight repressive efficiency more heavily than the coup threat in its objective function because the type of military less strongly affects the probability of a coup. As $\theta_D \rightarrow \overline{\theta}_D$, the probability of a coup attempt goes to 0 regardless of θ_M, and the dictator's objective in Equation 4 becomes equivalent to maximizing efficiency (Equation 1). Graphically, as $\theta_D \rightarrow \overline{\theta}_D$, the black region in which the dictator prefers the national military in Figure 6 converges to the gray region in Figure 5 in which the national military exhibits higher repressive efficiency. This provides further intuition for the $\tilde{\theta}'_D$ threshold discussed above.

Second, lowering the coup threat does not necessarily cause the dictator to choose a national military. The revised loyalty-efficiency logic explained by the model (see Figure 5) implies that when facing a non-radical threat, the ethnic military exhibits higher repressive efficiency regardless of the strength of the threat. Therefore, if $r < \tilde{r}$, then a low coup threat does not cause the dictator to switch to a "more unitary and socially inclusive apparatus" (Greitens 2016, 18). The existing argument is true only if the threat is radical, $r > \tilde{r}$, which generates a loyalty-efficiency tradeoff for the dictator.

To formalize this logic, define the difference between equilibrium repressive efficiency and the dictator's equilibrium survival probability (for a given choice of θ_M):

$$\Delta \equiv E^*\big(\theta_M, \theta_T, r\big) - S^*\big(\theta_M, \theta_T, \theta_D, r\big) \tag{8}$$

Visually, Figure 6 shows that for fixed r, Δ equals the distance between $\tilde{\theta}'_T$ and $\tilde{\theta}_T$, i.e., the gray region.

Proposition 4 (Effects of dictator popularity)**.**

Part a. *If the national military exhibits higher repressive efficiency than the ethnic military ($r > \tilde{r}$ and $\theta_T > \tilde{\theta}_T$; see Lemma 4), then for Δ defined in Equation 8:*

- *An increase in θ_D raises the dictator's likelihood of choosing the national military: $\frac{d\Delta}{d\theta_D} < 0$.*
- *As the dictator becomes perfectly able to prevent coups, the dictator chooses the national military: $\lim_{\theta_D \to \overline{\theta}_D} \Delta = 0$.*

Part b. *If the ethnic military exhibits higher repressive efficiency than the national military ($r < \tilde{r}$ or $\theta_T < \tilde{\theta}_T$), then the dictator chooses the ethnic military regardless of the value of θ_D.*

6 Alternative Loyalty Mechanisms

The baseline model proposes an opportunity-based loyalty mechanism by which the ethnic military is less likely than the national military to stage a coup. Substantively, the underlying assumption follows from more prevalent coup-proofing institutions inherent in ethnic militaries that diminish opportunities to successfully stage a coup. But other differences between national and ethnic militaries also affect coup propensity, which this section analyzes. Appendix B proves the formal statements and presents several additional extensions.

6.1 Inherent Loyalty

To capture *inherent loyalty*, I alter the model so that the ethnic military enjoys higher expected consumption under the incumbent regime. Appendix Table B.1 provides the revised game tree. Now, at every information set for the military, it can attempt a coup that succeeds with probability 1. The function $q(\theta_M, \theta_D)$ instead determines the military's consumption under the incumbent regime. With probability $q(\cdot)$, the military's valuation of the incumbent regime is low, $\omega_D = \underline{\omega}_D \in (0, 1)$, and with complementary probability it is high, $\omega_D = \overline{\omega}_D > 1$. The same assumptions as in the baseline model apply to $q(\cdot)$, most important, that it strictly increases in θ_M. The difficulty of governing and of maintaining a hierarchical command chain while exerting political influence can cause a military to prefer civilian rule (Finer 2002), which corresponds with $\omega_D > 1$. By contrast, in the baseline model, the military necessarily prefers military dictatorship over the incumbent regime, and only the lack of a coup opportunity prevents either type of military from installing a military dictatorship.

The military's optimal choices and the associated probabilities with which it chooses each are unchanged from the baseline model (see Table 3).[16] If $\omega_D = \underline{\omega}_D$, then the military prefers a coup over repression because it consumes more in a military dictatorship than in the incumbent regime. This is strategically identical to the information set in the baseline model in which the military has a coup opportunity. By contrast, if $\omega_D = \overline{\omega}_D$, then the military prefers repression to a coup—even though a coup attempt succeeds with probability 1—because it consumes more under the incumbent regime than in a military dictatorship. This is strategically identical to the information sets in the baseline model in which the military lacks a coup opportunity. Consequently, despite the different motivation for several parameters in this alteration, the dictator's survival objective function in Equation 4 is unchanged.

Compared to the national military, I assume that the ethnic military exhibits a higher expected valuation for the incumbent dictatorship, which implies a lower probability of staging a coup because $\omega_D = \overline{\omega}_D$ is necessary (and sufficient) for choosing to defend the regime. This result captures the idea of inherent loyalty because the ethnic military's stronger preferences to uphold the incumbent regime increase the dictator's probability of surviving. Existing research suggests many possible sources of higher inherent loyalty for narrowly constructed ethnic militaries. One possibility is that officers gain some type of "warm glow" from co-ethnic governance. For example, Quinlivan's (1999, 135) section "The Exploitation of Special Loyalties" begins by stating: "The building block of political action in Saudi Arabia, Iraq, and Syria is the 'community of trust' that is willing to act together." Another is that members of different ethnic groups exhibit similar preferences over public goods, and higher expected ω_D expresses in reduced form that the ethnic military consumes more because the dictator provides public goods that the military values more highly (Alesina et al. 1999). Other possibilities relate to the dictator's ability to commit to deliver spoils to the military. The descent-based characteristics of ethnic groups make it easier to commit to reward co-ethnics because it is difficult to hide or to change one's ethnicity (Caselli and Coleman 2013). Alternatively, co-ethnics may more effectively solve the coordination problems inherent in compelling a dictator to compensate subordinates after defending the autocrat in battle (Myerson 2008).

[16] Appendix Table B.1 changes the appropriate labels from Table 3 for the present extension.

6.2 STRATEGIC LOYALTY

The second alteration yields a strategic endogenous loyalty mechanism: the ethnic military's lower reservation value to outsider rule lowers its likelihood of staging a coup. Appendix Figure B.2 presents the revised game tree. I retain the assumption from the inherent loyalty extension that, conditional on effective repression, the military always has a coup opportunity, but I revert to the assumption from the baseline model that both types of military consume the same amount $\omega_D < 1$ under the incumbent authoritarian regime. To generate a strategic loyalty mechanism, I assume that a coup yields a military dictatorship with probability $\sigma \in (0, 1)$.[17] With complementary probability, a coup engenders outsider rule. In between the dictator's and military's moves, Nature draws σ from a smooth distribution $G(\cdot)$ with full support over $[0, 1]$. As with the other Nature moves, this ensures an ex ante positive probability of the military choosing any of its options. Notably, the military does not know whether or not a coup attempt will engender a military dictatorship, but it knows the specific probability σ whereas the dictator does not. The bounds of the support for $G(\cdot)$ are strategically equivalent to different information sets in the baseline model: $\sigma = 0$ is equivalent to lacking a coup opportunity because the military cannot establish a military dictatorship, and $\sigma = 1$ is equivalent to having a coup opportunity because a coup attempt ensures a military dictatorship.

Various substantive considerations motivate the additional Nature move following the military's decision to stage a coup that determines whether the military or the outsider controls the next regime. Empirically, within several years after staging a coup, militaries often hold elections. This pattern is even more prevalent since the Cold War ended. This fits with the present conceptualization of transitioning to outsider rule because the generals did not create a consolidated military dictatorship. In some cases, the military may indeed have planned to hand power over to civilians from the beginning, whereas in other cases the military may have gambled that it could hold on—but instead ended up negotiating a transition because of concerted domestic or international pressure.

The first necessary condition in the baseline model for the military to defend the regime is unchanged: the cost of repression μ is low enough that the military prefers repressing to transitioning. However, the second

[17] Although this parameter resembles $q(\cdot)$ from the baseline model and the inherent loyalty extension because it affects the military's coup option, I use new notation to avoid confusion because σ is not a function of θ_M and θ_D.

necessary condition—the military prefers repression to a coup—is met if and only if:

$$\underbrace{\omega_D - \mu}_{\text{Military's utility to defending regime}} > \underbrace{\sigma + (1-\sigma)\cdot\omega_T\big(\theta_M, r\big) - \mu}_{\text{Military's utility to coup}} \tag{9}$$

Solving Equation 9 for σ and imposing the assumed probability distribution for σ implies that the dictator's survival objective function is:

$$S^*_{sl}\big(\theta_M, \theta_T, \theta_D, r\big) \equiv \underbrace{G\left(\frac{\omega_D - \omega_T\big(\theta_M, r\big)}{1-\omega_T\big(\theta_M, r\big)}\right)}_{\text{Prefers repression to a coup}} \cdot \underbrace{p\big(\theta_M, \theta_T\big)\cdot F\Big(\omega_D - \omega_T\big(\theta_M, r\big)\Big)}_{\text{Repressive efficiency}}, \tag{10}$$

and the subscript sl expresses strategic loyalty. Although the efficiency component of the survival function is identical to that in the baseline model, the coup component differs. Now, whether or not the military will repress to defend the dictator rather than stage a coup depends not only on the draw of σ, but also on its reservation value to outsider rule, $\omega_T\big(\theta_M, r\big)$. The ethnic military is less likely to stage a coup than the national military—despite equally valuing the incumbent regime and facing identical opportunities to overthrow the dictator—because it more greatly fears the possibility of outsider rule, consistent with higher *strategic loyalty*.[18] Appendix Proposition B.1 formalizes this logic.

6.3 Repressive Loyalty of Mercenaries

Many polities earlier in history lacked standing armies, or augmented a small corps of professional officers with other troops. In such cases, military coups are not a pressing concern for the ruler. Instead, the only aspect of loyalty that perplexes leaders is what—to clearly distinguish the dictator's tradeoff between insider and outsider threats—I have defined as an element of repressive efficiency, the probability that the military exercises repression if possible. For example, Finer (1975, 93-5) posits this alternative formulation of the loyalty-efficiency tradeoff in his study of early modern Europe. He contrasts foreign mercenary troops—whose availability enabled rulers to import troops with cutting-edge military technology, such as Swiss pikemen in the fifteenth century—with more loyal paid domestic troops, popular militias, and feudal levies. The biggest concern with mercenary troops is not a coup, but rather that they will refuse to fight and return

[18]Unlike in the baseline game, the coup term in the dictator's objective function, $G\left(\frac{\omega_D-\omega_T(\theta_M,r)}{1-\omega_T(\theta_M,r)}\right)$, does not equal the equilibrium probability of a coup attempt, as Appendix Section B.2 discusses.

home. The model provides a useful framework for studying this alternative formulation of the loyalty-efficiency tradeoff. To do so, I rename the national military as the "mercenary" army and the ethnic military as the "militia." I eliminate the possibility of either type of military staging a coup, and instead assume that the mercenary army (but not the militia) can choose to flee and consume $\gamma > 0$.[19] Appendix Figure B.3 provides the associated game tree.

The main change is that a radical threat no longer pins the mercenary army into a corner in which it effectively has no choice but to fight for the regime. Instead, the mercenary army can exercise its outside option to flee, which decreases its reliability against highly radical threats. Formally, Equation 1 characterizes the dictator's probability of survival if it chooses the militia, but γ rather than $\omega_T(\theta_M, r)$ may constrain the mercenary army's willingness to exercise repression. This implies that the mercenary army is more likely than the militia to successfully defend the dictator if and only if:

$$p(\overline{\theta}_M, \theta_T) \cdot F\Big(\omega_D - \max\big\{\omega_T(\overline{\theta}_M, r), \gamma\big\}\Big) > p(\underline{\theta}_M, \theta_T) \cdot F\Big(\omega_D - \omega_T(\underline{\theta}_M, r)\Big) \tag{11}$$

Figure 8 shows how the tradeoff between mercenary armies and militias changes relative to the baseline model. It uses the same parameter values as Figure 5 (the original efficiency figure) while additionally fixing γ. If ω_T is the binding constraint on the mercenary army's choice to exercise repression, then regions 1a* and 1b* collectively comprise the region in which the dictator's survival is higher under the mercenary army, which is identical to region 1 in dark gray in Figure 5.[20] If instead γ is the binding constraint, then the mercenary army yields a higher survival probability in regions 1a* and 2*. The intersection of these two constraints, region 1a*, provides the only parameter values in which the mercenary army is regime-enhancing, since the mercenary can either flee or transition to outsider rule rather than exercise repression.

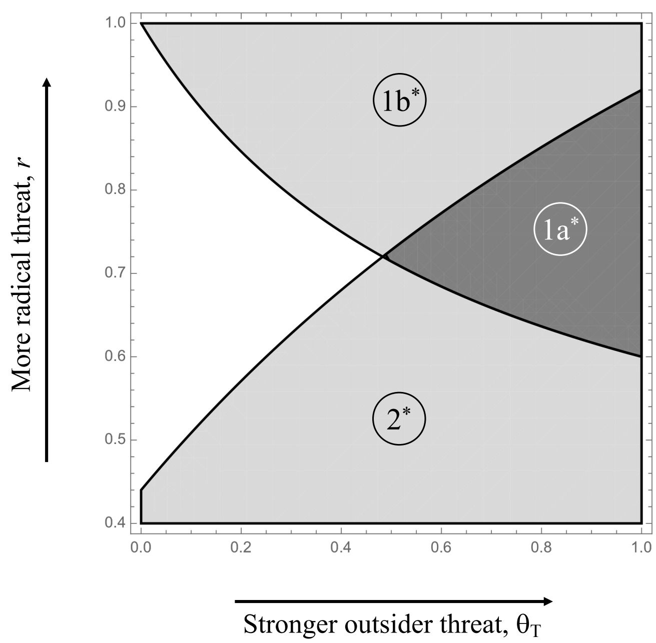
The discussion of Figure 5—in which transitioning to outsider rule constrains the military's repression choice—explains that the national military (here, mercenary army) is more efficient in equilibrium than the ethnic military (here, militia) if and only if the threat is strong and highly radical. In the baseline game, an increase in r raises the national military's probability of exercising repression relative to that of the

[19]The results would be qualitatively identical if the militia could also flee but this option was lower-valued than for the mercenary army.

[20]Figure 8 denotes these regions with asterisks to differentiate them from regions 1a and 1b in Figure 6.

ethnic military by narrowing the gap between the value of their exit option to outsider rule. However, in this extension, if the flee constraint γ binds, then the mercenary army is better than the militia if and only if the threat is strong and *non*-radical. An increase in r diminishes the mercenary army's probability of exercising repression relative to that of the militia by increasing the probability with which the mercenary flees. Combining these two constraints implies that the dictator prefers the mercenary army if and only if the threat is strong and *intermediate* radical. Appendix Proposition B.2 formalizes this logic.

Figure 8: Survival Prospects under Mercenary Army versus Militia



Notes: Figure 8 uses the same parameter values and functional form assumptions as Figure 5 and sets $\gamma = 0.08$.

7 CONCLUSION

This paper presents a model in which a dictator facing an outsider threat chooses between an ethnic and a national military, and the military can either defend the regime by exercising repression, or defy the regime by staging a coup or transitioning to outsider rule. The main results challenge two important premises regarding whether dictators trade off between military loyalty and efficiency, and the consequences of this choice for the guardianship dilemma. The primary contribution of the paper is to theoretically analyze these foundational authoritarian strategies in the context of military agency problems and heterogeneous types of outsider threats. But the analysis also helps to make sense of outcomes in many existing cases for which the traditional loyalty-efficiency logic cannot account, and generates additional important empirical implications.

Broad literatures on civil conflict, contentious politics, and authoritarian regimes analyze various challenges that dictators face. One important takeaway from the present analysis is that different types of challengers create differential incentives for rulers. Scholars such as Bellin (2012) studying urban protests emphasize the importance of ethnic militaries for defeating the threats. By contrast, scholars such as Quinlivan (1999), Powell (2014), Talmadge (2015), and Roessler (2016) that focus on counterinsurgency highlight the drawbacks of ethnic militaries as effective fighting units. The present analysis suggests that an underemphasized difference between these types of cases is how the radicalism of the outsider threat affects the military's incentives to fight, as opposed to the tactics of the opposition or the geography of rebellion per se. Nonviolent urban protests, especially successful ones, often feature broad-based and diverse membership (Chenoweth and Stephan 2011), therefore posing a non-radical threat. By contrast, violent insurgent groups are more likely to pose an existential radical threat. Focusing on the interaction between the type of military and type of threat can also explain variance within these modes of contention. Many civil wars end with negotiated pacts rather than outright defeat, but this should only be possible when facing a non-radical threat. By contrast, largely peaceful urban protests often fail when facing a military whose fate is intimately tied to that of the incumbent regime, as occurred in Burma in 1988 and China in 1989. Future empirical work could operationalize threat radicalism and assess whether these patterns generalize across a broader sample.

These considerations also highlight a dictator's perilous position if its main threat changes. For example, Egypt's international competition with Israel and regional influence created benefits from developing a nationally recruited and competent army. Yet this made the regime more vulnerable to downfall in 2011 when mass, non-radical protests broke out in Cairo and Alexandria. By contrast, many African rulers favored more ethnically oriented militaries after independence to mitigate their coup risk—therefore leaving these regimes more susceptible to facing credible outsider rebellions even amid considerable foreign aid from superpowers (Roessler 2016). However, when this aid diminished considerably after the Cold War, outsider rebellions toppled many of these regimes (e.g., Rwanda, Zaire) because the militaries lacked basic capacity—suggesting that these dictators would have benefitted from creating broader-based militaries during the Cold War period.[21]

[21] Angola and Zimbabwe, mentioned in the introduction, differed because their revolutionary roots en-

Beyond the loyalty-efficiency tradeoff, analyzing the radicalism of outsider threats also yields new implications for related research questions. To highlight some possibilities, Appendix Section B.6 alters the model to study gambling for resurrection: costly gambles that increase a ruler's probability of survival despite worsening its fate conditional on losing power, such as mass repression or initiating international wars. I alter the model to (1) allow dictators to vary in their fates under outsider rule, as opposed to consuming 0, and (2) shift the dictator's strategic focus from choosing the type of military to choosing the radicalism of the outsider threat. Although, historically, dictators have faired poorly following overthrow by extremist movements, the baseline model highlights a selection effect that explains why a dictator might prefer a *more* radical opposition: higher incentives for the military to exercise repression. This strategic effect can cause dictators with a narrow support base to gamble for resurrection by provoking extremist opposition leaders, whereas more popular dictators do not hang onto power at all costs because of their higher reservation value under non-radical outsider rule.

Overall, scrutinizing the logical relationships implied by the model and their empirical implications will hopefully spur productive future research on central questions for understanding authoritarian stability: how dictators craft their militaries, how this choice affects regime survival, and how the nature of the opposition affects dictators' broader survival strategies.

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A Proofs for Baseline Model

Table A.1: Summary of Parameters and Choice Variables

Aspect of game	Variables/description
Coercive endowments	• Dictator: θ_D, with maximum value $\overline{\theta}_D$ • Outsider threat: θ_T, with maximum value $\overline{\theta}_T$ • Military: $\underline{\theta}_M$ for ethnic and $\overline{\theta}_M$ for national
Military's utility to defending the regime	• ω_D: Military's consumption under incumbent dictator • $p(\theta_M, \theta_T)$: Probability repression is effective • μ: Military's cost of repression with maximum value $\overline{\mu}$ • $F(\cdot)$: Distribution function for repression cost, with pdf $f(\cdot)$
Military's utility to coup	• $q(\theta_M, \theta_D)$: Probability the military has a coup opportunity
Military's utility to negotiated transition	• r: Radicalism of outsider threat • $\omega_T\big(\theta_M, r\big)$: Military's consumption under outsider rule

Lemmas 1 through 3 follow trivially from the assumptions. I use the following to prove Lemma 4.

Lemma A.1. *For E^* defined in Equation 1:*

Part a.
$$\frac{d^2E^*}{d\theta_M dr} > 0$$

Part b.
$$\frac{d^2E^*}{d\theta_M d\theta_T} > 0$$

Proof. The first derivative is:

$$\frac{dE^*}{d\theta_M} = \underbrace{\overbrace{\frac{\partial p}{\partial \theta_M} \cdot F\big(\omega_D - \omega_T(\theta_M, r)\big)}^{1}}_{(+)\uparrow \text{Pr(effective repression)}} - \underbrace{\overbrace{p(\theta_M, \theta_T) \cdot f\big(\omega_D - \omega_T(\theta_M, r)\big) \cdot \frac{\partial \omega_T}{\partial \theta_M}}^{2}}_{(-)\downarrow \text{Pr(defend regime | effective rep.)}} >< 0 \tag{A.1}$$

Part a.

$$\frac{d^2E^*}{d\theta_M dr} = \underbrace{\frac{\partial p}{\partial \theta_M} \cdot f\big(\omega_D - \omega_T(\theta_M, r)\big) \cdot \left(-\frac{\partial \omega_T}{\partial r}\right)}_{(+)\uparrow \text{ magnitude of } 1 \text{ by } \uparrow \text{Pr(defend regime | effective rep.)}}$$

$$\underbrace{+p(\theta_M, \theta_T) \cdot f\big(\omega_D - \omega_T(\theta_M, r)\big) \cdot \left(-\frac{\partial^2 \omega_T}{\partial \theta_M \partial r}\right)}_{(+)\downarrow \text{ magnitude of } 2 \text{ by } \downarrow \text{ effect of } \theta_M \text{ on } \omega_T}$$

$$\underbrace{+p(\theta_M, \theta_T) \cdot \Big[-f'\big(\omega_D - \omega_T(\theta_M, r)\big)\Big] \cdot \left(-\frac{\partial \omega_T}{\partial r}\right) \cdot \frac{\partial \omega_T}{\partial \theta_M}}_{(+)\downarrow \text{ magnitude of } 2 \text{ by } \uparrow \text{Pr(defend regime | effective rep.)}} > 0$$

Part b.

$$\frac{d^2E^*}{d\theta_M d\theta_T} = \underbrace{\frac{\partial^2 p}{\partial\theta_M \partial\theta_T} \cdot F\big(\omega_D - \omega_T(\theta_M, r)\big)}_{(+)\uparrow \text{ magnitude of } \textcircled{1} \text{ by } \uparrow \text{ effect of } \theta_M \text{ on Pr(effective rep.)}}$$

$$+ \underbrace{\left(-\frac{\partial p}{\partial\theta_T}\right) \cdot f\big(\omega_D - \omega_T(\theta_M, r)\big) \cdot \frac{\partial\omega_T}{\partial\theta_M}}_{(+)\downarrow \text{ magnitude of } \textcircled{2} \text{ by } \downarrow \text{ Pr(effective rep.)}} > 0$$

■

The first derivative shows the two countervailing effects of an increase in θ_M on equilibrium repressive efficiency. Mechanism 1 is positive because higher θ_M raises the probability that the military can repress effectively (this is efficiency mechanism #1 in Table 4). Mechanism 2 is negative because higher θ_M decreases the probability that the military defends the regime conditional on effective repression, which follows from $\frac{\partial\omega_T}{\partial\theta_M} > 0$ (this is efficiency mechanism #2 in Table 4). The effects encompassed in the second derivatives are:

- **Part a.** An increase in r increases the magnitude of $\frac{dE^*}{d\theta_M}$ if that term is positive, and decreases its magnitude if it negative, through three effects:
  - Increases the magnitude of mechanism 1 by increasing the probability that the military defends the regime conditional on effective repression because $\frac{\partial\omega_T}{\partial r} < 0$.
  - Decreases the magnitude of mechanism 2 by decreasing the magnitude of the positive effect of θ_M on ω_T because $\frac{\partial^2\omega_T}{\partial\theta_M\partial r} < 0$.
  - Decreases the magnitude of mechanism 2 by increasing the probability that the military defends the regime conditional on effective repression because $\frac{\partial\omega_T}{\partial r} < 0$.
- **Part b.** An increase in θ_T increases the magnitude of $\frac{dE^*}{d\theta_M}$ if that term is positive, and decreases its magnitude if it negative, through two effects:
  - Increases the magnitude of mechanism 1 by increasing the magnitude of the positive effect of θ_M on the probability the military can repress effectively because $\frac{\partial^2 p}{\partial\theta_M\partial\theta_T} > 0$.
  - Decreases the magnitude of mechanism 2 by decreasing the probability the military can repress effectively because $\frac{\partial p}{\partial\theta_T} < 0$.

Proof of Lemma 4, part a. Part b of Lemma A.1 implies that if $E^*\big(\underline{\theta}_M, \overline{\theta}_T, r\big) > E^*\big(\overline{\theta}_M, \overline{\theta}_T, r\big)$, then this inequality holds for all $\theta_T \in \big(0, \overline{\theta}_T\big)$. Showing that the conditions for the intermediate value theorem hold establishes the existence of at least one $\tilde{r} \in (0, 1)$ as defined in Equation 2:

- $E^*\big(\underline{\theta}_M, \underline{\theta}_T, 0\big) = p\big(\underline{\theta}_M, \overline{\theta}_T\big) \cdot \underbrace{F\Big(\omega_D - \omega_T\big(\underline{\theta}_M, r\big)\Big)}_{>0} < p\big(\overline{\theta}_M, \overline{\theta}_T\big) \cdot \underbrace{F(0)}_{=0} = E^*\big(\overline{\theta}_M, \overline{\theta}_T, 0\big)$

follows from assuming $\omega_T\left(\overline{\theta}_M, 0\right) = \omega_D$.

- $E^*\left(\underline{\theta}_M, \overline{\theta}_T, 1\right) = p\left(\underline{\theta}_M, \overline{\theta}_T\right) \cdot F\left(\omega_D\right) < p\left(\overline{\theta}_M, \overline{\theta}_T\right) \cdot F\left(\omega_D\right) = E^*\left(\overline{\theta}_M, \overline{\theta}_T, 1\right)$ follows from assuming $\omega_T\left(\theta_M, 1\right) = 0$ for $\theta_M \in \left\{\underline{\theta}_M, \overline{\theta}_M\right\}$.
- Continuity trivially holds.

The unique threshold claim for $\tilde{r}$ follows from $\frac{d^2 E^*}{d\theta_M dr} > 0$ (part a of Lemma A.1).

Part b. Showing that the conditions for the intermediate value theorem hold establishes that for any $r < \tilde{r}$, then there exists at least one $\tilde{\theta}_T(r) \in \left(0, \overline{\theta}_T\right)$ as defined in Equation 3:

- $E\left(\underline{\theta}_M, 0, r\right) = F\left(\omega_D - \omega_T\left(\underline{\theta}_M, r\right)\right) > F\left(\omega_D - \omega_T\left(\overline{\theta}_M, r\right)\right) = E\left(\overline{\theta}_M, 0, r\right)$ follows from assuming $p(\theta_M, 0) = 1$ for $\theta_M \in \left\{\underline{\theta}_M, \overline{\theta}_M\right\}$.
- $E\left(\underline{\theta}_M, \overline{\theta}_T, r\right) < E\left(\overline{\theta}_M, \overline{\theta}_T, r\right)$ follows from assuming $r < \tilde{r}$ (see the proof for part a).
- Continuity trivially holds.

The unique threshold claim for $\tilde{\theta}_T$ follows from $\frac{d^2 E^*}{d\theta_M d\theta_T} > 0$ (part b of Lemma A.1). ■

The following assumption characterizes the lower bounds for the magnitude of two second derivatives mentioned in the text.

Assumption A.1. *The proof for Lemma A.2 defines the following thresholds.*

Part a.
$$-\frac{\partial^2 \omega_T}{\partial \theta_M \partial r} > \underline{\partial^2 \omega_T}$$

Part b.
$$-\frac{\partial^2 q}{\partial \theta_M \partial \theta_D} > \underline{\partial^2 q}$$

I use the following technical lemma to prove the propositions.

Lemma A.2. *For S^* defined in Equation 4:*

Part a.
$$\frac{d^2 S^*}{d\theta_M dr} > 0$$

Part b.
$$\frac{d^2 S^*}{d\theta_M d\theta_D} > 0$$

Part c.
$$\frac{d^2 S^*}{d\theta_M d\theta_T} > 0$$

Proof. The first derivative is:

$$\frac{dS^*}{d\theta_M} = \left[1 - q(\theta_M, \theta_D)\right] \cdot \underbrace{\frac{dE^*}{d\theta_M}}_{\text{(+/–) Lemma A.1}} \overbrace{\underbrace{-\frac{\partial q}{\partial \theta_M} \cdot p(\theta_M, \theta_T) \cdot F\Big(\omega_D - \omega_T(\theta_M, r)\Big)}_{\text{(–)} \uparrow \text{Pr(coup opportunity)}}}^{3} >< 0 \quad \text{(A.2)}$$

Part a.

$$\frac{d^2S^*}{d\theta_M dr} = \left[1 - q(\theta_M, \theta_D)\right] \cdot \underbrace{\frac{d^2E^*}{d\theta_M dr}}_{\text{(+) Lemma A.1}} \underbrace{-\frac{\partial q}{\partial \theta_M} \cdot p(\theta_M, \theta_T) \cdot f\Big(\omega_D - \omega_T(\theta_M, r)\Big) \cdot \Big(-\frac{\partial \omega_T}{\partial r}\Big)}_{\text{(–)} \downarrow \text{magnitude of } 3 \text{ by} \uparrow \text{Pr(defend regime | effective rep.)}} >< 0$$

Eliding the terms in parentheses, substituting in terms for $\frac{d^2E^*}{d\theta_M dr}$ from the Lemma A.1 proof shows that the overall term is strictly positive if and only if:

$$-\frac{\partial^2 \omega_T}{\partial \theta_M \partial r} > \underline{\partial^2 \omega_T} \equiv$$

$$\left\{(1-q) \cdot \left[\frac{\partial q}{\partial \theta_M} \cdot p \cdot f(\cdot) - \frac{\partial p}{\partial \theta_M} \cdot f(\cdot) + p \cdot \Big[-f'(\cdot)\Big] \cdot \frac{\partial \omega_T}{\partial \theta_M}\right]\right\} \cdot \Big(-\frac{\partial \omega_T}{\partial r}\Big) \cdot \frac{1}{(1-q) \cdot p \cdot f(\cdot)}, \quad \text{(A.3)}$$

which part a of Assumption A.1 assumes is true.

Part b.

$$\frac{d^2S^*}{d\theta_M d\theta_D} = \underbrace{\Big(-\frac{\partial q}{\partial \theta_D}\Big) \cdot \frac{dE^*}{d\theta_M}}_{\text{(+/–)} \uparrow \text{magn. of } \frac{dE^*}{d\theta_M} \text{ by} \downarrow \text{Pr(coup opp.)}} + \underbrace{\Big(-\frac{\partial^2 q}{\partial \theta_M \partial \theta_D}\Big) \cdot p(\theta_M, \theta_T) \cdot F\big(\omega_D - \omega_T(\theta_M, r)\big)}_{\text{(+)} \downarrow \text{magnitude of } 3 \text{ by} \downarrow \text{effect of } \theta_M \text{ on } q} >< 0$$

Eliding the terms in parentheses, substituting in terms for $\frac{dE^*}{d\theta_M}$ from the Lemma A.1 proof shows that the overall term is strictly positive if and only if:

$$-\frac{\partial^2 q}{\partial \theta_M \partial \theta_D} > \underline{\partial^2 q} \equiv \Big(-\frac{\partial q}{\partial \theta_D}\Big) \cdot \left[p \cdot f(\cdot) \cdot \frac{\partial \omega_T}{\partial \theta_M} - \frac{\partial p}{\partial \theta_M} \cdot F(\cdot)\right] \cdot \frac{1}{p \cdot F(\cdot)}, \quad \text{(A.4)}$$

which part b of Assumption A.1 assumes is true.

Part c.

$$\frac{d^2S^*}{d\theta_M d\theta_T} = \left[1 - q(\theta_M, \theta_D)\right] \cdot \underbrace{\frac{d^2E^*}{d\theta_M d\theta_T}}_{\text{(+) Lemma A.1}} + \underbrace{\frac{\partial q}{\partial \theta_M} \cdot \Big(-\frac{\partial p}{\partial \theta_T}\Big) \cdot F\big(\omega_D - \omega_T(\theta_M, r)\big)}_{\text{(+)} \downarrow \text{magnitude of } 3 \text{ by} \downarrow \text{Pr(effective rep.)}} > 0$$

■

Remark A.1 simplifies the complementarity thresholds from Assumption A.1 using the functional form assumptions from figures in the text. This explains the minimum value $r = 0.5$ in Figures 5 and 6.

Remark A.1 (Illustration of complementarity thresholds)**.** *Assume the following functional forms:*

- $p(\theta_M, \theta_T) = 1 - \theta_T \cdot (1 - \theta_M)$
- $\omega_T(\theta_M, r) = \left(\theta_M / \overline{\theta}_M\right) \cdot (1 - r) \cdot \omega_D$
- $\mu \sim U(0, 1 - \omega_D)$
- $q(\theta_M, \theta_D) = \left(\theta_M / \overline{\theta}_M\right) \cdot (1 - \theta_D)$

Part a. *If $\theta_D > \frac{1}{2}$, then Part a of Assumption A.1 holds for all $\theta_T \in \left(0, \overline{\theta}_T\right)$ and $\theta_M \in \left\{\underline{\theta}_M, \overline{\theta}_M\right\}$.*

Part b. *If $r > \frac{1}{2}$, then Part b of Assumption A.1 holds for all $\theta_T \in \left(0, \overline{\theta}_T\right)$ and $\theta_M \in \left\{\underline{\theta}_M, \overline{\theta}_M\right\}$.*

Proof. The following preliminary result shows that the right-hand side of Equations A.3 and A.4 reach their upper bound at $\theta_T = 0$:

$$\frac{d}{d\theta_T}\left[-\frac{\partial p}{\partial \theta_M} \cdot \frac{1}{p(\theta_M, \theta_T)}\right] = -\left[\frac{\partial^2 p}{\partial \theta_M \partial \theta_T} \cdot \frac{1}{p} + \frac{\partial p}{\partial \theta_M} \cdot \frac{-\frac{\partial p}{\partial \theta_T}}{p^2}\right] < 0$$

Therefore, if the inequalities hold at $\theta_T = 0$, then they hold for all $\theta_T \in \left(0, \overline{\theta}_T\right)$.

Part a. Substituting the functional form assumptions and $\theta_T = 0$ into Equation A.3 yields:

$$\frac{\omega_D}{\overline{\theta}_M} > \frac{1 - \theta_D}{\overline{\theta}_M} \cdot \frac{1}{1 - \frac{\theta_M}{\overline{\theta}_M} \cdot (1 - \theta_D)} \cdot \frac{\theta_M}{\overline{\theta}_M} \cdot \omega_D,$$

which simplifies to:

$$\theta_D > 1 - \frac{1}{2} \cdot \frac{\overline{\theta}_M}{\theta_M}$$

Because the right-hand side achieves its upper bound at $\theta_M = \overline{\theta}_M$, substituting in $\theta_M = \overline{\theta}_M$ yields the claim.

Part b. Substituting the functional form assumptions and $\theta_T = 0$ into Equation A.4 yields:

$$\frac{1}{\overline{\theta}_M} > \frac{1}{\omega_D - \frac{\theta_M}{\overline{\theta}_M} \cdot (1 - r) \cdot \omega_D} \cdot \frac{1}{\overline{\theta}_M} \cdot (1 - r) \cdot \omega_D \cdot \frac{\theta_M}{\overline{\theta}_M},$$

which simplifies to:

$$r > 1 - \frac{1}{2} \cdot \frac{\overline{\theta}_M}{\theta_M}$$

Because the right-hand side achieves its upper bound at $\theta_M = \overline{\theta}_M$, substituting in $\theta_M = \overline{\theta}_M$ yields the claim. ∎

Proof of Proposition 1, part a. Parts a and c of Lemma A.2 imply that if $S^*\big(\underline{\theta}_M, \overline{\theta}_T, 1, \theta_D\big) > S^*\big(\overline{\theta}_M, \overline{\theta}_T, 1, \theta_D\big)$, then this inequality holds for all $\theta_T \in \big(0, \overline{\theta}_T\big)$ and $r \in \big(0, 1\big)$. Showing that the conditions for the intermediate value theorem hold establishes the existence of at least one $\tilde{\theta}'_D \in \big(0, \overline{\theta}_D\big)$ that satisfies Equation 5:

- If $\theta_D = 0$, then $q\big(\underline{\theta}_M, 0\big) < q\big(\overline{\theta}_M, 0\big) = 1$, which implies $S^*\big(\underline{\theta}_M, \overline{\theta}_T, 1, 0\big) > S^*\big(\overline{\theta}_M, \overline{\theta}_T, 1, 0\big) = 0$.
- If $\theta_D = \overline{\theta}_D$, then $q\big(\underline{\theta}_M, \overline{\theta}_D\big) = q\big(\overline{\theta}_M, \overline{\theta}_D\big) = 0$. This implies that $S^*\big(\underline{\theta}_M, \overline{\theta}_T, 1, \overline{\theta}_D\big) = E^*\big(\underline{\theta}_M, \overline{\theta}_T, 1\big)$ and $S^*\big(\overline{\theta}_M, \overline{\theta}_T, 1, \overline{\theta}_D\big) = E^*\big(\overline{\theta}_M, \overline{\theta}_T, 1\big)$. The proof for part b of Lemma 4 shows that $E^*\big(\underline{\theta}_M, \overline{\theta}_T, 1\big) < E^*\big(\overline{\theta}_M, \overline{\theta}_T, 1\big)$.
- Continuity is trivially satisfied.

The unique threshold claim for $\tilde{\theta}'_D$ follows from $\frac{d^2 S^*}{d\theta_M d\theta_D} > 0$, which part b of Lemma A.2 establishes.

Part b. Part c of Lemma A.2 implies that if $S^*\big(\underline{\theta}_M, \overline{\theta}_T, r, \theta_D\big) > S^*\big(\overline{\theta}_M, \overline{\theta}_T, r, \theta_D\big)$, then this inequality holds for all $\theta_T \in \big(0, \overline{\theta}_T\big)$. Showing that the conditions for the intermediate value theorem hold establishes that if $\theta_D > \tilde{\theta}'_D$, then there exists at least one $\tilde{r}' \in \big(\tilde{r}, 1\big)$ that satisfies Equation 6:

- $S^*\big(\underline{\theta}_M, \overline{\theta}_T, \tilde{r}, \theta_D\big) > S^*\big(\overline{\theta}_M, \overline{\theta}_T, \tilde{r}, \theta_D\big)$ simplifies to $q\big(\overline{\theta}_M, \theta_D\big) > q\big(\underline{\theta}_M, \theta_D\big)$, which follows from Equation 2.
- $S^*\big(\underline{\theta}_M, \overline{\theta}_T, 1, \theta_D\big) < S^*\big(\overline{\theta}_M, \overline{\theta}_T, 1, \theta_D\big)$ follows from assuming $\theta_D > \tilde{\theta}'_D$.
- Continuity trivially holds.

The unique threshold claim for $\tilde{r}'$ follows from $\frac{d^2 S^*}{d\theta_M dr} > 0$, which part a of Lemma A.2 establishes.

Part c. Showing that the conditions for the intermediate value theorem hold establishes that if $\theta_D > \tilde{\theta}'_D$ and $r > \tilde{r}'$, then there exists at least one $\tilde{\theta}'_T \in \big(\tilde{\theta}_T, \overline{\theta}_T\big)$ that satisfies Equation 7:

- $S^*\big(\underline{\theta}_M, \tilde{\theta}_T, r, \theta_D\big) > S^*\big(\overline{\theta}_M, \tilde{\theta}_T, r, \theta_D\big)$ simplifies to $q\big(\overline{\theta}_M, \theta_D\big) > q\big(\underline{\theta}_M, \theta_D\big)$, which follows from Equation 3.
- $S^*\big(\underline{\theta}_M, \overline{\theta}_T, r, \theta_D\big) < S^*\big(\overline{\theta}_M, \overline{\theta}_T, r, \theta_D\big)$ follows from assuming $\theta_D > \tilde{\theta}'_D$ and $r > \tilde{r}'$.
- Continuity trivially holds.

The unique threshold claim for $\tilde{\theta}'_T$ follows from $\frac{d^2 S^*}{d\theta_M d\theta_T} > 0$, which part c of Lemma A.2 establishes. ■

Proof of Proposition 2. The equilibrium probability of a coup is:

$$Pr(coup) = \begin{cases} q\big(\underline{\theta}_M, \theta_D\big) \cdot p\big(\underline{\theta}_M, \theta_T\big) & \text{if } \theta_T < \tilde{\theta}'_T \\ q\big(\overline{\theta}_M, \theta_D\big) \cdot p\big(\overline{\theta}_M, \theta_T\big) & \text{if } \theta_T > \tilde{\theta}'_T \end{cases}$$

Assuming $\frac{\partial p}{\partial \theta_T} < 0$ implies that this function strictly decreases at all $\theta_T \in \big(0, \tilde{\theta}'_T\big) \cup \big(\tilde{\theta}'_T, \overline{\theta}_T\big)$. Lemma 2 implies that the function exhibits a discrete increase at $\theta_T = \tilde{\theta}'_T$. ■

Proof of Proposition 4, part a. Equation 8 simplifies to $q\big(\theta_M, \theta_D\big) \cdot p\big(\theta_M, \theta_T\big) \cdot F\Big(\omega_D - \omega_T\big(\theta_M, r\big)\Big)$. By assumption, $\frac{\partial q}{\partial \theta_D} < 0$ and $\lim\limits_{\theta_D \to \overline{\theta}_D} q\big(\theta_M, \theta_D\big) = 0$, which establishes the claim.

Part b. Follows because the dictator chooses the ethnic military if $E^*\big(\underline{\theta}_M, \theta_T, r\big) > E^*\big(\overline{\theta}_M, \theta_T, r\big)$, and θ_D does not affect this inequality. ■

B Extensions

B.1 Inherent Loyalty

Figure B.1: Game Tree for Inherent Loyalty Extension

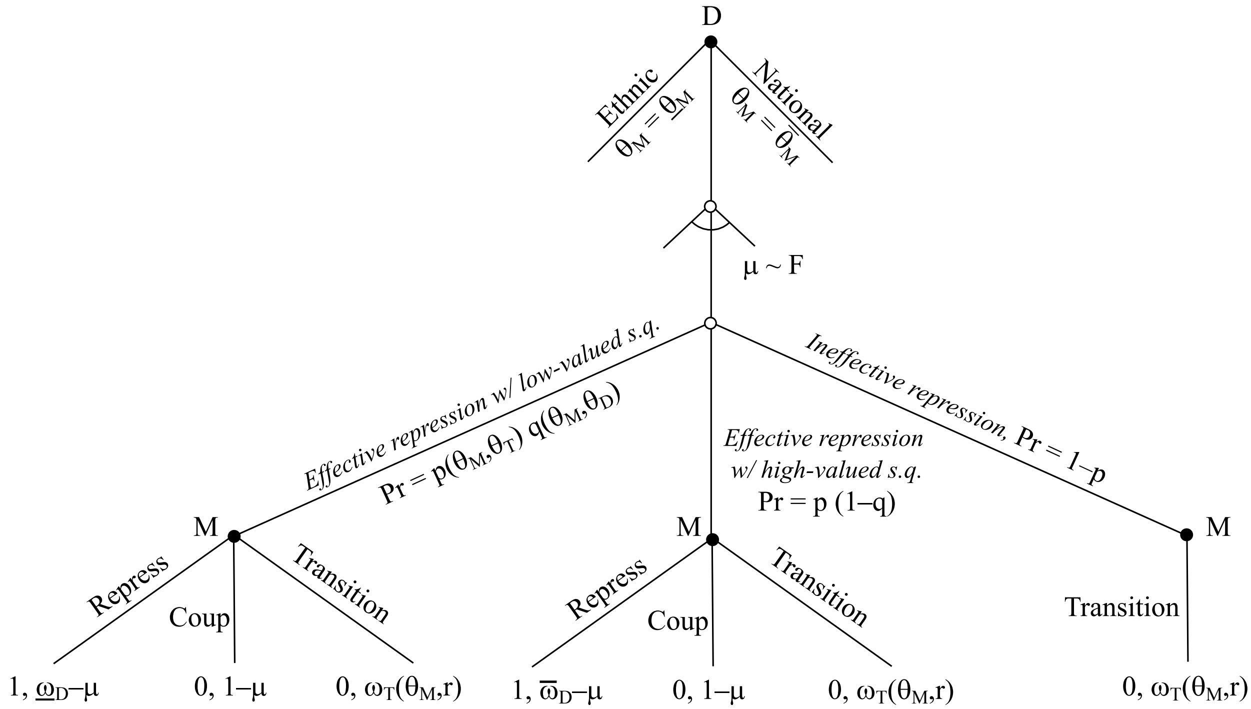


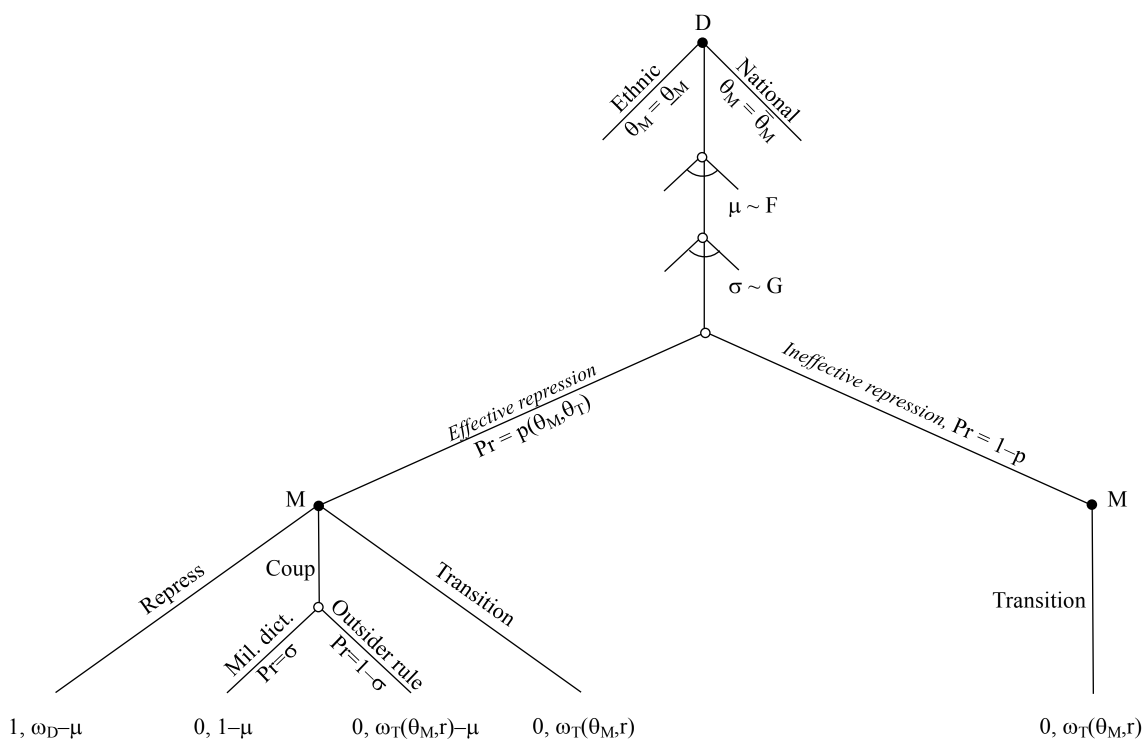
Table B.1 is identical to Table 3 except it changes the description of the columns to correspond with the terminology from the inherent loyalty extension.

Table B.1: Military's Optimal Choice with Inherent Loyalty

Effective repression with low-valued s.q. Pr=$p(\theta_M, \theta_T) \cdot q(\theta_M, \theta_D)$	**Effective repression with high-valued s.q.** Pr=$p \cdot (1-q)$	**Ineffective repression** Pr=$1-p$
Coup	Repress if μ is low Transition if μ is high	Transition

B.2 Strategic Loyalty

Figure B.2: Game Tree for Strategic Loyalty Extension



Proposition B.1 formalizes the discussion from the text regarding the conditions under which the military prefers defending the regime over attempting a coup.

> **Proposition B.1** (Strategic loyalty mechanism)**.** *The coup component of the dictator's survival objective function,* $G\left(\frac{\omega_D - \omega_T(\theta_M, r)}{1 - \omega_T(\theta_M, r)}\right)$ *in Equation 10, strictly decreases in* θ_M *through the effect of* θ_M *on* ω_T*.*

Proof. Expressing $G\left(\frac{\omega_D - \omega_T(\theta_M, r)}{1 - \omega_T(\theta_M, r)}\right)$ as G:

$$\frac{dG}{d\theta_M} = \frac{\partial G}{\partial \theta_M} + \frac{\partial G}{\partial \omega_T} \cdot \frac{d\omega_T}{d\theta_M}$$

$$\frac{\partial G}{\partial \theta_M} = 0$$

$$\frac{\partial G}{\partial \omega_T} = -\frac{1 - \omega_D}{(1 - \omega_T)^2}$$

$$\frac{d\omega_T}{d\theta_M} > 0 \text{ by assumption}$$

This implies that:

$$\frac{\partial G}{\partial \omega_T} \cdot \frac{d\omega_T}{d\theta_M} < 0$$

■

Unlike in the baseline setup, the coup term in the dictator's objective function for the strategic loyalty setup, $G\left(\frac{\omega_D - \omega_T(\theta_M, r)}{1-\omega_T(\theta_M, r)}\right)$, does not equal the equilibrium probability of a coup attempt. For low enough σ, the military prefers a negotiated transition to a coup for any draw of μ. This creates the possibility that the military prefers negotiated transition to a coup for parameter values in which the military strictly prefers a coup to repression, which is not possible in the baseline model. This consideration is irrelevant for the dictator's objective function—conditional on the military choosing not to repress, its coup/transition choice does not affect the dictator's consumption—but does affect the equilibrium probability of a coup attempt. Instead, this probability equals:

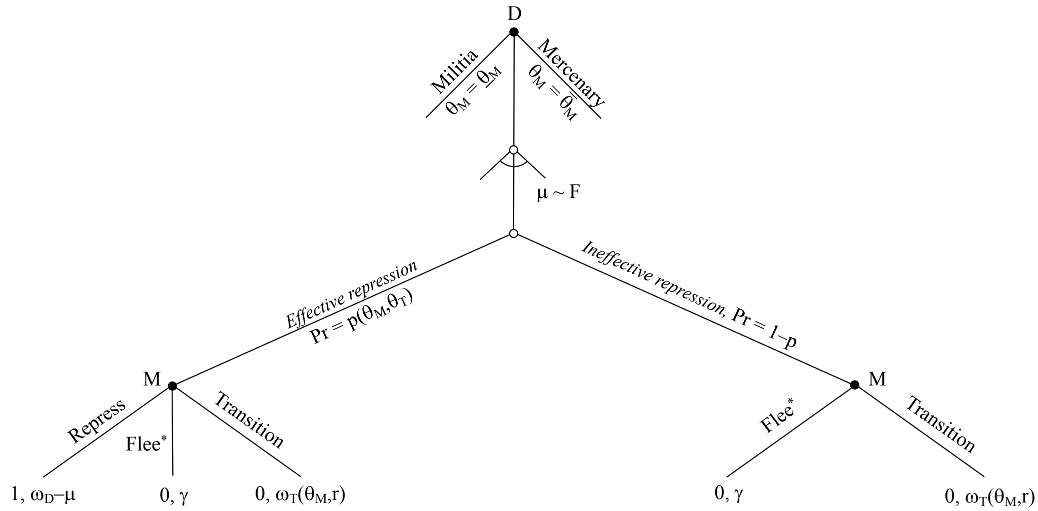
$$\underbrace{\int_0^{\omega_D - \omega_T} \overbrace{\int_{\frac{\omega_D - \omega_T}{1-\omega_T}}^1 dG(\sigma)}^{\text{Prefers coup to repression}} \cdot dF(\mu)}_{\text{Prefers repression to transition}} + \underbrace{\int_{\omega_D - \omega_T}^{\overline{\mu}} \overbrace{\int_{\frac{\mu}{1-\omega_T}}^1 dG(\sigma)}^{\text{Prefers coup to transition}} \cdot dF(\mu)}_{\text{Prefers transition to repression}}$$

The outer integral for each term expresses the probability that the military prefers repression to transitioning or vice versa, and the inner integral expresses the probability that the military prefers a coup to the more-preferred alternative. The range of the outer integrals is the same as in the baseline model: the military prefers repression to transition if $\mu \in \big(0, \omega_D - \omega_T\big)$, and prefers transition to repression if $\mu \in \big(\omega_D - \omega_T, \overline{\mu}\big)$. The range of the inner integral in the first term expresses that the military prefers a coup over repression if $\sigma \in \big(\frac{\omega_D - \omega_T}{1-\omega_T}, 1\big)$, which follows from the discussion in the text. The range of the inner integral in the second term expresses that the military prefers a coup over transition if $\sigma \in \big(\frac{\mu}{1-\omega_T}, 1\big)$, which follows from solving $\sigma + (1-\sigma) \cdot \omega_T - \mu > \omega_T$ for μ. The entire expression simplifies to:

$$\left[1 - G\left(\frac{\omega_D - \omega_T}{1-\omega_T}\right)\right] \cdot F\big(\omega_D - \omega_T\big) + \int_{\omega_D - \omega_T}^{\overline{\mu}} \left[1 - G\left(\frac{\mu}{1-\omega_T}\right)\right] \cdot dF(\mu)$$

B.3 REPRESSIVE LOYALTY OF MERCENARIES

Figure B.3: Game Tree for Mercenary Loyalty Extension



*Only the mercenary army can exercise the flee option.

Lemma B.1 (Military choice if flee constraint binds)**.** *Assume the mercenary army consumes γ if it does not repress the threat. There exist unique thresholds $\tilde{\gamma}^\dagger \in \big(0, \omega_D\big)$, $\tilde{r}^\dagger > 0$, and $\tilde{\theta}_T^\dagger \in \big(0, \overline{\theta}_T\big)$ with the following properties:*

Part a. High-valued flee option. *If $\gamma > \tilde{\gamma}^\dagger$, then the dictator chooses the militia.*

Part b. Radical threat. *If $\gamma < \tilde{\gamma}^\dagger$ and $r > \tilde{r}^\dagger$, then the dictator chooses the militia.*

Part c. Non-radical threat. *If $\gamma < \tilde{\gamma}^\dagger$ and $r < \tilde{r}^\dagger$, then:*

- *If $\theta_T < \tilde{\theta}_T^\dagger$, then the dictator chooses the militia.*
- *If $\theta_T > \tilde{\theta}_T^\dagger$, then the dictator chooses the mercenary army.*

Proof of part a. Implicitly define $\tilde{\gamma}^\dagger$ as:

$$p\big(\overline{\theta}_M, \overline{\theta}_T\big) \cdot F\big(\omega_D - \tilde{\gamma}^\dagger\big) = p\big(\underline{\theta}_M, \overline{\theta}_T\big) \cdot F\Big(\omega_D - \omega_T\big(\underline{\theta}_M, 0\big)\Big) \tag{B.1}$$

It is straightforward to establish (1) if $\gamma = 0$, then the left-hand side exceeds the right-hand side, (2) if $\gamma = \omega_D$, then the right-hand side exceeds the left-hand side, and (3) the left-hand side is continuous and strictly decreases in γ. The right-hand side achieves its lower bound at $r = 0$, which implies that the claim holds for all $r > 0$.

Part b. Implicitly define $\tilde{r}^\dagger$ as:

$$p\big(\overline{\theta}_M, \overline{\theta}_T\big) \cdot F\big(\omega_D - \gamma\big) = p\big(\underline{\theta}_M, \overline{\theta}_T\big) \cdot F\Big(\omega_D - \omega_T\big(\underline{\theta}_M, \tilde{r}^\dagger\big)\Big) \tag{B.2}$$

It is straightforward to establish (1) if $r = 0$, then $\gamma < \tilde{\gamma}^\dagger$ implies that the left-hand side exceeds the right-hand side and (2) the right-hand side is continuous and strictly increases in r. A decrease in θ_T

strictly increases the right-hand side relative to the left-hand side, which implies that if the claim holds at $\theta_T = \overline{\theta}_T$, then it holds for all $\theta_T < \overline{\theta}_T$.

Part c. Implicitly define $\theta_T = \tilde{\theta}_T^{\dagger}$ as:

$$p(\overline{\theta}_M, \tilde{\theta}_T^{\dagger}) \cdot F(\omega_D - \gamma) = p(\underline{\theta}_M, \tilde{\theta}_T^{\dagger}) \cdot F\Big(\omega_D - \omega_T(\underline{\theta}_M, r)\Big)$$

If $\theta_T = 0$, then the right-hand side exceeds the left-hand side. This follows because if γ is the binding constraint for the mercenary army, then $\gamma > \omega_T(\overline{\theta}_M, r)$, and in turn we have $\omega_T(\overline{\theta}_M, r) > \omega_T(\underline{\theta}_M, r)$. If $\theta_T = \overline{\theta}_T$, then the left-hand side exceeds the right-hand side because $\gamma < \tilde{\gamma}^{\dagger}$ and $r < \tilde{r}^{\dagger}$. It is straightforward to establish that both sides are continuous in θ_T and that increases in θ_T increase the left-hand side relative to the right-hand side. ■

Proposition B.2 (Mercenary army or militia?)**.**

Part a. *The conditions in both Lemma 4 and Lemma B.1 must hold for the dictator to choose the mercenary army: $r > \tilde{r}$ and $\theta_T > \tilde{\theta}_T$; and $\gamma < \tilde{\gamma}^{\dagger}$, $r < \tilde{r}^{\dagger}$ and $\theta_T > \tilde{\theta}_T^{\dagger}$.*

Part b. *There exists a unique $\tilde{\gamma}^{\dagger\dagger} \in (0, \tilde{\gamma}^{\dagger})$ such that this set is non-empty if and only if $\gamma < \tilde{\gamma}^{\dagger\dagger}$.*

Proof of part a. Equation 11 states the conditions under which the mercenary army is more efficient than the militia. This inequality is equivalent to:

$$\min\left\{p(\overline{\theta}_M, \theta_T) \cdot F\Big(\omega_D - \omega_T(\overline{\theta}_M, r)\Big), p(\overline{\theta}_M, \theta_T) \cdot F(\omega_D - \gamma)\right\} >$$

$$p(\underline{\theta}_M, \theta_T) \cdot F\Big(\omega_D - \omega_T(\underline{\theta}_M, r)\Big)$$

The first term in the min function is the mercenary army's efficiency if ω_T is the binding constraint, and the second term is the mercenary army's efficiency if γ is the binding constraint, which implies that all the conditions in both lemmas must hold for the dictator to choose the mercenary army.

Part b. The set is non-empty if and only if $\tilde{r} < \tilde{r}^{\dagger}$. It is useful to restate the implicit definition of $\tilde{r}$ from Equation 2 and of $\tilde{r}^{\dagger}$ from Equation B.2:

$$\underbrace{p(\overline{\theta}_M, \overline{\theta}_T) \cdot F\Big(\omega_D - \omega_T(\overline{\theta}_M, \tilde{r})\Big)}_{1} = \underbrace{p(\underline{\theta}_M, \overline{\theta}_T) \cdot F\Big(\omega_D - \omega_T(\underline{\theta}_M, \tilde{r})\Big)}_{2}$$

$$\underbrace{p(\overline{\theta}_M, \overline{\theta}_T) \cdot F(\omega_D - \gamma)}_{3} = \underbrace{p(\underline{\theta}_M, \overline{\theta}_T) \cdot F\Big(\omega_D - \omega_T(\underline{\theta}_M, \tilde{r}^{\dagger})\Big)}_{4}$$

We can implicitly define $\tilde{r}^{\dagger}(\tilde{\gamma}^{\dagger\dagger}) = \tilde{r}$. To establish the bounds, if $\gamma = 0$, then 3 > 1, which implies 4 > 2, and therefore $\tilde{r}^{\dagger} > \tilde{r}$. If $\gamma = \tilde{\gamma}^{\dagger}$, then Equation B.1 implies that $\tilde{r}^{\dagger} = 0$, and therefore $\tilde{r}^{\dagger} < \tilde{r}$. Finally, 3 is continuous and strictly decreasing in γ. ■

B.4 Probabilistic Repression Success

To simplify the exposition, the baseline model assumes that the military knows whether it was effective at repression (wins with probability 1) or ineffective (wins with probability 0) at its information set. If instead the military does not know the outcome of the Nature draw at its information set, then one aspect of the military's calculus changes. Specifically, if the military defends the regime, then with probability $p(\theta_M, \theta_T)$ it succeeds and the military consumes $\omega_D - \mu$, and with complementary probability repression fails and the military consumes $-\mu$. To highlight the main difference that arises with this alteration, here I focus only on repressive efficiency (i.e., examining the military's choice between repression and negotiated transition), which now equals:

$$E_p^*\big(\theta_M, \theta_T, r\big) \equiv \underbrace{p(\theta_M, \theta_T)}_{\text{Pr(repression succeeds)}} \cdot \underbrace{F\Big(p(\theta_M, \theta_T) \cdot \omega_D - \omega_T\big(\theta_M, r\big)\Big)}_{\text{Pr(defend regime)}}, \tag{B.3}$$

where the subscript p in E_p^* expresses probabilistic repression success. Equation B.3 differs from Equation 1 in one way, highlighted in blue: the possibility that repression can fail affects the equilibrium probability with which the military defends the regime. This term now equals $F\big(p(\theta_M, \theta_T) \cdot \omega_D - \omega_T\big(\theta_M, r\big)\big)$, as opposed to $F\big(\omega_D - \omega_T(\theta_M, r)\big)$ in the baseline model.

The key difference in this extension is that, for some parameter values, the national military is more likely to choose to repress. The higher probability of success increases its expected utility to repressing relative to the ethnic military. By contrast, in the baseline model, the ethnic military was more likely to defend the regime—conditional on Nature drawing effective repression—because its reservation value to outsider rule is lower. Formally, for some parameter values, we have

$$F\big(p(\overline{\theta}_M, \theta_T) \cdot \omega_D - \omega_T(\overline{\theta}_M, r)\big) > F\big(p(\underline{\theta}_M, \theta_T) \cdot \omega_D - \omega_T(\underline{\theta}_M, r)\big).$$

Despite this difference, the overall logic is similar to that in the baseline model. Recovering an analogy for Lemma 4 requires imposing sufficient assumptions for an analogy of Lemma A.1 to hold. Specifically, if Assumption B.1 holds, then the magnitude of the cross-partials is large enough that the direct effects that yield Lemma 4 exceed in magnitude the indirect effects created by assuming that the military is uncertain about repression success. Lemma B.2, the analogy for Lemma A.1, proves this result. Note that the threshold in part a of Assumption B.1 differs from the corresponding threshold in part a of Assumption A.1, and that the baseline model does not require an assumption about the magnitude of the cross-partial for $p(\cdot)$.

Assumption B.1. *The proof for Lemma B.2 defines the following thresholds.*

Part a.
$$-\frac{\partial^2 \omega_T}{\partial \theta_M \partial r} > \overline{\partial^2 \omega_T}$$

Part b.
$$-\frac{\partial^2 p}{\partial \theta_M \partial \theta_T} > \overline{\partial^2 p}$$

Lemma B.2. *For E_p^* defined in Equation B.3:*

Part a.
$$\frac{d^2 E_p^*}{d\theta_M dr} > 0$$

Part b.
$$\frac{d^2 E_p^*}{d\theta_M d\theta_T} > 0$$

Proof. The structure of the proof is identical to that for Lemma A.1. The terms in blue are the additional terms that arise from probabilistic repression success. The first derivative is:

$$\frac{dE_p^*}{d\theta_M} = \overbrace{\underbrace{\frac{\partial p}{\partial \theta_M} \cdot F\big(p(\theta_M,\theta_T) \cdot \omega_D - \omega_T(\theta_M, r)\big)}_{(+)\uparrow \text{Pr(repression succeeds)}}}^{1}$$

$$\overbrace{\underbrace{-p(\theta_M,\theta_T) \cdot f\big(p(\theta_M,\theta_T) \cdot \omega_D - \omega_T(\theta_M, r)\big) \cdot \left[\frac{\partial \omega_T}{\partial \theta_M} - \frac{\partial p}{\partial \theta_M} \cdot \omega_D\right]}_{(-)\downarrow \text{Pr(defend regime) if term in brackets} > 0}}^{2} >< 0$$

Part a.

$$\frac{d^2 E_p^*}{d\theta_M dr} = \underbrace{\frac{\partial p}{\partial \theta_M} \cdot f\big(p(\theta_M,\theta_T) \cdot \omega_D - \omega_T(\theta_M, r)\big) \cdot \left(-\frac{\partial \omega_T}{\partial r}\right)}_{(+)\uparrow \text{ magnitude of } 1 \text{ by } \uparrow \text{Pr(defend regime)}}$$

$$\underbrace{+p(\theta_M,\theta_T) \cdot f\big(p(\theta_M,\theta_T) \cdot \omega_D - \omega_T(\theta_M, r)\big) \cdot \left(-\frac{\partial^2 \omega_T}{\partial \theta_M \partial r}\right)}_{(+)\downarrow \text{ magnitude of } 2 \text{ by } \downarrow \text{ effect of } \theta_M \text{ on } \omega_T}$$

$$\underbrace{+p(\theta_M,\theta_T) \cdot \left[-f'\big(p(\theta_M,\theta_T) \cdot \omega_D - \omega_T(\theta_M, r)\big)\right] \cdot \left(-\frac{\partial \omega_T}{\partial r}\right) \cdot \left[\frac{\partial \omega_T}{\partial \theta_M} - \frac{\partial p}{\partial \theta_M} \cdot \omega_D\right]}_{(+)\downarrow \text{ magnitude of } 2 \text{ by } \uparrow \text{Pr(defend regime) if term in brackets} > 0} >< 0$$

Eliding the terms in parentheses, the overall term is strictly positive if and only if:

$$-\frac{\partial^2 \omega_T}{\partial \theta_M \partial r} > \overline{\partial^2 \omega_T} \equiv \left\{\frac{\partial p}{\partial \theta_M} \cdot f(\cdot) \cdot \left(-\frac{\partial \omega_T}{\partial r}\right) + p \cdot \left[-f'(\cdot)\right] \cdot \left(-\frac{\partial \omega_T}{\partial r}\right) \cdot \left[\frac{\partial \omega_T}{\partial \theta_M} - \frac{\partial p}{\partial \theta_M} \cdot \omega_D\right]\right\} \cdot \frac{1}{p \cdot f(\cdot)},$$

which part a of Assumption B.1 assumes is true.

Part b.

$$\frac{d^2 E_p^*}{d\theta_M d\theta_T} = \underbrace{\frac{\partial^2 p}{\partial \theta_M \partial \theta_T} \cdot F\big(p(\theta_M,\theta_T) \cdot \omega_D - \omega_T(\theta_M, r)\big)}_{(+)\uparrow \text{ magnitude of } 1 \text{ by } \uparrow \text{ effect of } \theta_M \text{ on Pr(repression succeeds)}}$$

$$+\underbrace{\left(-\frac{\partial p}{\partial \theta_T}\right) \cdot f\big(p(\theta_M,\theta_T) \cdot \omega_D - \omega_T(\theta_M, r)\big) \cdot \left[\frac{\partial \omega_T}{\partial \theta_M} - \frac{\partial p}{\partial \theta_M} \cdot \omega_D\right]}_{(+)\downarrow \text{ magnitude of } 2 \text{ by } \downarrow \text{Pr(repression succeeds) if term in brackets} > 0}$$

$$\underbrace{-\frac{\partial p}{\partial \theta_M} \cdot f\big(p(\theta_M,\theta_T) \cdot \omega_D - \omega_T(\theta_M, r)\big) \cdot \left(-\frac{\partial p}{\partial \theta_T}\right) \cdot \omega_D}_{(-)\downarrow \text{ magnitude of } 1 \text{ b/c diminish Pr(defend)}}$$

$$
\underbrace{+p(\theta_M,\theta_T)\cdot f\big(p(\theta_M,\theta_T)\cdot\omega_D-\omega_T(\theta_M,r)\big)\cdot\frac{\partial^2 p}{\partial\theta_M\partial\theta_T}\cdot\omega_D}_{(+)\downarrow\text{ magnitude of }2\text{ because }\uparrow\text{ effect of }\theta_M\text{ on Pr(defend)}}
$$

$$
\underbrace{-p(\theta_M,\theta_T)\cdot\Big[-f'\big(p(\theta_M,\theta_T)\cdot\omega_D-\omega_T(\theta_M,r)\big)\Big]\cdot\Big(-\frac{\partial p}{\partial\theta_T}\Big)\cdot\omega_D\cdot\Big[\frac{\partial\omega_T}{\partial\theta_M}-\frac{\partial p}{\partial\theta_M}\cdot\omega_D\Big]}_{(-)\uparrow\text{ magnitude of }2\text{ by }\uparrow\text{ Pr(defend regime) if term in brackets}>0}><0
$$

Eliding the terms in parentheses, the overall term is strictly positive if and only if:

$$
\frac{\partial^2 p}{\partial\theta_M\partial\theta_T}>\overline{\partial^2 p}\equiv
$$

$$
\left\{\Big(-\frac{\partial p}{\partial\theta_T}\Big)\cdot\Big[\frac{\partial\omega_T}{\partial\theta_M}-\frac{\partial p}{\partial\theta_M}\cdot\omega_D\Big]\cdot\Big[f(\cdot)-p\cdot[-f'(\cdot)]\cdot\omega_D\Big]-\frac{\partial p}{\partial\theta_M}\cdot f(\cdot)\cdot\Big(-\frac{\partial p}{\partial\theta_T}\Big)\cdot\omega_D\Big]\right\}\cdot\frac{1}{F(\cdot)+p\cdot f(\cdot)\cdot\omega_D},
$$

which part b of Assumption B.1 assumes is true. ■

How do the expressions in the proof for Lemma A.2 differ from those in the proof for Lemma A.1? To explain the differences, the following copy and pastes the text that follows the proof of Lemma A.1 in black, and the blue text comments on the differences in Lemma A.2. To avoid repetition, I do not comment on the change that arises within every $F(\cdot)$, $f(\cdot)$, and $f'(\cdot)$ term because each ω_D term is multiplied by $p\big(\theta_M,\theta_T\big)$ in this extension.

The first derivative shows the two countervailing effects of an increase in θ_M on equilibrium repressive efficiency. Mechanism 1 is positive because higher θ_M raises the probability that the military can repress effectively. This term is unchanged, although is now phrased as the probability that the military succeeds at repression. Mechanism 2 is negative because higher θ_M decreases the probability that the military defends the regime conditional on effective repression, which follows from $\frac{\partial\omega_T}{\partial\theta_M}>0$. This mechanism no longer requires the qualifying statement about effective repression. More important, this mechanism is not necessarily negative because of an additional effect of θ_M on the military's probability of defending the regime: θ_M increases the probability that repression succeeds, which increases the military's incentive to defend the regime. Mechanism 2 is negative if and only if:

$$
\frac{\partial\omega_T}{\partial\theta_M}>\frac{\partial p}{\partial\theta_M}\cdot\omega_D. \tag{B.4}
$$

If Equation B.4 does not hold, then the national military's higher probability of winning dominates the ethnic military's lower value to outsider rule to yield a higher probability of defending the incumbent for national militaries.

The effects encompassed in the second derivatives are:

- **Part a.** An increase in r increases the magnitude of $\frac{dE^*}{d\theta_M}$ if that term is positive, and decreases its magnitude if it negative, through three effects:
  - Increases the magnitude of mechanism 1 by increasing the probability that the military defends the regime conditional on effective repression because $\frac{\partial\omega_T}{\partial r}<0$. The sign of this term is

unchanged, although does not require the phrase about effective repression.

- Decreases the magnitude of mechanism (2) by decreasing the magnitude of the positive effect of θ_M on ω_T because $\frac{\partial^2 \omega_T}{\partial \theta_M \partial r} < 0$. Unchanged.

- Decreases the magnitude of mechanism (2) by increasing the probability that the military defends the regime conditional on effective repression because $\frac{\partial \omega_T}{\partial r} < 0$. This mechanism no longer requires the qualifying statement about effective repression. More important, this effect is positive if Equation B.4 holds, and negative otherwise. In the latter case, $-\frac{\partial^2 \omega_T}{\partial \theta_M \partial r}$ must be large enough in magnitude for the overall derivative in part a to be positive.

  NB: An alternative sufficient condition for part a, which does not require an assumption about the magnitude of $-\frac{\partial^2 \omega_T}{\partial \theta_M \partial r}$, is $f'(\cdot) = 0$. The uniform distribution satisfies this assumption.

- **Part b.** An increase in θ_T increases the magnitude of $\frac{dE^*}{d\theta_M}$ if that term is positive, and decreases its magnitude if it negative, through two effects:

  - Increases the magnitude of mechanism (1) by increasing the magnitude of the positive effect of θ_M on the probability the military can repress effectively because $\frac{\partial^2 p}{\partial \theta_M \partial \theta_T} > 0$. This term is unchanged, although is now phrased as the probability that the military succeeds at repression.

  - Decreases the magnitude of mechanism (2) by decreasing the probability the military can repress effectively because $\frac{\partial p}{\partial \theta_T} < 0$. This mechanism no longer requires the qualifying statement about effective repression. More important, this effect is positive if Equation B.4 holds and negative otherwise. In the latter case, $\frac{\partial^2 p}{\partial \theta_M \partial \theta_T}$ must be large enough in magnitude for the overall derivative in part b to be positive. The intuition for the countervailing effect is as follows. If Equation B.4 does not hold, then mechanism (2) is positive. In this case, a decrease in the probability that repression succeeds caused by higher θ_T diminishes the magnitude of a positive effect on repressive efficiency, hence the negative sign.

  - Three additional expressions (lines 3 through 5 in the proof for part b) arise from assuming probabilistic repression success.

B.5 Continuous Military Choice

Although the main results are easier to express if the dictator's military choice is binary, Proposition B.3 presents a set of straightforward sufficient conditions for the dictator's objective function in Equation 4 to yield a unique optimal solution if it chooses $\theta_M \in \left[0, \overline{\theta}_M\right]$, with the same boundary conditions applying to $\overline{\theta}_M$ as stated in the text.

Proposition B.3 (Continuous military choice)**.** *Assume $p(\cdot)$ and $\omega_T(\cdot)$ are each strictly concave in θ_M and the magnitude of the second derivative for $\omega_T(\cdot)$ is large, $-\frac{\partial^2 \omega_T}{\partial \theta_M^2} > \underline{\underline{\partial^2 \omega_T}}$; and $q(\cdot)$ is strictly convex in θ_M. Then there exists a unique maximizer for Equation 4 over $\theta_M \in \left[0, \overline{\theta}_M\right]$.*

Proof. It suffices to prove $\frac{d^2 S^*}{d\theta_M^2} < 0$. From Equation A.2, we can calculate:

$$\frac{d^2 S^*}{d\theta_M^2} = \underbrace{-\frac{\partial^2 q}{\partial \theta_M^2} \cdot E^*}_{1} + \underbrace{\left[1 - q(\theta_M, \theta_D)\right] \cdot \frac{d^2 E^*}{d\theta_M^2}}_{2} + \underbrace{-2\frac{\partial q}{\partial \theta_M} \cdot \frac{dE^*}{d\theta_M}}_{3} \tag{B.5}$$

The strict negativity of 1 follows from the strict convexity of $q(\cdot)$ and the strict positivity of E^*.

For 2, need to calculate:

$$\begin{aligned} \frac{d^2 E^*}{d\theta_M^2} &= \frac{\partial^2 p}{\partial \theta_M^2} \cdot F\big(\omega_D - \omega_r(\theta_M, r)\big) - 2\frac{\partial p}{\partial \theta_M} \cdot f\big(\omega_D - \omega_T(\theta_M, r)\big) \cdot \frac{\partial \omega_T}{\partial \theta_M} \\ &+ p(\theta_M, \theta_T) \cdot \left[f'(\cdot) \cdot \left(\frac{\partial \omega_T}{\partial \theta_M}\right)^2 + f(\cdot) \cdot \frac{\partial^2 \omega_T}{\partial \theta_M^2}\right] < 0 \end{aligned}$$

That each of these terms are negative follows from the assumptions about $p(\cdot)$ and $q(\cdot)$, assuming $q'(\cdot) < 0$, and the weak positivity of cdfs and pdfs.

The sign of 3 is ambiguous and equals $\text{sgn}\left\{\frac{dE^*}{d\theta_M}\right\}$. Substituting in $\frac{dE^*}{d\theta_M}$ from Equation A.1 shows that Equation B.5 is strictly negative if:

$$-\frac{\partial^2 \omega_T}{\partial \theta_M^2} > \underline{\underline{\partial^2 \omega_T}} \equiv \left\{\left[-\frac{\partial^2 q}{\partial \theta_M^2} \cdot E^* - 2\frac{\partial q}{\partial \theta_M} \cdot \frac{dE^*}{d\theta_M}\right] \cdot \frac{1}{1 - q\big(\theta_M, \theta_D\big)}\right.$$

$$\left. + \frac{\partial^2 p}{\partial \theta_M^2} \cdot F(\cdot) - 2\frac{\partial p}{\partial \theta_M} \cdot f(\cdot) \cdot \frac{\partial \omega_T}{\partial \theta_M} + p(\theta_M, \theta_T) \cdot f'(\cdot) \cdot \left(\frac{\partial \omega_T}{\partial \theta_M}\right)^2\right\} \cdot \frac{1}{p(\cdot) \cdot f(\cdot)}$$

■

B.6 Gambling for Resurrection by Radicalizing the Opposition

Beyond the loyalty-efficiency tradeoff, analyzing the radicalism of outsider threats yields new implications for related research questions. Some study incentives to gamble for resurrection in the form of mass repression or fighting international wars: costly gambles that increase a ruler's probability of survival despite worsening its fate conditional on losing power. However, we know little about how attributes of the opposition affect the ruler's calculus. I alter the model to (1) allow dictators to vary in their fates under outsider rule and (2) shift the dictator's strategic focus from choosing the type of military to choosing the radicalism of the outsider threat. Therefore, this extension also incorporates the consideration that although rulers prioritize surviving in office—the dictator's only objective in the baseline model—the institutional basis of regimes also affects rulers' decisions.

A casual survey of dictators' fates following radical overthrow highlights the perils of extremist movements. French king Louis XVI faced the guillotine in 1793, Bolshevik guards executed Russian emperor Nicholas II in 1918, and rebels in Libya publicly displayed Muammar Gaddafi's body for four days after finding and assassinating him in 2011. However, the baseline model highlights a selection effect that explains why a dictator might prefer a *more* radical opposition: higher incentives for the military to exercise repression. This section shows how this strategic effect can cause dictators with a narrow support base to gamble for resurrection by provoking extremist opposition leaders, whereas more popular regimes do not hang onto power at all costs.

Assume that the dictator consumes $\omega_T(\theta_D, r)$ under outsider rule, rather than 0 as in the baseline model. This is the same $\omega_T(\cdot)$ function as the the military's consumption under outsider rule in the baseline model, but for the dictator I switch the coercive capacity parameter from θ_M to θ_D.[22] I also take out elements of the baseline model that are not essential for studying the dictator's preferences to gamble for resurrection: military type is fixed at an exogenous θ_M, and the probability of a coup opportunity, $q(\cdot)$, equals 0. Instead, I add another strategic choice for the dictator: determining the radicalism of the outsider threat by setting $r \in [0, 1]$. Substantively, the choice to raise r is reduced form for actions such as using the secret police and intelligence agencies to spy on the mainstream opposition and arrest their leaders, therefore driving mainstream movements underground and leaving only extremists to openly oppose the regime.[23] Figure B.4 depicts the revised game tree.

Equation B.6 presents the dictator's optimization problem under these alterations:[24]

$$\max_{r\in[0,1]} \quad \underbrace{p\big(\theta_M, \theta_T\big) \cdot F\Big(\omega_D - \omega_T\big(\theta_M, r\big)\Big)}_{\text{Military more likely to repress}} + \Big[1 - p(\cdot) \cdot F(\cdot)\Big] \cdot \underbrace{\omega_T(\theta_D, r)}_{\text{Lower payoff under outsider rule}} \tag{B.6}$$

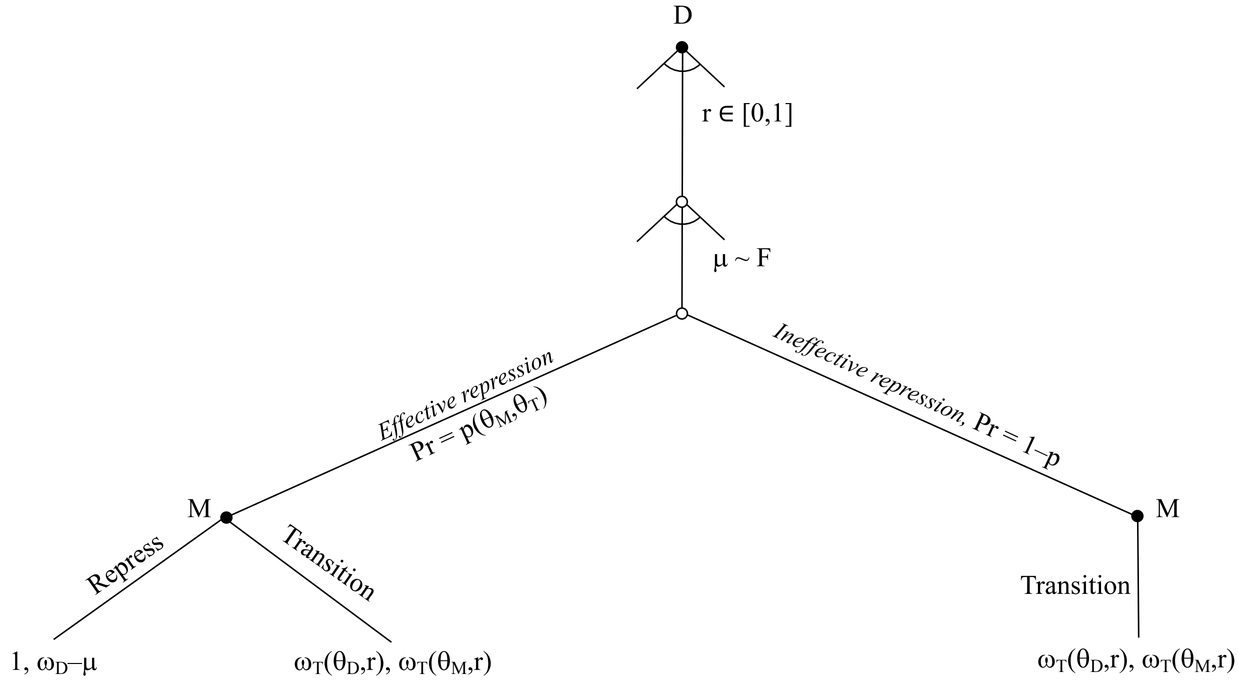
In addition to switching the dictator's choice variable from θ_M to r and setting the probability of a coup to 0, this objective function differs from that in Equation 4 because the dictator consumes a positive amount even if it loses power—and therefore its objective is not equivalent to maximizing the probability of political survival. Equation B.6 highlights two countervailing effects of r. The first is the beneficial indirect strategic effect from the baseline model: higher r raises the military's probability of exercising repression, which

[22] I impose two boundary conditions. First, the most popular type of dictator consumes the same under an ideal type non-radical opposition as in the incumbent regime: $\omega_T\big(\overline{\theta}_D, 0\big) = 1$. This requires assuming $\overline{\theta}_D > \overline{\theta}_M$. Second, the least popular type of dictator consumes 0 under outsider rule regardless of the opposition's radicalism: $\omega_T\big(0, r\big) = 0$.

[23] Although it is implausible in the real world that such actions would not also affect θ_T, empirically, the effect could go in either direction: either narrowing the opposition and reducing θ_T (perhaps reinforced by moderate opposition figures not wanting to contribute to an extremist-led movement), or raising θ_T by sparking widespread discontent. I fix θ_T to focus specifically on the dictator's tradeoff regarding r.

[24] The military's optimal choice is identical to that in the baseline model, although there is 0 probability of the information set at which it has a coup opportunity.

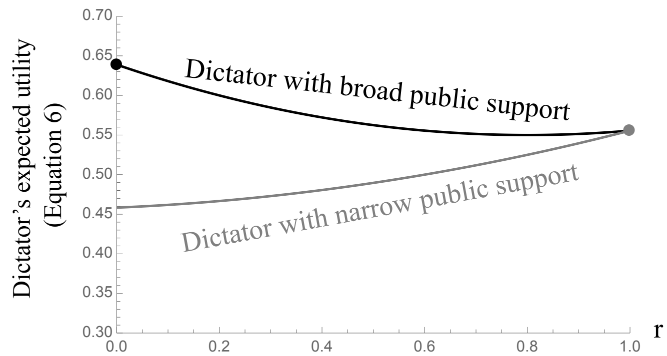
Figure B.4: Game Tree for Gambling for Resurrection Extension



raises the probability that the dictator will consume 1 rather than $\omega_T(\theta_D, r) < 1$. This effect arises because higher r decreases the military's utility under outsider rule, $\omega_T\big(\theta_M, r\big)$, and therefore increases the range of μ values low enough that the military optimally exercises repression (at the information set in which Nature enables the military to repress effectively). Second, the deleterious direct effect of r arises from assuming that $\omega_T(\cdot)$ strictly decreases in r. That is, conditional on outsider rule occurring, the dictator prefers a less radical opposition.

Figure B.5 provides the intuition for the main result by distinguishing the calculus for a dictator with high θ_D (black line) from that for a dictator with low θ_D (gray line). Whereas a popular dictator maximizes its utility by setting $r^* = 0$ (black dot), an unpopular dictator maximizes utility with $r^* = 1$ (gray dot). That is, unpopular dictators prefer an extreme opposii to maximize the probability of regime survival, whereas popular dictators sacrifice a lower survival probability for greater consumption under outsider rule. Narrowly based rulers prioritize surviving at all costs—hence gambling for resurrection—because they expect a bad fate even if non-radical actors take control. Formally, the negative cross-partial between θ_D and r implies that the direct effect of r on lowering $\omega_T(\theta_D, r)$ (see Equation B.6) is small in magnitude if θ_D is low, causing the dictator to prioritize the indirect effect by which higher r increases the probability that the military exercises repression. By contrast, a more popular dictator expects a relatively favorable post-tenure fate under a non-radical outsider. Higher θ_D diminishes the dictator's incentives to raise r by increasing the extent to which r decreases $\omega_T(\theta_D, r)$. Figure B.5 also shows that the utility lines converge at $r = 1$ because of the boundary condition that any actor consumes 0 if the opposition is ideal-type radical. Proposition B.4 formalizes this logic.

Figure B.5: Gambling for Resurrection: Popular vs. Unpopular Dictators



Notes: Figure B.5 uses the parameter values $\theta_T = 0.8$, $\theta_M = 1$, $\omega_D = 0.5$, and assumes $p(\theta_M, \theta_T) = \frac{\theta_M}{\theta_M + \theta_T}$ and $\omega_T(\theta, r) = \left(\theta / \overline{\theta}\right) \cdot (1-r) \cdot \omega_D$. For the black curve, $\theta_D = 1$, and for the gray curve, $\theta_D = 0.5$.

Proposition B.4 (Gambling for resurrection)**.** *Assume that $\omega_T(\cdot)$ is linear in r and μ is uniformly distributed. There exists a unique threshold $\hat{\theta}_D \in \left(0, \overline{\theta}_D\right)$ such that if $\theta_D < \hat{\theta}_D$, then the dictator sets $r^* = 1$; and $r^* = 0$ otherwise.*

Proof. The dictator's objective function is continuous in r with a compact constraint set, which implies that a maximizer exists. To demonstrate that there is no interior maximizer, it suffices to prove that the objective function is strictly convex. The first partial derivative with respect to r is:

$$\begin{aligned}\frac{\partial}{\partial r} &= -p\left(\theta_M, \theta_T\right) \cdot f\Big(\omega_D - \omega_T\left(\theta_M, r\right)\Big) \cdot \frac{\partial \omega_T\left(\theta_M, r\right)}{\partial r} \cdot \left[1 - \omega_T\left(\theta_D, r\right)\right] \\ &\quad + \Big[1 - p(\cdot) \cdot F(\cdot)\Big] \cdot \frac{\partial \omega_T\left(\theta_D, r\right)}{\partial r}.\end{aligned}$$

The second partial derivative with respect to r is:

$$\begin{aligned}\frac{\partial^2}{\partial r^2} &= p\left(\theta_M, \theta_T\right) \cdot f'\Big(\omega_D - \omega_T\left(\theta_M, r\right)\Big) \cdot \left[\frac{\partial \omega_T\left(\theta_M, r\right)}{\partial r}\right]^2 \cdot \left[1 - \omega_T\left(\theta_D, r\right)\right] \\ &\quad - p\left(\theta_M, \theta_T\right) \cdot f\Big(\omega_D - \omega_T\left(\theta_M, r\right)\Big) \cdot \frac{\partial^2 \omega_T\left(\theta_M, r\right)}{\partial r^2} \cdot \left[1 - \omega_T\left(\theta_D, r\right)\right] \\ &\quad + 2p\left(\theta_M, \theta_T\right) \cdot f\Big(\omega_D - \omega_T\left(\theta_M, r\right)\Big) \cdot \frac{\partial \omega_T\left(\theta_M, r\right)}{\partial r} \cdot \frac{\partial \omega_T\left(\theta_D, r\right)}{\partial r} \\ &\quad + \Big[1 - p\left(\theta_M, \theta_T\right) \cdot f\Big(\omega_D - \omega_T\left(\theta_M, r\right)\Big)\Big] \cdot \frac{\partial^2 \omega_T\left(\theta_D, r\right)}{\partial r^2}.\end{aligned}$$

The linearity and uniformity assumptions imply $f'(\cdot) = 0$ and $\frac{\partial^2 \omega_T(\theta_M, r)}{\partial r^2} = \frac{\partial^2 \omega_T(\theta_D, r)}{\partial r^2} = 0$, which reduces the second derivative to:

$$\frac{\partial^2}{\partial r^2} = 2\underbrace{p\left(\theta_M, \theta_T\right) \cdot f\Big(\omega_D - \omega_T\left(\theta_M, r\right)\Big)}_{+} \cdot \underbrace{\frac{\partial \omega_T\left(\theta_M, r\right)}{\partial r}}_{-} \cdot \underbrace{\frac{\partial \omega_T\left(\theta_D, r\right)}{\partial r}}_{-} > 0.$$

This implies that the objective function is strictly convex and no interior solution exists, leaving $r = 0$ and $r = 1$ as the remaining possible solutions.

We can implicitly define $\hat{\theta}_D$ as:

$$p\big(\theta_M, \theta_T\big) \cdot F\Big(\omega_D - \omega_T\big(\theta_M, 0\big)\Big) + \Big[1 - p\big(\theta_M, \theta_T\big) \cdot F\Big(\omega_D - \omega_T\big(\theta_M, 0\big)\Big)\Big] \cdot \omega_T(\hat{\theta}_D, 0) =$$

$$p\big(\theta_M, \theta_T\big) \cdot F\Big(\omega_D - \omega_T\big(\theta_M, 1\big)\Big) + \underbrace{\Big[1 - p\big(\theta_M, \theta_T\big) \cdot F\Big(\omega_D - \omega_T\big(\theta_M, 1\big)\Big)\Big] \cdot \omega_T(\hat{\theta}_D, 1)}_{=0}$$

To establish the boundary claims, if $\theta_D = 0$, then $\omega_T(0, r) = 0$ for all r, which implies that the right-hand side of the equation strictly exceeds the left-hand side. If $\theta_D = \overline{\theta}_D$, then the assumption that $\omega_T\big(\overline{\theta}_D, 0\big) = 1$ simplifies the left-hand side to 1, which strictly exceeds the right-hand side. The strict threshold claim follows from:

$$\begin{aligned}\frac{\partial^2}{\partial r \partial \theta_D} &= p\big(\theta_M, \theta_T\big) \cdot f\Big(\omega_D - \omega_T\big(\theta_M, r\big)\Big) \cdot \frac{\partial \omega_T\big(\theta_M, r\big)}{\partial r} \cdot \frac{\partial \omega_T(\theta_D, r)}{\partial \theta_D} \\ &\quad + \Big[1 - p(\cdot) \cdot F(\cdot)\Big] \cdot \frac{\partial \omega_T^2\big(\theta_D, r\big)}{\partial r \partial \theta_D} < 0\end{aligned}$$

■