Junior

Saturday October 11, 2014

1:00 pm - 3:00 pm

Each problem is graded on a basis of 0 to 10 points. All the necessary work to justify an answer and all the necessary steps of a proof must be shown clearly to obtain full credit. Some partial credit may be given, but only when a contestant has shown significant and substantial progress toward a solution.

1. Call a triangle simple if each of its sides has length $4, 6$, or 8. Find all different simple triangles.

2. In the following, there are 8 possible paths to spell the word MATH:

   M
   A A
   T T T
   H H H H

   In each move you can only go up or down by one row. Determine the number of paths that spell the word LEVEL in the following

   L
   E E
   V V V
   E E
   L

3. In the Fibonacci sequence, $1, 1, 2, 3, 5, \ldots$, each term after the second is the sum of the previous two terms. How many of the first 100 terms of the Fibonacci sequence are odd?

4. Four people A, B, C, D are enrolled in some number of courses. A is taking 8 courses and B is taking 5 courses. It is known that A is taking more courses than anyone else and B is taking less courses than anyone else. It is also known that exactly three of these four people are enrolled in each math course. How many courses are offered?

5. Out of six children, exactly two were known to have been stealing apples. But who? Helen said "Christine and George". Jane said "Donald and Tom". Donald said "Tom and Christine". George said "Helen and Christine". Christine said "Donald and Jane". Tom couldn't be found. Four of the children actually named one of the culprits but lied about the other. The fifth child lied about both. Who stole the apples?

Senior

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1. A sequence $\{a_n\}$ is defined recursively by

$$a_1 = 1, a_2 = 4, \text{ and } a_n = 2a_{n-1} - a_{n-2} + 2 \text{ for } n \geq 3$$

   Find the closed form formula for a_n.

2. Let x and y be positive integers satisfying

$$xy = 2014x + 2014y$$

   Prove that $x \leq 4058210$.

3. In a convex quadrilateral, the lengths of the diagonals are 100 and 1. Given that the perimeter is an integer, find all possible perimeters.

4. Determine if there exists a polynomial f with integer coefficients such that

$$f(3) = 1, f(5) = 3, \text{ and } f(7) = 9$$

5. Find all non-negative integer solutions to $x!^{y!} = z!$.