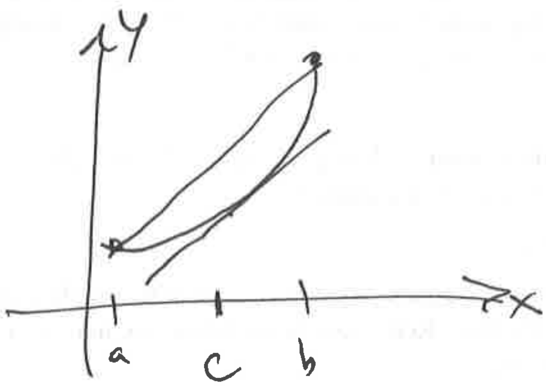
Calc 2

Mean Value Thm

if f cont's on $[a,b]$ & diff'ble (a,b) there is a c in (a,b) where $f'(c) = \dfrac{f(b) - f(a)}{b-a}$



this is used to prove the fundamental Thm of Calc.

Anti-derivative

if $y = x^2$ $\quad y' = 2x$

given $y' = 2x$ what is y

we denote this by an integral

$\int 2x\,dx = x^2 + c$ there's an entire list of standard integrals

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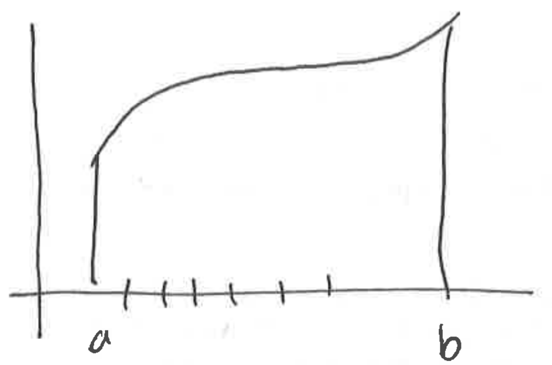
<u>Next</u> Area Under Curve - Riemann Sums 2-7



(1) subdivide interval

$$a = x_0 < x_1 < x_2 \cdots < x_i < x_{i+1} < \cdots x_n = b$$

create Rectangle

(2) thickness - $\Delta x_i = x_i - x_{i-1}$

height $f(x_i^*)$ when x_i^* in $[x_i, x_{i+1}]$

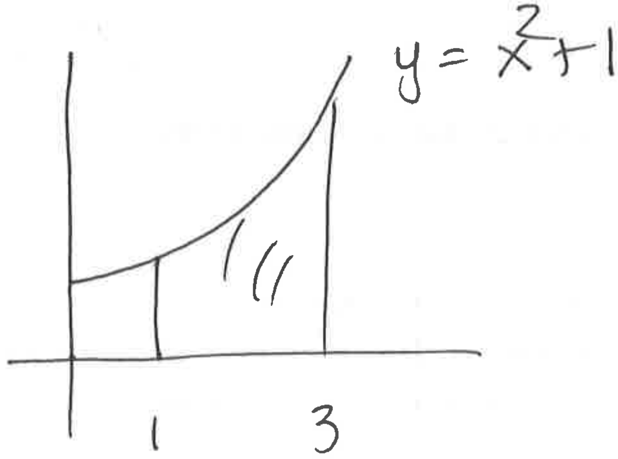
Area $f(x_i)\Delta x_i$

Sum $\displaystyle\sum_{i=1}^{n} f(x_i)\Delta x_i$

limit $\displaystyle\lim_{\substack{\Delta x_i \to 0 \\ n \to \infty}} \sum_{i=1}^{n} f(x_i)\Delta x_i = \int_a^b f(x)dx$

Definite integral

Ex



$$A = \int_1^3 (x^2+1)\,dx = \frac{x^3}{3} + x \Big|_1^3$$

$$= \left(\frac{3^3}{3}+3\right) - \left(\frac{1}{3}+1\right)$$

$$= 12 - \frac{1}{3} - 1 = \frac{22}{3}$$

Application

- area between curves
- volume of revolution
- arc length
- surface area
- center of mass
- work
- fluid pressure

$\Rightarrow$ leads to the subject of differential eq^n's.

L'Hôpital's Rule (Briggs et al section 4.7)

Suppose f & g are differentiable on a open interval I containing $x=a$ & $g'(x) \neq 0$ on I when $x \neq a$

If $\lim\limits_{x \to a} f(x) = \lim\limits_{x \to a} g(x) = 0$

then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)} \quad \text{provide Limit exists}^{*}$$

Ex 1 $\lim\limits_{x \to 1} \dfrac{x^2-1}{x-1} =$ " $\dfrac{0}{0}$ " so LH

$$\lim_{x \to 1} \frac{2x}{1} = 2$$

* Note limit " $\dfrac{0}{0}$ " or " $\dfrac{\infty}{\infty}$ "

Ex 2 $\lim_{x \to 0} \frac{\sqrt{x+1} - 1}{x} = \text{"}\frac{0}{0}\text{"}$ LH

$$\lim_{x \to 0} \frac{1}{2\sqrt{x+1}} = \frac{1}{2}$$

Ex 3 $\lim_{x \to 0} \frac{e^x - 1}{x} = \text{"}\frac{0}{0}\text{"}$ LH

$$\lim_{x \to 0} \frac{e^x}{1} = 1$$

Ex 4 $\lim_{x \to \infty} \frac{x}{e^x} = \text{"}\frac{\infty}{\infty}\text{"}$ LH

$$\lim_{x \to \infty} \frac{1}{e^x} \to 0$$

Ex5 $\displaystyle\lim_{x\to\infty} \frac{\ln x}{x} = \text{"}\frac{\infty}{\infty}\text{"}$ LH

$$\lim_{x\to\infty} \frac{\frac{1}{x}}{1} = 0$$

Ex6 $\displaystyle\lim_{x\to 2} \frac{x^3-3x^2+4}{3x^3-21x^2+48x-36} = \text{"}\frac{0}{0}\text{"}$

LH $\displaystyle\lim_{x\to 2} \frac{3x^2-6x}{9x^2-42x+48} = \text{"}\frac{0}{0}\text{"}$ again

$$\lim_{x\to 2} \frac{6x-6}{18x-42} = \lim_{x\to 2} \frac{6(x-1)}{6(3x-7)} = \frac{1}{-1} = -1$$

Indeterminate Form

"$0\cdot\infty$", "1^∞", "0^0", "$\infty-\infty$"

so now we have to manipulate the limit 2-7 first before applying L'H

Ex $\lim_{x\to\infty} e^{-x}\sqrt{x} = "0\cdot\infty"$

$\lim_{x\to\infty} \frac{\sqrt{x}}{e^x} = "\frac{\infty}{\infty}"$ now L'H

$\lim_{x\to\infty} \frac{\frac{1}{2\sqrt{x}}}{e^x} = \lim_{x\to\infty} \frac{1}{2\sqrt{x}\,e^x} = 0$

Ex $\lim_{x\to 0^+} x\ln x = "0\cdot -\infty"$

$\lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} = "\frac{-\infty}{\infty}"$ ok

$\lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-1}{x^2}} = \lim_{x\to 0^+} \frac{-x^2}{x} = -\lim_{x\to 0^+} x = 0$

Ex $\lim_{x \to 0^+} x^x = $ "0^0"

let $y = x^x$ $\ln y = x \ln x$

so new limit

$\lim_{x \to 0^+} x \ln x = 0$ previous ex.

so $x \to 0^+$. $x \ln x \to 0$

$\ln y \to 0 \Rightarrow y \to 1$

So $\lim_{x \to 0^+} x^x = 1$

Ex $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = $ "1^∞"

Need to move power

$y = \left(1 + \frac{1}{x}\right)^x$ $\ln y = x \ln\left(1 + \frac{1}{x}\right)$

New limit

$$\lim_{x\to\infty} x \ln\left(1+\frac{1}{x}\right) = \text{"}0\cdot\infty\text{"}$$

$$= \lim_{x\to\infty} \frac{\ln 1+\frac{1}{x}}{\frac{1}{x}} = \text{"}\frac{0}{0}\text{"} \quad \text{LH}$$

$$= \lim_{x\to\infty} \frac{1}{1+\frac{1}{x}} \frac{\cancel{\left(-\frac{1}{x^2}\right)}}{\cancel{-\frac{1}{x^2}}}$$

$$= \lim_{x\to\infty} \frac{1}{1+\frac{1}{x}} = \frac{1}{1+0} = 1$$

so $x\to\infty$ $\quad x\ln\left(1+\frac{1}{x}\right) \to 1$

$\ln y \to 1$ so $y \to e^1$

so $\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x = e$