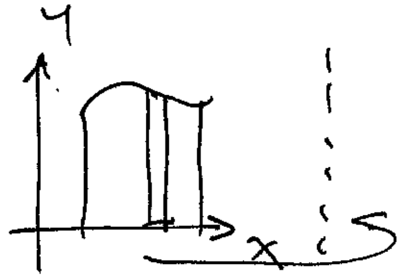
Volumes of Revolution - 3

We now consider volumes of revolution about lines that are not $x=0$ (y axis) or $y=0$ (x axis). Here we will consider only 1 curve $y=f(x)$ but extends easily to 2 curves

Ex 1



$X=L$

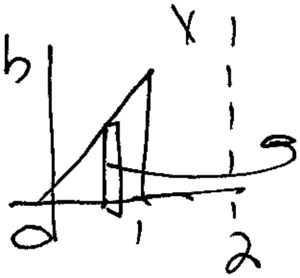
Here L is on the right side of the region. We will use the method of shell

$$V = 2\pi \int_0^b r\, f(x)\,dx$$

Here $r = L-x$ so

$$V = 2\pi \int_0^b (L-x)\, f(x)\,dx$$

Ex 1b

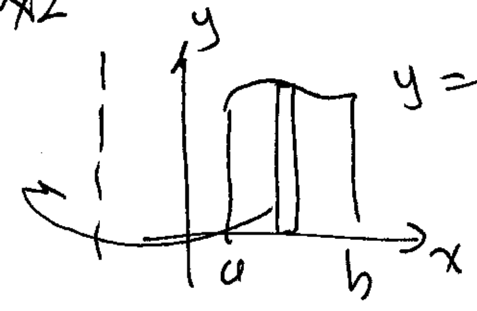


$$V = 2\pi \int_0^1 (2-x)\,x\,dx$$

$$= 2\pi \int_0^1 (2x-x^2)\,dx$$

$$= 2\pi \left. x^2 - \frac{x^3}{3} \right|_0^1 = 2\pi \cdot \frac{2}{3} = \frac{4\pi}{3}$$

Ex2



$X=L$

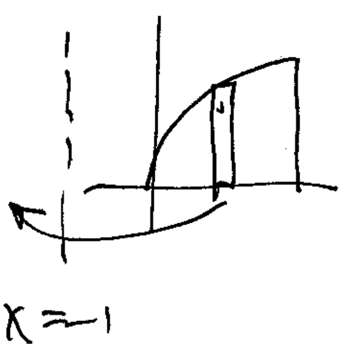
$y=f(x)$ Here L is now on the left side of the region we still use

$$V = 2\pi \int_a^b r\, f(x)\,dx$$

but now $r = x - L$

$$V = 2\pi \int_a^b (x-L)\, f(x)\,dx$$

Ex2b Find the volume of the region bound by $y = \sqrt{x}$, $y = 0$, $x = 1$ when revolved about $x = -1$



$x=-1$

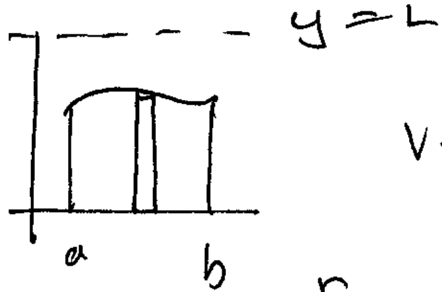
$$V = 2\pi \int_0^1 (x+1)\sqrt{x}\,dx$$

$$= 2\pi \int_0^1 \left(x^{3/2} + x^{1/2}\right) dx$$

$$= 2\pi \left(\frac{2x^{5/2}}{5} + \frac{2x^{3/2}}{3} \right) \Big|_0^1$$

$$= 2\pi \left(\frac{2}{5} + \frac{2}{3} \right) = 2\pi \, \frac{6+10}{15} = \frac{32\pi}{15}$$

Ex 3

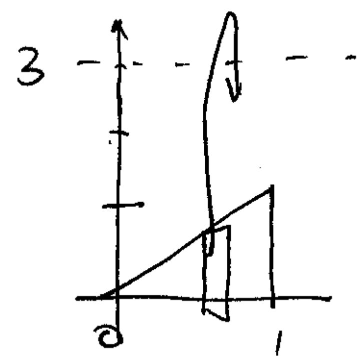


$$V = \pi \int_a^b (r_o^2 - r_i^2)\,dx$$

$$r_o = L - 0, \quad r_i = L - f(x)$$

$$V = \pi \int_a^L L^2 - (L - f(x))^2\,dx$$

Ex 3b $y = x$, $y = 0$, $x = 1$ about $y = 3$



$$V = \pi \int_0^1 3^2 - (3-x)^2\,dx$$

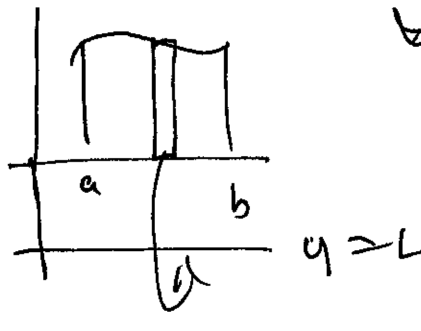
$$= \pi \int_0^1 (\cancel{9} - \cancel{9} + 6x - x^2)\,dx$$

$$= \pi \quad 3x^2 - \frac{x^3}{3} \Big|_0^1$$

$$= \pi \left\{ \left(3 - \frac{1}{3}\right) - 0 \right\}$$

$$= \frac{8\pi}{3}$$

ex 3



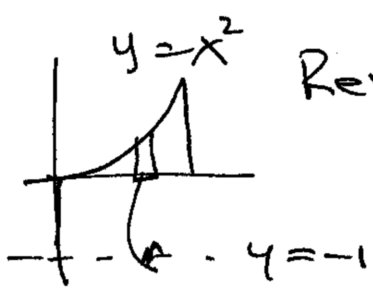
In this case $y = L$ is below the region. We still use

$$V = \pi \int_a^b (r_o^2 - r_i^2)\,dx$$

instead $r_o = f(x) - L \quad r_i = L$

$$V = \pi \int_a^b (f(x) - L)^2 - L^2 \; dx$$

Ex 4b



Revolve the region bound by $y = x^2,\ y = 0,\ x = 2$ about $y = -1$

$r_o = x^2 + 1, \quad r_i = 1$

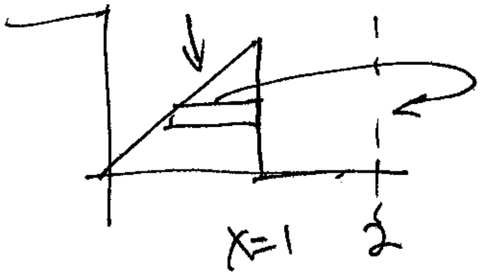
$$V = \pi \int_0^2 (x^2+1)^2 - 1^2 \, dx$$

$$= \pi \int_0^2 (x^4 + 2x^2)\,dx = \pi \left(\frac{x^5}{5} + \frac{2x^3}{3} \Big|_0^2 \right)$$

$$= \pi \left(\frac{32}{5} + \frac{16}{3} \right) = \frac{176}{15} \pi$$

Here, we redo Ex 1 – 4 but use y

Ex 1 $y=x$



$$V=\pi\int_0^1 (2-y)^2-1^2$$

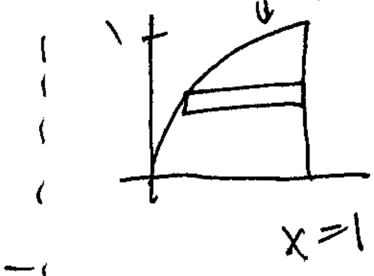
$$=\pi\int_0^1 (3-4y+y^2)\,dy$$

$$=\pi\left(3y-2y^2+\frac{y^3}{3}\Big|_0^1\right)$$

$$=\pi\left(3-2+\frac{1}{3}\right)=4\pi/3$$

Ex 2

$y=\sqrt{x}$ $x=y^2$



$$V=\pi\int_0^1 2^2-(y^2+1)^2\,dy$$

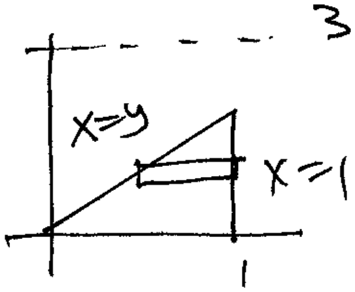
$$=\pi\int_0^1 (3-2y^2-y^4)\,dy$$

$$=\pi\left(3y-\frac{2}{3}y^3-\frac{y^5}{5}\Big|_0^1\right)$$

$$=\pi\left(3-\frac{2}{3}-\frac{1}{5}\right)$$

$$=\pi\left(\frac{45-10-3}{15}\right)=\frac{32\pi}{15}$$

Ex3



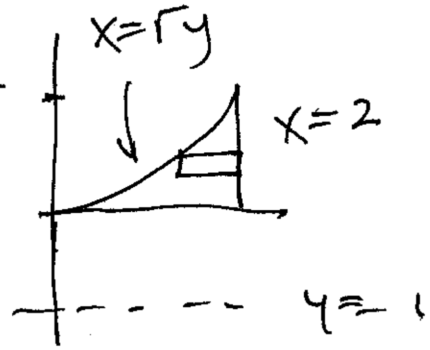
$$V = 2\pi \int_0^1 (3-y)(1-y)\,dy$$

$$= 2\pi \int_0^1 (3-4y+y^2)\,dy$$

$$= 2\pi \left(3y - 2y^2 + \frac{y^3}{3} \Big|_0^1 \right)$$

$$= 2\pi \left(3 - 2 + \frac{1}{3} \right) = 2\pi \cdot \frac{4}{3} = \frac{8\pi}{3}$$

Ex4



$$V = 2\pi \int_0^4 (y+1)(2-\sqrt{y})\,dy$$

$$= 2\pi \int_0^4 (2y - y^{3/2} + 2 - y^{1/2})\,dy$$

$$= 2\pi \left(y^2 - \frac{2}{5} y^{5/2} + 2y - \frac{2y^{3/2}}{3} \right) \Big|_0^4$$

$$= 2\pi \cdot \left(16 - \frac{2}{5} \cdot 32 + 8 - \frac{2}{3} \cdot 8 \right)$$

$$= \frac{176\pi}{15}$$