

Public List of Theorems for the ICPS 9-10 Entrance Exam (2019)

1. For perpendicular lines with slopes m_1 and m_2 , the product $m_1 \cdot m_2 = -1$.
2. $0.999 \dots = 1$
3. $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$.
4. A negative number times a negative number equals a positive number.
5. An even number plus an odd number equals an odd number.
6. When you multiply both sides of an inequality by a negative number, you flip the direction of the inequality, *i.e.*, $-x > y \leftrightarrow x < -y$.
7. The area of a triangle is one-half the base times the height, *i.e.*, $\frac{1}{2}bh$.
8. The sum of the interior angles of a triangle = 180° .
9. A number is divisible by 3 if the sum of the digits of the number is divisible by 3.
10. The area of a trapezoid = $\frac{1}{2}h(b_1 + b_2)$.
11. The circumference of a circle = $2\pi r$.
12. Pythagorean theorem: for right triangles, $a^2 + b^2 = c^2$.
13. Quadratic formula: For $ax^2 + bx + c = 0$, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
14. Multiplication: $x^n x^k = x^{n+k}$
15. Negative exponent: $x^{-n} = \frac{1}{x^n}$
16. Power of a power: $(x^n)^k = x^{nk}$
17. Division: $\frac{x^n}{x^k} = x^{n-k}$.
18. Fractional exponent: $x^{k/n} = \sqrt[n]{x^k}$

The ICPS application packet is available at:

<http://institutecreativeproblemsolving.org/nomination.htm>

(important instructions on the other side of this sheet)

Instructions and recommendations for the ICPS 9-10 Entrance Exam

1. The ICPS 9-10 exam is based on the ICPS public list of eighteen math theorems, and the proofs of those 18 theorems which you'll have learned, practiced, and enhanced (see **Section II**, below).
2. **Section I** of the exam consists of five theorems from the public list. For maximum credit, you will write out a complete proof of each theorem, strictly from your memory. No notes or other written materials are to be brought to the exam, and no calculators, phones or other technology may be used during the exam.
3. You are allowed and encouraged to get help from anyone and any source you have access to, including friends, teachers, parents, the Internet, books, and so on, while preparing for the exam. No giving or receiving of help is allowed during the exam itself.
4. You are encouraged to study, practice, and collaborate with friends to learn the proofs and create ideas about them, along with your individual work.
5. Each proof you provide should contain: math statements and diagrams in a logical sequence; math reasons and propositions (e.g., the distributive property) justifying the statements or steps; **and** explanation and discussion in English. Use your explanations to show that you **understand** the proofs.

Below, to help you prepare, is **Section II** exactly as it appears on the exam.

Section II. You may provide multiple responses to any item, and each response you give is eligible for credit. Do as many of these items as you can. [n] is the maximum credit possible for each response to an item.

1. Provide additional proofs for the Pythagorean Theorem and/or the area of a trapezoid, proofs of other theorems from the ICPS exam public list, and/or proofs of other theorems that you know. Include reasons, verbal explanation, and discussion, as with **Section I**. [15]
2. Create questions and conjectures inspired by the theorems and proofs. Conceptual, open-ended questions earn the most credit. [15]
3. Find connections (patterns, themes, etc.) inside a proof, and between proofs. For example, show which principles of logic are used in a proof. [10]
4. Identify the crucial step in a proof and explain why it's important. [10]
5. Name and discuss the infinite sets involved with the theorems and their proofs. [10]
6. Identify your favorite proof that you learned, and explain why it's a favorite. [15]
7. Explain techniques you used to help you learn and understand the proofs for this exam. [10].
8. Describe how learning the proofs has changed how you learn, do, and understand math for school. [15]

(end of **Section II**)

Recommendations for preparation for the exam (and for learning math):

1. Work on your proofs at least once a day, **every** day, for at least 20 minutes, from **right now** until exam day.
2. Practice by **testing** and checking yourself, just as if you were doing the exam on exam day. Be able to write out the entire proof, 100% correct, strictly from your memory.
3. Find/create, and then practice, at least one proof of each theorem on the public list. If there are numerous proofs of a theorem, learn more than one for that theorem. (see **Section II**, #1 above)
4. Keep a notebook just for theorems, proofs, and your ideas about them.
5. Make an index card deck with a theorem on one side of each card and a proof plus your comments on the other side. Carry the deck with you and practice in free moments.
6. Make note of your **Section II** responses (connections, comments, questions, etc.) as they occur to you. Include reviewing these in your daily practice, as well as improving on them and making more.
7. Make a game of it, have fun, and enjoy the feeling of getting stronger with math every day!

(list of theorems on the other side of this sheet)