Math 1496 - Calc 1

u - Substitutions

The idea is to make a substitution to make your integral one that you know how to do

Ex 1 $\int \frac{x}{x^2+1}\,dx$

Note quite 2 steps but x^2+1 is harder than x

let $u = x^2+1$

$du = 2x\,dx$ so $x\,dx = \frac{du}{2}$

so $\int \frac{x}{x^2+1}\,dx = \int \frac{du/2}{u} = \frac{1}{2}\int \frac{du}{u}$

$= \frac{1}{2}\ln|u| + c = \frac{1}{2}\ln|x^2+1| + c$

Ex 2 $\displaystyle\int_1^{e^2} \frac{\ln x}{x}\,dx$

let $u = \ln x$ $\qquad$ $x = 1 \quad u = \ln 1 = 0$

$du = \dfrac{dx}{x}$ $\qquad$ $x = e^2 \quad u = \ln e^2 = 2$

$$\int_0^2 u\,du = \frac{u^2}{2}\Big|_0^2 = \frac{2^2}{2} - \frac{0^2}{2} = 2$$

Ex 3 $\displaystyle\int \frac{x}{\sqrt{x+1}}\,dx$ $\quad$ a $\quad$ $\displaystyle\int x\sqrt{x+1}\,dx$

let $u = x+1$ $\qquad$ $\displaystyle\int \frac{x\,du}{\sqrt{u}}$ still has x

$du = dx$

$x = u - 1$ so $\displaystyle\int \frac{u-1}{\sqrt{u}}\,du = \int \frac{u}{\sqrt{u}} - \frac{1}{\sqrt{u}}\,du$

$$= \int (u^{1/2} - u^{-1/2})\,du = \frac{u^{3/2}}{3/2} - \frac{u^{1/2}}{1/2} + c$$

$$= \frac{2}{3}(x+1)^{3/2} - 2(x+1)^{1/2} + c$$

Ex 4 $\displaystyle\int \frac{x^3\,dx}{x^2+1}$

let $u = x^2+1$ $\quad du = 2x\,dx$

(A) $\displaystyle\int \frac{x^2 \cdot x\,dx}{x^2+1}$

$\displaystyle\int \frac{x^2\,\frac{du}{2}}{u}$

$\displaystyle\frac{1}{2}\int \frac{(u-1)\,du}{u}$

$\displaystyle\frac{1}{2}\int \left(1-\frac{1}{u}\right)du$ same

(B) $dx = \dfrac{du}{2x}$

$\displaystyle\int \frac{x^3\,\frac{du}{2x}}{x^2+1}$

$\displaystyle\frac{1}{2}\int \frac{x^2\,du}{x^2+1} \leftarrow x^2 = u-1$

$\displaystyle\frac{1}{2}\int \frac{u-1\,du}{u}$

$\frac{1}{2}\left(u - \ln|u|\right) + C$

$\frac{1}{2}\left(x^2+1 - \ln|x^2+1|\right) + C$

$\frac{1}{2}x^2 - \frac{1}{2}\ln|x^2+1| + C + \frac{1}{2}$

$\leftarrow$ absorb

Ex 5

$$\int \frac{x\,dx}{1+x^4}$$

$u = 1+x^4 \qquad du = 4x^3 dx \qquad dx = \frac{du}{4x^3}$

$$\int \frac{x \frac{du}{4x^3}}{1+x^4} = \frac{1}{4}\int \frac{du}{x^2(1+x^4)}$$

↑ problem

If $\quad u = ? \qquad du = x\,dx$

$u = x^2 \qquad du = 2x\,dx \qquad x\,dx = \frac{du}{2}$

$$\int \frac{\frac{du}{2}}{1+u^2} = \frac{1}{2}\int \frac{du}{1+u^2}$$

$$= \frac{1}{2}\tan^{-1} u + C = \frac{1}{2}\tan^{-1}(x^2) + C$$