Balanced Image Restoration with ADMM-B Algorithm

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Abstract- The balanced regularization approach in image restoration bridges the synthesis-based and analysis-based approaches and balances the fidelity, sparsity and smoothness of the solution. This paper here introducing an efficient algorithm which solves the balanced regularization problem in the frame-based image restoration. Here the proposed algorithm is based on a variable splitting strategy and the classical alternating direction method (ADM) for solving the balanced optimal problem This paper shows that the for solving the standard image restoration with balanced regularization proposed algorithm is fast and efficient. The efficiency of proposed algorithm is represented by numerical simulations in the frame-based image restoration with balanced regularization.

Keywords- Alternating direction method, balanced regularization, synthesis -based approach, analysis-based approach, image restoration.

I. INTRODUCTION

Image restoration is a classical and important research topic in image processing, which is often formulated as an inverse problem [1]. It continues to attract attention of many researchers and engineers who are working in imaging applications from astronomy to consumer imaging systems. In general, the goal is to recover an unknown true image $u \in R^n$ from a noisy measurement $y \in R^m$ that is often modeled as

$$y = Bu + n \qquad \text{Eqn. (1)}$$

where B is a linear operator, typically a convolution operator in image deconvolution, a projection in image inpainting and the identity map in image denoising, n is a white Gaussian noise with variance σ2. It is well known that this problem is ill-posed, but many algorithms for this inverse problem are known and acceptable by adopting a variety of signal prior information. Among these, a recently emerging technique exploits efficiently the sparse and redundant representations of the signals . A signal u is said to have a sparse representation over a known frame $W \in R^{n\times d}$, if there exists a sparse vector $x \in R^d$ such that u = Wx. In general, the frame may be redundant. In this paper, the redundant and normalized tight frame (Parseval frame) is used, i.e., $WW^T = I$, where I denotes the identity matrix. Thus , $u=W(W^T u)$ for every vector $u \in R^n$. The components of the vector W^Tu are called the canonical coefficients representing u. Hence the framebased image restoration can be described as: the coefficient vector x is estimated from the noisy image under the sparsity assumption first, then a linear combination of a few columns of frame W represent an unknown image u., The mapping from the image u to its coefficients is not one-to-one since tight wavelet frame systems are redundant i.e., the representation of u in the frame domain is not unique. The sparseness of the frame coefficients are utilized by three formulations namely analysis-based approach, synthesisbased approach and balanced regularization approach. The balanced regularization approach can be formulated as

$$\min_x (1/2)\|BWx - y\|_2^2 + (\gamma/2)\ \|(I - W^TW)x\|_2^2 + \lambda^T\ |x|_1 \qquad \text{Eqn. 2}$$

where γ > 0 and λ are given nonnegative weight vectors $\|.\|_2$, denotes the l_2- and $|z|_1$ denotes the vector obtained from z by taking absolute values of its elements. Penalty on the data fidelity represent first term and the last term penalizes the sparsity of coefficient vector, the second term penalizes the distance between the frame coefficients x and the range of W^T , i.e., the distance to the canonical frame coefficients of u. The larger γ makes the frame coefficients x closer to the range of W^T , that is to say, the frame coefficients x is closer to the canonical frame coefficients of u for the larger γ.

When γ = 0, the problem (2) is reduced to

$$\min_x (1/2)\ )\|BWx - y\|_2^2 + \lambda^T\ |x|_1 \qquad \text{Eqn.3}$$

This is called the synthesis-based approach, and in this only the sparsity of the frame coefficients is penalized and the sparsest coefficients synthesized the estimated image . On the other extreme, when γ = ∞, the term $\|(I - W^TW)x\|_2^2$ must be 0 if the problem (2) has a finite solution. This implies that x is in the range of W^T , i.e., $x = W^T u$ for some $u \in R^n$. Thus the problem (2) can be rewritten as

$$\min_{x\in \mathrm{Range}(W^T)} \frac{1}{2}\|BWx - y\|_2^2 + \lambda^T|x|_1 = \min_{u\in \mathbf{R}^n} \frac{1}{2}\|Bu - y\|_2^2 + \lambda^T|W^Tu|_1. \qquad \text{Eqn. 4}$$

This is called analysis-based approach because the coefficient is in the range of the analysis operator W^T. It is noted that in (4) only the sparsity of the canonical frame coefficients is penalized, which corresponds to the smoothness of the underlying image.

II. COMPARATIVE REVIEW

In this era, researchers have used number of techniques for balanced image restoration.

[1] Jian-feng et.al, introduces Split Bregman methods which is proven more efficient tools for solving total variation norm minimization problems, arising from partial differential equation based image restoration such as image denoising and magnetic resonance imaging reconstruction from sparse samples. In this paper, the convergence of the split Bregman iterations are proved, where the number of inner iterations is fixed to be one. It also shows that minimization problems arising from the analysis based approach can be solved by split Bregman iterations for image restoration .

[2] Yue Hu et.al, introduces a novel image regularization termed as multiple degree total variation (MDTV). The main aim is to combines the first and second degree directional derivatives by regularization which provides a good balance between region smoothness and preservation of edges. An iteratively overweighted majorize minimize algorithm is proposed to solve the resulting optimization problem. Then there is comparison of the proposed method with the standard TV, including higher degree total variation (HDTV) and total generalized variation (TGV) based schemes. Numerical results indicate that the MDTV penalty provides improved image recovery performance.

[3] Yutang Fu et.al, introduces Total variation (TV) based method which is used in image compressive sensing and also achieves great success in under-sampled recovery due to its virtue for edges preserving. However, since TV provides piecewise constant solution, it shows bad performance in fine structure recovery. The paper proposed a novel image compressive sensing scheme as the balanced sparsity model is introduced into the conventional TV based compressive sensing to overcome the above problem. Experimental results demonstrate that the novel scheme outperforms the compared compressive sensing methods in both of objective indicator and visualization..

[4] Javier Portilla et.al, proposed a novel formulation for relaxed analysis-based sparsity in multiple dictionaries as a general type of prior for images, and it is applied for Bayesian estimation in image restoration problems. The resulting constrained dynamic method is not just fast and effective, but also highly robust and flexible. It provide an tradeoff between computational load and performance, Second, the performance benchmark can be easily adapted to specific types of degradation, image classes, and even performance criteria. Third, it allows for using several dictionaries simultaneously with complementary features. This unique combination makes a highly practical deconvolution method.

[5] Bin Dong et.al, used mathematical tools in image restoration, where wavelet frame based approach is one of the successful examples. In this paper, a generic wavelet frame based image restoration model, called the "general model", is introduced which includes most of the existing wavelet frame based models as special cases. Moreover, the general model also includes examples that are new to the literature. An asymptotic analysis of the general model as image resolution establishes a connection between the general model in discrete setting and a new variatonal model in continuum setting. The variational model also includes some of the existing variational models as special cases, such as the total generalized variational model proposed by . In the end, an algorithm solving the general model is introduced.

Table 1. shows techniques used by other authors

Year of publication	Technique used	Pros	Cons
2009	S_B	E_{ff} tool for solving min. problems	Less flx
2016	MDTV	a good blc btw region smoothness and p_e	Requires imp in image recovery
2016	TV	provides pcs	Bad performance in F_{SR}
2015	A_{BS}	E_{ff} , R_b	S_p
2016	W_{FM}	R_b	Requires imp

S_B : Split Bregman technique, MDTV : Multiple degree total variation , TV : Total Variation, A_{BS} : Analysis-based sparsity Model, W_{FM} : Wavelet frame based approach, E_{ff} : efficient, min. : minimization problems, blc: balance, btw: between, p_e : preservation of edges, pcs: piecewise constant solution, R_b : robust, flx: flexible, F_{SR} fine structure recovery, imp: improvement, S_p : speed.

III. METHOD BASED ON BALANCED IMAGE RESTORATION

A. STANDARD ALTERNATING DIRECTION METHOD

In this paper, ADMM will be employed to solve the regularization problem in the image restoration, hence, the standard ADMM should be overviewed first. Consider an unconstrained optimization problem of the form

$$\min_{u \in R^n} f_1(u) + f_2(Gu) \quad \text{Eqn. 5}$$

where f1 and f2 are closed, proper convex functions, and $G \in R^{d\times n}$. Variable splitting consists in creating a new variable, say v, to serve as the argument of f2, under the constraint that Gu = v. This leads to the constrained problem:

$$\min_{u\in R^n} f_1(u) + f_2(v), \qquad \text{subject to } Gu = v$$

Eqn.6

which is clearly equivalent to the unconstrained problem. Using the classical augmented Lagrangian (AL) approach, also known as the method of multiplier (MM), to deal with the problem , following iterative algorithm can be obtained

$$(u_{k+1}, v_{k+1}) \in \arg\min_{u,v} f_1(u) + f_2(v) + \frac{\mu}{2}\|Gu - v - d_k\|_2^2$$

Eqn. 7

$$d_{k+1} = d_k - (Gu_{k+1} - v_{k+1}).$$

Eqn. 8

Here $\|.\|_2$ denotes the l2-norm, $\mu \geq 0$ is called AL penalty parameter and dk corresponds to the vector of Lagrange multipliers at the iteration k. The above minimization problem is not trivial since it involves a non-separable quadratic as well as possibly nonsmooth terms. A natural way to address is to minimize it alternatingly with respect to u and v while keeping the other variable fixed. This leads to the so-called alternating direction method of multipliers (ADMM) .

Algorithm ADMM:

1) Set $k = 0$, choose $\mu > 0$, v_0 and d_0.
2) *repeat*
3) $u_{k+1} \in \arg\min_u f_1(u) + \frac{\mu}{2}\|Gu - v_k - d_k\|_2^2$.
4) $v_{k+1} \in \arg\min_v f_2(v) + \frac{\mu}{2}\|Gu_{k+1} - v - d_k\|_2^2$.
5) $d_{k+1} = d_k - (Gu_{k+1} - v_{k+1})$.
6) $k \leftarrow k + 1$.
7) *until stopping criterion is satisfied.*

Now standard ADMM is used to solve the balanced regularization problem in frame-based standard image restoration. This problem can be rewritten as the constrained optimization problem by variable splitting first:

$$\min_{x,v\in R^n} \frac{1}{2}\|BWx - y\|_2^2 + \frac{\gamma}{2}\|(I - W^TW)x\|_2^2 + \lambda^T|v|_1 \quad \text{subject to } x = v.$$

Eqn . 9

Define

$$f_1(x) = \frac{1}{2}\|BWx - y\|_2^2 + \frac{\gamma}{2}\|(I - W^TW)x\|_2^2$$

Eqn. 10

$$f_2(v) = \lambda^T|v|_1, \qquad G = I.$$

Eqn. 11

If the ADMM is applied to solve the above constrained optimization problem (9), the steps 3) – 5) in Algorithm ADMM should be replaced with

3a) $x_{k+1} = \arg\min_{x,v} \|BWx - y\|_2^2 + \gamma\|(I - W^TW)x\|_2^2 + \mu\|u - v_k - d_k\|_2^2$.
4a) $v_{k+1} = \arg\min_v \lambda^T|v|_1 + \frac{\mu}{2}\|x_{k+1} - v - d_k\|_2^2$.
5a) $d_{k+1} = d_k - (x_{k+1} - v_{k+1})$.

The minimization problem in the step 4a) with respect to v can be solved by the soft thresholding method [25] which has a closed form:

$$v_{k+1} = \text{soft}(v_k', \frac{\lambda}{\mu})$$

Eqn.12

Where $v_k' = x_{k+1} - d_k$ and

$$\text{soft}(x, \tau) = \text{sign}(x) \odot \max\{|x| - \tau, 0\}$$

Eqn. 13

′ k = xk+1 − dk and soft(x,τ) = sign(x) ⊙ max{|x| − τ, 0} (13)

with ⊙ denoting the component-wise product, i.e., $(x\odot y)_i = x_iy_i$ and sign being the signum function, which is defined as

$$\text{sign}(x) = \begin{cases} +1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0. \end{cases}$$

Eqn. 14

Note that the step 3a) is a strictly convex quadratic minimization problem with respect to x, hence it can be reduced to the following linear system:

$$\begin{aligned} x_{k+1} &= [W^TB^TBW + \gamma(I - W^TW) + \mu I]^{-1} \\ &\quad \cdot(W^TB^Ty + \mu(v_k + d_k)) \\ &= \mathcal{A}^{-1}(W^TB^Ty + \mu(v_k + d_k)) \end{aligned}$$

Eqn.15

Where

$$\mathcal{A} = W^TB^TBW + \gamma(I - W^TW) + \mu I$$

Eqn. 16

denotes the regularized version of Hessian matrix W^TB^TBW.

The matrix A can be seen as a regularized version of the Hessian of $1/2\|BWx-y\|^2_2$ by adding the terms $\gamma(I-W^T W)$ and μI. In general, the computations of this matrix and its inverse are not affordable for large-size matrices B and W, we may just take a steepest descent step instead, and that leads to the linear system being solved inexactly. This is the gradient-based algorithms such as ISTA and FISTA. However, in many standard image restorations like deblurring and inpainting problems, the matrix B has a special structure and the frame W is a tight wavelet, so the matrix-vector products can be computed quite efficiently. For example, if W is a tight wavelet frame, any matrix-vector multiplications can be performed by a fast transform algorithm .Similarly, if B represents a convolution, the matrix-vector products can be performed with the help of fast Fourier transform (FFT). Hence these facts have motivated that these special structures can be exploit to solve the linear system fast and exactly, in which the operations involving matrices are only matrix-vector products with fast algorithms. However, since the last two terms is added into A, it is not straightforward to obtain the inverse of A such that the fast computations can be employed explicitly. Then a formula is derived that can compute the inverse of A efficiently. Hence in the solution, the proposed algorithm use the second-order information of the data-fidelity function, not like gradient-based algorithms that only use the first-order information. Using the Sherman-Morrison-Woodbury matrix inversion lemma and $WW^T = I$, the following formula can be obtained,

$$\mathcal{A}^{-1} = \frac{1}{\mu}[\alpha I + (1-\alpha)W^T W - W^T \mathcal{F} W] \quad \text{Eqn. 17}$$

Where

$$\alpha = \frac{\mu}{\mu+\gamma} \quad \text{Eqn. 18}$$

$$\mathcal{F} = B^T(\mu I + BB^T)^{-1}B. \quad \text{Eqn. 19}$$

It shows that the inverse of regularized matrix A involves the tight frame W and its transpose which can be efficiently calculated by a fast wavelet transform algorithm. In view of (12), (19), (17) and (15), the following algorithm to solve the balanced regularization optimization problem in the frame-based image restoration can be obtained.

Algorithm ADMM for balanced regularization approach (ADMM-B):

1) Set $k = 0$, choose $\mu > 0$, v_0 and d_0.
2) *repeat*
3) $r_k = W^T B^T y + \mu(v_k + d_k)$.
4) $x_{k+1} = \frac{1}{\mu}(\alpha r_k + (1-\alpha)W^T W r_k - W^T \mathcal{F} W r_k)$.
5) $v_{k+1} = \text{soft}(v'_k, \frac{\lambda}{\mu})$.
6) $d_{k+1} = d_k - (x_{k+1} - v_{k+1})$.
7) $k \leftarrow k+1$.
8) *until stopping criterion is satisfied.*

IV. SUMMARY

An efficient ADMM-based algorithm for solving the balanced regularization problem in the frame-based image restoration has been presented. In this paper the analysis-based and synthesis-based approaches in image restoration are equalised by the balanced regularization approach. To solve this optimization problem more efficiently, the proposed ADMM-B algorithm is used and it exploits the fast tight frame transform algorithm and the special structures of observation matrices in the standard image restoration problems. Theoretical results have shown that the proposed ADMMB algorithm is much faster than the previous methods on a set of standard image restoration problems such as the image deblurring and inpainting.

V. REFERENCES

[1]. Cai, J. F., Osher, S., & Shen, Z., "Split Bregman methods and frame based image restoration", Multiscale modeling & simulation, ISSN no.1540-3467, vol.8, no.2, pp.337-369, Dec 9, 2009.

[2]. Hu, Y., Lu, X., & Jacob, M. "Multiple degree total variation (MDTV) regularization for image restoration", IEEE International Conference on Image Processing, ISSN no.2381-8549, pp.1958-1962, Aug 19,2016.

[3]. Fu, Y., Feng, W., & Peng, W., "Image Compressive Sensing via Multiple Constraints", International Conference on In Measuring Technology and Mechatronics Automation, ISSN no.2157-1481, pp. 327-330, June 13,2016.

[4]. Portilla, J., Tristán-Vega, A., & Selesnick, I. W. , "Efficient and Robust Image Restoration Using Multiple-Feature L2-Relaxed Sparse Analysis Priors", IEEE Transactions on Image Processing, ISSN no.1941-0042, vol.24, no.12, pp. 5046-5059, Sept 14,2015.

[5]. Dong, B., Shen, Z., & Xie, P., "Image Restoration: A General Wavelet Frame Based Model and Its Asymptotic Analysis", Feb 17, 2016.

[6]. Xie, S., Guan, C., Huang, W., & Lu, Z., "Frame-based compressive sensing MR image reconstruction with balanced regularization", IEEE International Conference on Engineering in Medicine and Biology Society, ISSN no.1558-4615, pp. 7031-7034, Nov 5, 2015.

[7]. Fan, Q., Jiang, D., & Jiao, Y., "A multi-parameter regularization model for image restoration", Signal Processing, ISSN no.0165-1684,vol.114, pp.131-142, 2015.

[8]. Mourya, R., Denis, L., Becker, J. M., & Thiébaut, E., "A blind deblurring and image decomposition approach for astronomical image restoration", In Signal Processing Conference, ISSN no.2076-1465, pp.1636-1640), Dec 28, 2015.

[9]. Bi, C., Zhang, Q., Bao, R., & Wang, H., "SAR image restoration and change detection based on game theory", International Conference on Intelligent Computing and Internet of Things, pp. 55-58, Jan, 2015.

[10]. He, C., Hu, C., Zhang, W., & Shi, B., " A fast adaptive parameter estimation for total variation image restoration", IEEE Transactions on Image Processing, ISSN no.1941-0042, vol.23, no.12, pp.4954-4967, Sept 24, 2014.

[11]. Dong, B., & Shen, Z., "Image restoration: a data-driven perspective", International Congress of Industrial and Applied Mathematics, pp. 65-108, Aug, 2015.

[12]. Sankaran, K. S., Ammu, G., & Nagarajan, V., "Non local image restoration using iterative method", International Conference on Communications and Signal Processing, ISBN no.978-1-4799-3358-7, pp. 1740-1744, Nov 10, 2014.

[13]. Dong, W., Shi, G., & Li, X., "Nonlocal image restoration with bilateral variance estimation: a low-rank approach", IEEE transactions on image processing, ISSN no.1941-0042, vol.22, no.2, pp.700-711, Oct 2, 2012.