Best Practices in Math: Modeling and Practice Word Problems (Grades K-2)





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December 2022	Best Practices in Math: Math Language and Fluency
January 24, 2023	Best Practices in Math: Modeling and Practice Word-Problem Solving
TBD	Best Practices in Math: Use of Multiple Representations



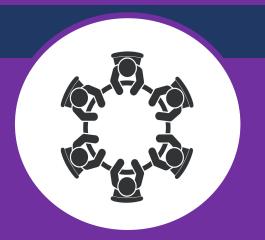


What are some of the difficulties your students have with math?



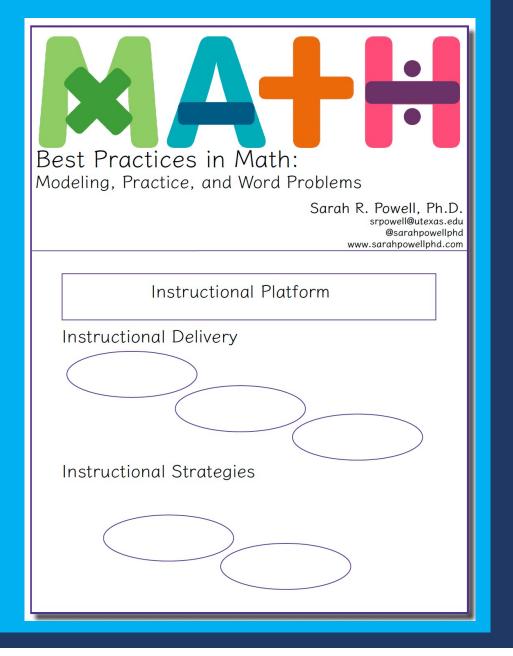


What is your mathematical language goal for January?

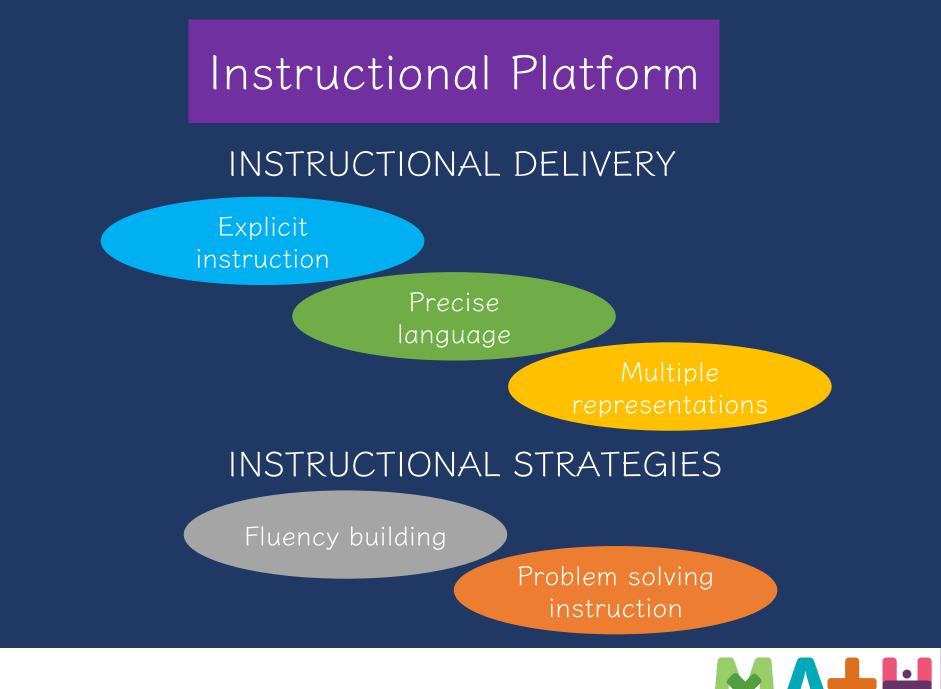


What is your fluency goal for January?









Explicit Instruction		
Research and Information		
What are your strengths?		
What are your opportunities for growth?		
What are your immediate next steps?		



Explicit Instruction: Modeling and Practice





Over a half century of research supports explicit (i.e., direct, systematic) instruction. (Stockard et al., 2018)

When compared to discovery approaches, explicit instruction demonstrates higher outcomes.(Alfieri et al., 2011; Kroesbergen et al., 2004; Poncy et al., 2010)

Numerous meta-analyses and large-scale studies have identified explicit instruction as essential for the teaching and learning of mathematics. (Chodura et al., 2015; Ennis & Losinski, 2019; Jitendra et al., 2018; Kong et al., 2021; Morgan et al., 2015; Nelson & McMaster, 2019; Powell et al., 2021).



Explicit Instruction (teaching in a direct and structured way)

foundational knowledge

Inquiry (seeking information)

expertise and transfer

XA+H

Explicit Instruction			
MODELING	PRACTICE		
SUPPORTS			



Step-by-step explanation

PRACTICE

Guided practice

Independent practice

Planned examples

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback



Modeling is a dialogue between the teacher and students.

MODELING

Step-by-step explanation

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Modeling includes a step-by-step explanation of how to do a math problem.

> A teacher may do 1 modeled problem or several.

MODELING	PRACTICE			
Step-by-step explanation	Guided practice Independent practice			
Planned examples				
SUPPORTS Ask high-level and low-level questions				
Eliciting frequent responses				
Providing affirmative and corrective feedback				





"Today, we are learning about addition. This is important because sometimes you have different amounts – like money – and you want to know how much money you have altogether."







"Let's solve this problem. What's the problem?"

"26 plus 79."

"To solve 26 plus 79, first decide about the operation. Should we add, subtract, multiply, or divide?"

"How did you know we want to add?" "Add."

"There's a plus sign."





"The plus sign tells us we want to add. To add, let's use the partial sums strategy. What strategy?"

"What might partial mean?"

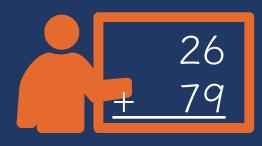
"We'll find parts – or partial sums – then add them together. With the partial sums strategy, we start adding in the greatest place value. What's the greatest place value in this problem?" "Partial sums."

"Part of, only some of."









"So, let's add the tens. What's 20 plus 70? Work with a partner and use your base-10 blocks or other tools."

"20 plus 70 equals 90. Let's write 90 right here below the equal line. What will we write?"

"90 is the partial sum when you add the tens. What does 90 represent?"

"Now, let's add the ones. What should we add?" "90."

"90."

"It's the partial sum of adding 20 plus 70."

XA++

"6 plus 9."



"6 plus 9 equals what? Work with a partner. Use your base-10 blocks or other tools."

"How did you get 15?"

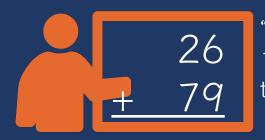
"Let's write 15 below the 90. Where do we write the 15?"

"15 is the partial sum when you add the ones. Now, let's add the partial sums together. What will we add?" "15."

"We knew we had 9, then we added on 6."

> "Below the 90."





"What's 90 plus 15? Use your goto strategy."

"How did you add those addends?"

"So, when you add 26 plus 79, the sum is 105. Who can share how we solved this problem?"



"I added 90 plus 10 then added 5 more."

"We used the partial sums strategy. We added the tens then added the ones. Then we added the partial sums."







What did you observe?

How would you improve this example?



Modeling needs to include planned examples.

These examples should be sequenced so easier skills lead to more difficult skills.

MODELING PRACTICE Step-by-step Guided practice explanation Independent practice Planned examples **SUPPORTS** Ask high-level and low-level questions Eliciting frequent responses Providing affirmative and corrective feedback



Step-by-step explanation

PRACTICE

Guided practice

Independent practice

Planned examples

SUPPORTS Ask high-level and low-level questions

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Share some of the math content that is important to model step-by-step.



Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

Practice continues as a dialogue between the teacher and students.



MODELING Step-by-step explanation Planned examples	PRACTICE Guided practice Independent practice	Guided practice is practice in which the teacher and
SUPPORTS		students
Ask high-level and low-level questions		practice
Eliciting frequent responses		problems
Providing affirmative and corrective feedback		together.





Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback



"Now, you'll practice a problem on your own. Use your attack strategy!" Independent practice is practice in which the students practice independently with teacher support.



Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

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Describe how you would engage students in practice.



Step-by-step explanation

PRACTICE

Guided practice

Independent practice

Planned examples

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

These **Supports** should be used in both **Modeling** and **Practice**.



Step-by-step explanation

PRACTICE

Guided practice

Independent practice

Planned examples

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

During **Modeling** and **Practice**, it is essential to engage students and check for understanding.



Step-by-step explanation

PRACTICE

Guided practice

Independent practice

Planned examples

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

"What is 7 times 9?"

"63."



Step-by-step explanation

PRACTICE

Guided practice

Independent practice

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SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

"Because a positive 1 and a negative 1 equal 0. I use the zero pair to help me subtract."



"Why do you use zero pairs?"

Step-by-step explanation

PRACTICE

Guided practice

Independent practice

Planned examples

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

During **Modeling** and **Practice**, students should frequently respond. The frequent responses keeps student attention and keeps student learning active.



Step-by-step explanation

PRACTICE

Guided practice

Independent practice

Planned examples

SUPPORTS

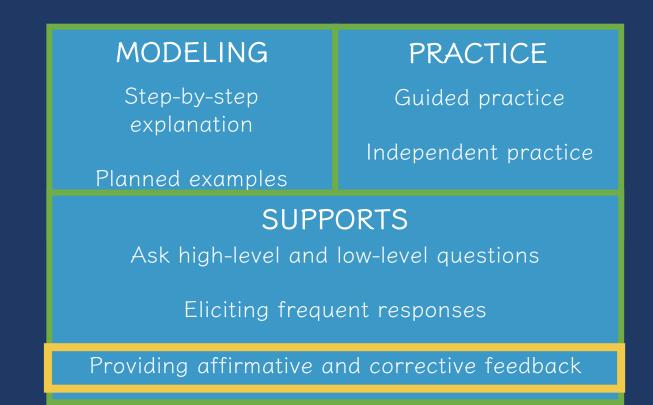
Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback







During **Modeling** and **Practice**, students should receive immediate feedback on their responses.



MODELING

Step-by-step explanation

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Planned examples

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

"Nice work using your word problem attack strategy."

XA+H

MODELING

Step-by-step explanation

PRACTICE

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Planned examples

SUPPORTS Ask high-level and low-level questions

Eliciting frequent responses

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"Let's look at that again. Tell me how you added in the hundreds column."



MODELING

Step-by-step explanation

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Explicit Instru	ction
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Provide several types of questions you ask. Provide examples of your feedback.



Explicit Instruction

Research and Information

What are your strengths?

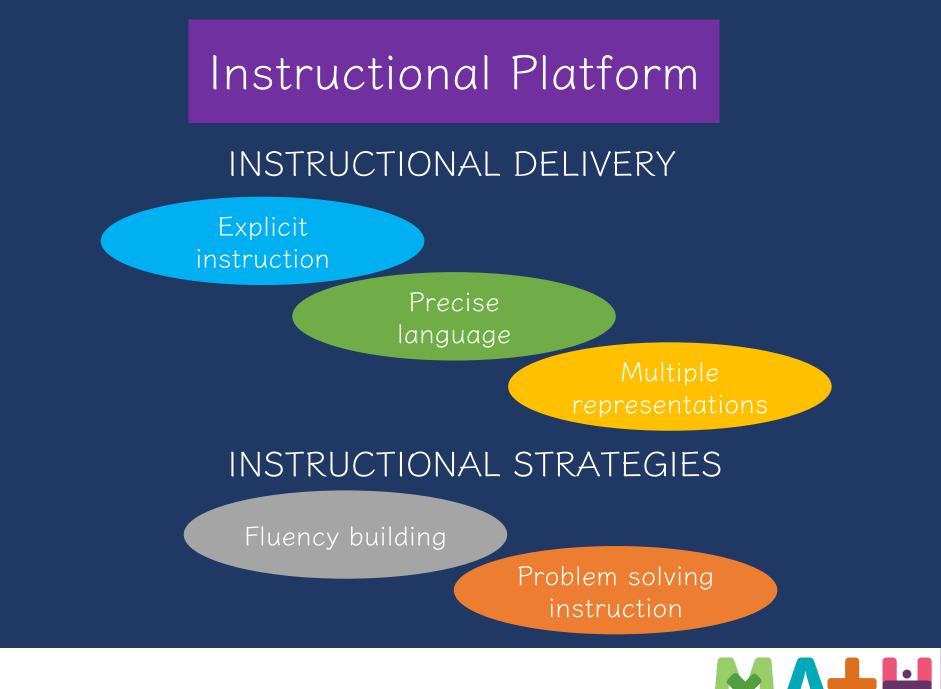
What are your opportunities for growth?

What are your immediate next steps?



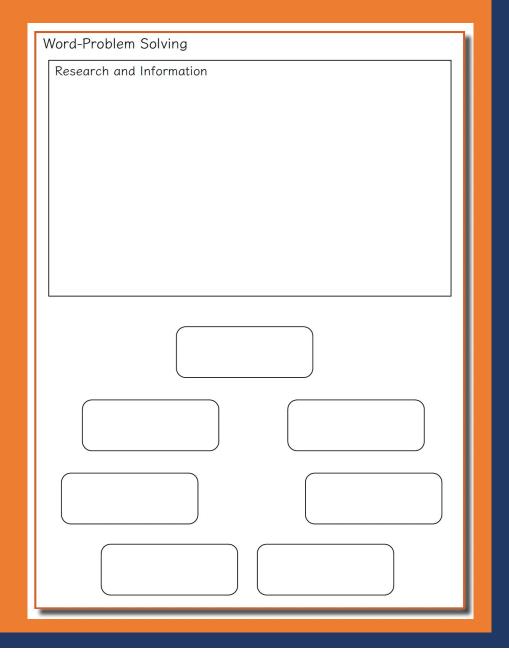
What are strengths and opportunities for growth?





Word-Problem Solving









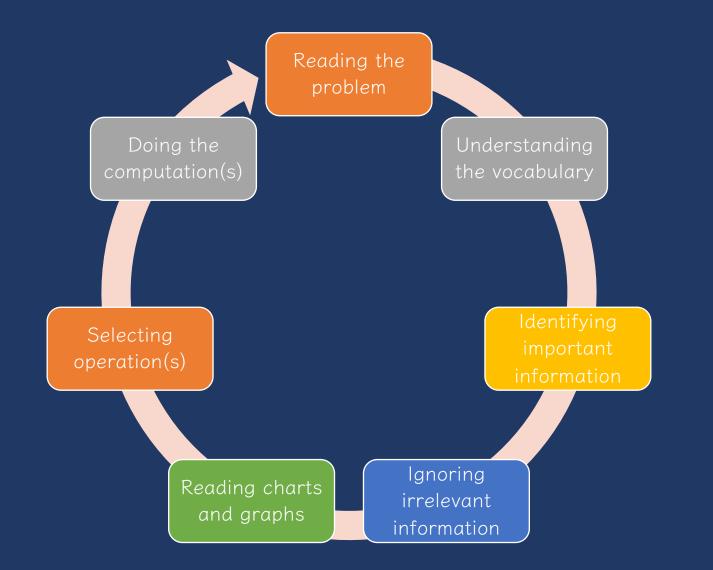
Key words tied to operations is an ineffective word-problem strategy. (Karp et al., 2019; Powell et al., 2022)

Using a meta-cognitive strategy improves wordproblem performance. (Freeman-Green et al., 2015; Krawec et al., 2012; Montague et al., 2011; Swanson et al., 2014)

A focus on schemas improves word-problem performance.

(Alghamdi et al., 2020; Cook et al., 2020; Flores et al., 2016; Fuchs et al., 2021; Griffin et al., 2019; Jitendra et al., 2013; Lein et al., 2020; Peltier et al., 2020; Powell et al., 2022; Xin & Xhang, 2009; Zheng et al., 2013)







Ineffective Strategies



1.Keywords tied to operations

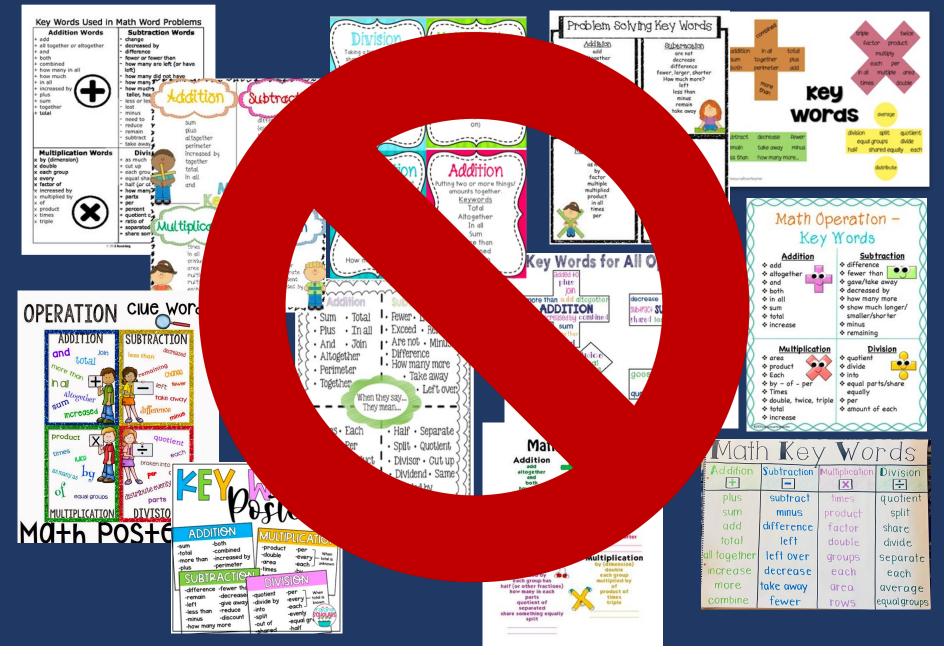




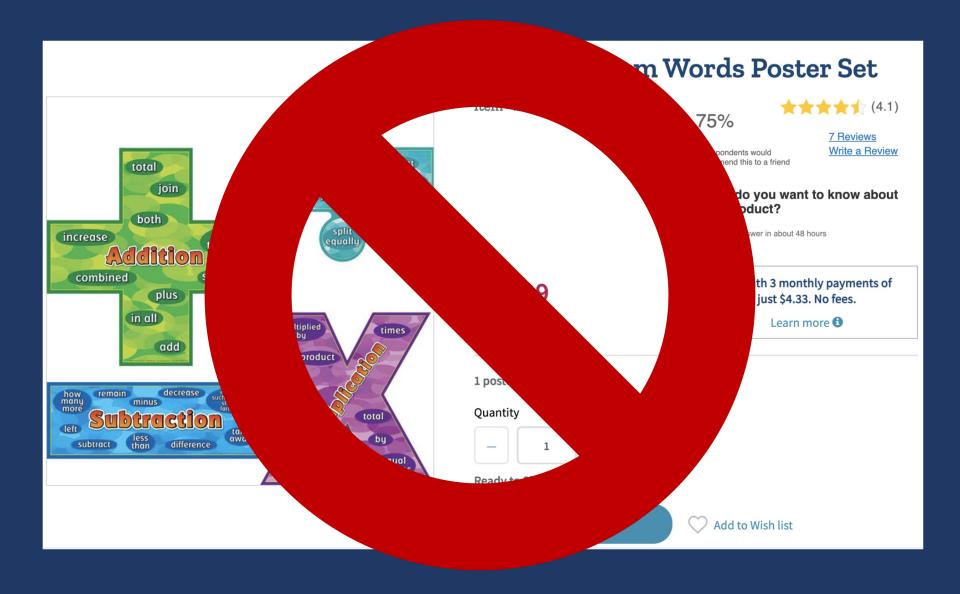
Lincoln had 8 pencils **fewer** than Roscoe. If Roscoe had 18 pencils, how many pencils did Lincoln have?

Lincoln had 8 pencils **fewer** than Roscoe. If Lincoln had 18 pencils, how many pencils did Roscoe have?





XA+H







Description of Single-Step Word Problems (n = 132)											
				Schema-				Keyword(s) led			
	Occurrence of schema		Any		specific		Multiple		to correct		
			keyword		keywords ^a		keywords ^a		solution		
Schema	n	%	n	%	n	%	n	%	n	%	
Total	27	20.5	26	96.3	23	88.5	5	19.2	21	80.8	Π
Difference	17	12.9	17	100.0	14	82.4	2	11.8	12	70.6	
Change	11	8.3	7	63.6	5	71.4	5	71.4	2	28.6	
Equal groups	29	22.0	26	89.7	22	84.6	18	69.2	8	30.8	
Comparison	10	7.6	9	90.0	9	100.0	4	44.4	5	55.6	
Ratios or proportions	29	22.0	23	79.3	9	39.1	9	39.1	6	26.1	
Product of measures	9	6.8	9	100.0	8	88.9	1	11.1	5	55.6	
^a When a problem featured a keyword.											

×A++



Description of Multi-Step Word Problems (n = 84)							
	_	Any keywor	ď	Keyword(s) led to correct solution ^b			
n	%	n	%	n	%		
40	47.6	39	97.5	3	7.7		
11	13.1	11	100.0	1	9.1		
21	23.8	19	95.0	1	5.3		
49	58.3	48	98.0	1	2.1		
7	8.3	7	100.0	0	0.0		
22	25.0	16	76.2	1	6.3		
7	8.3	7	100.0	2	28.6		
	Occurren schem n 40 11 21 49 7	Occurrence of schema* <u>n %</u> 40 47.6 11 13.1 21 23.8 49 58.3 7 8.3 22 25.0	Occurrence of schema* Any keywor n n 40 47.6 39 11 13.1 11 21 23.8 19 49 58.3 48 7 8.3 7 22 25.0 16	Occurrence of schema* Any keyword n % n % 40 47.6 39 97.5 11 13.1 11 100.0 21 23.8 19 95.0 49 58.3 48 98.0 7 8.3 7 100.0 22 25.0 16 76.2	Occurrence of schema* Any keyword Keyword(s) I correct solu n % n 40 47.6 39 97.5 3 11 13.1 11 100.0 1 21 23.8 19 95.0 1 49 58.3 48 98.0 1 7 8.3 7 100.0 0 22 25.0 16 76.2 1		

^aSum across schemas does not equal 100 because each word problem featured more than one schema.

^bWhen a problem featured a keyword.



Mr. Rivera's taxable income is \$20 each hour before taxes are taken out. Mr. Rivera worked a total of 40 hours each week for 50 weeks.

What is the dollar amount, to the nearest dollar, taken out for taxes based on Mr. Rivera's taxable income?

Jessica rented 1 video game and 3 movies for a total of \$11.50.

- The video game cost \$4.75 to rent.
- The movies cost the same amount each to rent.

What amount, in dollars, did Jessica pay to rent each movie?

The temperature of a substance decreased by 24°C per minute for 3 minutes. What was the overall change of the temperature of the substance?



Keywords are important to identify and understand

Keywords are the mathematical vocabulary that help an students understand what the story is about and what they need to do Talk about keywords ("What does *more than* tell you about?")

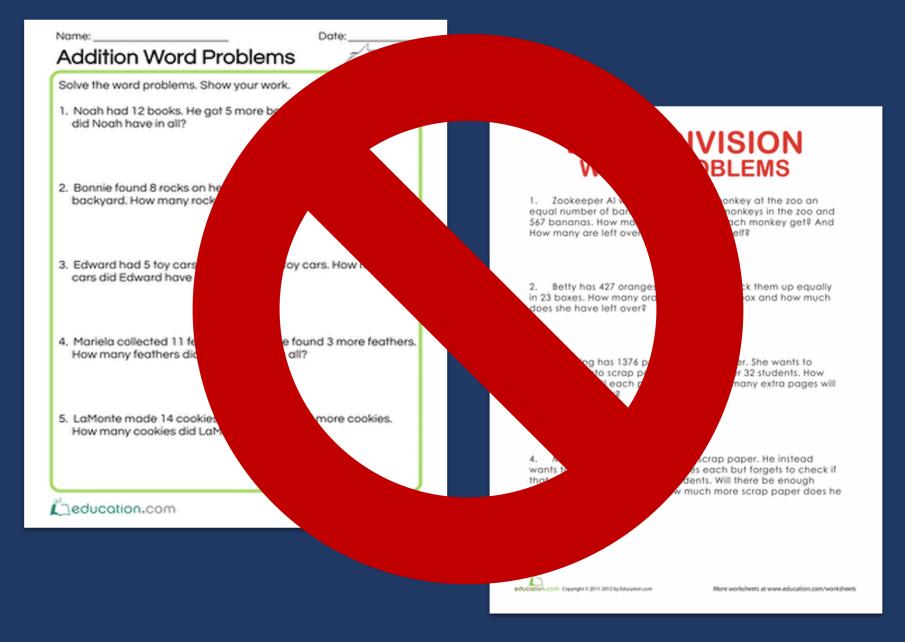


But, *do not* tie a keyword to a specific operation!



2. Presenting problems by operation







Effective Strategies



Teach an attack strategy

Teach about schemas



Word-Problem Solving

SOLVE

Study the problem.

Organize the facts.

Line up the plan.

Verify the plan with computation.

Examine the answer.

R-CUBES

Read the problem. Circle key numbers. Underline the question. Box action words. Evaluate steps. Solve and check.



PLAN How will you solve the problem?



CHECK Does your answer make sense?



RIDE

Read the problem. Identify the relevant information. Determine the operation and unit for the answer. Enter the correct numbers and calculate, then check the answer.

RIDGES

Read the problem. I know statement. Draw a picture. Goal statement. Equation development. Solve the equation.



STAR

Stop and read the problem carefully.

Think about your plan and the strategy you will use. Act. Follow your plan and solve

the problem.

Review your answer.

RICE

Read and record the problem. Illustrate your thinking. Compute. Explain your thinking.



SUPER

Slowly read the story problem twice. Underline the question and circle the numbers you need. Picture it. Draw the scenario to show what is happening. Explain the problem with a number sentence. Rewrite the answer in a sentence.

SHINES

Slowly and carefully read the problem. Highlight or underline key information. Identify the question by drawing a circle around it. Now solve the problem. Show your work. Examine your work for precision, accuracy, and clarity. Share your answer by writing a sentence.



SOLVE

Study the problem.

Organize the facts.

Line up the plan.

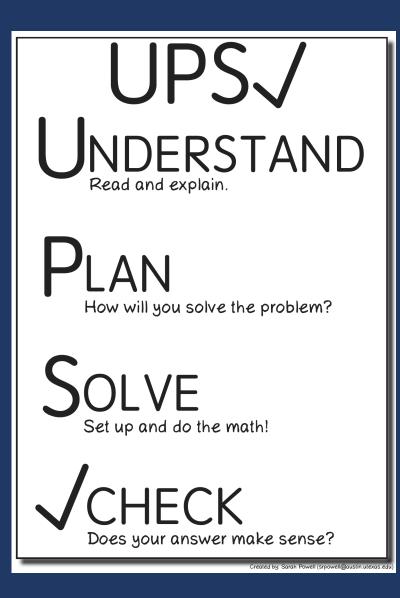
Verify the plan with computation.

Examine the answer.

R-CUBES

Read the problem. Circle key numbers. Underline the question. Box action words. Evaluate steps. Solve and check.









Share your favorite attack strategy.



Teach an attack strategy

Teach about schemas





Difference

Change

Equal Groups

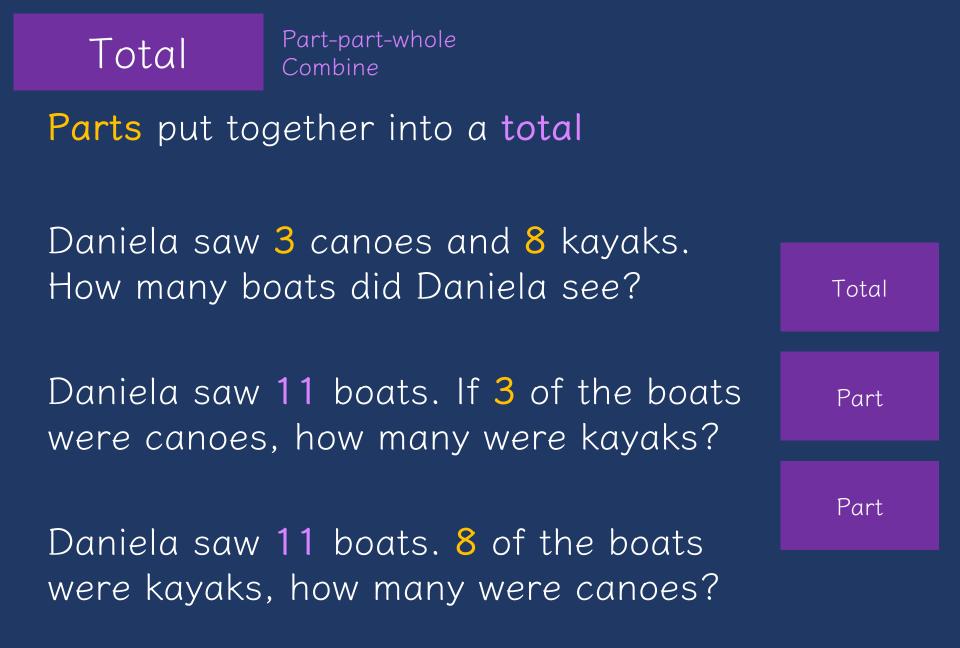
Comparison

Ratios/Proportions



Schema and Definition	Equations and Graphic Organizers	Examples			Variations
Total (Combine; Part-part- whole) Parts combined for a sum	P1 + P2 = T (part + part = total) (total) (part) (part)	Sum unknown: Lyle has 11 red apples and 18 green apples. How many apples does Lyle have altogether?	Part unknown: Lyle has 29 red and green apples. If 11 of the apples are red, how many green apples does Lyle have?		More than two parts: Lyle has 34 apples. Of the apples, 11 are red, 18 are green, and the rest are yellow. How many yellow apples does Lyle have?
Difference (Compare) Sets compared for a difference	B - S = D (bigger - ansiler = difference) G - L = D (prester - less = difference) (greater) (greater) (lesser) (difference)	Difference un- known: Sasha wrote 85 words in her essay, and Tabitha wrote 110 words. How many fewer words did Sasha write than Tabitha?	Bigger/greater unknown: Tabitha wrote 25 more words than Sasha. If Sasha wrote 85 words, how many words did Tabitha write?	Smaller/lesser unknown: Tabitha wrote 110 words in her essay. Sasha wrote 25 words fewer than Tabitha. How many words did Sasha write?	(None)
Change (Join; Separate) An amount that increases or decreases	ST +/- C = E (start +/- change = end)	End (increase) unknown: Jorge had \$52. Then, he earned \$16 babysitting. How much money does Jorge have now?	Change (increase) unknown: Jorge had \$52. Then, he earned some money babysitting. Now, Jorge has \$68. How much did Jorge earn babysitting?	Start (increase) unknown: Jorge has some money, and then he earned \$16 for babysitting. Now, Jorge has \$68. How much money did he have to start with?	Multiple changes: Jorge had \$78. He stopped and bought a pair of shoes for \$42 and then he spent \$12 at the grocery. How much money does Jorge have now?
	(beginning) (end)	End (decrease) unknown: Jorge had \$52. Then, he spent \$29 at the ballpark. How much money does Jorge have now?	Change (decrease) unknown: Jorge had \$52 but spent some money when he went to the ballpark. Now, Jorge has \$23. How much did Jorge spend at the ballpark?	Start (decrease) unknown: Jorge had some money. Then, he spent \$29 at the ballpark and has \$23 left. How much money did Jorge have before going to the ballpark?	



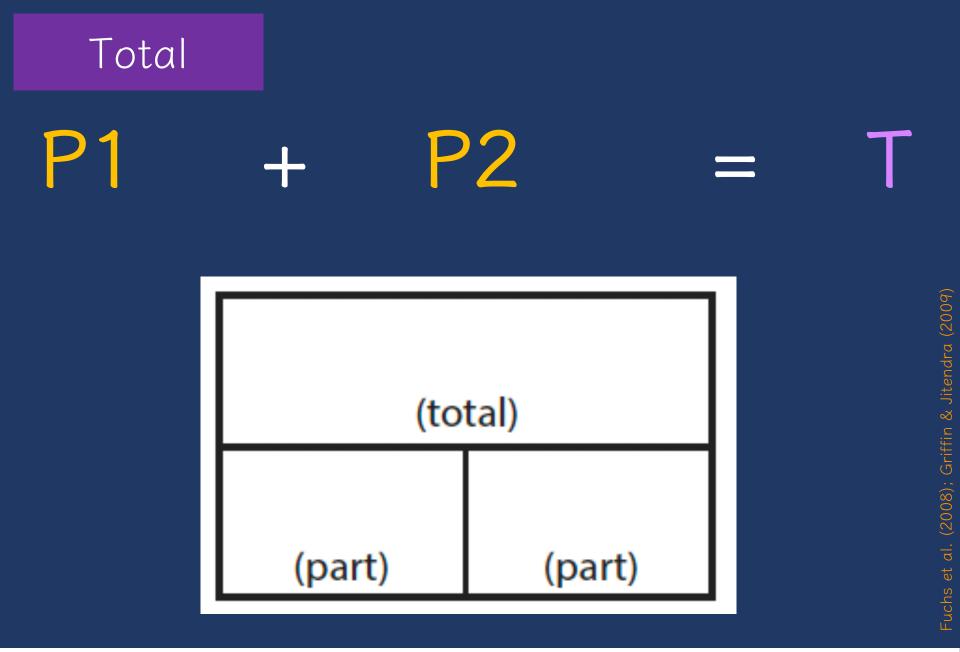


XA+H



"Are parts put together for a total?"







Total

Β.

In March and April, it rained a total of 11.4 inches. If it rained 3.9 inches in March, how many inches did it rain in April?

PI + P2 = TU 3.9 + ? = 11.4 Pr SV ?= 7.5 inches



Total



Share a Total problem.



Difference Compare

Greater and **lesser** amounts compared for a **difference**

Adrianna has 10 pencils. Tracy has 4 pencils. How many more pencils does Adrianna have?

Adrianna has 6 more pencils than Tracy. If Tracy has 4 pencils, how many does Adrianna have?

Tracy has 6 fewer pencils than Adrianna. Adrianna has 10 pencils. How many pencils does Tracy have? Difference

Greater amount

Lesser amount





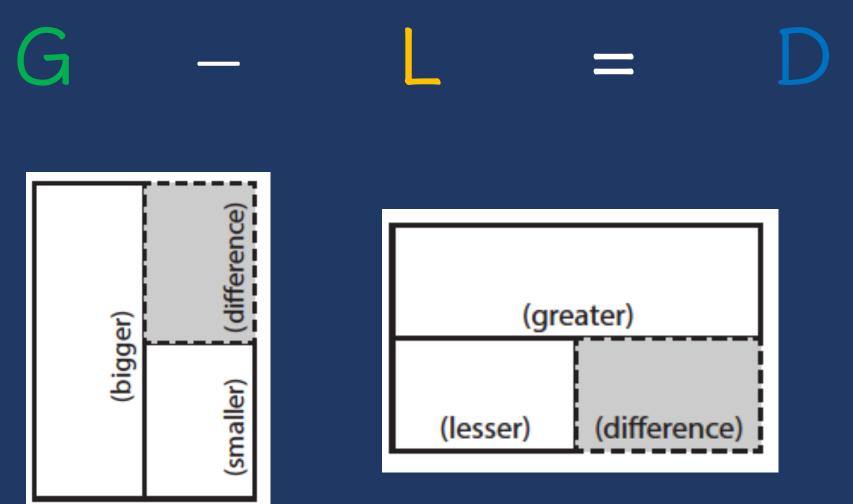
"Are parts put together for a total?"

Difference

"Are amounts compared for a difference?"



Difference



chs et al. (2008); Griffin & Jitendra (2009)

Difference

Jana has 107 wooden beads and 68 glass beads. How many more wooden beads than glass beads does Jana have?

Enter your answer in the response box.



Difference



Share a Difference problem.



An amount that increases or decreases

Nickole had 6 notebooks. Then, she bought 3 notebooks. How many notebooks does Nickole have now?

Nickole had 6 notebooks. Then, she bought a few more notebooks. Now, Nickole has 9 notebooks. How many notebooks did she buy?

Nickole had some notebooks. Then, she bought 3 notebooks. Now, Nickole has 9 notebooks. How many notebooks did she have to start with? Change amount

Start amount



Separate

An amount that increases or decreases

Samantha baked 20 cookies. Then, she ate 3 of the cookies. How many cookies does Samantha have now?

Samantha baked 20 cookies. Then, she ate some of the cookies. Now, she has 17 cookies. How many cookies did Samantha eat?

Samantha baked some cookies. She ate 3 of the cookies and has 17 cookies left. How many cookies did Samantha bake?

End amount

Change amount

Start amount



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Change



"Are parts put together for a total?"

Difference

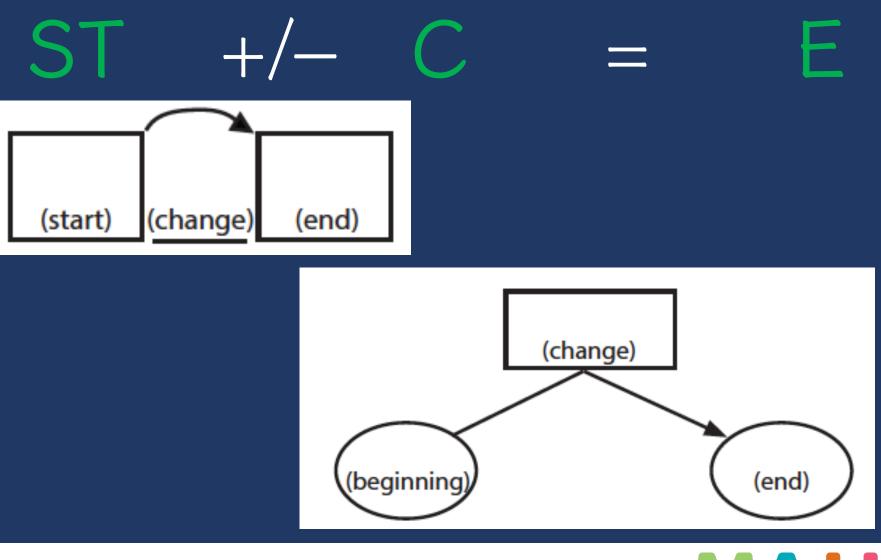
"Are amounts compared for a difference?"

Change

'Does an amount increase or decrease?"

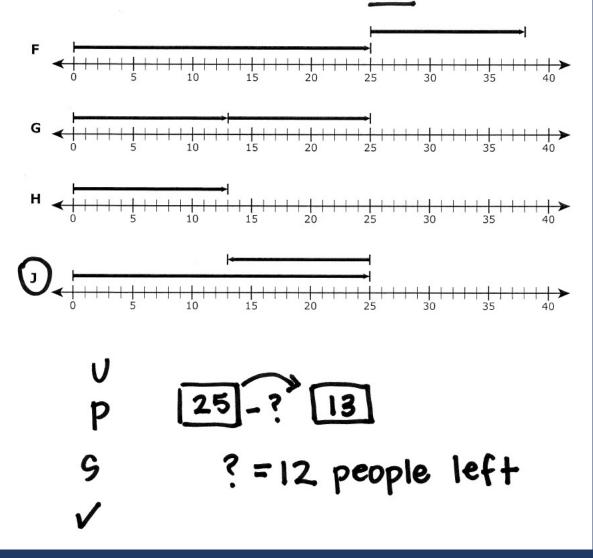






Change

28 There were 25 people in a library. Some people left the library and went home. Then there were 13 people remaining in the library. Which number line represents one way to determine the number of people who left the library?





Change



Share a Change problem.





Schema Check!





Pablo goes to a stamp show where he can share, buy, and sell stamps.

26. Part A

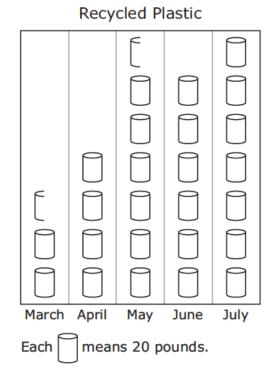
The first day, Pablo starts with 744 stamps. He buys 27 stamps from his friend. He then sells 139 stamps.

What is the total number of stamps that Pablo has after the first day of the stamp show?



Difference

The graph below shows the number of pounds of plastic the Keller family recycled for five months.



Based on the graph, how many more pounds of plastic did the family recycle in July than in April?

0

s adde

Total

Mr. Conley delivers packages. The bar graph shows the total number of packages he delivered on five days last week.



Package Delivery

10. Part A

What is the total number of packages Mr. Conley delivered on Monday and Tuesday?

- A 300
- B 340
- © 350
- 360
 360



	-	-			
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Difference

Change

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Comparison

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Teach an attack strategy

Teach about schemas



Grades K-2 Word Problems					
A. Ali delivered 12 boxes of cookies on Friday and 25 boxes of cookies on Saturday. How many boxes of cookies did Ali deliver?	B. Hui saved \$70 in January. In February, she spent \$64 of the money she saved. She saved \$92 more in March. How much has Hui saved by the end of March?				
C. A plant was 3 inches tall at the beginning of June. By the end of July, the plant was 9 inches tall. How many inches did the plant grow in 2 months?	D. Audrey has 62 wooden beads and 95 glass beads. What is the difference between Audrey's wooden beads and glass beads?				





What is your modeling and practice goal for this winter?



What is your word-problem goal for this winter?





December 2022	Best Practices in Math: Math Language and Fluency
January 24, 2023	Best Practices in Math: Modeling and Practice Word-Problem Solving
TBD	Best Practices in Math: Use of Multiple Representations



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