## Best Practices in Math:

 Modeling and Practice Word Problems (Grades K-2)

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## Schedule for This Year

| December 2022 | Best Practices in Math: <br> Math Language and Fluency |
| :--- | :--- |
| January 24, 2023 | Best Practices in Math: <br> Modeling and Practice <br> Word-Problem Solving |
| TBD | Best Practices in Math: <br> Use of Multiple Representations |

What are some of the difficulties your students have with math?

What is your mathematical language goal for January?

What is your fluency goal for January?


Best Practices in Math: Modeling, Practice, and Word Problems

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Instructional Delivery


Instructional Strategies


## Instructional Platform

INSTRUCTIONAL DELIVERY


INSTRUCTIONAL STRATEGIES
Fluency building

Explicit Instruction
Research and Information

What are your strengths?

What are your opportunities for growth?

What are your immediate next steps?

Explicit Instruction: Modeling and Practice
x $\mathrm{A}+\dot{1}$

Over a half century of research supports explicit (i.e., direct, systematic) instruction. (Stockard et al., 2018)

When compared to discovery approaches, explicit instruction demonstrates higher outcomes. (Alfieri et al., 2011; Kroesbergen et al., 2004; Poncy et al., 2010)

Numerous meta-analyses and large-scale studies have identified explicit instruction as essential for the teaching and learning of mathematics. (Chodura et al., 2015; Ennis \& Losinski, 2019; Jitendra et al.. 2018; Kong et al., 2021; Morgan et al., 2015; Nelson \& McMaster, 2019; Powell et al., 2021).

## Explicit Instruction (teaching in a direct and structured way)

foundational knowledge

Inquiry (seeking information)


## MODELING

Step-by-step explanation

## PRACTICE

Guided practice

Independent practice
Planned examples

## SUPPORTS

Ask high-level and low-level questions
Eliciting frequent responses
Providing affirmative and corrective feedback

| Modeling is a dialogue between the teacher and students. | MODELING <br> Step-by-step explanation <br> Planned examples | PRACTICE <br> Guided practice <br> Independent practice |
| :---: | :---: | :---: |
|  | SU <br> Ask high-level <br> Eliciting fre <br> Providing affirmativ | RTS <br> w-level questions <br> t responses <br> corrective feedback |


| Modeling |
| ---: | :---: | :---: |
| includes a |$\quad$ MODELING $\quad$ PRACTICE

"Today, we are learning about addition. This is important because sometimes you have different amounts - like money - and you want to know how much money you have altogether."

## 9

"Let's solve this problem. What's the problem?"
"To solve 26 plus
79 , first decide
about the operation.
Should we add, subtract, multiply, or divide?"
"How did you know we want to add?"
"There's a plus, sign."
"The plus sign tells us we want to add. To add, let's use the partial sums strategy. What strategy?"
"What might partial mean?"
"We'll find parts - or partial sums - then add them together. With the partial sums strategy, we start adding in the greatest place value. What's the greatest place value in this problem?"
"Part of, only some of."
"The tens."
$x A+H$
"So, let's add the tens.
What's 20 plus $70 ?$
Work with a partner and use your base-10 blocks or other tools."

"20 plus 70 equals 90 . Let's write 90 right here below the equal line. What will we write?"

" 90 is the partial sum when you add the tens. What does 90 represent?"
"It's the partial sum of adding 20 plus 70 ."
"Now, let's add the ones. What should we add?"

"We knew we had 9, then we added on 6."

"15 is the partial sum when you add the ones. Now, let's add the partial sums together.
What will we add?"
"How did you add those addends?"
"So, when you add 26 plus 79 , the sum is 105. Who can share how we solved this problem?"
"I added 90 plus 10 then added 5 more.
"We used the partial sums strategy. We added the tens then added the ones. Then we added the
tial sums." partial sums."
 "105." $\square \square \square$


## What did you observe?

How would you improve this example?


## MODELING

Step-by-step explanation

Planned examples

## SUPPORTS

Ask high-level and low-level questions
Eliciting frequent responses
Providing affirmative and corrective feedback


## MODELING <br> Step-by-step explanation <br> Planned examples <br> PRACTICE <br> Guided practice <br> Independent practice <br> SUPPORTS <br> Ask high-level and low-level questions <br> Eliciting frequent responses

Providing affirmative and corrective feedback
"Let's work on a problem together."

## MODELING <br> Step-by-step explanation <br> Planned examples <br> SUPPORTS <br> Ask high-level and low-level questions <br> Eliciting frequent responses <br> Providing affirmative and corrective feedback <br> "Now, you'll practice a problem on your own. Use your attack strategy!"

## PRACTICE

Guided practice

Independent practice

Independent practice is practice in which the students practice independently with teacher support.

## MODELING

Step-by-step explanation

Planned examples

## SUPPORTS

Ask high-level and low-level questions
Eliciting frequent responses
Providing affirmative and corrective feedback


Describe how you would engage students in practice.

## MODELING <br> Step-by-step explanation <br> PRACTICE <br> Guided practice <br> Independent practice <br> Planned examples <br> SUPPORTS <br> Ask high-level and low-level questions <br> Eliciting frequent responses <br> Providing affirmative and corrective feedback

These Supports should be used in both Modeling and Practice.

## MODELING

Step-by-step explanation

## PRACTICE

Guided practice

Independent practice

## Planned examples

## SUPPORTS

Ask high-level and low-level questions
Eliciting frequent responses
Providing affirmative and corrective feedback

During Modeling and Practice, it is essential to engage students and check for understanding.

## MODELING

Step-by-step explanation

## PRACTICE

Guided practice

Independent practice

Planned examples

## SUPPORTS

Ask high-level and low-level questions
Eliciting frequent responses
Providing affirmative and corrective feedback
"What is 7 times 9?"

## MODELING

Step-by-step explanation

## PRACTICE

Guided practice

Independent practice

## Planned examples

## SUPPORTS

Ask high-level and low-level questions

## Eliciting frequent responses

Providing affirmative and corrective feedback
"Why do you use zero pairs?"
"Because a positive 1 and a negative 1 봅ㅂ equal 0 . I use the zero pair to help me subtract."
$x A+H$

## MODELING

Step-by-step explanation

## PRACTICE

Guided practice
Independent practice
Planned examples

## SUPPORTS

Ask high-level and low-level questions

## Eliciting frequent responses

## Providing affirmative and corrective feedback

During Modeling and Practice, students should frequently respond. The frequent responses keeps student attention and keeps student learning active.

## MODELING

Step-by-step explanation

## PRACTICE

Guided practice

Independent practice

Planned examples

## SUPPORTS

Ask high-level and low-level questions

## Eliciting frequent responses

Providing affirmative and corrective feedback


## MODELING

Step-by-step explanation

## PRACTICE

Guided practice

Independent practice
Planned examples

## SUPPORTS

Ask high-level and low-level questions
Eliciting frequent responses

## Providing affirmative and corrective feedback

During Modeling and Practice, students should receive immediate feedback on their responses.

## MODELING

Step-by-step explanation

## PRACTICE

Guided practice

Independent practice

Planned examples

## SUPPORTS

Ask high-level and low-level questions
Eliciting frequent responses

## Providing affirmative and corrective feedback

"Nice work using your word problem attack strategy."

## MODELING

Step-by-step explanation

## PRACTICE

Guided practice

Independent practice

Planned examples

## SUPPORTS

Ask high-level and low-level questions
Eliciting frequent responses

## Providing affirmative and corrective feedback

"Let's look at that again.
Tell me how you added in the hundreds column."

## MODELING

Step-by-step explanation

## PRACTICE

Guided practice

Independent practice

## Planned examples

## SUPPORTS

Ask high-level and low-level questions
Eliciting frequent responses
Providing affirmative and corrective feedback


Provide several types of questions you ask. Provide examples of your feedback.

Explicit Instruction
Research and Information

What are your strengths?

What are your opportunities for growth?

What are your immediate next steps?

## Instructional Platform

INSTRUCTIONAL DELIVERY


INSTRUCTIONAL STRATEGIES
Fluency building



Key words tied to operations is an ineffective word-problem strategy. (Karp et al., 2019; Powell et al., 2022)

Using a meta-cognitive strategy improves wordproblem performance.
(Freeman-Green et al., 2015; Krawec et al., 2012; Montague et al., 2011; Swanson et al., 2014)

A focus on schemas improves word-problem performance.
(Alghamdi et al., 2020; Cook et al., 2020; Flores et al., 2016; Fuchs et al., 2021; Griffin et al., 2019; Jitendra et al., 2013; Lein et al., 2020; Peltier et al., 2020; Powell et al., 2022; Xin \& Xhang, 2009; Zheng et al., 2013)



## 1.Keywords tied to operations



> Lincoln had 8 pencils fewer than Roscoe. If Roscoe had 18 pencils, how many pencils did Lincoln have?

Lincoln had 8 pencils fewer than Roscoe. If Lincoln had 18 pencils, how many pencils did Roscoe have?




Description of Single-Step Word Problems ( $n=132$ )

| Schema | Occurrence of schema |  | Any keyword |  | Schemaspecific keywords ${ }^{\text {a }}$ |  | Multiple keywords ${ }^{3}$ |  | Keyword(s) led to correct solution ${ }^{\text {a }}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $n$ | \% | $n$ | \% | $n$ | \% | $n$ | \% | $n$ | \% |
| Total | 27 | 20.5 | 26 | 96.3 | 23 | 88.5 | 5 | 19.2 | 21 | 80.8 |
| Difference | 17 | 12.9 | 17 | 100.0 | 14 | 82.4 | 2 | 11.8 | 12 | 70.6 |
| Change | 11 | 8.3 | 7 | 63.6 | 5 | 71.4 | 5 | 71.4 | 2 | 28.6 |
| Equal groups | 29 | 22.0 | 26 | 89.7 | 22 | 84.6 | 18 | 69.2 | 8 | 30.8 |
| Comparison | 10 | 7.6 | 9 | 90.0 | 9 | 100.0 | 4 | 44.4 | 5 | 55.6 |
| Ratios or proportions | 29 | 22.0 | 23 | 79.3 | 9 | 39.1 | 9 | 39.1 | 6 | 26.1 |
| Product of measures | 9 | 6.8 | 9 | 100.0 | 8 | 88.9 | 1 | 11.1 | 5 | 55.6 |
| ${ }^{3}$ When a problem featured a keyword. |  |  |  |  |  |  |  |  |  |  |



| Schema | Occurrence of schema ${ }^{\text {a }}$ |  | Any keyword |  | Keyword(s) led to correct solution ${ }^{\text {b }}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $n$ | \% | $n$ | \% | $n$ | \% |
| Total | 40 | 47.6 | 39 | 97.5 | 3 | 7.7 |
| Difference | 11 | 13.1 | 11 | 100.0 | 1 | 9.1 |
| Change | 21 | 23.8 | 19 | 95.0 | 1 | 5.3 |
| Equal groups | 49 | 58.3 | 48 | 98.0 | 1 | 2.1 |
| Comparison | 7 | 8.3 | 7 | 100.0 | 0 | 0.0 |
| Ratios or proportions | 22 | 25.0 | 16 | 76.2 | 1 | 6.3 |
| Product of measures | 7 | 8.3 | 7 | 100.0 | 2 | 28.6 |

${ }^{3}$ Sum across schemas does not equal 100 because each word problem featured more than one schema.
${ }^{\text {b }}$ When a problem featured a keyword.

Mr. Rivera's taxable income is $\$ 20$ each hour before taxes are taken out. Mr. Rivera worked a total of 40 hours each week for 50 weeks.

What is the dollar amount, to the nearest dollar, taken out for taxes based on Mr. Rivera's taxable income?

Jessica rented 1 video game and 3 movies for a total of $\$ 11.50$.

- The video game cost $\$ 4.75$ to rent.
- The movies cost the same amount each to rent.

What amount, in dollars, did Jessica pay to rent each movie?

The temperature of a substance decreased by $24^{\circ} \mathrm{C}$ per minute for 3 minutes. What was the overall change of the temperature of the substance?

Keywords are important to identify and understand

## Keywords are the mathematical

 vocabulary that help an students understand what the story is about and what they need to doTalk about keywords
("What does more than tell you about?")


But, do not tie a keyword to a specific operation!
2. Presenting problems by operation

## Addition Word Problems




## Teach an attack strategy

## Teach about schemas

$x A+1$


## RIDE

Read the problem.
Identify the relevant information.
Determine the operation and unit for the answer.
Enter the correct numbers and calculate, then check the answer.

## RIDGES

Read the problem.
I know statement.
Draw a picture.
Goal statement.
Equation development. Solve the equation.

## RICE

Read and record the problem.
Illustrate your thinking.

## STAR

Stop and read the problem carefully.
Think about your plan and the strategy you will use.
Act. Follow your plan and solve the problem.
Review your answer.

## SUPER

Slowly read the story problem twice.
Underline the question and circle the numbers you need.
Picture it. Draw the scenario to show what is happening.
Explain the problem with a number sentence.
Rewrite the answer in a sentence.

## SHINES

Slowly and carefully read the problem. Highlight or underline key information. Identify the question by drawing a circle around it. Now solve the problem. Show your work.
Examine your work for precision, accuracy, and clarity. Share your answer by writing a sentence.

## R-CUBES

## Read the problem.

Circle key numbers.
Underline the question.

## SOLVE

Study the problem.
Organize the facts.
Line up the plan.
Verify the plan with computation.
Examine the answer.
Box action words.
Evaluate steps.
Solve and check.

## Share your favorite attack strategy.

## Teach an attack strategy

## Teach about schemas

$x A+1$

## Total

## Difference

## Change

## Equal Groups

## Comparison

## Ratios/Proportions

| Schema and Definition | Equations and Graphic Organizers | Examples |  |  | Variations |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Total (Combine: Part-partwhole) Parts combined for a sum | $\begin{aligned} & P 1+\mathrm{P} 2=\mathrm{T} \\ & (\text { part }+ \text { part }=\text { total) } \end{aligned}$ | Sum unknown: Lyle has 11 red apples and 18 green apples. How many apples does Lyle have altogether? | Part unknown: Lyle has 29 red and green apples. If 11 of the apples are red, how many green apples does Lyle have? |  | More than two parts: Lyle has 34 apples. Of the apples, 11 are red, 18 are green, and the rest are yellow. How many yellow apples does Lyle have? |
| Difference <br> (Compare) <br> Sets <br> compared <br> for a <br> difference |  | Difference unknown: <br> Sasha wrote 85 words in her essay. and Tabitha wrote 110 words. How mary fewer words did Sasha write than Tabitha? | Bigger/greater unknown: <br> Tabitha wrote 25 more words than Sasha. If Sasha wrote 85 words, how many words did Tabitha write? | Smaller/lesser unknown: <br> Tabitha wrote 110 words in her essay. Sasha wrote 25 words fewer than Tabitha. How many words did Sasha write? | (None) |
| Change <br> (Join: <br> Separate) <br> An amount <br> that <br> increases <br> or <br> decreases | $\mathrm{ST}+/-\mathrm{C}=\mathrm{E}$ <br> (start $+\mathcal{L}$ change $=$ end) | End (increase) unknown: Jorge had \$52. Then, he earned $\$ 16$ babysitting. How much money does Jorge have now? <br> End (decrease) unknown: <br> Jorge had \$52. <br> Then, he spent \$29 at the ballpark. How much money does Jorge have now? | Change (increase) unknown: <br> Jorge had \$52. <br> Then, he eamed <br> some money babysitting. <br> Now, Jorge has $\$ 68$. How much did Jorge earn babysitting? <br> Change <br> (decrease) unknown: <br> Jorge had $\$ 52$ but spent some money when he went to the ballpark. Now, Jorge has \$23. How much did Jorge spend at the ballpark? | Start (increase) unknown: Jorge has some money, and then he earned $\$ 16$ for babysitting. Now, Jorge has $\$ 68$. How much money did he have to start with? <br> Start (decrease) unknown: <br> Jorge had some money. Then, he spent $\$ 29$ at the ballpark and has $\$ 23$ left. How much money did Jorge have before going to the ballpark? | Multiple changes: Jorge had \$78. He stopped and bought a pair of shoes for \$42 and then he spent $\$ 12$ at the grocery. How much money does Jorge have now? |

Parts put together into a total

Daniela saw 3 canoes and 8 kayaks. How many boats did Daniela see?

Total

Part

Part

Daniela saw 11 boats. 8 of the boats were kayaks, how many were canoes?

## Total

"Are parts put together for a total?"

Total
$P 1+P 2=$ T

## (total)

## (part)

## (part)

## Total

## B.

In March and April, it rained a total of 11.4 inches. If it rained 3.9 inches in March, how many inches did it rain in April?

$x A+1 \cdot$

## Total

## Share a Total problem.

## Difference

Greater and lesser amounts compared for a difference

Adrianna has 10 pencils. Tracy has 4 pencils. How many more pencils does Adrianna have?

Adrianna has 6 more pencils than Tracy. If Tracy has 4 pencils, how many does Adrianna have?

Tracy has 6 fewer pencils than Adrianna. Adrianna has 10 pencils. How many pencils does

## Difference

Tracy have?

## Total

"Are parts put together for a total?"

## Difference

"Are amounts compared for a difference?"

## Difference



## Difference

Jana has 107 wooden beads and 68 glass beads. How many more wooden beads than glass beads does Jana have?

Enter your answer in the response box.

|  | $U$ | $G-L=D$ |
| ---: | :---: | :---: |
|  | $P$ | $107-68=?$ |
|  | $S$ | $?=39$ more |
| wooden beads |  |  |

## Difference

Share a Difference problem.

## Change

An amount that increases or decreases

Nickole had 6 notebooks. Then, she bought 3 notebooks. How many notebooks does Nickole have now?

End amount
Nickole had 6 notebooks. Then, she bought a few more notebooks. Now, Nickole has 9 notebooks. How many notebooks did she buy?

Nickole had some notebooks. Then, she bought 3

Change amount

Start amount notebooks. Now, Nickole has 9 notebooks. How many notebooks did she have to start with?

## Change

## An amount that increases or decreases

Samantha baked 20 cookies. Then, she ate 3 of the cookies. How many cookies does Samantha have now?

## End amount

Samantha baked 20 cookies. Then, she ate some of the cookies. Now, she has 17 cookies. How many cookies did Samantha eat?

Samantha baked some cookies. She ate 3 of the

Change amount

Start amount cookies and has 17 cookies left. How many cookies did Samantha bake?

## Total

"Are parts put together for a total?"

## Difference

"Are amounts compared for a difference?"

Change
"Does an amount increase or decrease?"

## Change



$$
+1-
$$


(start)
(change)
(end)


## Change

28 There were 25 people in a library. Some people left the library and went home. Then there were 13 people remaining in the library. Which number line represents one way to determine the number of people who left the library?


## $?=12$ people left

## Change

Share a Change problem.

## - <br>  - <br>  $\square$ <br> Schema Check!

## Change

Pablo goes to a stamp show where he can share, buy, and sell stamps.

## 26. Part A

The first day, Pablo starts with 744 stamps. He buys 27 stamps from his friend. He then sells 139 stamps.

What is the total number of stamps that Pablo has after the first day of the stamp show?

## Difference

The graph below shows the number of pounds of plastic the Keller family recycled for five months.

Recycled Plastic


Each $\square$ means 20 pounds.
Based on the graph, how many more pounds of plastic did the family recycle in July than in April?

Mr. Conley delivers packages. The bar graph shows the total number of packages he delivered on five days last week.

10. Part A

What is the total number of packages Mr. Conley delivered on Monday and Tuesday?
(4) 300
(8) 340
(c) 350
(2) 360

| Schema and Definition | Equations and Graphic Organizers |  |  | Examples |  |  | Variations |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Total (Combine; Part-partwhole) Parts combined for a sum | $\begin{aligned} & \mathrm{P} 1+\mathrm{P} 2=\mathrm{T} \\ & (\text { part }+ \text { part }=\text { total }) \end{aligned}$ |  |  | Sum unknown: Lyle has 11 red apples and 18 green apples. How many apples does Lyle have altogether? | Part unknown: Lyle has 29 red and green apples. If 11 of the apples are red, how many green apples does Lyle have? |  | More than two parts: Lyle has 34 apples. Of the apples, 11 are red, 18 are green, and the rest are yellow. How many yellow apples does Lyle have? |
|  |  |  |  |  |  |  |  |
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## Total

## Difference

## Change

## Equal Groups

## Comparison

## Ratios/Proportions

## Teach an attack strategy

## Teach about schemas

$x A+1$

$\left.$| A. |
| :--- | :--- |
| Ali delivered 12 boxes of cookies on Friday |
| and 25 boxes of cookies on Saturday. How |
| many boxes of cookies did Ali deliver? | | B. |
| :--- |
| Hui saved $\$ 70$ in January. In February, she |
| spent $\$ 64$ of the money she saved. She saved |
| $\$ 92$ more in March. How much has Hui saved |
| by the end of March? | \right\rvert\, |  |
| :--- | :--- |

## What is your modeling and practice goal for this winter?

What is your word-problem goal for this winter?

## Schedule for This Year

| December 2022 | Best Practices in Math: <br> Math Language and Fluency |
| :--- | :--- |
| January 24, 2023 | Best Practices in Math: <br> Modeling and Practice <br> Word-Problem Solving |
| TBD | Best Practices in Math: <br> Use of Multiple Representations |

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