Emergent Photons and Gravitons: The Problem of Vacuum Structure

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We discuss vacuum condensates associated with emergent QED and with torsion, as well as the possible role of the Kodama wave function in quantum cosmology.

1. Emergent QED and Gravity

The idea that the photon might be a Goldstone boson of a theory of spontaneously broken Lorentz covariance goes back a long way. I myself made a try, [1] copying closely the Nambu-Jona-Lasinio formalism [2] for the Goldstone pion. A few years ago, I revisited the subject, [3] and guessed that the leading gauge-variant term might be a Mexican hat potential, with a huge, GUT-scale vacuum expectation value $M$ for the gauge potential. I chose the quartic coupling constant to be extremely small, of order $10^{-30}$, in such a way that it would vanish in the limit of vanishing dark energy. When the dust settled, at tree level the effect of adding the extra Mexican-hat term amounted to fixing the gauge—although the gauge one gets is a curiously nonlinear one. If gauge invariance is broken, as above, then there is a preferred gauge, in terms of which the theory most closely follows the dynamics of the underlying, hidden degrees of freedom. In general, it makes sense to guess the "most probable gauge". My choice is temporal gauge, and the above scenario is a specific way to express this choice. In temporal gauge, the longitudinal-photon degrees of freedom are, in a sense, dynamical, because they have non-vanishing canonical momenta. However, in practice the Gauss-law constraint makes these degrees of freedom act like a Bose condensate, described by only a few classical degrees of freedom. It is interesting that, in CPT2001, Nambu discussed just this point in his talk, [4]
and ascribed this idea to work of Dirac [5] in the 1950’s.

To me it would be very interesting if somehow this longitudinal-photon condensate might somehow be activated. So recently I gave it a try. The game is to stay with the Mexican-hat picture above, but to assume that the vacuum gauge-potential condensate has spacetime dependence. A very simple, cosmological type of behavior is to assume

$$F_{\mu\nu} = 0, \quad A_\mu = \partial_\mu \Lambda(r, t).$$

(1)

Our previous example set $\Lambda = M t$. If we choose instead

$$\Lambda = M \tau \quad \tau^2 = t^2 - \frac{r^2}{c^2},$$

(2)

it follows that

$$\Phi = \frac{Mt}{\tau}, \quad A = -\frac{Mr}{c^2 \tau}, \quad \Phi^2 - c^2 A^2 = M^2.$$  

(3)

This can be constructed from the same Mexican-hat potential as before, provided that $c = 1$. I put in the Lorentz violation mostly (but not entirely) for fun, because the solution admits so easily the generalization. The net result of this construction is a vacuum which will become, or which has been, unstable, depending upon whether we live in the past or future ”lightcone” associated with the gauge function $\Lambda$. It seems to me that this might be a mechanism for catalyzing the cosmological ”reheating” transition, because the onset of the instability outraces even the accelerated expansion of the universe.

What about emergent gravitons? The idea goes back to Sakharov, [6] and the Einstein-Hilbert action is arguably easy to obtain via radiative loops. Again the problem is what else, if anything, comes along for the ride. A general attack can quickly lead to quite a mess. [7] At the opposite extreme, I might guess that the most important violating term is a potential $V(g)$, depending only upon the determinant $g$ of the metric. The Einstein equations are then easy to obtain, and they will make trouble unless $V'$ vanishes. This leads to a fixed value of the determinant, and a consequent ”emergent unimodular gravity”. So at this level I only see gauge fixing as the output consequence. Quite a lot more can be said about this approach, [8] but Alan is better equipped than I to say it [9].

2. An Axial-Vector Condensate

Just as I did for the Goldstone photon, I have tried to ”activate” torsion degrees of freedom in as simple a way as possible, in order to see how
they might enter into phenomenology. This led to presuming that there might exist, for some fermionic degrees of freedom (either standard-model or beyond-the-standard-model), a Lorentz-violating vacuum condensate of axial vector current:

\[ < \Psi \gamma_5 \gamma_\mu \Psi > = \eta_\mu \rho_A. \] (4)

Here \( \eta_\mu \) is a unit timelike vector, at rest in the CMB rest frame. This provides a source of torsion. In the context of FRW cosmology, this does not lead to a modification of the FRW cosmological evolution equations. But it does lead to a renormalization of the cosmological constant:

\[ H^2 = H^2_{cc} - \frac{(4\pi \gamma \rho_A)^2}{M^4_{pl}(1 + \gamma^2)}. \] (5)

Here \( H_{cc} \) is evidently the value the Hubble parameter would take in the absence of torsion and the axial condensate. And \( \gamma \) is the Barbero-Immirzi parameter, prominent in the loop quantum gravity formalism. If this renormalization of the dark energy scale is of order unity, one has

\[ \frac{4\pi \gamma \rho_A}{\sqrt{1 + \gamma^2}} \sim H M^2_{pl} \sim 10^{-60} M^3_{pl} \sim (10^{-20} M_{pl})^3 \sim \Lambda^3_{QCD}. \] (6)

This is what I call the Zeldovich relation: in natural units the cube of the QCD scale is of order the Hubble scale. It was noticed by Zeldovich [10] in 1967 and has been occasionally been rediscovered in the interim. [11] I encounter it often in my speculative excursions into trying to understand the dark energy problem, and I now take it seriously. I find that this is a minority viewpoint. Most people seem to dismiss the Zeldovich relation as a numerical coincidence.

This axial condensate has another consequence. Because all spinor degrees of freedom couple to gravity, they must all feel the effect of the vacuum torsion. This leads to a Lorentz-violating term in the effective action, one which is prominent in the SME catalog: [12]

\[ L' = b_\mu \sqrt{\gamma} \gamma_\mu \gamma_5 \Psi \] (7)

The condensate contribution to this Kostelecky’ \( b \) - parameter is

\[ b_\mu = \eta_\mu \frac{2\pi \gamma^2 \rho_A}{M^2_{pl}(1 + \gamma^2)}. \] (8)

If the Zeldovich relation holds, then

\[ b_\mu \leq 10^{-33} eV. \] (9)
The effect is a billion times smaller than the experimental limit, unless the condensate density is taken to be much higher than its "natural" value. Such behavior would be appropriate for scenarios in which there is a fine-tuned cancellation of the torsion contribution with a much larger "primordial" dark energy. At this meeting I learned of closely related work of Poplawski. He uses the QCD quark vacuum condensates instead of a Lorentz-violating axial condensate to arrive at a very similar endpoint.

3. Vacuum Phase Density

Consider a finite box of spatially flat FRW ΛCDM universe, with periodic boundary conditions [14] applied ("compactification on a torus"). As time goes on, this box will expand. The dimensions of the box are controlled by the FRW scale factor, which evolves according to the Einstein equations of cosmology. If the box contains only pure dark energy, it will expand exponentially. The problem of what is going on at the microscopic level within such a box is the fundamental problem of dark energy. The semi-classical wave function of this box of dark energy is the exponential of a phase factor, given by the classical action. It turns out to be proportional to the volume of the box. The coefficient of this phase factor is linear in the Hubble parameter, in natural units. This leads to the conclusion that the characteristic volume, for which the "phase density" is of order $2\pi$, is of order the QCD scale-the Zeldovich relation again applies.

There is an interesting subplot to this story, which originates in a variant of first-order gravity invented [15] by MacDowell and Mansouri and elaborated recently by Freidel and Starodubtsev. [16] The idea is to synthesize the tetrad and connection variables $(e, \omega)$ of the first order theory into a single grand connection $A$ which lives in an internal O(4,1) space. This way of expressing gravity is provocative and certainly invites its use as a starting point for enlarging the theory in some way to encompass standard model degrees of freedom. [17] However, that is not the issue here. Instead it is easy to find that imposing a "gauge condition" $F = 0$ for the field strength associated with the connection $A$ leads to nontrivial solutions. In particular, deSitter space, which describes our expanding box of dark energy, is such a solution. According to this interpretation, the vacuum phase density, given by the exponential of the MacDowell-Mansouri action (which is quadratic in the field strength $F$) should vanish. The resolution of this paradox is that the Gauss-Bonnet term, although pure topological, does contribute vacuum phase. And the MacDowell-Mansouri construction guarantees that this topological contribution to the phase density cancels out.
the contribution given by the standard metric theory. This Gauss-Bonnet term, complete with a remarkably large coefficient of $10^{120}$, is essentially what is known in the loop-gravity community as the Kodama wave function. \[18\] However, there it plays a different–and controversial–role. \[19\] In any case, what is suggested here is that vacuum topology might be an important ingredient in the understanding of the Zeldovich relation, of course assuming–as I always do–that it is more than a numerical accident.

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References