Math 1496 Calculus I − Final 2016 Name

This test consists of 2 parts. You are required to do all of part A #1-6 and any 4 of part B. The point value for each question is in the parenthesis. Please show all of your work for full credit!

1. (15) Calculate the following limits.

(i) $\lim_{x\to 9} \dfrac{\sqrt{x}-3}{x-9}$

(ii) $\lim_{x\to 0} \dfrac{\sin 4x}{\sin 3x}$

(iii) $\lim_{x\to \infty} \dfrac{3x^3-4x^2+x-1}{2x^3-x+5}$

(iv) $\lim_{x\to 2} \dfrac{x^2+x-6}{x-2}$

2. (15) Calculate the first derivative (either $f'(x)$ or y') of the following. Do not simplify your answer

(i) $y = \dfrac{\ln x}{x}$

(ii) $y = x^{\tan x}$

(iii) $xy^4 - xy = x + y + 3$

(iv) $y = e^x \cos 5x$

3. (15) Integrate the following.

(i) $\int \sin^3 x \cos x \, dx$

(ii) $\int x\sqrt{x+1}dx$

(iii) $\int_0^1 \frac{e^x}{e^x+1} \, dx$

(iv) $\int_0^2 x^2 e^{x^3} dx$

4. (5) Find the equation of the tangent to $y = x^2 - \frac{4}{x^2}$ and $x = 2$.

5. (5) Find the absolute minimum and absolute maximum of $f(x) = x^3 - 12x + 1$ on $[1, 5]$.

6. (5) Find the area between $y = x^2$ and $y = 2x - x^2$. Sketch the region.

Part B - Choose any 4 of the following 5

7. (10) Consider the function

$$f(x) = \begin{cases} 2x^2 - 1, & x \leq 1 \\ 2x - 1, & x > 1 \end{cases}$$

Is $f(x)$ continuous at $x = 1$? Is $f(x)$ differentiable at $x = 1$. Provide a sketch of $f(x)$

8. (10) Consider the function

$$f(x) = x^3 - 6x^2 + 9x$$

Determine:

(i) The critical points

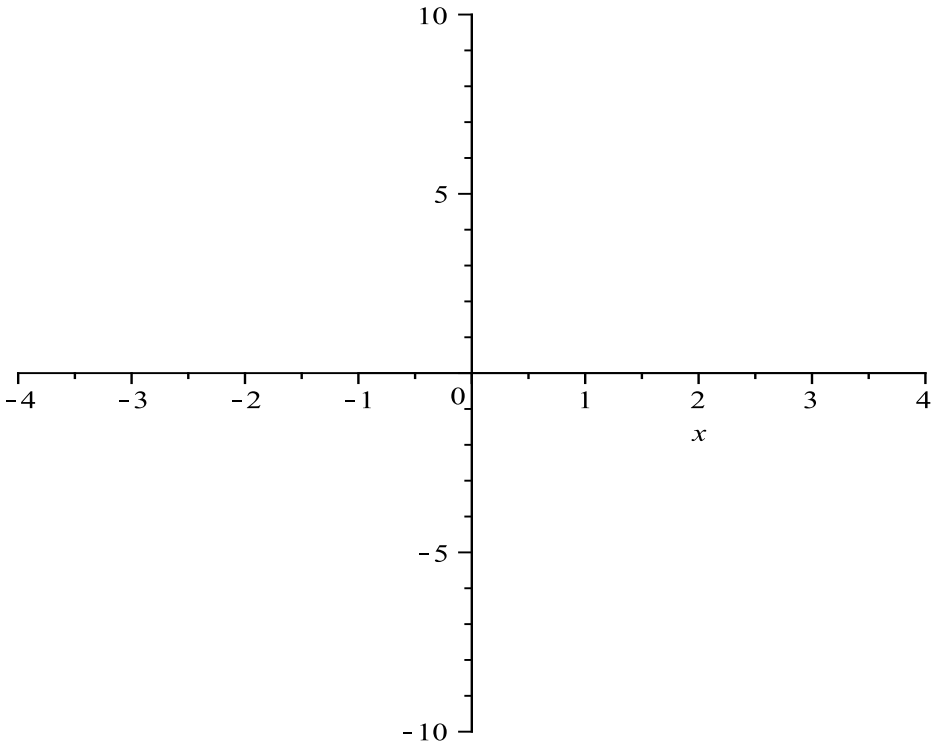
(ii) When y is increasing and decreasing

(iii) Determine whether any of the critical points are minimum or maximum

(iv) Inflection points

(iv) When y is concave up and down

(v) Then sketch the curve and label.



9. (10) Car A is traveling west at 50 mi/hr and car B is traveling north at 60 mi/hr. Both are headed for the intersection of the two roads. At what rate are the cars approaching each other when car A is 0.3 mi and car B is 0.4 mi from the intersection?

10. (10) A rectangular box with a square bottom is to be constructed using 96 sq. ft. of material. Find the dimensions of the box that maximizes volume.

11. (10) Find the volumes of revolution when the region bound by the following curve

$$y = x, \quad y = 2 - x, \quad y = 0$$

is revolved about the x and y axis.

Basic Formulas

Areas

$$\text{circle}\quad A=\pi r^2 \qquad\qquad \text{rectangle}\quad A=l\,w$$

$$\text{sphere}\quad A=4\pi r^2 \qquad\qquad \text{triangle}\quad A=\frac{1}{2}\,b\,h$$

$$\text{box}\quad A=2\,l\,w+2\,l\,h+2\,w\,h \qquad\qquad \text{cylinder}\quad A=2\pi r^2+2\pi r\,h$$

Volumes

$$\text{cylinder}\quad V=\pi r^2 h \qquad\qquad \text{box}\quad V=l\,w\,h$$

$$\text{sphere}\quad V=\frac{4}{3}\pi r^3 \qquad\qquad \text{cone}\quad V=\frac{1}{3}\pi r^2 h$$

$$\text{Pythagorean Thm.}\quad a^2+b^2=c^2$$