

# Mathematical Modeling of Rayleigh Waves by GA-SVM Approach

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**ABSTRACT:** In this paper the effect of temperature and initial hydrostatic stress has been shown on the propagation of Rayleigh Waves in a Half-Space with variable rigidity and density. It has been depicted in a graph how the phase velocity of the Rayleigh waves varies with respect to the magnitude of the initial hydrostatic stress of the half-space under certain practical assumptions. It has been analytically explained that this velocity is also influenced by the parameters connected with the variable rigidity and density of the half-space even if the initial hydrostatic stress is not considered.

## I. INTRODUCTION

Recently the propagation of thermo elastic waves has been discussed by many authors as Chandrasekharaiyah[1], Sharma, Singh, Kumar [2] and Misra, Chattopadhyay, Chakravorty [3]. They discussed the subject based on different theories and considered different types of media. But they all ignored the presence of initial stress in the media. But our planet earth itself is an initially stressed medium. Hence it should be of interest to see how the initial stress may influence the propagation of elastic waves along with the temperature and other physical properties of a medium [4].

This paper has discussed the effect of the temperature as well as the initial hydrostatic stress on the propagation of Rayleigh waves in a half-space with variable rigidity and density [5, 6].

When an earthquake or an artificial explosion occurs near the surface of the earth, Rayleigh waves arrive after the faster irrotational and equivoluminal elastic waves. But their relative effect increases with an increasing distance from the center of the disturbance[7, 8 and 9].

In this paper a new frequency equation of the Rayleigh waves has been derived , which involves the parameters connected with the temperature, initial hydrostatic stress and variable rigidity and density of the medium taken as half-space. The values of the phase velocities of the Rayleigh waves have been computed for different values of the initial hydrostatic stress of the half-space in the dimensionless form for certain cases. The result is plotted in a graph. This graph shows that the phase velocity of the Rayleigh waves changes remarkably with respect to magnitudes of the initial hydrostatic stress of the half-space. The frequency equation

also shows that the phase velocity of the Rayleigh waves depends on the parameters involving the variable rigidity and density without the initial hydrostatic stress of the half-space [10, 11 and 12].

## II. FORMULATION AND SOLUTION OF THE PROBLEM

Let us consider a half-space  $Y>0$ , the boundary of which  $Y=0$ , is free from any mechanical load, but does permit heat exchange with surroundings. The rigidity and density of the half-space are supposed to vary according to Meissner's formula[13, 14 and 15]. The half-space is under an initial hydrostatic stress  $H$  at an initial temperature  $T_0$ . When the temperature of the half-space is changed, incremental thermal stress  $s_{ij}$  together with incremental strains  $e_{ij}$  are produced in it, which are measured with reference to rotated axes as explained by Biot [4].

The dynamical equations of equilibrium matrix is given by [4].

$$\sum = \begin{pmatrix} S_y^2 & S_{yx} & S_{yz} \\ S_{xy} & S_y^2 & S_{xz} \\ S_{zy} & S_{zx} & S_y^2 \end{pmatrix} \quad (1)$$

Here  $y$  is the survey variable and  $x, z$  are two auxiliary variables. We consider a particular situation where  $\mu_x$  is unknown but  $\mu_z$  is known exactly and we seek to estimate  $\mu_y$  using a two-phase sampling mechanism. The stress- strain relations with incremental isotropy are given by [16 and 17]

$$b_{yx} = \hat{a}_{\bar{il}s} \frac{(y_i - y')(x_i - x')}{(x_i - x')^2} \quad (2)$$

where  $y$  ( $x$  being the coefficient of linear expansion) and  $x'$  is the incremental change of temperature from the initial state.

$$X' m_z / z', \quad x' \hat{=} y_R \quad (3)$$

The incremental strain component are given by [18]

$$\overline{Y_{CR}} = y \frac{-x' m}{x z} \quad (4)$$

Similarly, considering a regression estimator

$$x' - b'_{xz}(z' - mz)$$

$$\text{where } b'_{xz} = \hat{a}_{i1s'} \frac{(x_i - x')(z_i - z')}{(z - z_i)^2} \quad (5)$$

$$y_{CRG} = y - b'_{yx}[x - b'_{xz}(z' - m_z)] \text{ from } y_{RG} \quad (6)$$

The equations (2) with the help of (3), (4) and (5) and (6) change to (7), (8), (9) and (10).

$$t_x = x' - d(z - z'), \quad t_x' = x' - d(z' - m_z) \quad (7)$$

$$t_R^* = y \frac{x' - d(z' - m_z)}{x' - d(z - z')} \quad (8)$$

$$t_P^* = y' \frac{x' - d(z' - m_z)}{x' - d(z - z')} \quad (9)$$

$$t_{RG}^* = y - b'_{yx}[\{x' - d(z - z')\} - \{x' - d'(z' - m_z)\}] \quad (10)$$

The displacement components  $t_x$ ,  $t_R^*$  and  $t_P^*$  may be expressed in terms of the functions in terms of the function  $d$ ,  $z'$  and  $\mu$  as follows:

$$w = \frac{t_x}{t_x'} \text{ and } h = (y, w) \quad (11)$$

$$t = l(y, w) = l(h) \quad (12)$$

The equations (11) and (12) shows that the functions satisfies the equations (1) and (2).

Equations (13) and (14) are derived from (11) and (12).

$$t_1 = y + a(w - 1) \quad (13)$$

$$t_{RG} = y - m_{xy,z}(x - x') - m_{yz,x} \quad (14)$$

The heat conduction derived equation is given by

$$t_x = y - m_{xy,z}(y - x') - y_{yz,x} \quad (15)$$

where  $\mu$  is the specific heat and  $z$  is thermal conductivity of the medium.

Estimating  $x'$  from (14) and (15), and retaining the equation (15) we have

$$\min V(y_f) = [(f' - f')(1 - h_{yx}^2) + f]S_y^2 \quad (16)$$

Where  $S$  is the specific heat and  $\emptyset$  is thermal conductivity of the medium. Eliminating  $\emptyset$  from (14), (15) and (16).

We have

$$\min V(l_p) = [(f')(1 - h_{yx}^2)] S_y^2 + b_{yz}(z - m) \quad (17)$$

which are plane harmonic waves moving along the  $x$  axis.

Introducing (11) into (9), we obtain the following equations:

$$\min V(t) \hat{=} \min V(l_h) \hat{=} \min V(y_f) \quad (18)$$

For different values of  $z$ , the value of  $\emptyset$  are calculated and these are plotted in a graph. This graph shows how the vlocity of the Rayleigh waves varies with respect to magnitude of the initial hydrostatic stress present in the half space[19, 20 and 21].

If we take the magnitude of the initial hydrostaic stress  $H$  to be equal to the rigidity of the half space near the boundary, then  $z$  becomes  $1/2$ . In this case, the equation (18) can be derived from equation (17)[22, 23, 24 and 25].

### III. RESULTS

The solution of this equation shows that the maximum value of  $z$  in consistent of the conditions of the problem is 1. This value is 0.2352 for the same value of  $z$  if we ignore the presence of the quantity  $H$  in the half space. This is very interesting result worth mentioning. If we take the magnitude of the initial hydrostatic stress  $H$  to be equal to twice the rigidity of the half-space near the boundary, then  $z$  becomes 1. In this case,  $z$  becomes zero showing that the Rayleigh waves cease to propagate.

The equation (17) shows that the velocity of the Rayleigh waves depends on the parameters  $a$  and  $b$  connected with the varying rigidity and density of the medium even if we do not consider the initial hydrostatic stress of the half space.

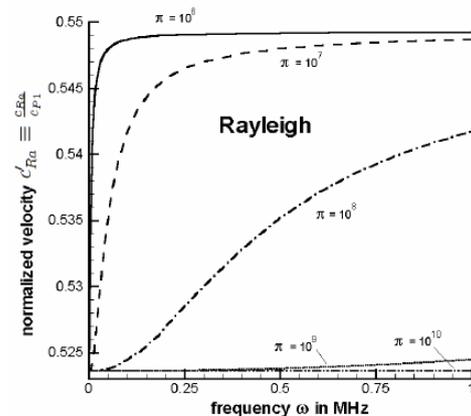


Fig 1. Rayleigh graph plotted between frequency and normalized velocity. Table 1 shows the GA-SVM estimated values for different temperatures. These results are analyzed

for different types  $c_{11}$  to  $c_{15}$ . These types are derived from the equations (1) to (3).

| GA-SVM Estimator1 |             |          |               |     |    |    |
|-------------------|-------------|----------|---------------|-----|----|----|
| Pop. No.          | Temperature | Type     | Derived Value | n   | E1 | E2 |
| 1                 | 100         | $C_{11}$ | A             | 130 | 81 | 72 |
| 2                 | 200         | $C_{12}$ | S             | 230 | 75 | 80 |
| 3                 | 300         | $C_{13}$ | N             | 340 | 89 | 81 |
| 4                 | 400         | $C_{14}$ | L             | 320 | 79 | 63 |
| 5                 | 500         | $C_{15}$ | R             | 243 | 81 | 91 |

Table 1. Spherical inclusion with Rayleigh wave

| GA-SVM Estimator2 |             |          |               |     |    |    |
|-------------------|-------------|----------|---------------|-----|----|----|
| Pop. No.          | Temperature | Type     | Derived Value | p   | E2 | E3 |
| 1                 | 600         | $C_{16}$ | A             | 131 | 73 | 82 |
| 2                 | 700         | $C_{17}$ | S             | 142 | 52 | 81 |
| 3                 | 800         | $C_{18}$ | N             | 131 | 99 | 82 |
| 4                 | 900         | $C_{19}$ | L             | 180 | 73 | 80 |
| 5                 | 1000        | $C_{20}$ | R             | 109 | 99 | 69 |

Table 2. Strain field inclusion with Rayleigh wave

| GA-SVM Estimator3 |             |           |               |    |    |    |
|-------------------|-------------|-----------|---------------|----|----|----|
| Pop. No.          | Temperature | Cell Type | Derived Value | q  | E3 | E4 |
| 1                 | 1100        | $C_{21}$  | A             | 91 | 89 | 91 |
| 2                 | 1200        | $C_{22}$  | S             | 92 | 90 | 93 |
| 3                 | 1300        | $C_{23}$  | N             | 89 | 92 | 92 |
| 4                 | 1400        | $C_{24}$  | L             | 90 | 93 | 89 |
| 5                 | 1500        | $C_{25}$  | R             | 89 | 94 | 91 |

Table 3. Stress field inclusion with Rayleigh wave

| GA-SVM Estimator4 |             |          |               |     |    |    |
|-------------------|-------------|----------|---------------|-----|----|----|
| Pop. No.          | Temperature | Type     | Derived Value | r   | E4 | E5 |
| 1                 | 1600        | $C_{31}$ | A             | 110 | 92 | 79 |
| 2                 | 1700        | $C_{32}$ | S             | 143 | 91 | 81 |
| 3                 | 1800        | $C_{33}$ | N             | 162 | 89 | 72 |

|   |      |          |   |     |    |    |
|---|------|----------|---|-----|----|----|
| 4 | 1900 | $C_{34}$ | L | 132 | 91 | 53 |
| 5 | 2000 | $C_{35}$ | R | 156 | 94 | 74 |

Table 4. Potential field inclusion with Rayleigh wave.

Single Nucleotide Polymorphisms

Table2 shows the GA-SVM estimated values for different temperatures varies from 600 °C to 1000 °C .These results are analyzed for different types  $c_{16}$  to  $c_{20}$ . These types are derived from the equations (4) to (7). Table3 shows the GA-SVM estimated values for different temperatures varies from 1100 °C to 15000 °C .These results are analyzed for different types  $c_{21}$  to  $c_{25}$ . These types are derived from the equations (8) to (11). Table4 shows the GA-SVM estimated values for different temperatures varies from 1600 °C to 2000 °C .These results are analyzed for different types  $c_{31}$  to  $c_{35}$ . These types are derived from the equations (12) to (15).

#### IV. CONCLUSION

In this study, all datasets are analyzed with four types of GA-SVM estimators. All the GA-SVM estimators are thoroughly tested under different temperatures. Genetic algorithm- Support vector machine estimator1, Spherical inclusion with Rayleigh wave are included. Genetic algorithm- Support vector machine estimator2, Strain field inclusion with Rayleigh wave are included. Genetic algorithm- Support vector machine estimator3, Stress field inclusion with Rayleigh wave is included. Genetic algorithm- Support vector machine estimator4, Potential field inclusion with Rayleigh wave is included. Overall accuracy rate is increased in the case of Genetic Algorithm- Support Vector machine estimator4. The accuracy of this estimator is 96%.This accuracy is achieved with the help of mathematical model formulated in section2. All the derived values named as A, S, N, L and R are derived from the equations (1) to (17). These are the values which plays crucial role in enhancing the accuracy rate of the estimator4.

#### 6. REFERENCES

- [1] Chandrasekharaiah, D.S., one dimensional wave propagation in the Linear Theory of Thermoelasticity without energy dissipation, Thermal Stresse, vol.19, pp. 695-710, 2006.
- [2] Dr. M Raja Sekar *et al*, " Mammogram Images Detection Using Support Vector Machines," International Journal of Advanced Research in Computer Science “,Volume 8, No. 7 pp. 329-334, July – August 2017.
- [3] Smith, T.M.F., “Post-stratificatio, “, The Statisticians, pp 31-39, 1991.

- [4] Dr. M Raja Sekar, "Diseases Identification by GA-SVMs", International Journal of Innovative Research in Science, Engineering and Technology, Vol 6, Issue 8, pp. 15696-15704, August 2017.
- [5] Dr. M Raja Sekar., "Classification of Synthetic Aperture Radar Images using Fuzzy SVMs", International Journal for Research in Applied Science & Engineering Technology (IJRASET) , Volume 5 Issue 8, pp. 289-296 , Vol 45, August 2017.
- [6] Sharma, J.N., Devinder Singh and Rajneesh Kumar, Propagation of Thermoelastic Waves in Homogeneous Isotropic Planes, Indian J.Pure and Applied Math, 31 (9), pp. 1167-1181, 2017.
- [7] Dr. M Raja Sekar , "Software Metrics in Fuzzy Environment " , International Journal of Computer & Mathematical Sciences(IJCMS) , Volume 6, Issue 9, September 2017.
- [8] T.S. Furey, et al., "Support Vector classification and validation of cancer tissue sampling using micro array expression data", Bioinformatics, pp 906-914, 2000.
- [9] Dr. M Raja Sekar,"Optimization of the Mixed Model Processor Scheduling", International Journal of Engineering Technology and Computer Research (IJETCR), Volume 5, Issue 5, pp 74-79, September-October: 2017.
- [10] Dr. M Raja Sekar, "Fuzzy Approach to Productivity Improvement", International Journal of Computer & Mathematical Sciences Volume 6, Issue 9, pp 145-149, September 2017.
- [11] Dr. M Raja Sekar *et al.*, "An Effective Atlas-guided Brain image identification using X-rays", International Journal of Scientific & Engineering Research, Volume 7, Issue 12,pp 249-258, December-2016.
- [12] Dr. M Raja Sekar, " *Fractional* Programming with Joint Probability Constraints", International Journal of Innovations & Advancement in Computer Science, Volume 6, Issue 9, pp 338-342, September 2017.
- [13] Dr. M Raja Sekar, "Solving Mathematical Problems by Parallel Processors", " Current Trends in Technology and Science", Volume 6, Issue 4, PP 734-738.
- [14] Misra, J.C., N.C. Chattopadhyaya and A.Chakravarty, Study of Thermoelastic wave Propagation in a Half-Space Using GN Theory, Journal of Thermal Stresses, Vol.23, pp. 372-351, 2014.
- [15] Seth, G.S., N.Matho and Sigh, S.K., Presented in National seminar on Advances in mathematical, statistical and computational methods in science and Technology, Nov. 29-30, Dept of Applied Maths., I.S.M. Dhanbad.
- [16] Biot, M.A., "Mechanics of Incremental Deformations", John Wiley and Sons, Inc, New York, 2014.
- [17] Dork, "Selection of scale invariant parts for object class recognition. In IEEE International Conference on Computer Vision, Vol 1, pp. 634-639.
- [18] P.Ganesh Kumar, " Design of Fuzzy Expert Systems for Microarray Data Classification using Novel Genetic Swam Algorithm", Expert Systems with Applications, pp 1811-1812, 2012.
- [19] Dr. M Raja Sekar, "Interactive Fuzzy Mathematical Modeling in Design of Multi-Objective Cellular Manufacturing Systems", International Journal of Engineering Technology and Computer Research (IJETCR), Volume 5, Issue 5, pp 74-79, September-October: 2017.
- [20] Dr. M Raja Sekar, "Breast Cancer Detection using Fuzzy SVMs", International Journal for Research in Applied Science & Engineering Technology (IJRASET)", Volume 5 Issue 8, pp. 525-533, Aug ,2017.
- [21] Shukhatme et.al., "Sampling Theory of Surveys with Applications", Iowa State University Press, Ames, Iowa, USA, PP 34-39, 1984.
- [22] Dr. M Raja Sekar *et al.*, " Areas categorization by operating support Vector machines", ARPN Journal of Engineering and Applied Sciences", Vol. 12, No.15 , pp.4639-4647 , Aug 2017..
- [23] Jeffreys, H., "The Earth, its Origin History and Physical Constitution", Cambridge at the University Press 2015.
- [24] Singh, D. and Choudhary, F.S., "Theory and anlysis of sample survey designs", Wiley Eastern Limited, New Delhi, 1986.
- [25] Dr. M Raja Sekar *et al.*, " Tongue Image Analysis For Hepatitis Detection Using GA-SVM," *Indian Journal of computer science and Engineering*, Vol 8 No 4 , pp. , August 2017.