Chapter 4
Polynomial Functions

Section 4-3
Dividing Polynomials

Long Division of Polynomials

When you divide a polynomial $f(x)$ by a nonzero polynomial divisor $d(x)$, you get a quotient polynomial $q(x)$ and a remainder polynomial $r(x)$.

$$\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$$

The degree of the remainder must be less than the degree of the divisor. When the remainder is 0, the divisor *divides evenly* into the dividend. Also, the degree of the divisor is less than or equal to the degree of the dividend $f(x)$. One way to divide polynomials is called **polynomial long division**.

EXAMPLE 1 Using Polynomial Long Division

Divide $2x^4 + 3x^3 + 5x - 1$ by $x^2 + 3x + 2$.

Divide using polynomial long division.

1. $(x^3 - x^2 - 2x + 8) \div (x - 1)$

2. $(x^4 + 2x^2 - x + 5) \div (x^2 - x + 1)$

Synthetic Division

There is a shortcut for dividing polynomials by binomials of the form $x - k$. This shortcut is called **synthetic division**. This method is shown in the next example.

EXAMPLE 2 Using Synthetic Division

Divide $-x^3 + 4x^2 + 9$ by $x - 3$.

EXAMPLE 3 Using Synthetic Division

Divide $3x^3 - 2x^2 + 2x - 5$ by $x + 1$.

Divide using synthetic division.

3. $(x^3 - 3x^2 - 7x + 6) \div (x - 2)$

4. $(2x^3 - x - 7) \div (x + 3)$

Core Concept

The Remainder Theorem

If a polynomial $f(x)$ is divided by $x - k$, then the remainder is $r = f(k)$.

The Remainder Theorem tells you that synthetic division can be used to evaluate a polynomial function. So, to evaluate $f(x)$ when $x = k$, divide $f(x)$ by $x - k$. The remainder will be $f(k)$.

EXAMPLE 4 Evaluating a Polynomial

Use synthetic division to evaluate $f(x) = 5x^3 - x^2 + 13x + 29$ when $x = -4$.

Use synthetic division to evaluate the function for the indicated value of x.

5. $f(x) = 4x^2 - 10x - 21;\ x = 5$

6. $f(x) = 5x^4 + 2x^3 - 20x - 6;\ x = 2$

Section 4-3 Homework #5-17odd and #23, 25,29,31,35,37,39