

**Edexcel GCE
Core Mathematics C4
Gold Level G3
(Mark Scheme)**

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Question Number	Scheme	Marks
1. (a)	<p>** represents a constant</p> $f(x) = (3 + 2x)^{-3} = \underline{(3)^{-3}} \left(1 + \frac{2x}{3}\right)^{-3} = \underline{\frac{1}{27}} \left(1 + \frac{2x}{3}\right)^{-3}$ $= \frac{1}{27} \left\{ 1 + (-3)(**x) + \frac{(-3)(-4)}{2!} (**x)^2 + \frac{(-3)(-4)(-5)}{3!} (**x)^3 + \dots \right\}$ <p>with ** ≠ 1</p> $= \frac{1}{27} \left\{ 1 + (-3)\left(\frac{2x}{3}\right) + \frac{(-3)(-4)}{2!} \left(\frac{2x}{3}\right)^2 + \frac{(-3)(-4)(-5)}{3!} \left(\frac{2x}{3}\right)^3 + \dots \right\}$ $= \frac{1}{27} \left\{ 1 - 2x + \frac{8x^2}{3} - \frac{80}{27}x^3 + \dots \right\}$ $= \frac{1}{27} - \frac{2x}{27} + \frac{8x^2}{81} - \frac{80x^3}{729} + \dots$	<p>Takes 3 outside the bracket to give any of (3)⁻³ or $\frac{1}{27}$.</p> <p>See note below.</p> <p>Expands (1 + ** x)⁻³ to give a simplified or an un-simplified 1 + (-3)(** x);</p> <p>A correct simplified or an un-simplified {.....} expansion with candidate's followed thro' (** x)</p> <p>Anything that cancels to $\frac{1}{27} - \frac{2x}{27}$; Simplified $\frac{8x^2}{81} - \frac{80x^3}{729}$</p> <p>B1</p> <p>M1;</p> <p>A1 ✓</p> <p>A1;</p> <p>A1</p> <p>[5]</p> <p>5 marks</p>

2.	$\frac{du}{dx} = -\sin x$ $\int \sin x e^{\cos x+1} dx = -\int e^u du$ $= -e^u$ $= -e^{\cos x+1}$ $\left[-e^{\cos x+1}\right]_0^{\frac{\pi}{2}} = -e^1 - (-e^2)$ $= e(e-1) *$	<p>B1</p> <p>M1 A1</p> <p>A1ft</p> <p>ft sign error</p> <p>or equivalent with u</p> <p>M1</p> <p>A1</p> <p>cso</p>	<p>(6)</p> <p>[6]</p>
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Question Number	Scheme	Marks	
3.	$\frac{9x^2 + 20x - 10}{(x+2)(3x-1)} \equiv A + \frac{B}{x+2} + \frac{C}{3x-1}$ $A = 3$ $9x^2 + 20x - 10 \equiv A(x+2)(3x-1) + B(3x-1) + C(x+2)$ <p>Either $x^2: 9 = 3A, \quad x: 20 = 5A + 3B + C$ constant: $-10 = -2A - B + 2C$</p> <p>or</p> $x = -2 \Rightarrow 36 - 40 - 10 = -7B \Rightarrow -14 = -7B \Rightarrow B = 2$ $x = \frac{1}{3} \Rightarrow 1 + \frac{20}{3} - 10 = \frac{7}{3}C \Rightarrow -\frac{7}{3} = \frac{7}{3}C \Rightarrow C = -1$	<p>their constant term = 3</p> <p>Forming a correct identity.</p> <p>Attempts to find the value of either one of their B or their C from their identity.</p> <p>Correct values for their B and their C, which are found using a correct identity.</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p>

[4]

Question Number	Scheme	Marks
4	<p>(a) $\mathbf{d}_1 = -2\mathbf{i} + \mathbf{j} - 4\mathbf{k}$, $\mathbf{d}_2 = q\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$</p> <p>As $\left\{ \mathbf{d}_1 \bullet \mathbf{d}_2 = \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} \bullet \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix} \right\} = \underline{(-2 \times q) + (1 \times 2) + (-4 \times 2)}$</p> <p>$\mathbf{d}_1 \bullet \mathbf{d}_2 = 0 \Rightarrow -2q + 2 - 8 = 0$ $-2q = 6 \Rightarrow \underline{q = -3}$ AG</p> <p>(b) Lines meet where:</p> <p>$\begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} = \begin{pmatrix} -5 \\ 11 \\ p \end{pmatrix} + \mu \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix}$</p> <p>i: $11 - 2\lambda = -5 + q\mu$ (1) First two of j: $2 + \lambda = 11 + 2\mu$ (2) k: $17 - 4\lambda = p + 2\mu$ (3)</p> <p>(1) + 2(2) gives: $15 = 17 + \mu \Rightarrow \mu = -2$</p> <p>(2) gives: $2 + \lambda = 11 - 4 \Rightarrow \lambda = 5$</p> <p>(3) $\Rightarrow 17 - 4(5) = p + 2(-2)$ $\Rightarrow p = 17 - 20 + 4 \Rightarrow \underline{p = 1}$</p> <p>(c) $\mathbf{r} = \begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + 5 \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} -5 \\ 11 \\ 1 \end{pmatrix} - 2 \begin{pmatrix} -3 \\ 2 \\ 2 \end{pmatrix}$</p> <p>Intersect at $\mathbf{r} = \begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix}$ or $\underline{(1, 7, -3)}$</p>	<p>Apply dot product calculation between two direction vectors, ie. $\underline{(-2 \times q) + (1 \times 2) + (-4 \times 2)}$</p> <p>M1</p> <p>Sets $\mathbf{d}_1 \bullet \mathbf{d}_2 = 0$ and solves to find $\underline{q = -3}$</p> <p>A1 cso</p> <p>(2)</p> <p>Need to see equations (1) and (2). Condone one slip. (Note that $q = -3$.)</p> <p>M1</p> <p>Attempts to solve (1) and (2) to find one of either λ or μ</p> <p>dM1</p> <p>Any one of $\underline{\lambda = 5}$ or $\underline{\mu = -2}$</p> <p>A1</p> <p>Both $\underline{\lambda = 5}$ and $\underline{\mu = -2}$</p> <p>A1</p> <p>Attempt to substitute their λ and μ into their k component to give an equation in p alone.</p> <p>ddM1</p> <p>$\underline{p = 1}$</p> <p>A1 cso</p> <p>(6)</p> <p>Substitutes their value of λ or μ into the correct line l_1 or l_2.</p> <p>M1</p> <p>$\begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix}$ or $\underline{(1, 7, -3)}$</p> <p>A1</p> <p>(2)</p>

Question Number	Scheme	Marks
(d)	<p>Let $\overline{OX} = \mathbf{i} + 7\mathbf{j} - 3\mathbf{k}$ be point of intersection</p> $\overline{AX} = \overline{OX} - \overline{OA} = \begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix} - \begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} = \begin{pmatrix} -8 \\ 4 \\ -16 \end{pmatrix}$ <p>Finding vector \overline{AX} by finding the difference between \overline{OX} and \overline{OA}. Can be ft using candidate's \overline{OX}.</p> $\overline{OB} = \overline{OA} + \overline{AB} = \overline{OA} + 2\overline{AX}$ $\overline{OB} = \begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} + 2 \begin{pmatrix} -8 \\ 4 \\ -16 \end{pmatrix} \qquad \begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} + 2 \begin{pmatrix} \text{their } \overline{AX} \end{pmatrix}$ <p>Hence, $\overline{OB} = \begin{pmatrix} -7 \\ 11 \\ -19 \end{pmatrix}$ or $\overline{OB} = \underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$ $\begin{pmatrix} -7 \\ 11 \\ -19 \end{pmatrix}$ or $\underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$ or $\underline{(-7, 11, -19)}$</p>	<p>M1 $\sqrt{\pm}$</p> <p>dM1 $\sqrt{}$</p> <p>A1</p> <p>(3)</p> <p>[13]</p>
5.	<p>(a) $\frac{dx}{dt} = -4 \sin 2t, \quad \frac{dy}{dt} = 6 \cos t$</p> $\frac{dy}{dx} = -\frac{6 \cos t}{4 \sin 2t} \quad \left(= -\frac{3}{4 \sin t} \right)$ <p>At $t = \frac{\pi}{3}, \quad m = -\frac{3}{4 \times \frac{\sqrt{3}}{2}} = -\frac{\sqrt{3}}{2}$ accept equivalents, awrt -0.87</p> <p>(b) Use of $\cos 2t = 1 - 2 \sin^2 t$</p> $\cos 2t = \frac{x}{2}, \quad \sin t = \frac{y}{6}$ $\frac{x}{2} = 1 - 2 \left(\frac{y}{6} \right)^2$ <p>Leading to $y = \sqrt{(18 - 9x)} \quad (= 3\sqrt{(2 - x)})$ cao</p> $-2 \leq x \leq 2 \qquad k = 2$ <p>(c) $0 \leq f(x) \leq 6$ either $0 \leq f(x)$ or $f(x) \leq 6$</p> <p>Fully correct. Accept $0 \leq y \leq 6, [0, 6]$</p>	<p>B1, B1</p> <p>M1</p> <p>A1 (4)</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>B1 (4)</p> <p>B1</p> <p>B1 (2)</p> <p>(10 marks)</p>

Question Number	Scheme	Marks
6. (a)	$\overline{OA} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} \quad \& \quad \overline{OB} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}$ $\overline{AB} = \overline{OB} - \overline{OA} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$	<p>Finding the difference between \overline{OB} and \overline{OA}. Correct answer.</p> <p>M1 ± A1 [2]</p>
(b)	$l_1: \mathbf{r} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \quad \text{or} \quad \mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ $l_1: \mathbf{r} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} \quad \text{or} \quad \mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$	<p>An expression of the form (vector) ± λ(vector) r = $\overline{OA} \pm \lambda$(their \overline{AB}) or r = $\overline{OB} \pm \lambda$(their \overline{AB}) or r = $\overline{OA} \pm \lambda$(their \overline{BA}) or r = $\overline{OB} \pm \lambda$(their \overline{BA}) (r is needed.)</p> <p>M1 A1 √ aef [2]</p>
(c)	$l_2: \mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \Rightarrow \mathbf{r} = \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$	
	$\overline{AB} = \mathbf{d}_1 = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}, \quad \mathbf{d}_2 = \mathbf{i} + 0\mathbf{j} + \mathbf{k} \quad \& \quad \theta \text{ is angle}$	
	$\cos \theta = \frac{\overline{AB} \cdot \mathbf{d}_2}{(\overline{AB} \cdot \mathbf{d}_2)} = \frac{\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}}{\left(\sqrt{(1)^2 + (-2)^2 + (2)^2} \cdot \sqrt{(1)^2 + (0)^2 + (1)^2}\right)}$	<p>← Considers dot product between \mathbf{d}_2 and their \overline{AB}. M1 √</p>
	$\cos \theta = \frac{1+0+2}{\sqrt{(1)^2 + (-2)^2 + (2)^2} \cdot \sqrt{(1)^2 + (0)^2 + (1)^2}}$	<p>Correct followed through expression or equation. A1 √</p>
	$\cos \theta = \frac{3}{3\sqrt{2}} \Rightarrow \theta = 45^\circ \text{ or } \frac{\pi}{4} \text{ or awrt } 0.79.$	<p>$\theta = 45^\circ \text{ or } \frac{\pi}{4} \text{ or awrt } 0.79$ A1 cao</p>
		[3]

This means that $\cos \theta$ does not necessarily have to be the subject of the equation. It could be of the form $3\sqrt{2} \cos \theta = 3$.

Question Number	Scheme	Marks
6. (d)	<p>If l_1 and l_2 intersect then: $\begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$</p> <p>i: $2 + \lambda = \mu$ (1) j: $6 - 2\lambda = 0$ (2) k: $-1 + 2\lambda = \mu$ (3)</p> <p>(2) yields $\lambda = 3$ Any two yields $\lambda = 3, \mu = 5$</p> <p>$l_1: \mathbf{r} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + 3 \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}$ or $\mathbf{r} = 5 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}$</p>	<p>Either seeing equation (2) written down correctly with or without any other equation or seeing equations (1) and (3) written down correctly. M1 $\sqrt{\quad}$</p> <p>Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find ... dM1</p> <p>either one of λ or μ correct. A1</p> <p>$\begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}$ or $5\mathbf{i} + 5\mathbf{k}$ A1 cso</p> <p>Fully correct solution & no incorrect values of λ or μ seen earlier.</p>

[4]

Statistics for C4 Practice Paper G3

Qu	Max score	Modal score	Mean %	Mean score for students achieving grade:							
				ALL	A*	A	B	C	D	E	U
1	5		78	3.88		4.59	4.06	3.64	3.00	2.30	1.38
2	6		64	3.81	5.84	5.13	4.00	2.69	1.71	0.94	0.36
3	4	1	57	2.26	3.53	2.49	2.09	1.73	1.58	1.50	1.15
4	13		61	7.94		10.15	7.19	4.59	3.25	1.74	0.58
5	10		54	5.38		7.41	5.34	3.97	2.72	1.64	0.63
6	11		57	6.30		8.66	5.80	4.15	3.11	1.68	1.27
7	12		54	6.42	10.86	8.23	6.15	4.39	3.01	2.02	1.02
8	14		36	5.09		8.99	3.90	1.81	0.80	0.35	0.09
	75		55	41.08		55.65	38.53	26.97	19.18	12.17	6.48

Question	Scheme	Marks
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7.	<p>(a) j components $3 + 2\lambda = 9 \Rightarrow \lambda = 3$ $(\mu = 1)$ Leading to $C : (5, 9, -1)$ accept vector forms</p>	<p>M1 A1 A1 (3)</p>
	<p>(b) Choosing correct directions or finding \overrightarrow{AC} and \overrightarrow{BC} $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} = 5 + 2 = \sqrt{6} \sqrt{29} \cos \angle ACB$ use of scalar product $\angle ACB = 57.95^\circ$ awrt 57.95°</p>	<p>M1 M1 A1 A1 (4)</p>
	<p>(c) $A : (2, 3, -4)$ $B : (-5, 9, -5)$ $\overrightarrow{AC} = \begin{pmatrix} 3 \\ 6 \\ 3 \end{pmatrix}, \quad \overrightarrow{BC} = \begin{pmatrix} 10 \\ 0 \\ 4 \end{pmatrix}$ $AC^2 = 3^2 + 6^2 + 3^2 \Rightarrow AC = 3\sqrt{6}$ $BC^2 = 10^2 + 4^2 \Rightarrow BC = 2\sqrt{29}$ $\Delta ABC = \frac{1}{2} AC \times BC \sin \angle ACB$ $= \frac{1}{2} 3\sqrt{6} \times 2\sqrt{29} \sin \angle ACB \approx 33.5$ $15\sqrt{5}$, awrt 34</p>	<p>M1 A1 A1 M1 A1 (5) [12]</p>

Question Number	Scheme	Marks
Number		
8. (c)	$\frac{dP}{dt} = \lambda P \cos \lambda t$ and $t=0, P = P_0$ (1)	
8. (a)	$\frac{dP}{dt} = kP$ and $t=0, P = P_0$ (1)	
	$\int \frac{dP}{P} = \int k dt$ <p>$\ln P = kt; (+ c)$</p> <p>When $t=0, P = P_0 \Rightarrow \ln P_0 = c$ (or $P = Ae^{kt} \Rightarrow P_0 = A$)</p> <p>$\ln P = kt + \ln P_0 \Rightarrow e^{\ln P} = e^{kt + \ln P_0} = e^{kt} \cdot e^{\ln P_0}$</p> <p>Hence, <u>$P = P_0 e^{kt}$</u></p>	<p>Separates the variables with $\int \frac{dP}{P}$ and $\int k dt$ on either side with integral signs not necessary. M1</p> <p>Must see $\ln P$ and kt; Correct equation with/without $+ c$. A1</p> <p>Use of boundary condition (1) to attempt to find the constant of integration. M1</p> <p><u>$P = P_0 e^{kt}$</u> A1</p>
(b)	<p>$P = 2P_0$ & $k = 2.5 \Rightarrow \underline{2P_0 = P_0 e^{2.5t}}$</p> <p>$e^{2.5t} = 2 \Rightarrow \underline{\ln e^{2.5t} = \ln 2}$ or $\underline{2.5t = \ln 2}$...or $e^{kt} = 2 \Rightarrow \underline{\ln e^{kt} = \ln 2}$ or $\underline{kt = \ln 2}$</p> <p>$\Rightarrow t = \frac{1}{2.5} \ln 2 = 0.277258872... \text{ days}$</p> <p>$t = 0.277258872... \times 24 \times 60 = 399.252776... \text{ minutes}$</p> <p>$t = \underline{399 \text{ min}}$ or $t = \underline{6 \text{ hr } 39 \text{ mins}}$ (to nearest minute)</p>	<p>Substitutes $P = 2P_0$ into an expression involving P M1</p> <p>Eliminates P_0 and takes \ln of both sides M1</p> <p>awrt $t = \underline{399}$ or <u>$6 \text{ hr } 39 \text{ mins}$</u> A1</p>

[4]

[3]

$\int \frac{dP}{P} = \int \lambda \cos \lambda t \, dt$ <p>$\ln P = \sin \lambda t; (+ c)$</p> <p>When $t=0, P = P_0 \Rightarrow \ln P_0 = c$ (or $P = Ae^{\sin \lambda t} \Rightarrow P_0 = A$)</p> <p>$\ln P = \sin \lambda t + \ln P_0 \Rightarrow e^{\ln P} = e^{\sin \lambda t + \ln P_0} = e^{\sin \lambda t} \cdot e^{\ln P_0}$</p> <p>Hence, <u>$P = P_0 e^{\sin \lambda t}$</u></p>	<p>Separates the variables with $\int \frac{dP}{P}$ and $\int \lambda \cos \lambda t \, dt$ on either side with integral signs not necessary. M1</p> <p>Must see $\ln P$ and $\sin \lambda t$; Correct equation with/without + c. A1</p> <p>Use of boundary condition (1) to attempt to find the constant of integration. M1</p> <p><u>$P = P_0 e^{\sin \lambda t}$</u> A1</p> <p style="text-align: right;">[4]</p>
<p>(d) $P = 2P_0$ & $\lambda = 2.5 \Rightarrow 2P_0 = P_0 e^{\sin 2.5t}$</p> <p>$e^{\sin 2.5t} = 2 \Rightarrow \underline{\sin 2.5t = \ln 2}$...or ... $e^{\lambda t} = 2 \Rightarrow \underline{\sin \lambda t = \ln 2}$</p> <p><u>$t = \frac{1}{2.5} \sin^{-1}(\ln 2)$</u></p> <p>$t = 0.306338477\dots$</p> <p>$t = 0.306338477\dots \times 24 \times 60 = 441.1274082\dots$ minutes</p> <p>$t = \underline{441\text{min}}$ or $t = \underline{7 \text{ hr } 21 \text{ mins}}$ (to nearest minute)</p>	<p>Eliminates P_0 and makes $\sin \lambda t$ or $\sin 2.5t$ the subject by taking \ln's M1</p> <p>Then rearranges to make t the subject. (must use \sin^{-1}) dM1</p> <p>awrt $t = \underline{441}$ or <u>$7 \text{ hr } 21 \text{ mins}$</u> A1</p> <p style="text-align: right;">[3]</p>
14 marks	