Math 1496 - Calc I

u - Substitution

Last class we considered u-substitution.

For example

$$\int \frac{x}{x^2+1}\,dx$$

if we let $u = x^2+1$ $\quad du = 2x\,dx$

$$\int \frac{\frac{du}{2}}{u} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + c$$

$$= \frac{1}{2}\ln|x^2+1| + c$$

Now we consider definite integrals

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$$\int_1^2 e^{1-x}\,dx = \int_{x=1}^{x=2} e^{1-x}\,dx$$

let $u = 1-x$ so $du = -dx$

$$= \int_{x=1}^{x=2} -e^u \, du$$

Now what about the limits – we switch them

& from our sub. $u=1-x$

$x=1 \Rightarrow u=1-1=0 \quad x=2 \quad u=1-2=-1$

so the integral becomes

$$\int_0^{-1} -e^u \, du = \int_{-1}^{0} e^u \, du = e^u \Big|_{-1}^{0} = e^0 - e^{-1} = 1 - \frac{1}{e}$$

flip

Ex 2 $\displaystyle\int_{x=0}^{\pi/6} \sin 3x \, dx \qquad u = 3x \qquad du = 3dx$

$x=0 \quad u=0 \qquad x=\frac{\pi}{6} \quad u=\frac{3\pi}{6}=\pi/2$

so $$\int_{u=0}^{u=\pi/2} \frac{\sin u \, du}{3} = -\frac{\cos u}{3}\Big|_0^{\pi/2}$$

$$= -\frac{\cos \pi/2}{3} + \frac{\cos 0}{3} = -0 + \frac{1}{3} = \frac{1}{3}$$

Ex $\displaystyle\int_0^{\pi/2} \frac{\cos x}{1+\sin^2 x}\,dx$ (From Krista King math.com)

let $u = \sin x$ $\quad x=0 \;\; u=\sin 0 = 0$

$du = \cos x\,dx$ $\quad x=\pi/2 \;\; u = \sin \pi/2 = 1$

$$\int_0^1 \frac{du}{1+u^2} = \tan^{-1}(u)\Big|_0^1 = \tan^{-1}1 - \tan^{-1}0 = \pi/4$$

ex $\displaystyle\int_0^2 -2x\sqrt{4-x^2}\,dx$ (from web. iit. edu)

Let $u = 4-x^2 \quad du = -2x\,dx$

$x=0 \;\; u=4, \quad x=2 \;\; u=0$

$$\int_4^0 \sqrt{u}\,du = \frac{2u^{3/2}}{3}\Big|_4^0 = 0 - \frac{2}{3}(4)^{3/2} = -\frac{16}{3}$$

Ex $\displaystyle\int_{-\ln 2}^0 \frac{2e^x}{e^x+1}\,dx$ (people.math.sc.edu)

let $u = e^x + 1$ $\quad x = -\ln 2 \quad u = e^{-\ln 2} + 1 = \frac{1}{2} + 1 = 3/2$

$du = e^x dx \quad x = 0 \quad u = 2$

so $\displaystyle\int_{-\ln 2}^{0} \frac{2e^x\,dx}{e^x+1} = 2\int_{3/2}^{2} \frac{du}{u} = 2\ln|u|\Big|_{3/2}^{2}$

$= 2(\ln|2| - \ln|3/2|) = 2\ln\left(2\cdot\frac{2}{3}\right) = 2\ln(4/3)$

ex

$\displaystyle\int_0^1 6x^2 e^{x^3}\,dx$

$u = x^3 \quad du = 3x^2\,dx \qquad \begin{matrix} x=0 & u=0 \\ x=1 & u=1 \end{matrix}$

$\displaystyle\int_0^1 2\,e^u\,du = 2e^u\Big|_0^1 = 2(e-1)$

ex $\displaystyle\int_1^2 x(x-1)^4\,dx \implies \int_0^1 (u+1)u^4\,du$

let $u = x-1 \quad x=1,\ u=0$
$du = dx \quad x=2\ \ u=1$
$\displaystyle = \int_0^1 (u^5+u^4)\,du$

$\displaystyle = \frac{u^6}{6} + \frac{u^5}{5}\Big|_0^1 = \left(\frac{1}{6}+\frac{1}{5}\right) - 0$

$= +\dfrac{11}{30}$

ex $\displaystyle\int_1^4 \frac{dx}{2\sqrt{x}(1+\sqrt{x})^2}$ (harder ex.)

let $u = 1+\sqrt{x} = 1+x^{1/2}$ $\quad du = \frac{1}{2}x^{-1/2}dx = \frac{dx}{2\sqrt{x}}$

$x=1 \quad u = 1+\sqrt{1} = 2$

$x=4 \quad u = 1+\sqrt{4} = 3$

so $\displaystyle\int_2^3 \frac{du}{u^2} = \int_2^3 u^{-2}du = -u^{-1}\Big|_2^3 = -\frac{1}{u}\Big|_2^3$

$= -\frac{1}{3} + \frac{1}{2} = \frac{1}{6}$

ex $\displaystyle\int_0^1 \frac{x\,dx}{1+x^4}$ (Hard $\int$)

let $u = x^2$ $\quad du = 2x\,dx$ $\quad x=0, u=0 \quad x=1, u=1$

$\displaystyle\int_0^1 \frac{\frac{1}{2}du}{1+u^2} = \frac{1}{2}\tan^{-1}u\Big|_0^1 = \frac{1}{2}\tan^{-1}(1) - \frac{1}{2}\tan^{-1}0$

$= \frac{1}{2}\cdot\frac{\pi}{4} = \frac{\pi}{8}$