

Attacking Word Problems with Schema Instruction



Sarah R. Powell, Ph.D.

Associate Professor
The University of Texas at Austin



srpowell@utexas.edu



[@sarahpowellphd](https://twitter.com/sarahpowellphd)





Introduce yourself.

Describe your role as an educator.

Describe the mathematics you support.



Word Problem Problems? Keys to Math Word-Problem Solving

Sarah R. Powell, Ph.D.

srpowell@utexas.edu

@sarahpowellphd

www.sarahpowellphd.com

Maya has 120 caramel apples to sell. Each caramel apple is covered with one topping.

- $\frac{1}{5}$ of the caramel apples are covered with peanuts.
- $\frac{1}{3}$ are covered with chocolate chips.
- $\frac{3}{10}$ are covered with coconut.
- The rest are covered with sprinkles.

How many caramel apples are covered with sprinkles?

- A 100
- B 33
- C 25
- D 20

Solve the problem

What skills are necessary to solve this problem?



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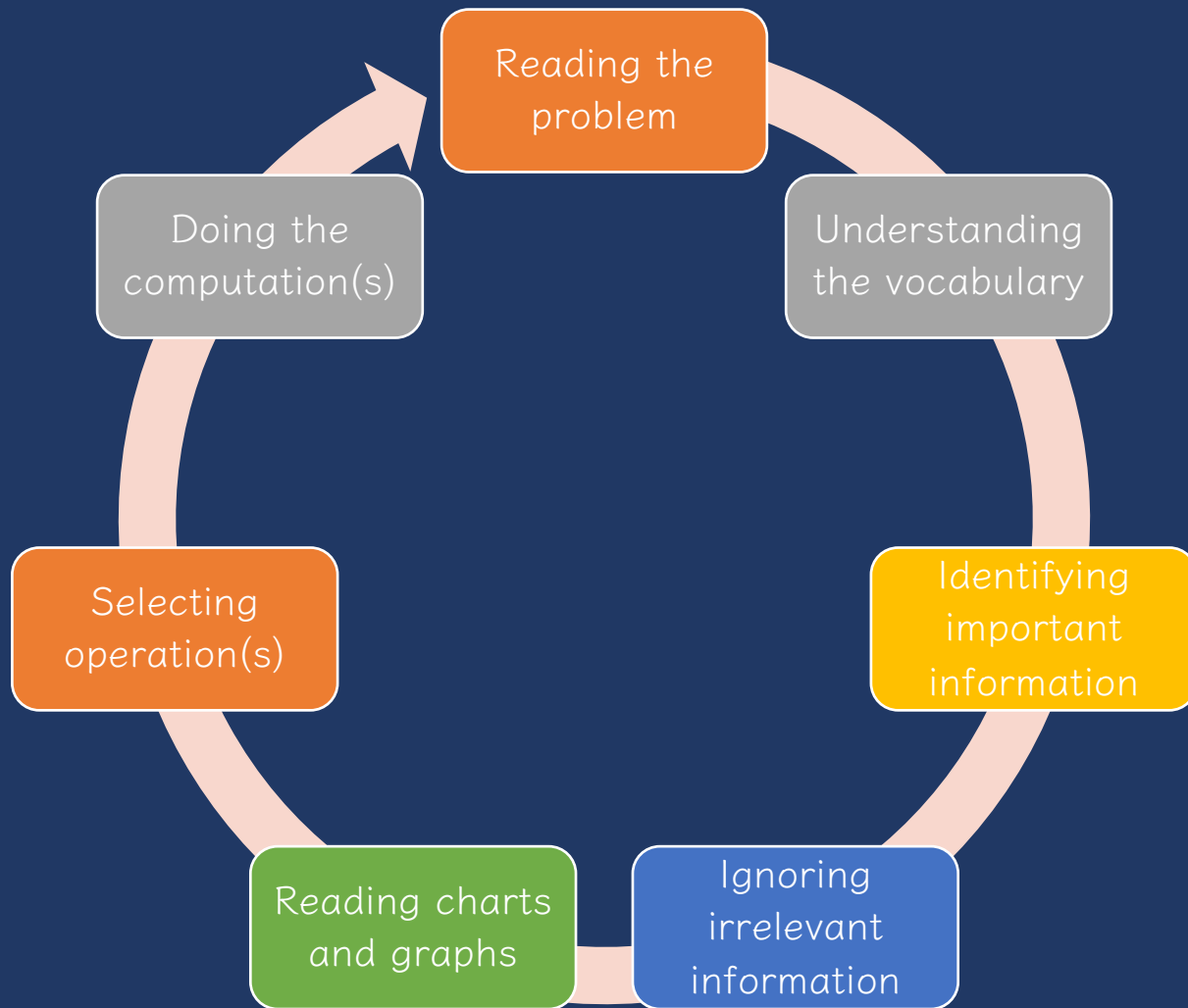


How would you solve this problem?
What skills are necessary to solve this problem?



Word-Problem Solving





Ineffective Strategies



~~1. Keywords tied to operations~~



Addition	Subtraction
• Sum • Total	• Fewer • Less than
• Plus • In all	• Exceed • Remain
• And • Join	• Are not • Minus
• Altogether	• Difference
• Perimeter	• How many more
• Together	• Take away
	• Left over
When they say... They mean...	
• Times • Each	• Half • Separate
• Twice • Per	• Split • Quotient
• Area • Product	• Division • Cut up
• In all • Multiple	• Dividend • Same
• Equal groups	• Divided by
• Multiplied by	• Cut up
Multiplication	Division

Lincoln had 8 pencils **fewer** than Roscoe.
If Roscoe had 18 pencils, how many
pencils did Lincoln have?

Lincoln had 8 pencils **fewer** than Roscoe.
If Lincoln had 18 pencils, how many
pencils did Roscoe have?

Key Words Used in Math Word Problems

Addition Words	Subtraction Words
<ul style="list-style-type: none"> + add + all together + and + both + combined + how many in all + how much + in all + increased by + plus + sum + together + total 	<ul style="list-style-type: none"> - change - decreased by - difference - fewer or fewer than - how many are left (or have) - how many did not have - how much - how much taller, heavier, less or less - lost - minus - need to - reduce - remain - subtract - take away

Multiplication Words	Division Words
<ul style="list-style-type: none"> x by (dimension) x double x each group x every x factor of x increased by x multiplied by x of x product x times x triple 	<ul style="list-style-type: none"> ÷ as much ÷ cut up ÷ each group ÷ equal share ÷ half (or at) ÷ how many parts ÷ per ÷ percent ÷ quotient ÷ ratio of ÷ separated ÷ share some

Division

Taking a total amount and sharing equally amongst groups or the number in each group.

Keywords
 how many will each receive
 How many in each group
 Shared
 Divided
 Equal/Equally
 Quotient

Multiplication

Putting things/objects into equal groups to find the total amount.

Keywords
 Altogether
 In all
 Product
 Equal groups
 At this rate
 Doubled, tripled (and so on)

Subtraction

Taking away from a total amount.

Keywords
 Different
 More than
 Less than
 How many more
 Fewer than
 How many left over

Addition

Putting two or more things/amounts together.

Keywords
 Total
 Altogether
 In all
 Sum
 More than
 Combined

Problem Solving Key Words

Addition	Subtraction
<ul style="list-style-type: none"> add altogether both combined How many in all? in all increase join plus sum total together 	<ul style="list-style-type: none"> are not decrease difference fewer, larger, shorter How much more? left less than minus remain take away

Multiplication	Division
<ul style="list-style-type: none"> area as much by factor multiple multiplied product in all 	<ul style="list-style-type: none"> average evenly equal parts divisor divided distribute quotient ratio some separate split

Key Words

addition: sum, both, total, together, plus, add, combined, in all, perimeter, more than

subtraction: difference, fewer, less than, minus, take away, how many more...

multiplication: product, equal groups, each, in all, multiple, area, double, times, distribute

division: quotient, split, equal groups, half, shared equally, each

OPERATION clue words

ADDITION	SUBTRACTION
<ul style="list-style-type: none"> total join more than in all altogether sum increased 	<ul style="list-style-type: none"> less than decreased remaining left fewer take away difference minus

MULTIPLICATION	DIVISION
<ul style="list-style-type: none"> product times twice as many as by of groups 	<ul style="list-style-type: none"> quotient each per into divided by split share equally

When they say... They mean...

Addition	Subtraction
<ul style="list-style-type: none"> Sum Total Plus In all And Altogether Perimeter Together 	<ul style="list-style-type: none"> Fewer Less than Exceed Remain Are not Minus Difference How many more Take away Left over

Multiplication	Division
<ul style="list-style-type: none"> Times Each Twice Per Product Multiple Groups led by 	<ul style="list-style-type: none"> Half Separate Split Quotient Divisor Cut up Dividend Same Divided by Cut up

Key Words for All Operations

addition: added to, plus, join, more than, add, altogether, and, increase, sum, together, total

subtraction: decrease, minus, less than, gave, subtract, share, less, take away, difference

multiplication: times, multiple, twice, equal groups, split, multiplied, increased by, product, of, total

division: divided by, parts, goes into, percent, quotient, share equally, split up, divided into, half

Math Operation - Key Words

Addition	Subtraction
<ul style="list-style-type: none"> add altogether and both in all sum total increase 	<ul style="list-style-type: none"> difference fewer than gave/take away decreased by how many more show much longer/smaller/shorter minus remaining

Multiplication	Division
<ul style="list-style-type: none"> area product Each by - of - per Times double, twice, triple total increase 	<ul style="list-style-type: none"> quotient divide into equal parts/share equally per amount of each

KEY WORD Posters

ADDITION	MULTIPLICATION
<ul style="list-style-type: none"> sum total more than plus both combined increased by perimeter 	<ul style="list-style-type: none"> product double area times per every each by

SUBTRACTION	DIVISION
<ul style="list-style-type: none"> difference remain left less than minus how many more fewer than decrease give away reduce discount 	<ul style="list-style-type: none"> quotient divide by into split out of shared per every each evenly equal groups half

Math Keywords

Addition
 + add, altogether, and, both, how many, how much, in all, increased by, plus, sum, together, total

Subtraction
 - are not, change, decreased by, difference, fewer, have left, how many did not have, how many more, how much more, less than, remain, subtract, take away, taller/shorter

Division
 ÷ as much, cut up, divided by, each group has half (or other fractions), how many in each parts, quotient of, separated, share something equally, split

Multiplication
 x by (dimension), double, each group, multiplied by, of, product of, times, triple

Math Key Words

Addition	Subtraction	Multiplication	Division
plus, sum, add, total, all together, increase, more, combine	subtract, minus, difference, left, left over, decrease, take away, fewer	times, product, factor, double, groups, each, area, rows	quotient, split, share, divide, separate, each, average, equal groups



Word-Problem Words Poster Set

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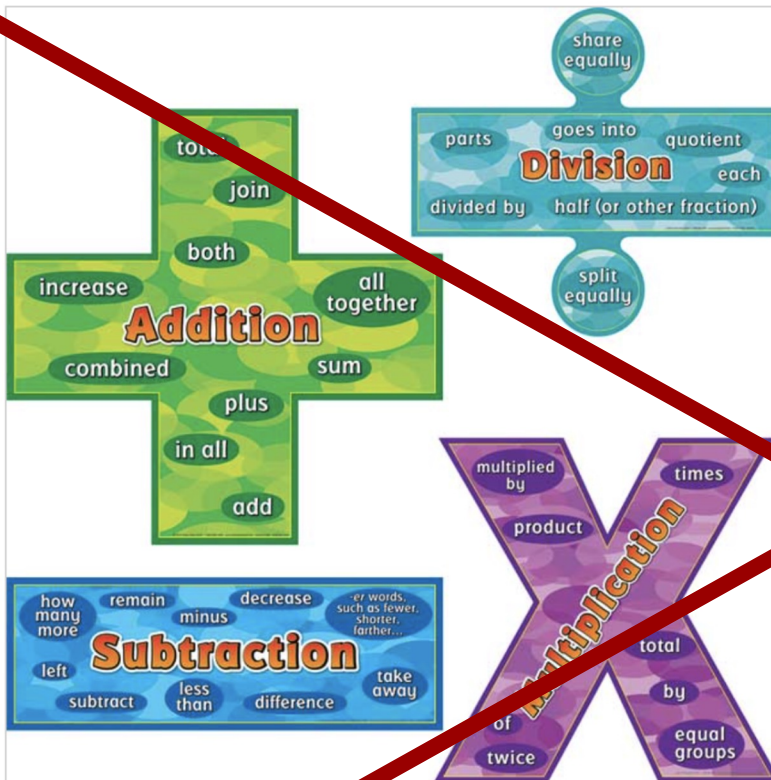
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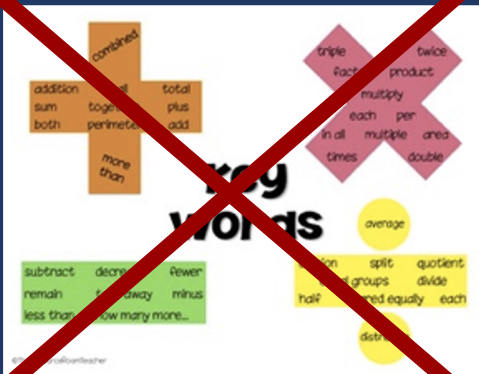




Description of Single-Step Word Problems (n = 132)

Schema	Occurrence of schema		Any keyword		Schema-specific keywords ^a		Multiple keywords ^a		Keyword(s) led to correct solution ^a	
	n	%	n	%	n	%	n	%	n	%
Total	27	20.5	26	96.3	23	88.5	5	19.2	21	80.8
Difference	17	12.9	17	100.0	14	82.4	2	11.8	12	70.6
Change	11	8.3	7	63.6	5	71.4	5	71.4	2	28.6
Equal groups	29	22.0	26	89.7	22	84.6	18	69.2	8	30.8
Comparison	10	7.6	9	90.0	9	100.0	4	44.4	5	55.6
Ratios or proportions	29	22.0	23	79.3	9	39.1	9	39.1	6	26.1
Product of measures	9	6.8	9	100.0	8	88.9	1	11.1	5	55.6

^aWhen a problem featured a keyword.



Description of Multi-Step Word Problems (n = 84)

Schema	Occurrence of schema ^a		Any keyword		Keyword(s) led to correct solution ^b	
	n	%	n	%	n	%
Total	40	47.6	39	97.5	3	7.7
Difference	11	13.1	11	100.0	1	9.1
Change	21	23.8	19	95.0	1	5.3
Equal groups	49	58.3	48	98.0	1	2.1
Comparison	7	8.3	7	100.0	0	0.0
Ratios or proportions	22	25.0	16	76.2	1	6.3
Product of measures	7	8.3	7	100.0	2	28.6

^aSum across schemas does not equal 100 because each word problem featured more than one schema.

^bWhen a problem featured a keyword.



Mr. Rivera's taxable income is \$20 each hour before taxes are taken out. Mr. Rivera worked a total of 40 hours each week for 50 weeks.

What is the dollar amount, to the nearest dollar, taken out for taxes based on Mr. Rivera's taxable income?

Jessica rented 1 video game and 3 movies for a total of \$11.50.

- The video game cost \$4.75 to rent.
- The movies cost the same amount each to rent.

What amount, in dollars, did Jessica pay to rent each movie?

The temperature of a substance decreased by 24°C per minute for 3 minutes. What was the overall change of the temperature of the substance?



Important notes about keywords

Keywords are important to identify and understand

Keywords are the mathematical vocabulary that help an students understand what the story is about and what they need to do

Talk about keywords
("What does *more than* tell you about?")



But, *do not* tie a keyword to a specific operation!



2. Presenting problems by operation



Name: _____

Date: _____

Addition Word Problems



Solve the word problems. Show your work.

- Noah had 12 books. He got 5 more books. How many books did Noah have in all?
- Bonnie found 8 rocks on her sidewalk and 7 rocks in her backyard. How many rocks did Bonnie find in all?
- Edward had 5 toy cars. He got 8 more toy cars. How many toy cars did Edward have in all?
- Mariela collected 11 feathers. Then she found 3 more feathers. How many feathers did Mariela have in all?
- LaMonte made 14 cookies. Then he made 5 more cookies. How many cookies did LaMonte have in all?



Subtraction word problems

Grade 2 Word Problem Worksheet

- Code had 87 marbles. He gave 18 to Dylan and 6 to Sam. How many does he have left?
- The aquarium has a lot of fish tanks. They bought 18 more fish and now the aquarium has 149 fish. How many fish did the aquarium have to begin with?
- Alyssa had 129 cookies. Aiyanna has 140 cookies and 34 crackers. How many more cookies does Aiyanna have than Alyssa?



LONG DIVISION WORD PROBLEMS

- Zookeeper Al wants to give each monkey in the zoo an equal number of bananas. There are 37 monkeys in the zoo and 567 bananas. How many bananas does each monkey get? And how many are left over for him to eat himself?
- Betty has 23 oranges and needs to pack them up equally in 23 boxes. How many oranges go in each box and how much does she have left over?
- Miss King has 1376 pages of scrap paper. She wants to make them into scrap paper packets for her 32 students. How many pages will each packet have? How many extra pages will she have left over?
- Mr. Chong has 1,440 pages of scrap paper. He instead wants to make packets of 40 pages each but forgets to check if that will be enough for his 37 students. Will there be enough packets per student? If not how much more scrap paper does he need?



Effective Strategies



Teaching Problem Solving

Have an attack strategy

Teach word-problem schemas

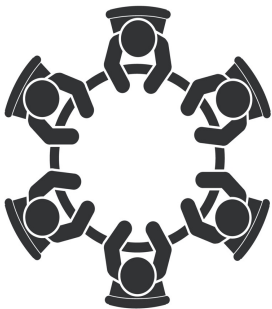


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- $\frac{3}{10}$ are covered with coconut.
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How many caramel apples are covered with sprinkles?

- A** 100
- B** 33
- C** 25
- D** 20



What was your process for working through this problem?



Attack Strategy

SOLVE

Study the problem.

Organize the facts.

Line up the plan.

Verify the plan with computation.

Examine the answer.

R-CUBES

Read the problem.

Circle key numbers.

Underline the question.

Box action words.

Evaluate steps.

Solve and check.

UPS✓

UNDERSTAND
Read and explain.

PLAN
How will you solve the problem?

SOLVE
Set up and do the math!

✓CHECK
Does your answer make sense?



Have an attack strategy

RIDE

Read the problem.

Identify the relevant information.

Determine the operation and unit for the answer.

Enter the correct numbers and calculate, then check the answer.

RIDGES

Read the problem.

I know statement.

Draw a picture.

Goal statement.

Equation development.

Solve the equation.



Have an attack strategy

STAR

Stop and read the problem carefully.

Think about your plan and the strategy you will use.

Act. Follow your plan and solve the problem.

Reverview your answer.

RICE

Read and record the problem.

Illustrate your thinking.

Compute.

Explain your thinking.



Have an attack strategy

SUPER

Slowly read the story problem twice.
Underline the question and circle the numbers you need.
Picture it. Draw the scenario to show what is happening.
Explain the problem with a number sentence.
Rewrite the answer in a sentence.

SHINES

Slowly and carefully read the problem.
Highlight or underline key information.
Identify the question by drawing a circle around it.
Now solve the problem. Show your work.
Examine your work for precision, accuracy, and clarity.
Share your answer by writing a sentence.



Have an attack strategy

SOLVE

Study the problem.

Organize the facts.

Line up the plan.

Verify the plan with computation.

Examine the answer.

R-CUBES

Read the problem.

Circle key numbers.

Underline the question.

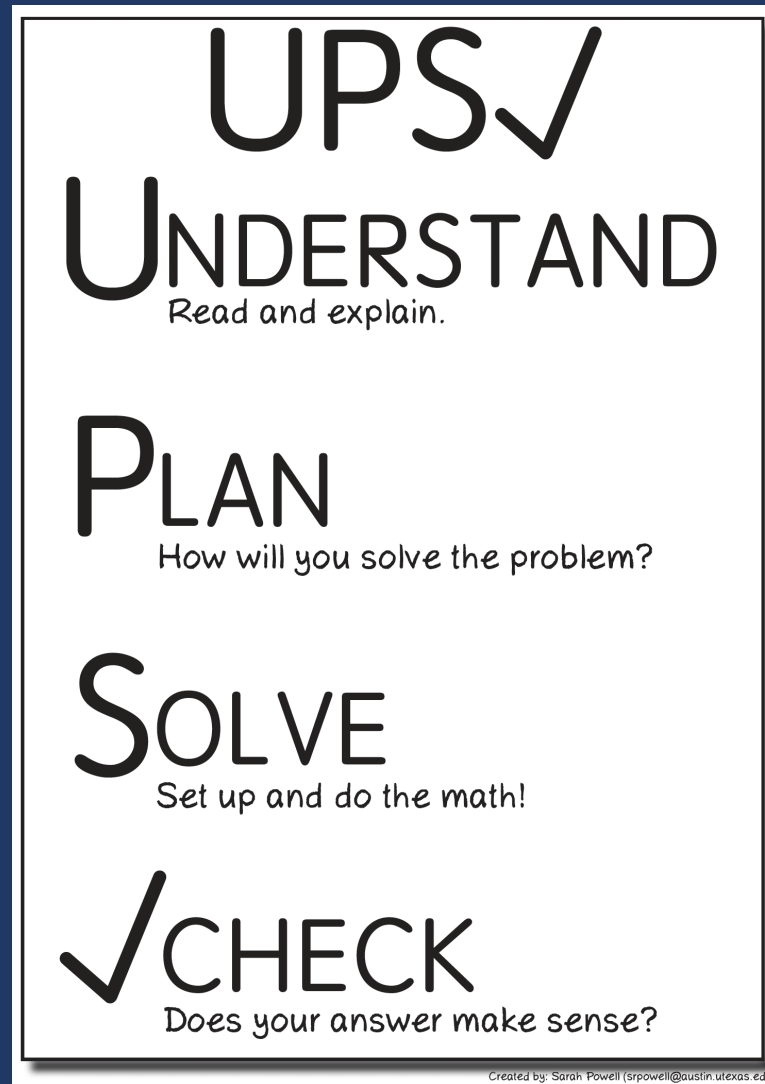
Box action words.

Evaluate steps.

Solve and check.



Have an attack strategy





Share your favorite attack strategy.



Combining an Attack Strategy with a Focus on Schemas



Teach word-problem schemas

Total

Difference

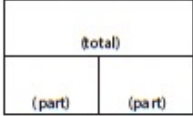
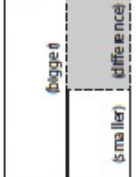


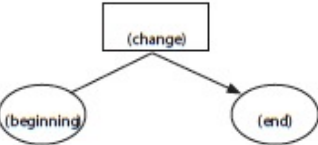
Change

Equal Groups

Comparison

Ratios/Proportions



Schema and Definition	Equations and Graphic Organizers	Examples			Variations
Total (Combine; Part-part-whole) Parts combined for a sum	$P1 + P2 = T$ (part + part = total) 	Sum unknown: Lyle has 11 red apples and 18 green apples. How many apples does Lyle have altogether?	Part unknown: Lyle has 29 red and green apples. If 11 of the apples are red, how many green apples does Lyle have?		More than two parts: Lyle has 34 apples. Of the apples, 11 are red, 18 are green, and the rest are yellow. How many yellow apples does Lyle have?
Difference (Compare) Sets compared for a difference	$B - s = D$ (bigger - smaller = difference)  $G - L = D$ (greater - less = difference) 	Difference unknown: Sasha wrote 85 words in her essay, and Tabitha wrote 110 words. How many fewer words did Sasha write than Tabitha?	Bigger/greater unknown: Tabitha wrote 25 more words than Sasha. If Sasha wrote 85 words, how many words did Tabitha write?	Smaller/lesser unknown: Tabitha wrote 110 words in her essay. Sasha wrote 25 words fewer than Tabitha. How many words did Sasha write?	(None)
Change (Join; Separate) An amount that increases or decreases	$ST \pm C = E$ (start \pm change = end)  	End (increase) unknown: Jorge had \$52. Then, he earned \$16 babysitting. How much money does Jorge have now? End (decrease) unknown: Jorge had \$52. Then, he spent \$29 at the ballpark. How much money does Jorge have now?	Change (increase) unknown: Jorge had \$52. Then, he earned some money babysitting. Now, Jorge has \$68. How much did Jorge earn babysitting? Change (decrease) unknown: Jorge had \$52 but spent some money when he went to the ballpark. Now, Jorge has \$23. How much did Jorge spend at the ballpark?	Start (increase) unknown: Jorge has some money, and then he earned \$16 for babysitting. Now, Jorge has \$68. How much money did he have to start with? Start (decrease) unknown: Jorge had some money. Then, he spent \$29 at the ballpark and has \$23 left. How much money did Jorge have before going to the ballpark?	Multiple changes: Jorge had \$78. He stopped and bought a pair of shoes for \$42 and then he spent \$12 at the grocery. How much money does Jorge have now?

Total

Part-part-whole
Combine

Parts put together into a **total**

Daniela saw **3** canoes and **8** kayaks.
How many boats did Daniela see?

Total

Daniela saw **11** boats. If **3** of the boats
were canoes, how many were kayaks?

Part

Daniela saw **11** boats. **8** of the boats
were kayaks, how many were canoes?

Part



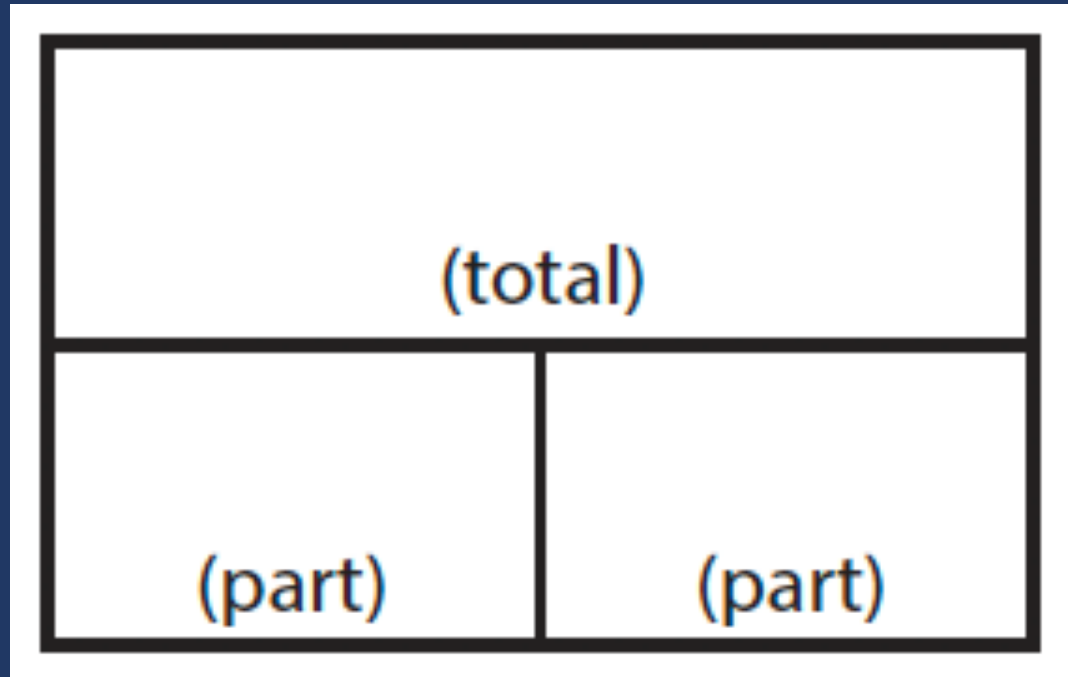
Total

“Are parts put together for a total?”



Total

$$P1 + P2 = T$$



Total

Additive Word Problems

A.

Ali delivered 12 boxes of cookies on Friday and 25 boxes of cookies on Saturday. How many boxes of cookies did Ali deliver?

B.

In March and April, it rained a total of 11.4 inches. If it rained 3.9 inches in March, how many inches did it rain in April?

C.

Audrey has 162 wooden beads and 95 glass beads. What is the difference between Audrey's wooden beads and glass beads?

D.

Damian's dog eats $9\frac{1}{2}$ cups of dog food each week. Monte's dog eats $4\frac{1}{4}$ cups less each week than Damian's dog. How much does Monte's dog eat in a week?



Total

B.

In March and April, it rained a total of 11.4 inches. If it rained 3.9 inches in March, how many inches did it rain in April?

U✓

P✓

S✓

✓✓

$$P1 + P2 = T$$

$$3.9 + ? = 11.4$$

$$? = 7.5 \text{ inches}$$



Total



Write a Total problem.

Difference

Compare

Greater and **lesser** amounts compared for a **difference**

Adrianna has **10** pencils. Tracy has **4** pencils.
How many more pencils does Adrianna have?

Difference

Adrianna has **6** more pencils than Tracy. If Tracy has **4** pencils, how many does Adrianna have?

Greater
amount

Tracy has **6** fewer pencils than Adrianna.
Adrianna has **10** pencils. How many pencils does Tracy have?

Lesser
amount



Total

“Are parts put together for a total?”

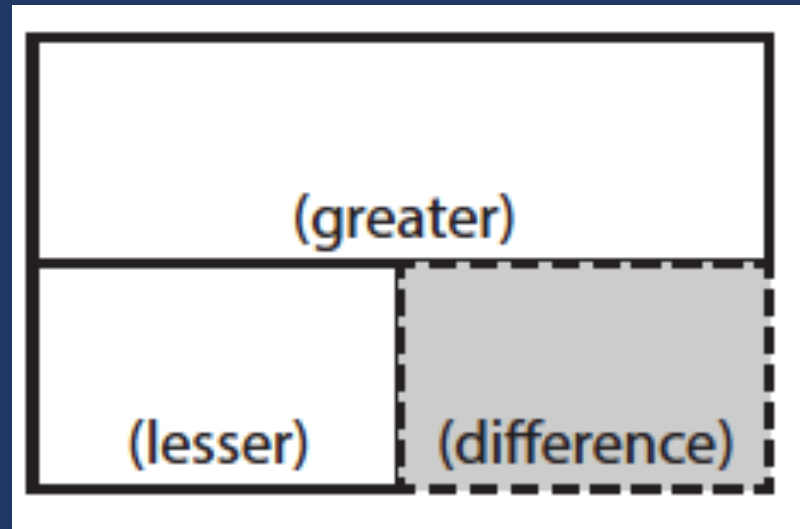
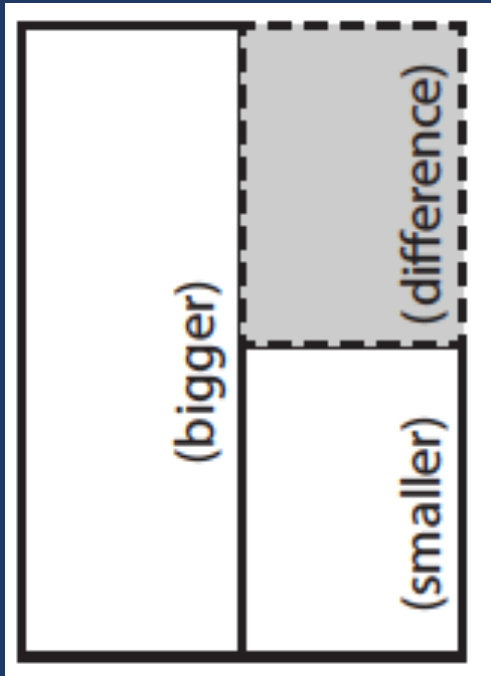
Difference

“Are amounts compared for a difference?”



Difference

$$G - L = D$$



Difference

Additive Word Problems

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Difference

Jana has 107 wooden beads and 68 glass beads. How many more wooden beads than glass beads does Jana have?

Enter your answer in the response box.

← → ↶ ↷ ✕

1	2	3
4	5	6
7	8	9
0	.	$\frac{\square}{\square}$

$$\begin{array}{l} U \\ P \\ S \\ \checkmark \end{array} \quad \begin{array}{l} G - L = D \\ 107 - 68 = ? \\ ? = 39 \text{ more} \\ \text{wooden beads} \end{array}$$

Difference



Write a
Difference
problem.

Change

Join

An amount that **increases** or decreases

Nickole had 6 notebooks. Then, she bought 3 notebooks. How many notebooks does Nickole have now?

End amount

Nickole had 6 notebooks. Then, she bought a few more notebooks. Now, Nickole has 9 notebooks. How many notebooks did she buy?

Change
amount

Nickole had some notebooks. Then, she bought 3 notebooks. Now, Nickole has 9 notebooks. How many notebooks did she have to start with?

Start
amount



Change

Separate

An amount that increases or **decreases**

Samantha baked **20** cookies. Then, she ate **3** of the cookies. How many cookies does Samantha have now?

End amount

Samantha baked **20** cookies. Then, she ate some of the cookies. Now, she has **17** cookies. How many cookies did Samantha eat?

Change
amount

Samantha baked some cookies. She ate **3** of the cookies and has **17** cookies left. How many cookies did Samantha bake?

Start
amount



Total

“Are parts put together for a total?”

Difference

“Are amounts compared for a difference?”

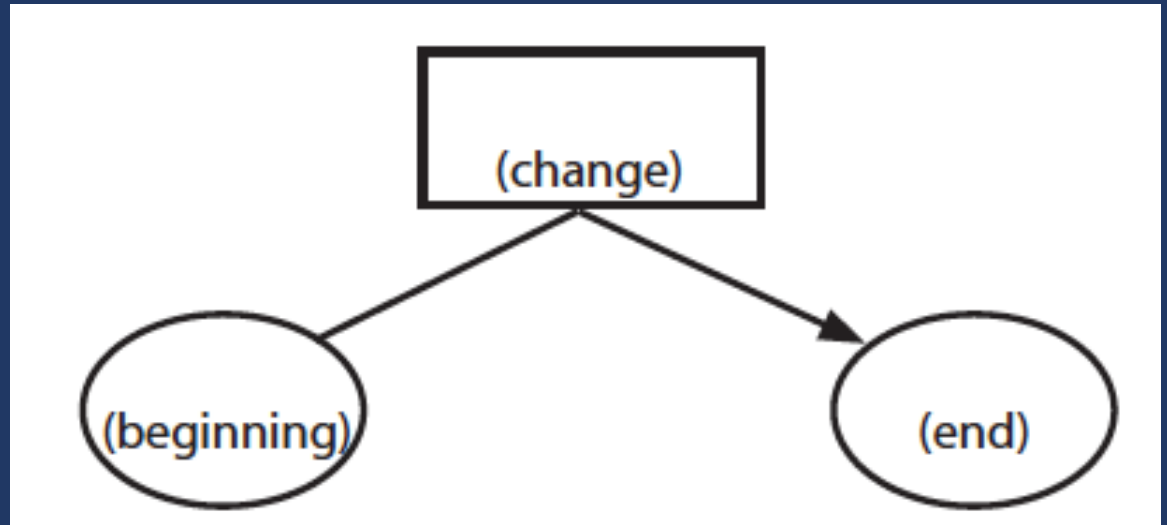
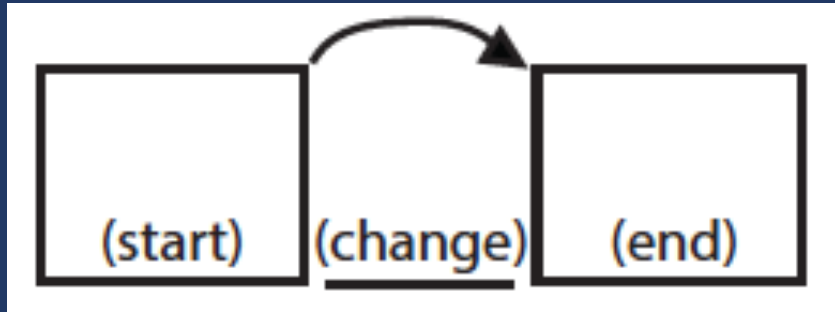
Change

“Does an amount increase or decrease?”



Change

$$ST + / - C = E$$



Change

Additive Word Problems

E.

A plant was $3\frac{3}{4}$ inches tall at the beginning of June. By the end of July, the plant was $9\frac{1}{8}$ inches tall. How many inches did the plant grow in 2 months?

F.

Martina has some money in her bank account. Then, she spent \$135.69 and has a balance of -\$24.80. How much money did Martina have to begin with?

G.

Sam mows lawns and made \$560 last week. She made \$95 on Monday, \$135 on Tuesday, and \$70 on Wednesday. How much did Sam make on Thursday and Friday?

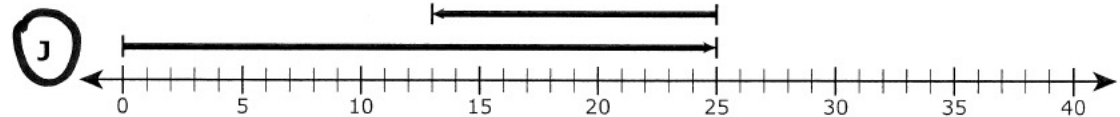
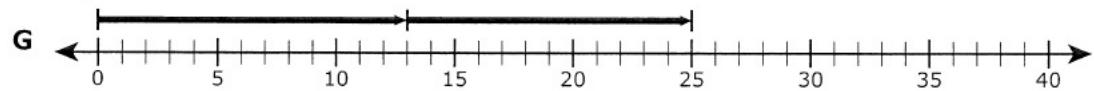
H.

Hui saved \$70 in January. In February, she spent \$64 of the money she saved. She saved \$92 more in March. How much has Hui saved by the end of March?



Change

- 28 There were 25 people in a library. Some people left the library and went home. Then there were 13 people remaining in the library. Which number line represents one way to determine the number of people who left the library?



U
P
S
✓

$$\boxed{25} - ? = \boxed{13}$$

? = 12 people left

Change



Write a Change
problem.

Additive Word Problems

E.

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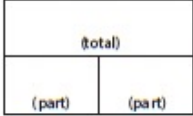
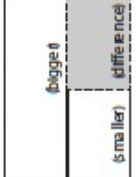

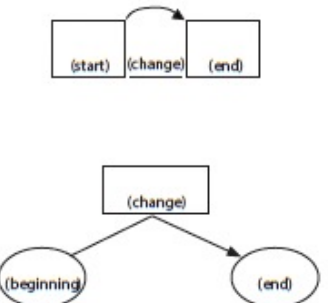
G.

Sam mows lawns and made \$560 last week. She made \$95 on Monday, \$135 on Tuesday, and \$70 on Wednesday. How much did Sam make on Thursday and Friday?

H.

Hui saved \$70 in January. In February, she spent \$64 of the money she saved. She saved \$92 more in March. How much has Hui saved by the end of March?



Schema and Definition	Equations and Graphic Organizers	Examples			Variations
Total (Combine; Part-part-whole) Parts combined for a sum	$P1 + P2 = T$ (part + part = total) 	Sum unknown: Lyle has 11 red apples and 18 green apples. How many apples does Lyle have altogether?	Part unknown: Lyle has 29 red and green apples. If 11 of the apples are red, how many green apples does Lyle have?		More than two parts: Lyle has 34 apples. Of the apples, 11 are red, 18 are green, and the rest are yellow. How many yellow apples does Lyle have?
Difference (Compare) Sets compared for a difference	$B - s = D$ (bigger - smaller = difference)  $G - L = D$ (greater - less = difference) 	Difference unknown: Sasha wrote 85 words in her essay, and Tabitha wrote 110 words. How many fewer words did Sasha write than Tabitha?	Bigger/greater unknown: Tabitha wrote 25 more words than Sasha. If Sasha wrote 85 words, how many words did Tabitha write?	Smaller/lesser unknown: Tabitha wrote 110 words in her essay. Sasha wrote 25 words fewer than Tabitha. How many words did Sasha write?	(None)
Change (Join; Separate) An amount that increases or decreases	$ST \pm C = E$ (start \pm change = end) 	End (increase) unknown: Jorge had \$52. Then, he earned \$16 babysitting. How much money does Jorge have now? End (decrease) unknown: Jorge had \$52. Then, he spent \$29 at the ballpark. How much money does Jorge have now?	Change (increase) unknown: Jorge had \$52. Then, he earned some money babysitting. Now, Jorge has \$68. How much did Jorge earn babysitting? Change (decrease) unknown: Jorge had \$52 but spent some money when he went to the ballpark. Now, Jorge has \$23. How much did Jorge spend at the ballpark?	Start (increase) unknown: Jorge has some money, and then he earned \$16 for babysitting. Now, Jorge has \$68. How much money did he have to start with? Start (decrease) unknown: Jorge had some money. Then, he spent \$29 at the ballpark and has \$23 left. How much money did Jorge have before going to the ballpark?	Multiple changes: Jorge had \$78. He stopped and bought a pair of shoes for \$42 and then he spent \$12 at the grocery. How much money does Jorge have now?

Teach word-problem schemas

Total

Difference

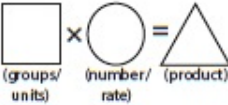
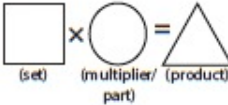
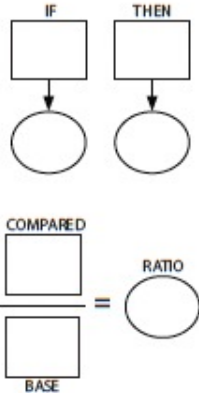
Change

Equal Groups

Comparison

Ratios/Proportions



Schema and Definition	Graphic Organizers	Examples			Variations
Equal Groups (Vary) A number of equal sets or units		Product unknown: Maria bought 5 cartons of eggs with 12 eggs in each carton. How many eggs did Maria buy?	Groups unknown: Maria bought 60 eggs. The eggs were sold in cartons with 12 eggs each. How many cartons of eggs did Maria buy?	Number unknown: Maria bought 5 cartons of eggs for a total of 60 eggs. How many eggs were in each carton?	With rate: Maria bought 5 cartons of eggs. Each carton cost \$2.95. How much did Maria spend on eggs?
Comparison One set as a multiple or part of another set		Product unknown: Malik picked 7 flowers. Danica picked 3 times as many flowers. How many flowers did Danica pick?	Set unknown: Danica picked 3 times as many flowers as Malik. If Danica picked 21 flowers, how many flowers did Malik pick?	Times unknown: Malik picked 7 flowers. Danica picked 21 flowers. How many times more flowers did Danica pick?	With fraction: Malik picked 25 red and yellow flowers. If 1/5 of the flowers were yellow, how many were red?
Proportions		Subject unknown: Sally typed 56 words in 2 minutes. How many words could Sally type in 7 minutes?	Object unknown: Sally typed 56 words in 2 minutes. How many minutes would it take Sally to type 192 words?	Base unknown: Justin baked cookies and brownies. The ratio of cookies to brownies was 3:5. If he baked 15 cookies, how many brownies did he bake?	With percentage: Watson received an 80% on his science quiz. If the test had 40 questions, how many questions did Watson answer correctly?
		Compared unknown: Justin baked cookies and brownies. The ratio of cookies to brownies was 3:5. If he baked 25 brownies, how many cookies did he bake?	Ratio unknown: Justin baked 15 cookies and 25 brownies. What's the ratio of cookies to brownies?	With unit rate: Paula bought 5 boxes of markers. She spent \$9.75. What is the price of one box of markers?	



Equal Groups

Array
Vary

Groups multiplied by **number in each group**
for a **product**

Toni has **2** boxes of crayons. There are **12** crayons in each box. How many crayons does Toni have altogether?

Groups

Toni has **24** crayons. They want to place them equally into **2** boxes. How many crayons will Toni place in each box?

Number in
each group

Toni has **24** crayons. They put them into boxes with **12** crayons each. How many boxes did Toni use?

Product



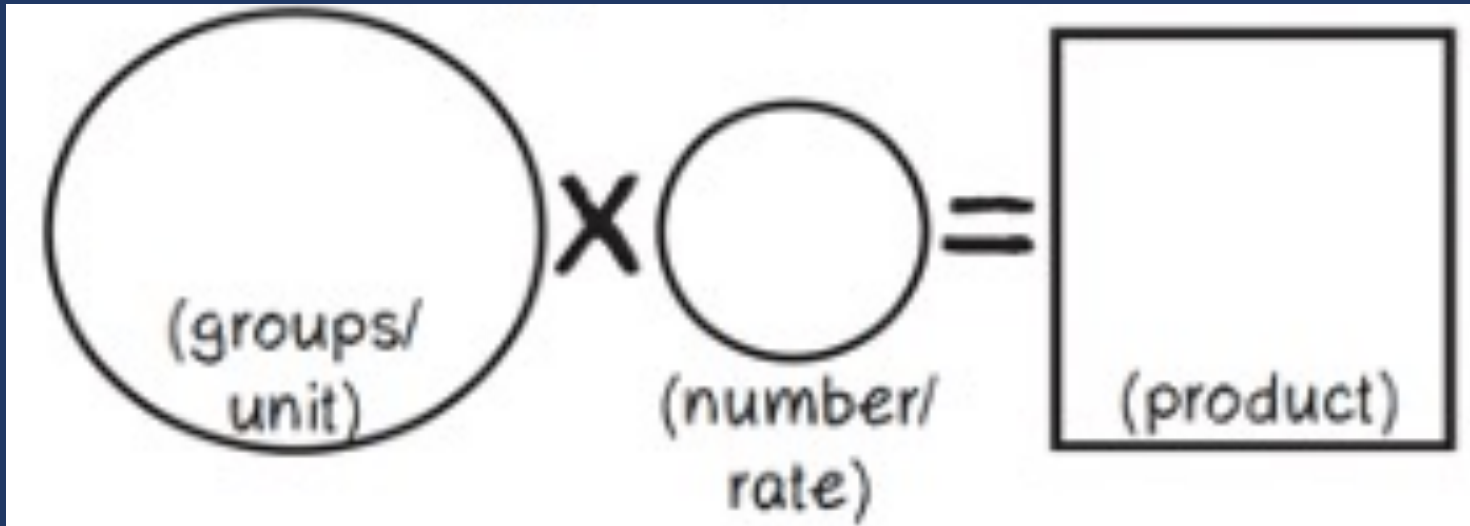
Equal Groups

“Are there groups with an equal number in each group?”



Equal Groups

$$GR \times N = P$$



Equal Groups

Multiplicative Word Problems

A.

Lola baked 6 pies. For each pie, Lola used 5 apples. How many apples did Lola use?

B.

Zachary has 3 feet of string. He makes bracelets, and each bracelet needs $5\frac{1}{4}$ inches of string. How many bracelets could Zachary make?

C.

Enrique has 2 times as many pencils as Ava. Ava has 6 pencils. How many pencils does Enrique have?

D.

Susan has 7 times as many books as Mo. Mo has 18 books. How many books Susan has?



Equal Groups

Jack has 24 fish. He puts them into 4 bowls. Each bowl has an equal number of fish.

How many fish are in each bowl?

Handwritten student work for the problem:

UP
S
✓

Calculator interface showing a 4x3 grid of numbers (1-9, 0, ., $\frac{\Box}{\Box}$) and operation buttons (\leftarrow , \rightarrow , \leftarrow , \rightarrow , \times).

Equation: $\boxed{4} \times \textcircled{?} = \triangle 24$

Division: $\begin{array}{r} 4 \\ \times ? \\ \hline 24 \end{array}$

Long Division: $4 \overline{)24}$

Answer: $? = 6$ fish

Equal Groups



Write an Equal Groups problem.

Comparison

Set multiplied by a number of **times**
for a **product**

Brooke ran **6** minutes. Shaleeni ran **4** times longer than Brooke. How many minutes did Shaleeni run?

Set

Number of
times

Product



Equal Groups

“Are there groups with an equal number in each group?”

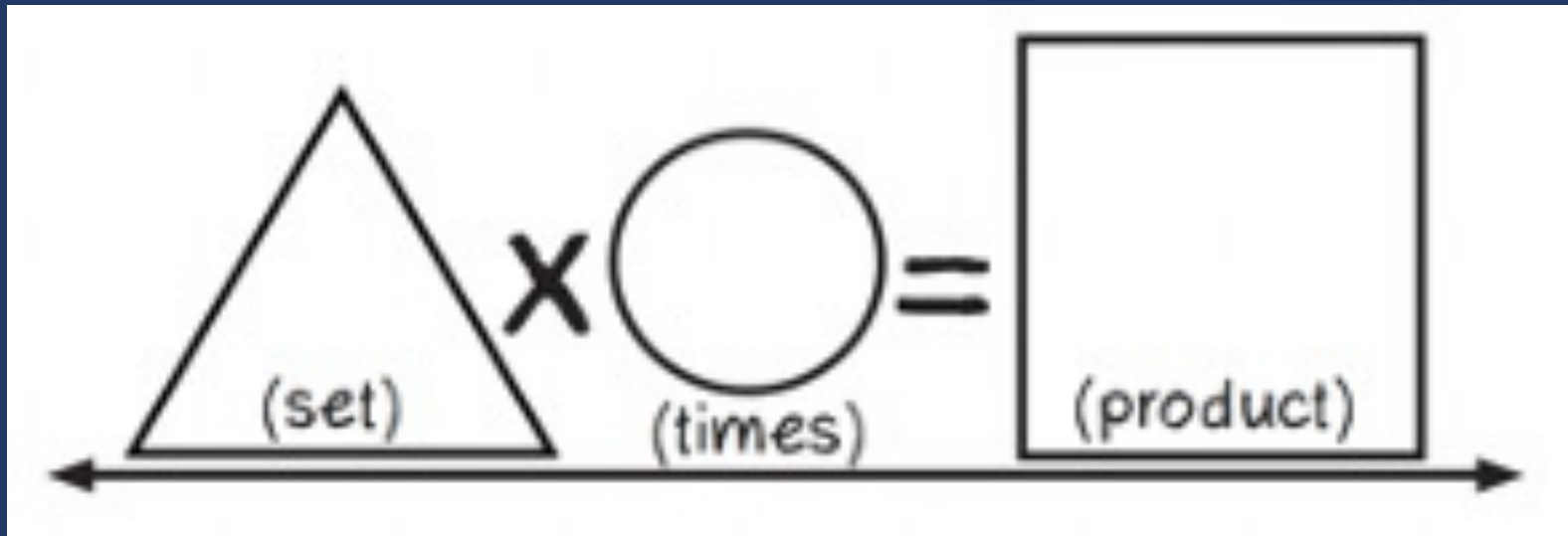
Comparison

“Is a set compared a number of times?”



Comparison

$$S \times T = P$$



Comparison

Multiplicative Word Problems	
A. Lola baked 6 pies. For each pie, Lola used 5 apples. How many apples did Lola use?	B. Zachary has 3 feet of string. He makes bracelets, and each bracelet needs $5\frac{1}{4}$ inches of string. How many bracelets could Zachary make?
C. Enrique has 2 times as many pencils as Ava. Ava has 6 pencils. How many pencils does Enrique have?	D. Susan has 7 times as many books as Mo. Mo has 18 books. How many books Susan has?



Comparison

Susan has 3 times as many books as Mary. Mary has 18 books. Which equation can be solved to figure out how many books Susan has?

(A) $\square - 3 = 18$

(B) $3 + 18 = \square$

(C) $18 \div \square = 3$

☒ (D) $3 \times 18 = \square$

U
P
S
✓

$$\boxed{18} \times \textcircled{3} = \triangle ?$$

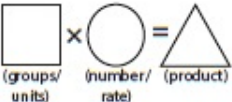
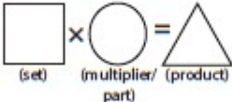
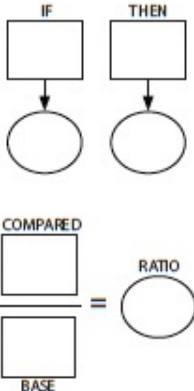
$$? = 54 \text{ books}$$



Comparison



Write a
Comparison
problem.

Schema and Definition	Graphic Organizers	Examples			Variations
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Teach word-problem schemas

Total

Difference

Change

Equal Groups

Comparison

Ratios/Proportions



Total

Difference

Change

Equal
Groups

Comparison



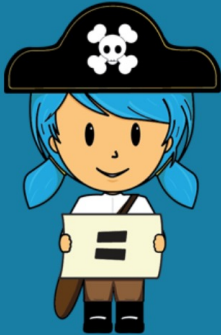


Pirate Math Equation Quest

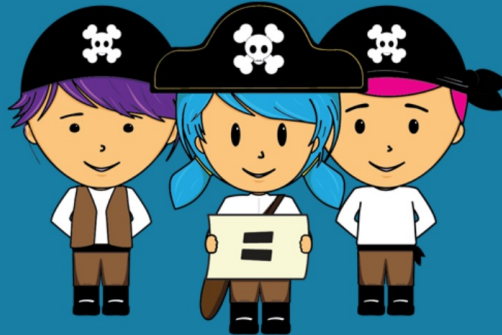
[About](#)[Research](#)[Individual](#)[Small Group](#)[STAAR](#)[Videos](#)

Welcome to Pirate Math Equation Quest!

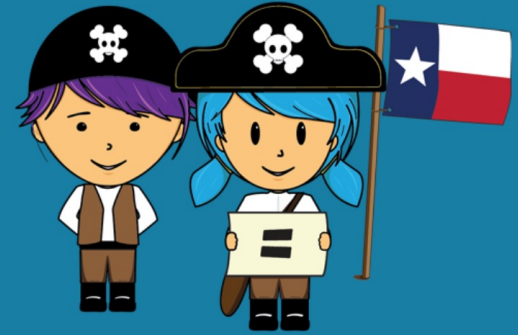
Individual Word-Problem Intervention



Small-Group Word-Problem Intervention



Small-Group Word-Problem Intervention for STAAR



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National Center on
INTENSIVE INTERVENTION
at American Institutes for Research

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Intensive Intervention in Mathematics Course Content

NCII, through a collaboration with the University of Connecticut, developed a set of course content focused on developing educators' skills in designing and delivering intensive mathematics instruction. This content is designed to support faculty and professional development providers with instructing pre-service and in-service educators who are developing and/or refining their implementation of intensive mathematics intervention.

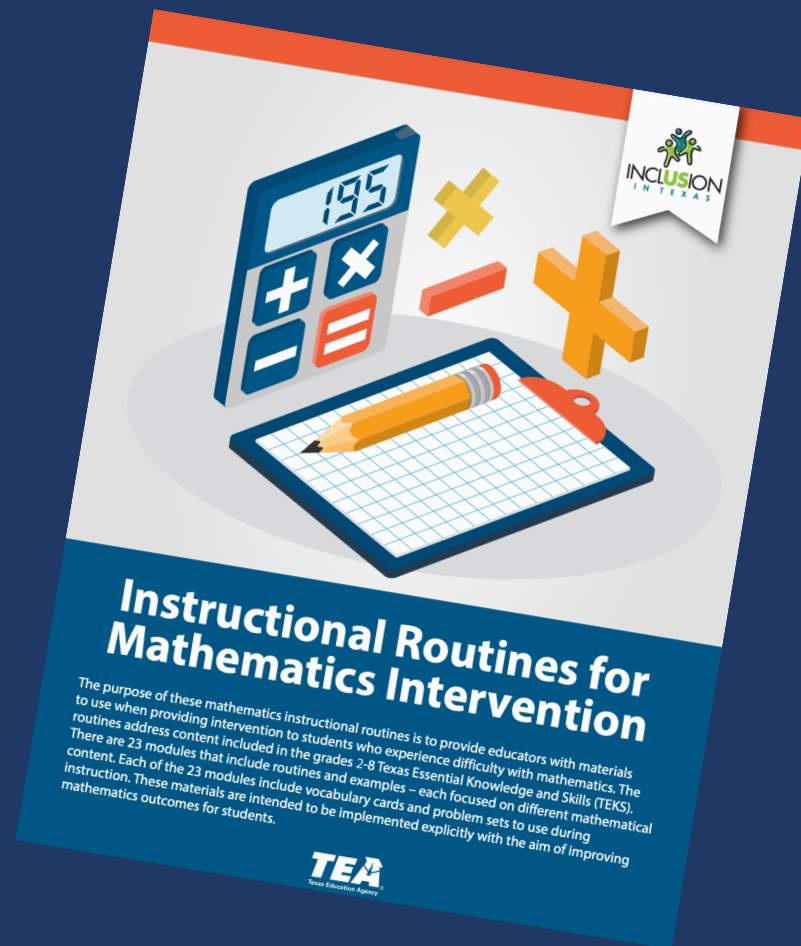
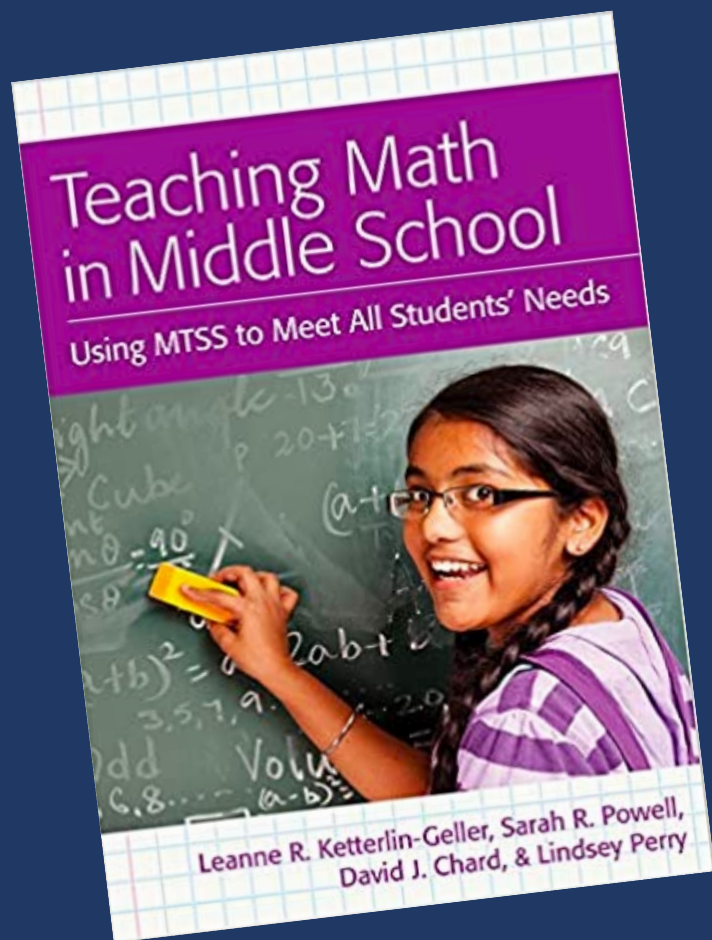
Intensive instruction was recently identified as a [high-leverage practice in special education](#), and DBI is a research based approach to delivering intensive instruction across content areas (NCII, 2013). This course provides learners with an opportunity to extend their understanding of intensive instruction through in-depth exposure to DBI in mathematics, complete with exemplars from actual classroom teachers.

NCII, through a collaboration with the University of Connecticut and the [National Center on Leadership in Intensive Intervention](#) and with support from the [CEEDAR Center](#), developed course content focused on enhancing educators' skills in intensive mathematics intervention. The course includes eight modules that can support faculty and professional development providers with instructing pre-service and in-service educators who are learning to implement intensive mathematics intervention through data-based individualization (DBI). The content in this course complements concepts covered in the [Features of Explicit Instruction Course](#) and so we suggest that users complete both courses.



MODULE 5: INTENSIVE MATHEMATICS INTERVENTION: INSTRUCTIONAL STRATEGIES

<https://www.amazon.com/Teaching-Math-Middle-School-Students/dp/1598572741>



https://www.inclusionintexas.org/apps/pages/index.jsp?uREC_ID=2155039&type=d&pREC_ID=2169859



Sarah R. Powell, Ph.D.

Associate Professor
The University of Texas at Austin



srpowell@utexas.edu



[@sarahpowellphd](https://twitter.com/sarahpowellphd)

