Math 1497 – Calc 2

Calculus of Parametric Eqn's

Derivative

$x = f(t)$, $y = g(t)$

$$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt}, \quad \frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dt}\Big/\frac{dx}{dt}\right)}{\frac{dx}{dt}}$$

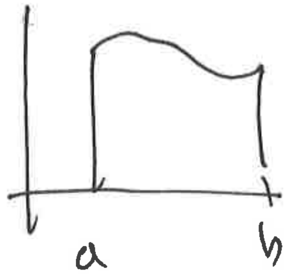
Ex $x = t^3 + t$, $y = t^5 + 1$

$$\frac{dx}{dt} = 3t^2 + 1, \quad \frac{dy}{dt} = 5t^4$$

so $$\frac{dy}{dx} = \frac{5t^4}{3t^2+1}$$

$$y'' = \frac{\frac{d}{dt}\left(\frac{5t^4}{3t^2+1}\right)}{3t^2+1} = \frac{\frac{20t^3(3t^2+1) - 6t \cdot 5t^4}{(3t^2+1)^2}}{3t^2+1} = \frac{30t^5 + 20t^3}{(3t^2+1)^3}$$

Areas



$$\int_a^b y\,dx$$

$x = f(t),\ y = g(t) \qquad dx = f'(t)\,dt$

and we suppose at $t = t_1,\ x = a,\quad t = t_2,\ y = b$

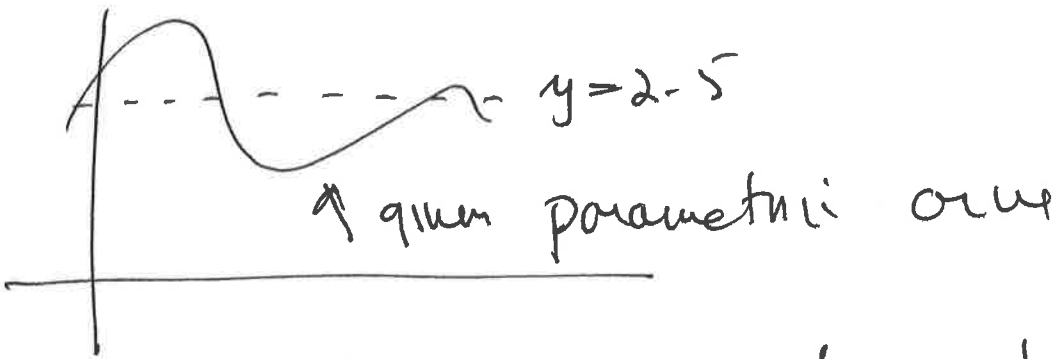
so $$\int_a^b y\,dx = \int_{t_1}^{t_2} g(t)\, f'(t)\,dt$$

important to note that $dx > 0$

From Patrick JMT

Find the area enclosed by $y = 2.5$
and the curve given by

$$x = t - \frac{1}{t},\quad y = t + \frac{1}{t}$$



to find intersection pt set

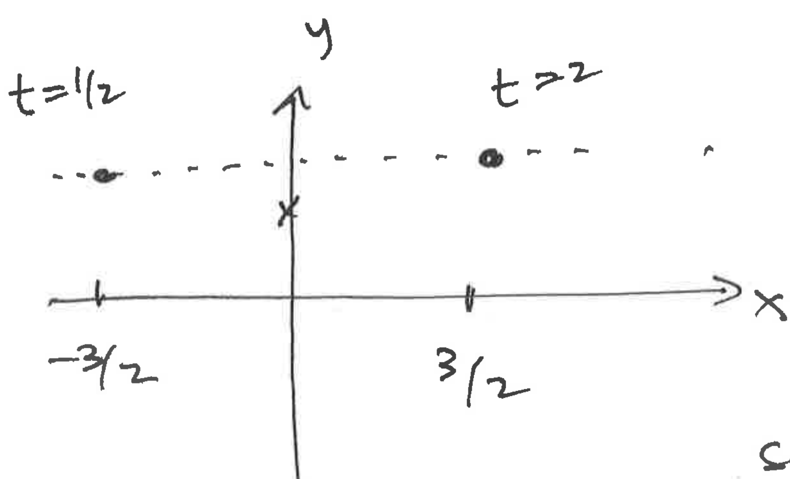
$$y = 2.5 = t + \frac{1}{t} \quad \text{or} \quad 2.5t = t^2 + 1$$

or $$\frac{5}{2}t = t^2 + 1 \Rightarrow t^2 - \frac{5}{2}t + 1 = 0$$

$$2t^2 - 5t + 2 = 0 \qquad (2t-1)(t-2) = 0$$

so $t = 1/2$ & $t = 2$

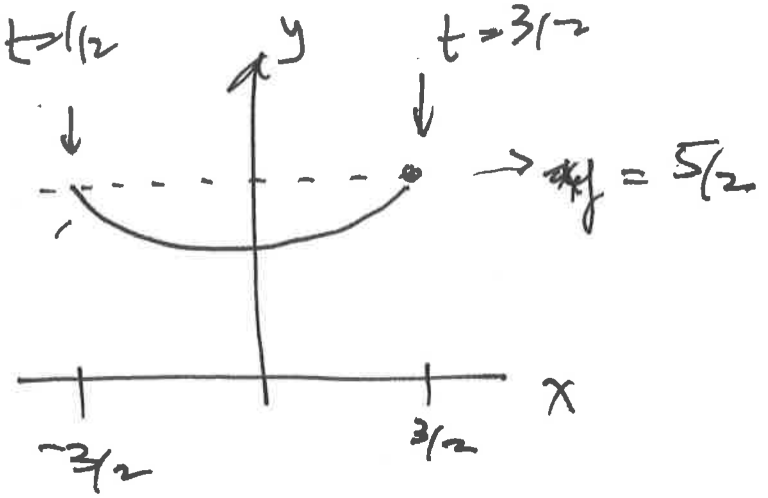
at these times $x = \frac{1}{2} - 2 = -3/2$, $\quad x = 2 - \frac{1}{2} = 3/2$



is curve above or below this line - sub $t = 0$

$x = 0, y = 2$

so below



$$x = t - \frac{1}{t}$$

$$y = t + \frac{1}{t}$$

$$dx = \left(1 + \frac{1}{t^2}\right) dt$$

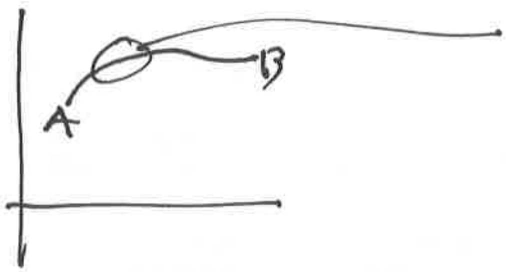
$$\int_{-3/2}^{3/2} (5/2 - y)\, dx = \int_{1/2}^{2} \left(\frac{5}{2} - t - \frac{1}{t}\right)\left(1 + \frac{1}{t^2}\right) dt$$

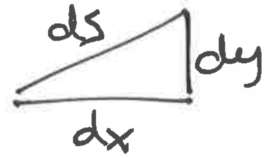
$$= \int_{1/2}^{2} \left(\frac{5}{2} - t - \frac{2}{t} + \frac{5}{2} \cdot \frac{1}{t^2} - \frac{1}{t^3}\right) dt$$

$$= \frac{5}{2}t - \frac{t^2}{2} - 2 \ln t - \frac{5}{2} \cdot \frac{1}{t} + \frac{1}{2t^2} \Big|_{1/2}^{2}$$

$$= \frac{15}{4} - 4 \ln 2$$

Arc Length





$$ds = \sqrt{dx^2 + dy^2}$$

$$\int_A^B ds$$

Now $\quad dx = \frac{dx}{dt}\,dt$

$\quad dy = \frac{dy}{dt}\,dt$

$$\sqrt{dx^2 + dy^2} = \sqrt{\left(\frac{dx}{dt}\right)^2 dt^2 + \left(\frac{dy}{dt}\right)^2 dt^2}$$

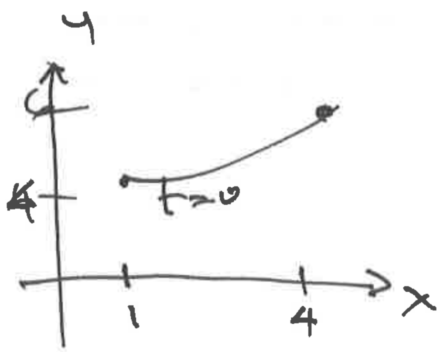
$$= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

$$= \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

Important to note that we only trace the curve at once.

Patrick JMT

$x = 1 + 3t^2, \quad y = 4 + 2t^3 \quad 0 \le t \le 1$



so $\frac{dx}{dt} = 6t, \quad \frac{dy}{dt} = 6t^2$

$$\int_0^1 \sqrt{(6t)^2+(6t^2)^2}\,dt$$

$$6\int_0^1 \sqrt{t^2+t^4}\,dt = 6\int_0^1 t\sqrt{1+t^2}\,dt$$

let $u = 1+t^2$ $du = 2t\,dt$

$t = 0$ $u = 1$

$t = 1$ $u = 2$

$$3\int_1^2 \sqrt{u}\,du = \cancel{3}\cdot\frac{2u^{3/2}}{\cancel{3}}\Big|_1^2$$

$$= 2\left(2^{3/2}-1\right)$$

Some notes $\frac{dx}{dt} = 6t$ & $0 \le t \le 1$

$\frac{dx}{dt} \geq 0$ also $\frac{dy}{dt} = 6t^2 \geq 0$

so curve go right & up