# Leveraging Diversity for Resilient Consensus in Sparse Networks


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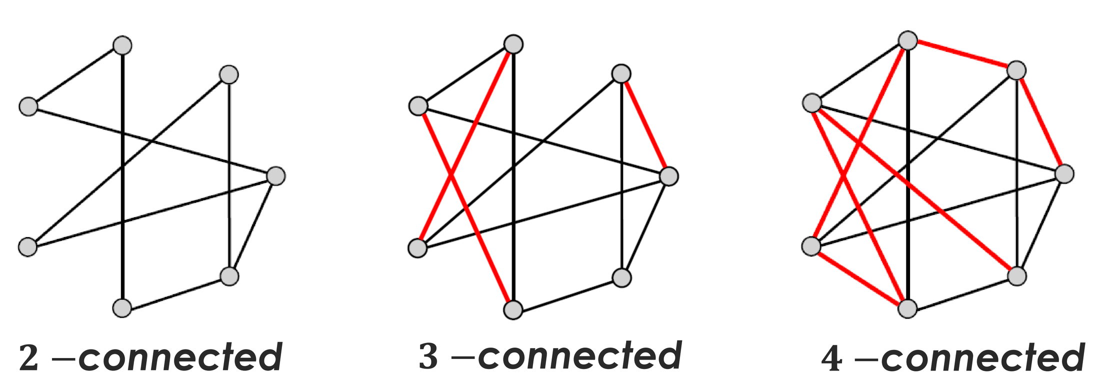
## Abstract

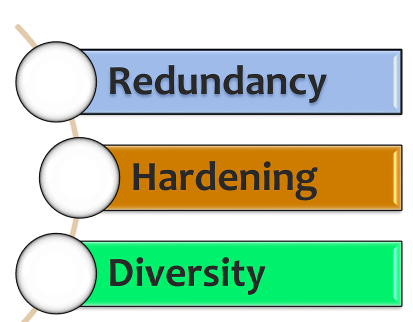
- We propose an alternative way of improving network's robustness by considering **heterogeneity of nodes**. Nodes in a network can be of different types having **disjoint sets of vulnerabilities**. Thus, an attacker can only compromise a particular type of nodes by exploiting a particular vulnerability.
- We show that, by such a diversification of nodes, attacker's ability to change the underlying network structure is significantly reduced. Consequently, even a **sparse network** with heterogeneous nodes can exhibit the properties of a **structurally robust network**.
- Using these ideas, we propose a distributed control policy that utilizes heterogeneity in the network to achieve resilient consensus in adversarial environment. By extending the notion of **$(r, s)$-robustness** to incorporate the diversity of nodes, we provide necessary and sufficient conditions to guarantee **resilient distributed consensus in heterogeneous networks**.


## Motivation
## Improving Robustness in Sparse Networks

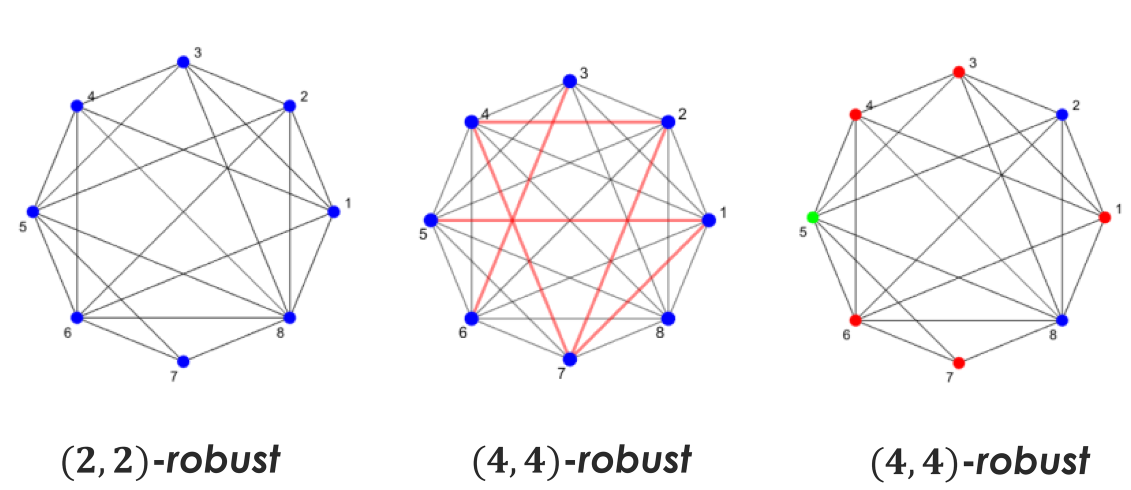
- A typical way of improving structural robustness in networks is by strategically adding edges.




- What can be alternate approaches, especially in sparse networks?

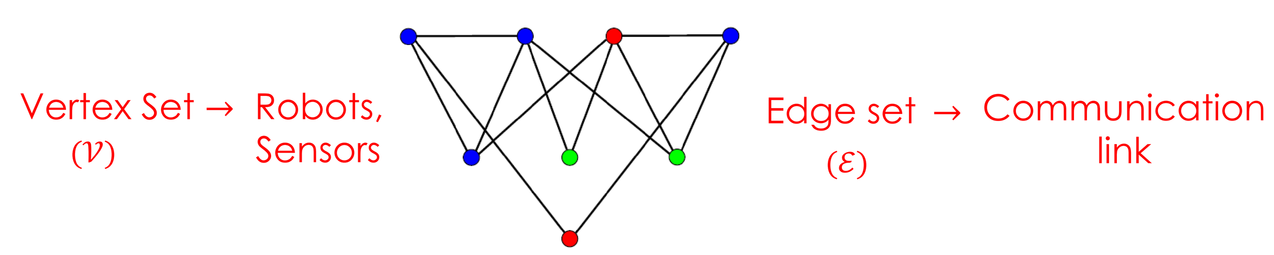



- **Diversification** can improve structural robustness in sparse heterogeneous networks without adding extra links.




## Resilient Consensus
## Network Model and Problem Formulation

- Multi-agent System → Undirected Graph




- Different types of nodes are represented by different colors. They share their states and color classes with each other.
- An attacker compromises nodes of the same type by exploiting a particular vulnerability.
- Two types of nodes:
  - Normal Nodes
  - Adversarial Nodes

**Threat Scope**

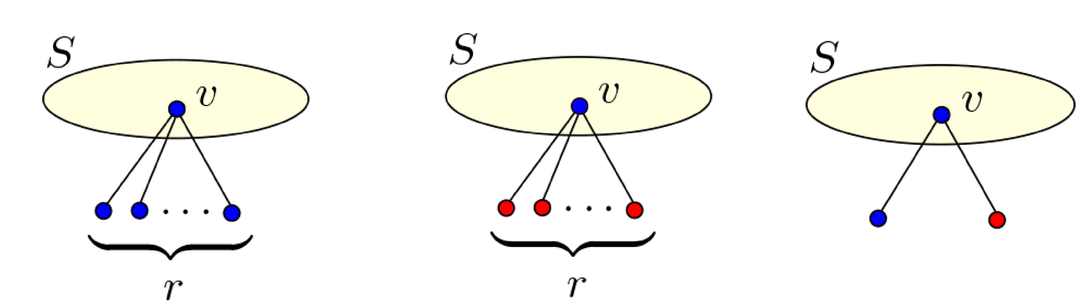
- F-Total : At most $F$ adversarial nodes of the same color in the overall network.
- F-Local : At most $F$ adversarial nodes of the same color in the neighborhood of a node.

**Objective : Resilient Consensus**

- Agreement : Asymptotic convergence of normal nodes' states to a common value .
- Safety : State of every normal node is within the interval defined by the maximum and minimum of the initial values

## Network Robustness with Coloring

**$r$-valid node**
A node $v \in S$ having at least $r$ monochromatic or two distinct color neighbours outside of $S$ .




**$(r, s)$-robustness with coloring**

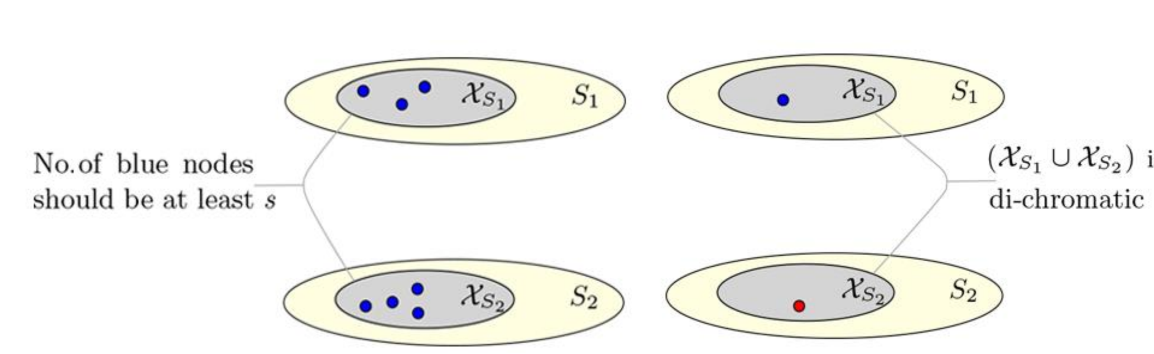
Let $r, s > 1$, and $S_1$ and $S_2$ be non-empty, disjoint subsets of $\mathcal{V}$. Let $\mathcal{X}_r^{S_1}$ and $\mathcal{X}_r^{S_1}$ be sets of $r$-valid nodes in $S_1$ and $S_2$ respectively. A graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ is $(r, s)$-robust with coloring if at least one is always satisfied:

$(i)$ $|\mathcal{X}_r^{S_1}| = |S_1|$

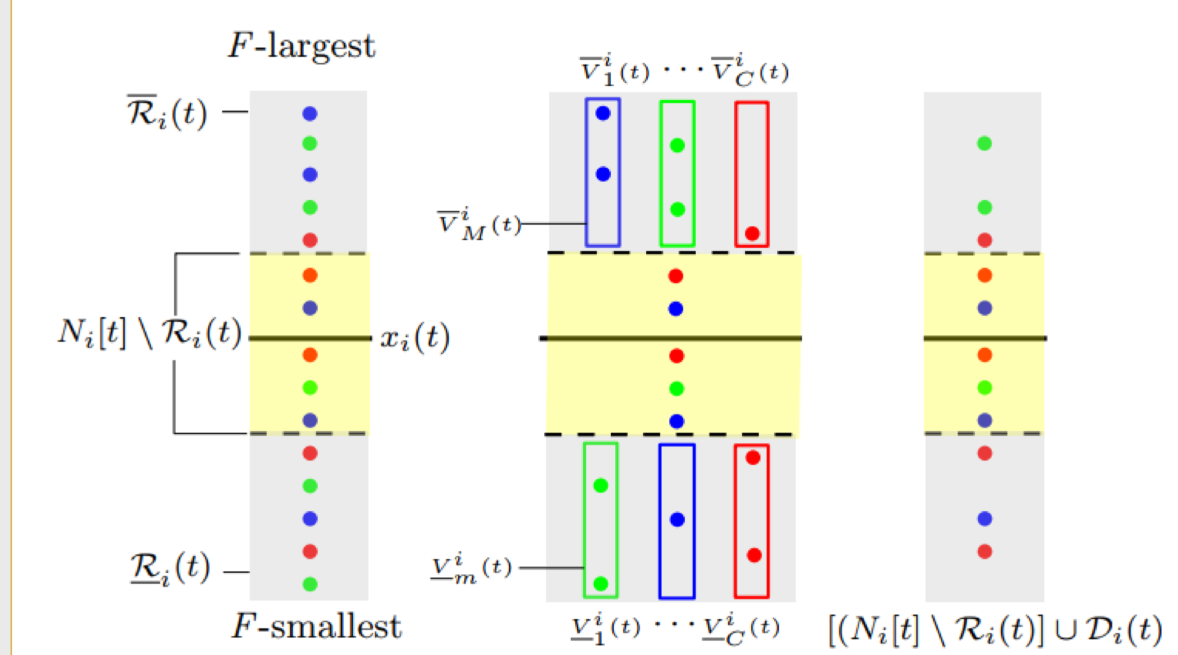
$(ii)$ $|\mathcal{X}_r^{S_2}| = |S_2|$

$(iii)$ $(\mathcal{X}_r^{S_1} \cup \mathcal{X}_r^{S_2})$ is polychromatic

$(iv)$ $(\mathcal{X}_r^{S_1} \cup \mathcal{X}_r^{S_2})$ is monochromatic and $|\mathcal{X}_r^{S_1} \cup \mathcal{X}_r^{S_2}| \geq s$




## Resilient Consensus Protocol with Coloring (RCP-C)




**Update Rule**

$$x_i(t+1) = \sum_{j \in [N_i[t] \setminus \mathcal{R}_i(t)] \cup \mathcal{D}_i(t)} w_{ij}(t) x_j(t)$$

where

$$\mathcal{R}_i(t) = \overline{\mathcal{R}}_i(t) \cup \underline{\mathcal{R}}_i(t)$$

$$\mathcal{D}_i(t) = \left(\overline{\mathcal{R}}_i(t) \setminus \overline{V_M^i}(t)\right) \cup \left(\underline{\mathcal{R}_i}(t) \setminus \underline{V_m^i}(t)\right)$$

**Theorem.** Let $\mathcal{G}(\mathcal{V}, \mathcal{E})$ be a colored network, in which normal nodes follow RCP-C. Then,

- ***F* -total model:** resilient asymptotic consensus is achieved if and only if the underlying graph topology is $(F+1, F+1)$-robust with colors.
- ***F* -local model**: resilient asymptotic consensus is achieved if the graph topology is $(2F+1)$-robust with colors.

## Simulations