

Lesson 1: Interpreting Division of a Fraction by a Whole

Number—Visual Models

Classwork

Opening Exercise

A

Write a division sentence to solve each problem.

1. 8 gallons of batter are poured equally into 4 bowls.
How many gallons of batter are in each bowl?
2. 1 gallon of batter is poured equally into 4 bowls.
How many gallons of batter are in each bowl?

Write a division sentence *and* draw a model to solve.

3. 3 gallons of batter are poured equally into 4 bowls.
How many gallons of batter are in each bowl?

B

Write a multiplication sentence to solve each problem.

1. One fourth of an 8-gallon pail is poured out.
How many gallons are poured out?
2. One fourth of a 1-gallon pail is poured out.
How many gallons are poured out?

Write a multiplication sentence *and* draw a model to solve.

3. One fourth of a 3-gallon pail is poured out.
How many gallons are poured out?

Example 1

$\frac{3}{4}$ gallon of batter is poured equally into 2 bowls. How many gallons of batter are in each bowl?

Example 2

$\frac{3}{4}$ pan of lasagna is shared equally by 6 friends. What fraction of the pan will each friend get?

Example 3

A rope of length $\frac{2}{5}$ m is cut into 4 equal cords. What is the length of each cord?

Exercises 1–6

Fill in the blanks to complete the equation. Then, find the quotient and draw a model to support your solution.

1. $\frac{1}{2} \div 3 = \frac{\square}{2} \times \frac{1}{2}$

2. $\frac{1}{3} \div 4 = \frac{1}{4} \times \frac{1}{\square}$

Find the value of each of the following.

3. $\frac{1}{4} \div 5$

4. $\frac{3}{5} \div 5$

5. $\frac{1}{5} \div 4$

Solve. Draw a model to support your solution.

6. $\frac{3}{5}$ pt. of juice is poured equally into 6 glasses. How much juice is in each glass?

Problem Set

Find the value of each of the following in its simplest form.

1.

a. $\frac{1}{3} \div 4$

b. $\frac{2}{5} \div 4$

c. $\frac{4}{7} \div 4$

2.

a. $\frac{2}{5} \div 3$

b. $\frac{5}{6} \div 5$

c. $\frac{5}{8} \div 10$

3.

a. $\frac{6}{7} \div 3$

b. $\frac{10}{8} \div 5$

c. $\frac{20}{6} \div 2$

4. 4 loads of stone weigh $\frac{2}{3}$ ton. Find the weight of 1 load of stone.

5. What is the width of a rectangle with an area of $\frac{5}{8} \text{ in}^2$ and a length of 10 inches?

6. Lenox ironed $\frac{1}{4}$ of the shirts over the weekend. She plans to split the remainder of the work equally over the next 5 evenings.

a. What fraction of the shirts will Lenox iron each day after school?

b. If Lenox has 40 shirts, how many shirts will she need to iron on Thursday and Friday?

7. Bo paid bills with $\frac{1}{2}$ of his paycheck and put $\frac{1}{5}$ of the remainder in savings. The rest of his paycheck he divided equally among the college accounts of his 3 children.

a. What fraction of his paycheck went into each child's account?

b. If Bo deposited \$400 in each child's account, how much money was in Bo's original paycheck?

Lesson 2: Interpreting Division of a Whole Number by a Fraction—Visual Models

Classwork

Example 1

Question # _____

Write it as a division expression.

Write it as a multiplication expression.

Make a rough draft of a model to represent the problem:

As you travel to each model, be sure to answer the following questions:

Original Question	Corresponding Division Expression	Corresponding Multiplication Expression	Write an Equation Showing the Equivalence of the Two Expressions.
1. How many $\frac{1}{2}$ miles are in 12 miles?			
2. How many quarter hours are in 5 hours?			
3. How many $\frac{1}{3}$ cups are in 9 cups?			
4. How many $\frac{1}{8}$ pizzas are in 4 pizzas?			
5. How many one-fifths are in 7 wholes?			

Example 2

Molly has 9 cups of flour. If this is $\frac{3}{4}$ of the number she needs to make bread, how many cups does she need?

a. Construct the tape diagram by reading it backward. Draw a tape diagram and label the unknown.

b. Next, shade in $\frac{3}{4}$.

c. Label the shaded region to show that 9 is equal to $\frac{3}{4}$ of the total.

d. Analyze the model to determine the quotient.

Exercises 1–5

1. A construction company is setting up signs on 2 miles of road. If the company places a sign every $\frac{1}{4}$ mile, how many signs will it use?
2. George bought 4 submarine sandwiches for a birthday party. If each person will eat $\frac{2}{3}$ of a sandwich, how many people can George feed?
3. Miranda buys 6 pounds of nuts. If she puts $\frac{3}{4}$ pound in each bag, how many bags can she make?

4. Margo freezes 8 cups of strawberries. If this is $\frac{2}{3}$ of the total strawberries that she picked, how many cups of strawberries did Margo pick?
5. Regina is chopping up wood. She has chopped 10 logs so far. If the 10 logs represent $\frac{5}{8}$ of all the logs that need to be chopped, how many logs need to be chopped in all?

Problem Set

Rewrite each problem as a multiplication question. Model your answer.

1. Nicole used $\frac{3}{8}$ of her ribbon to wrap a present. If she used 6 feet of ribbon for the present, how much ribbon did Nicole have at first?
2. A Boy Scout has 3 meters of rope. He cuts the rope into cords $\frac{3}{5}$ m long. How many cords will he make?
3. 12 gallons of water fill a tank to $\frac{3}{4}$ capacity.
 - a. What is the capacity of the tank?
 - b. If the tank is then filled to capacity, how many half-gallon bottles can be filled with the water in the tank?
4. Hunter spent $\frac{2}{3}$ of his money on a video game before spending half of his remaining money on lunch. If his lunch costs \$10, how much money did he have at first?
5. Students were surveyed about their favorite colors. $\frac{1}{4}$ of the students preferred red, $\frac{1}{8}$ of the students preferred blue, and $\frac{3}{5}$ of the remaining students preferred green. If 15 students preferred green, how many students were surveyed?
6. Mr. Scruggs got some money for his birthday. He spent $\frac{1}{5}$ of it on dog treats. Then, he divided the remainder equally among his 3 favorite charities.
 - a. What fraction of his money did each charity receive?
 - b. If he donated \$60 to each charity, how much money did he receive for his birthday?

Lesson 3: Interpreting and Computing Division of a Fraction by a Fraction—More Models

Classwork

Opening Exercise

Draw a model to represent $12 \div 3$.

Create a question or word problem that matches your model.

Example 1

$$\frac{8}{9} \div \frac{2}{9}$$

Write the expression in unit form, and then draw a model to solve.

Example 2

$$\frac{9}{12} \div \frac{3}{12}$$

Write the expression in unit form, and then draw a model to solve.

Example 3

$$\frac{7}{9} \div \frac{3}{9}$$

Write the expression in unit form, and then draw a model to solve.

Exercises 1–6

Write an expression to represent each problem. Then, draw a model to solve.

1. How many fourths are in 3 fourths?

2. $\frac{4}{5} \div \frac{2}{5}$

3. $\frac{9}{4} \div \frac{3}{4}$

4. $\frac{7}{8} \div \frac{2}{8}$

5. $\frac{13}{10} \div \frac{2}{10}$

6. $\frac{11}{9} \div \frac{3}{9}$

Lesson Summary

When dividing a fraction by a fraction with the same denominator, we can use the general rule $\frac{a}{c} \div \frac{b}{c} = \frac{a}{b}$.

Problem Set

For the following exercises, rewrite the division expression in unit form. Then, find the quotient. Draw a model to support your answer.

1. $\frac{4}{5} \div \frac{1}{5}$

2. $\frac{8}{9} \div \frac{4}{9}$

3. $\frac{15}{4} \div \frac{3}{4}$

4. $\frac{13}{5} \div \frac{4}{5}$

Rewrite the expression in unit form, and find the quotient.

5. $\frac{10}{3} \div \frac{2}{3}$

6. $\frac{8}{5} \div \frac{3}{5}$

7. $\frac{12}{7} \div \frac{12}{7}$

Represent the division expression using unit form. Find the quotient. Show all necessary work.

8. A runner is $\frac{7}{8}$ mile from the finish line. If she can travel $\frac{3}{8}$ mile per minute, how long will it take her to finish the race?

9. An electrician has 4.1 meters of wire.

- How many strips $\frac{7}{10}$ m long can he cut?
- How much wire will he have left over?

10. Saeed bought $21\frac{1}{2}$ lb. of ground beef. He used $\frac{1}{4}$ of the beef to make tacos and $\frac{2}{3}$ of the remainder to make quarter-pound burgers. How many burgers did he make?

11. A baker bought some flour. He used $\frac{2}{5}$ of the flour to make bread and used the rest to make batches of muffins. If he used 16 lb. of flour making bread and $\frac{2}{3}$ lb. for each batch of muffins, how many batches of muffins did he make?

Lesson 4: Interpreting and Computing Division of a Fraction by a Fraction—More Models

Classwork

Opening Exercise

Write at least three equivalent fractions for each fraction below.

a. $\frac{2}{3}$

b. $\frac{10}{12}$

Example 1

Molly has $1\frac{3}{8}$ cups of strawberries. She needs $\frac{3}{8}$ cup of strawberries to make one batch of muffins. How many batches can Molly make?

Use a model to support your answer.

Example 2

Molly's friend, Xavier, also has $\frac{11}{8}$ cups of strawberries. He needs $\frac{3}{4}$ cup of strawberries to make a batch of tarts. How many batches can he make? Draw a model to support your solution.

Example 3

Find the quotient: $\frac{6}{8} \div \frac{2}{8}$. Use a model to show your answer.

Example 4

Find the quotient: $\frac{3}{4} \div \frac{2}{3}$. Use a model to show your answer.

Exercises 1–5

Find each quotient.

1. $\frac{6}{2} \div \frac{3}{4}$

2. $\frac{2}{3} \div \frac{2}{5}$

3. $\frac{7}{8} \div \frac{1}{2}$

4. $\frac{3}{5} \div \frac{1}{4}$

5. $\frac{5}{4} \div \frac{1}{3}$

Problem Set

Calculate the quotient. If needed, draw a model.

1. $\frac{8}{9} \div \frac{4}{9}$

2. $\frac{9}{10} \div \frac{4}{10}$

3. $\frac{3}{5} \div \frac{1}{3}$

4. $\frac{3}{4} \div \frac{1}{5}$

Lesson 5: Creating Division Stories

Classwork

Opening Exercise

Tape Diagram:

$$\frac{8}{9} \div \frac{2}{9}$$

Number Line:

Molly's friend, Xavier, also has $\frac{11}{8}$ cups of strawberries. He needs $\frac{3}{4}$ cup of strawberries to make a batch of tarts. How many batches can he make? Draw a model to support your solution.

Example 1

$$\frac{1}{2} \div \frac{1}{8}$$

Step 1: Decide on an interpretation.

Step 2: Draw a model.

Step 3: Find the answer.

Step 4: Choose a unit.

Step 5: Set up a situation based upon the model.

Exercise 1

Using the same dividend and divisor, work with a partner to create your own story problem. You may use the same unit, but your situation must be unique. You could try another unit such as ounces, yards, or miles if you prefer.

Example 2

$$\frac{3}{4} \div \frac{1}{2}$$

Step 1: Decide on an interpretation.

Step 2: Draw a diagram.

Step 3: Find the answer.

Step 4: Choose a unit.

Step 5: Set up a situation based on the model.

Exercise 2

Using the same dividend and divisor, work with a partner to create your own story problem. You may use the same unit, but your situation must be unique. You could try another unit such as cups, yards, or miles if you prefer.

Lesson Summary

The method of creating division stories includes five steps:

Step 1: Decide on an interpretation (measurement or partitive). Today we used measurement division.

Step 2: Draw a model.

Step 3: Find the answer.

Step 4: Choose a unit.

Step 5: Set up a situation based on the model. This means writing a story problem that is interesting, realistic, and short. It may take several attempts before you find a story that works well with the given dividend and divisor.

Problem Set

Solve.

1. How many sixteenths are in $\frac{15}{16}$?
2. How many $\frac{1}{4}$ teaspoon doses are in $\frac{7}{8}$ teaspoon of medicine?
3. How many $\frac{2}{3}$ cups servings are in a 4 cup container of food?
4. Write a measurement division story problem for $6 \div \frac{3}{4}$.
5. Write a measurement division story problem for $\frac{5}{12} \div \frac{1}{6}$.
6. Fill in the blank to complete the equation. Then, find the quotient and draw a model to support your solution.
 - a. $\frac{1}{2} \div 5 = \frac{1}{\square} \text{ of } \frac{1}{2}$
 - b. $\frac{3}{4} \div 6 = \frac{1}{\square} \text{ of } \frac{3}{4}$
7. $\frac{4}{5}$ of the money collected from a fundraiser was divided equally among 8 grades. What fraction of the money did each grade receive?
8. Meyer used 6 loads of gravel to cover $\frac{2}{5}$ of his driveway. How many loads of gravel will he need to cover his entire driveway?

9. An athlete plans to run 3 miles. Each lap around the school yard is $\frac{3}{7}$ mile. How many laps will the athlete run?
10. Parks spent $\frac{1}{3}$ of his money on a sweater. He spent $\frac{3}{5}$ of the remainder on a pair of jeans. If he has \$36 left, how much did the sweater cost?

Lesson 6: More Division Stories

Classwork

Example 1

Divide $50 \div \frac{2}{3}$.

Step 1: Decide on an interpretation.

Step 2: Draw a model.

Step 3: Find the answer.

Step 4: Choose a unit.

Step 5: Set up a situation based upon the model.

Exercise 1

Using the same dividend and divisor, work with a partner to create your own story problem. You may use the same unit, dollars, but your situation must be unique. You could try another unit, such as miles, if you prefer.

Example 2

Divide $\frac{1}{2} \div \frac{3}{4}$.

Step 1: Decide on an interpretation.

Step 2: Draw a model.

Step 3: Find the answer.

Step 4: Choose a unit.

Step 5: Set up a situation based upon the model.

Exercise 2

Using the same dividend and divisor, work with a partner to create your own story problem. Try a different unit.

Problem Set

Solve.

1. $\frac{15}{16}$ is 1 sixteenth groups of what size?
2. $\frac{7}{8}$ teaspoons is $\frac{1}{4}$ groups of what size?
3. A 4-cup container of food is $\frac{2}{3}$ groups of what size?
4. Write a partitive division story problem for $6 \div \frac{3}{4}$.
5. Write a partitive division story problem for $\frac{5}{12} \div \frac{1}{6}$.
6. Fill in the blank to complete the equation. Then, find the quotient, and draw a model to support your solution.
 - a. $\frac{1}{4} \div 7 = \frac{1}{\square} \text{ of } \frac{1}{4}$
 - b. $\frac{5}{6} \div 4 = \frac{1}{\square} \text{ of } \frac{5}{6}$
7. There is $\frac{3}{5}$ of a pie left. If 4 friends wanted to share the pie equally, how much would each friend receive?
8. In two hours, Holden completed $\frac{3}{4}$ of his race. How long will it take Holden to complete the entire race?
9. Sam cleaned $\frac{1}{3}$ of his house in 50 minutes. How many hours will it take him to clean his entire house?
10. It took Mario 10 months to beat $\frac{5}{8}$ of the levels on his new video game. How many years will it take for Mario to beat all the levels?
11. A recipe calls for $1\frac{1}{2}$ cups of sugar. Marley only has measuring cups that measure $\frac{1}{4}$ cup. How many times will Marley have to fill the measuring cup?

Lesson 7: The Relationship Between Visual Fraction Models and Equations

Classwork

Example 1

Model the following using a partitive interpretation.

$$\frac{3}{4} \div \frac{2}{5}$$

Shade 2 of the 5 sections $\left(\frac{2}{5}\right)$.

Label the part that is known $\left(\frac{3}{4}\right)$.

Make notes below on the math sentences needed to solve the problem.

Example 2

Model the following using a measurement interpretation.

$$\frac{3}{5} \div \frac{1}{4}$$

Example 3

$$\frac{2}{3} \div \frac{3}{4}$$

Show the number sentences below.

Lesson Summary

Connecting models of fraction division to multiplication through the use of reciprocals helps in understanding the *invert and multiply* rule. That is, given two fractions $\frac{a}{b}$ and $\frac{c}{d}$, we have the following:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}.$$

Problem Set

Invert and multiply to divide.

1.

a. $\frac{2}{3} \div \frac{1}{4}$

b. $\frac{2}{3} \div 4$

c. $4 \div \frac{2}{3}$

2.

a. $\frac{1}{3} \div \frac{1}{4}$

b. $\frac{1}{8} \div \frac{3}{4}$

c. $\frac{9}{4} \div \frac{6}{5}$

3.

a. $\frac{2}{3} \div \frac{3}{4}$

b. $\frac{3}{5} \div \frac{3}{2}$

c. $\frac{22}{4} \div \frac{2}{5}$

4. Summer used $\frac{2}{5}$ of her ground beef to make burgers. If she used $\frac{3}{4}$ pounds of beef, how much beef did she have at first?

5. Alistair has 5 half-pound chocolate bars. It takes $1\frac{1}{2}$ pounds of chocolate, broken into chunks, to make a batch of cookies. How many batches can Alistair make with the chocolate he has on hand?

6. Draw a model that shows $\frac{2}{5} \div \frac{1}{3}$. Find the answer as well.

7. Draw a model that shows $\frac{3}{4} \div \frac{1}{2}$. Find the answer as well.

Lesson 8: Dividing Fractions and Mixed Numbers

Classwork

Example 1: Introduction to Calculating the Quotient of a Mixed Number and a Fraction

- a. Carli has $4\frac{1}{2}$ walls left to paint in order for all the bedrooms in her house to have the same color paint. However, she has used almost all of her paint and only has $\frac{5}{6}$ of a gallon left. How much paint can she use on each wall in order to have enough to paint the remaining walls?

- b. Calculate the quotient.

$$\frac{2}{5} \div 3\frac{4}{7}$$

Exercise

Show your work for the memory game in the boxes provided below.

A.
B.
C.
D.
E.
F.
G.
H.
I.
J.
K.
L.

Problem Set

Calculate each quotient.

1. $\frac{2}{5} \div 3\frac{1}{10}$

2. $4\frac{1}{3} \div \frac{4}{7}$

3. $3\frac{1}{6} \div \frac{9}{10}$

4. $\frac{5}{8} \div 2\frac{7}{12}$

Lesson 9: Sums and Differences of Decimals

Classwork

Example 1

$$25\frac{3}{10} + 376\frac{77}{100}$$

Example 2

$$426\frac{1}{5} - 275\frac{1}{2}$$

Exercises

Calculate each sum or difference.

1. Samantha and her friends are going on a road trip that is $245\frac{7}{50}$ miles long. They have already driven $128\frac{53}{100}$.
How much farther do they have to drive?
2. Ben needs to replace two sides of his fence. One side is $367\frac{9}{100}$ meters long, and the other is $329\frac{3}{10}$ meters long.
How much fence does Ben need to buy?
3. Mike wants to paint his new office with two different colors. If he needs $4\frac{4}{5}$ gallons of red paint and $3\frac{1}{10}$ gallons of brown paint, how much paint does he need in total?

4. After Arianna completed some work, she figured she still had $78\frac{21}{100}$ pictures to paint. If she completed another $34\frac{23}{25}$ pictures, how many pictures does Arianna still have to paint?

Use a calculator to convert the fractions into decimals before calculating the sum or difference.

5. Rahzel wants to determine how much gasoline he and his wife use in a month. He calculated that he used $78\frac{1}{3}$ gallons of gas last month. Rahzel's wife used $41\frac{3}{8}$ gallons of gas last month. How much total gas did Rahzel and his wife use last month? Round your answer to the nearest hundredth.

Problem Set

1. Find each sum or difference.

a. $381\frac{1}{10} - 214\frac{43}{100}$

b. $32\frac{3}{4} - 12\frac{1}{2}$

c. $517\frac{37}{50} + 312\frac{3}{100}$

d. $632\frac{16}{25} + 32\frac{3}{10}$

e. $421\frac{3}{50} - 212\frac{9}{10}$

2. Use a calculator to find each sum or difference. Round your answer to the nearest hundredth.

a. $422\frac{3}{7} - 367\frac{5}{9}$

b. $23\frac{1}{5} + 45\frac{7}{8}$

Lesson 10: The Distributive Property and the Products of Decimals

Classwork

Opening Exercise

Calculate the product.

a. 200×32.6

b. 500×22.12

Example 1: Introduction to Partial Products

Use partial products and the distributive property to calculate the product.

$$200 \times 32.6$$

Example 2: Introduction to Partial Products

Use partial products and the distributive property to calculate the area of the rectangular patio shown below.



Exercises

Use the boxes below to show your work for each station. Make sure that you are putting the solution for each station in the correct box.

Station One:

Station Two:

Station Three:

Station Four:

Station Five:

Problem Set

Calculate the product using partial products.

1. 400×45.2

2. 14.9×100

3. 200×38.4

4. 900×20.7

5. 76.2×200

Discussion

Record notes from the Discussion in the box below.

Exercises

1. Calculate the product. 324.56×54.82
2. Kevin spends \$11.25 on lunch every week during the school year. If there are 35.5 weeks during the school year, how much does Kevin spend on lunch over the entire school year? Remember to round to the nearest penny.

3. Gunnar's car gets 22.4 miles per gallon, and his gas tank can hold 17.82 gallons of gas. How many miles can Gunnar travel if he uses all of the gas in the gas tank?
4. The principal of East High School wants to buy a new cover for the sand pit used in the long-jump competition. He measured the sand pit and found that the length is 29.2 feet and the width is 9.8 feet. What will the area of the new cover be?

Problem Set

Solve each problem. Remember to round to the nearest penny when necessary.

1. Calculate the product. 45.67×32.58
2. Deprina buys a large cup of coffee for \$4.70 on her way to work every day. If there are 24 workdays in the month, how much does Deprina spend on coffee throughout the entire month?
3. Krego earns \$2,456.75 every month. He also earns an extra \$4.75 every time he sells a new gym membership. Last month, Krego sold 32 new gym memberships. How much money did Krego earn last month?
4. Kendra just bought a new house and needs to buy new sod for her backyard. If the dimensions of her yard are 24.6 feet by 14.8 feet, what is the area of her yard?

Lesson 12: Estimating Digits in a Quotient

Classwork

Discussion

Divide 150 by 30.

Exercises 1–5

Round to estimate the quotient. Then, compute the quotient using a calculator, and compare the estimation to the quotient.

1. $2,970 \div 11$

a. Round to a one-digit arithmetic fact. Estimate the quotient.

b. Use a calculator to find the quotient. Compare the quotient to the estimate.

2. $4,752 \div 12$

a. Round to a one-digit arithmetic fact. Estimate the quotient.

b. Use a calculator to find the quotient. Compare the quotient to the estimate.

3. $11,647 \div 19$

a. Round to a one-digit arithmetic fact. Estimate the quotient.

b. Use a calculator to find the quotient. Compare the quotient to the estimate.

4. $40,644 \div 18$

a. Round to a one-digit arithmetic fact. Estimate the quotient.

b. Use a calculator to find the quotient. Compare the quotient to the estimate.

5. $49,170 \div 15$

a. Round to a one-digit arithmetic fact. Estimate the quotient.

b. Use a calculator to find the quotient. Compare the quotient to the estimate.

Example 3: Extend Estimation and Place Value to the Division Algorithm

Estimate and apply the division algorithm to evaluate the expression $918 \div 27$.

Problem Set

Round to estimate the quotient. Then, compute the quotient using a calculator, and compare the estimate to the quotient.

1. $715 \div 11$
2. $7,884 \div 12$
3. $9,646 \div 13$
4. $11,942 \div 14$
5. $48,825 \div 15$
6. $135,296 \div 16$
7. $199,988 \div 17$
8. $116,478 \div 18$
9. $99,066 \div 19$
10. $181,800 \div 20$

Lesson 13: Dividing Multi-Digit Numbers Using the Algorithm

Classwork

Example 1

Divide $70,072 \div 19$.

a. Estimate:

b. Create a table to show the multiples of 19.

Multiples of 19

- c. Use the algorithm to divide $70,072 \div 19$. Check your work.

$$19 \overline{) 70072}$$

Example 2

Divide $14,175 \div 315$.

- a. Estimate:
- b. Use the algorithm to divide $14,175 \div 315$. Check your work.

Exercises 1–5

For each exercise,

- a. Estimate.
- b. Divide using the algorithm, explaining your work using place value.

1. $484,692 \div 78$

- a. Estimate:

- b.

2. $281,886 \div 33$

- a. Estimate:

- b.

3. $2,295,517 \div 37$

a. Estimate:

b.

4. $952,448 \div 112$

a. Estimate:

b.

5. $1,823,535 \div 245$

a. Estimate:

b.

Problem Set

Divide using the division algorithm.

1. $1,634 \div 19$

2. $2,450 \div 25$

3. $22,274 \div 37$

4. $21,361 \div 41$

5. $34,874 \div 53$

6. $50,902 \div 62$

7. $70,434 \div 78$

8. $91,047 \div 89$

9. $115,785 \div 93$

10. $207,968 \div 97$

11. $7,735 \div 119$

12. $21,948 \div 354$

13. $72,372 \div 111$

14. $74,152 \div 124$

15. $182,727 \div 257$

16. $396,256 \div 488$

17. $730,730 \div 715$

18. $1,434,342 \div 923$

19. $1,775,296 \div 32$

20. $1,144,932 \div 12$

Lesson 14: The Division Algorithm—Converting Decimal Division into Whole Number Division Using Fractions

Classwork

Opening Exercise

Divide $\frac{1}{2} \div \frac{1}{10}$. Use a tape diagram to support your reasoning.

Relate the model to the invert and multiply rule.

Example 1

Evaluate the expression. Use a tape diagram to support your answer.

$$0.5 \div 0.1$$

Rewrite $0.5 \div 0.1$ as a fraction.

Express the divisor as a whole number.

Exercises 1–3

Convert the decimal division expressions to fractional division expressions in order to create whole number divisors. You do not need to find the quotients. Explain the movement of the decimal point. The first exercise has been completed for you.

1. $18.6 \div 2.3$

$$\frac{18.6}{2.3} \times \frac{10}{10} = \frac{186}{23}$$
$$186 \div 23$$

I multiplied both the dividend and the divisor by ten, or by one power of ten, so each decimal point moved one place to the right because they grew larger by ten.

2. $14.04 \div 4.68$

3. $0.162 \div 0.036$

Example 2

Evaluate the expression. First, convert the decimal division expression to a fractional division expression in order to create a whole number divisor.

$25.2 \div 0.72$

Use the division algorithm to find the quotient.

Exercises 4–7

Convert the decimal division expressions to fractional division expressions in order to create whole number divisors. Compute the quotients using the division algorithm. Check your work with a calculator.

4. $2,000 \div 3.2$

5. $3,581.9 \div 4.9$

6. $893.76 \div 0.21$

7. $6.194 \div 0.326$

Example 3

A plane travels 3,625.26 miles in 6.9 hours. What is the plane's unit rate?

Represent this situation with a fraction.

Represent this situation using the same units.

Estimate the quotient.

Express the divisor as a whole number.

Use the division algorithm to find the quotient.

Use multiplication to check your work.

Problem Set

Convert decimal division expressions to fractional division expressions to create whole number divisors.

1. $35.7 \div 0.07$
2. $486.12 \div 0.6$
3. $3.43 \div 0.035$
4. $5,418.54 \div 0.009$
5. $812.5 \div 1.25$
6. $17.343 \div 36.9$

Estimate quotients. Convert decimal division expressions to fractional division expressions to create whole number divisors. Compute the quotients using the division algorithm. Check your work with a calculator and your estimates.

7. Norman purchased 3.5 lb. of his favorite mixture of dried fruits to use in a trail mix. The total cost was \$16.87. How much does the fruit cost per pound?
8. Divide: $994.14 \div 18.9$
9. Daryl spent \$4.68 on each pound of trail mix. He spent a total of \$14.04. How many pounds of trail mix did he purchase?
10. Mamie saved \$161.25. This is 25% of the amount she needs to save. How much money does Mamie need to save?
11. Kareem purchased several packs of gum to place in gift baskets for \$1.26 each. He spent a total of \$8.82. How many packs of gum did he buy?
12. Jerod is making candles from beeswax. He has 132.72 ounces of beeswax. If each candle uses 8.4 ounces of beeswax, how many candles can he make? Will there be any wax left over?
13. There are 20.5 cups of batter in the bowl. This represents 0.4 of the entire amount of batter needed for a recipe. How many cups of batter are needed?
14. Divide: $159.12 \div 6.8$
15. Divide: $167.67 \div 8.1$

Lesson 15: The Division Algorithm—Converting Decimal Division into Whole Number Division Using Mental Math

Classwork

Opening Exercise

Use mental math to evaluate the numeric expressions.

a. $99 + 44$

b. $86 - 39$

c. 50×14

d. $180 \div 5$

Example 1: Use Mental Math to Find Quotients

Use mental math to evaluate $105 \div 35$.

Exercises 1–4

Use mental math techniques to evaluate the expressions.

1. $770 \div 14$

2. $1,005 \div 5$

3. $1,500 \div 8$

4. $1,260 \div 5$

Example 2: Mental Math and Division of Decimals

Evaluate the expression $175 \div 3.5$ using mental math techniques.

Exercises 5–7

Use mental math techniques to evaluate the expressions.

5. $25 \div 6.25$

6. $6.3 \div 1.5$

7. $425 \div 2.5$

Example 3: Mental Math and the Division Algorithm

Evaluate the expression $4,564 \div 3.5$ using mental math techniques and the division algorithm.

Example 4: Mental Math and Reasonable Work

Shelly was given this number sentence and was asked to place the decimal point correctly in the quotient.

$$55.6875 \div 6.75 = 0.825$$

Do you agree with Shelly?

Divide to prove your answer is correct.

Problem Set

Use mental math, estimation, and the division algorithm to evaluate the expressions.

- | | | |
|-----------------------|-----------------------|----------------------------|
| 1. $118.4 \div 6.4$ | 2. $314.944 \div 3.7$ | 3. $1,840.5072 \div 23.56$ |
| 4. $325 \div 2.5$ | 5. $196 \div 3.5$ | 6. $405 \div 4.5$ |
| 7. $3,437.5 \div 5.5$ | 8. $393.75 \div 5.25$ | 9. $2,625 \div 6.25$ |
| 10. $231 \div 8.25$ | 11. $92 \div 5.75$ | 12. $196 \div 12.25$ |
| 13. $117 \div 6.5$ | 14. $936 \div 9.75$ | 15. $305 \div 12.2$ |

Place the decimal point in the correct place to make the number sentence true.

- | | | |
|-------------------------------|--------------------------------|--------------------------------|
| 16. $83.375 \div 2.3 = 3,625$ | 17. $183.575 \div 5,245 = 3.5$ | 18. $326,025 \div 9.45 = 3.45$ |
| 19. $449.5 \div 725 = 6.2$ | 20. $446,642 \div 85.4 = 52.3$ | |

Lesson 16: Even and Odd Numbers

Classwork

Opening Exercise

- a. What is an even number?

- b. List some examples of even numbers.

- c. What is an odd number?

- d. List some examples of odd numbers.

What happens when we add two even numbers? Do we always get an even number?

Exercises 1–3

1. Why is the sum of two even numbers even?
 - a. Think of the problem $12 + 14$. Draw dots to represent each number.

 - b. Circle pairs of dots to determine if any of the dots are left over.

 - c. Is this true every time two even numbers are added together? Why or why not?

2. Why is the sum of two odd numbers even?
 - a. Think of the problem $11 + 15$. Draw dots to represent each number.

 - b. Circle pairs of dots to determine if any of the dots are left over.

 - c. Is this true every time two odd numbers are added together? Why or why not?

3. Why is the sum of an even number and an odd number odd?
- Think of the problem $14 + 11$. Draw dots to represent each number.
 - Circle pairs of dots to determine if any of the dots are left over.
 - Is this true every time an even number and an odd number are added together? Why or why not?
 - What if the first addend is odd and the second is even? Is the sum still odd? Why or why not? For example, if we had $11 + 14$, would the sum be odd?

Let's sum it up:

-
-
-

6. The product of an even number and an odd number is even.

Lesson Summary

Adding:

- The sum of two even numbers is even.
- The sum of two odd numbers is even.
- The sum of an even number and an odd number is odd.

Multiplying:

- The product of two even numbers is even.
- The product of two odd numbers is odd.
- The product of an even number and an odd number is even.

Problem Set

Without solving, tell whether each sum or product is even or odd. Explain your reasoning.

1. $346 + 721$
2. $4,690 \times 141$
3. $1,462,891 \times 745,629$
4. $425,922 + 32,481,064$
5. $32 + 45 + 67 + 91 + 34 + 56$

Lesson 17: Divisibility Tests for 3 and 9

Classwork

Opening Exercise

Below is a list of 10 numbers. Place each number in the circle(s) that is a factor of the number. Some numbers can be placed in more than one circle. For example, if 32 were on the list, it would be placed in the circles with 2, 4, and 8 because they are all factors of 32.

24; 36; 80; 115; 214; 360; 975; 4,678; 29,785; 414,940

The diagram shows five large circles arranged in two rows. The top row contains three circles, and the bottom row contains two circles. Each circle has a small rectangular box at the top containing a number. The numbers are 2, 4, 5, 8, and 10, respectively, from left to right in the top row, and 8 and 10 in the bottom row. The circles are intended for students to place numbers from the list above based on their divisibility by these factors.

Discussion

- Divisibility rule for 2:

- Divisibility rule for 4:

- Divisibility rule for 5:

- Divisibility rule for 8:

- Divisibility rule for 10:

- Decimal numbers with fraction parts do not follow the divisibility tests.

- Divisibility rule for 3:

- Divisibility rule for 9:

Example 1

This example shows how to apply the two new divisibility rules we just discussed.

Explain why 378 is divisible by 3 and 9.

- a. Expand 378.

- b. Decompose the expression to factor by 9.
- c. Factor the 9.
- d. What is the sum of the three digits?
- e. Is 18 divisible by 9?
- f. Is the number 378 divisible by 9? Why or why not?
- g. Is the number 378 divisible by 3? Why or why not?

Example 2

Is 3,822 divisible by 3 or 9? Why or why not?

Exercises 1–5

Circle ALL the numbers that are factors of the given number. Complete any necessary work in the space provided.

1. 2,838 is divisible by

3

9

4

Explain your reasoning for your choice(s).

2. 34,515 is divisible by

3

9

5

Explain your reasoning for your choice(s).

3. 10,534,341 is divisible by

3

9

2

Explain your reasoning for your choice(s).

4. 4,320 is divisible by

3

9

10

Explain your reasoning for your choice(s).

5. 6,240 is divisible by

3

9

8

Explain your reasoning for your choice(s).

Lesson Summary

To determine if a number is divisible by 3 or 9:

- Calculate the sum of the digits.
- If the sum of the digits is divisible by 3, the entire number is divisible by 3.
- If the sum of the digits is divisible by 9, the entire number is divisible by 9.

Note: If a number is divisible by 9, the number is also divisible by 3.

Problem Set

1. Is 32,643 divisible by both 3 and 9? Why or why not?
2. Circle all the factors of 424,380 from the list below.
2 3 4 5 8 9 10
3. Circle all the factors of 322,875 from the list below.
2 3 4 5 8 9 10
4. Write a 3-digit number that is divisible by both 3 and 4. Explain how you know this number is divisible by 3 and 4.
5. Write a 4-digit number that is divisible by both 5 and 9. Explain how you know this number is divisible by 5 and 9.

Lesson 18: Least Common Multiple and Greatest Common Factor

Classwork

Opening

The *greatest common factor* of two whole numbers (not both zero) is the greatest whole number that is a factor of each number. The greatest common factor of two whole numbers a and b is denoted by $\text{GCF}(a, b)$.

The *least common multiple* of two whole numbers is the smallest whole number greater than zero that is a multiple of each number. The least common multiple of two whole numbers a and b is denoted by $\text{LCM}(a, b)$.

Example 1: Greatest Common Factor

Find the greatest common factor of 12 and 18.

- Listing these factor pairs in order helps ensure that no common factors are missed. Start with 1 multiplied by the number.
- Circle all factors that appear on both lists.
- Place a triangle around the greatest of these common factors.

$\text{GCF}(12, 18)$

12

18

Example 2: Least Common Multiple

Find the least common multiple of 12 and 18.

LCM (12, 18)

Write the first 10 multiples of 12.

Write the first 10 multiples of 18.

Circle the multiples that appear on both lists.

Put a rectangle around the least of these common multiples.

Exercises**Station 1: Factors and GCF**

Choose one of these problems that has not yet been solved. Solve it together on your student page. Then, use your marker to copy your work neatly on the chart paper. Use your marker to cross out your choice so that the next group solves a different problem.

GCF (30, 50)

GCF (30, 45)

GCF (45, 60)

GCF (42, 70)

GCF (96, 144)

Station 2: Multiples and LCM

Choose one of these problems that has not yet been solved. Solve it together on your student page. Then, use your marker to copy your work neatly on the chart paper. Use your marker to cross out your choice so that the next group solves a different problem.

LCM (9, 12)

LCM (8, 18)

LCM (4, 30)

LCM (12, 30)

LCM (20, 50)

Next, choose one of these problems that has not yet been solved. Solve it together on your student page. Then, use your marker to copy your work neatly on this chart paper and to cross out your choice so that the next group solves a different problem.

- a. Hot dogs come packed 10 in a package. Hot dog buns come packed 8 in a package. If we want one hot dog for each bun for a picnic with none left over, what is the least amount of each we need to buy? How many packages of each item would we have to buy?

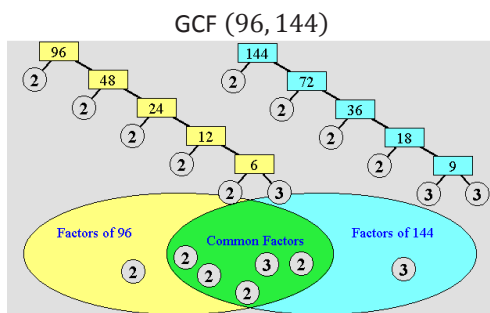
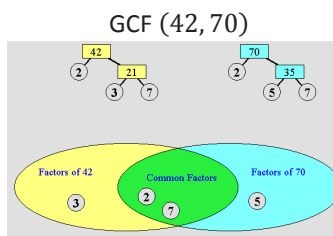
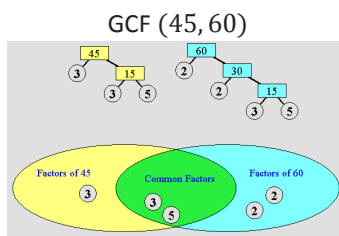
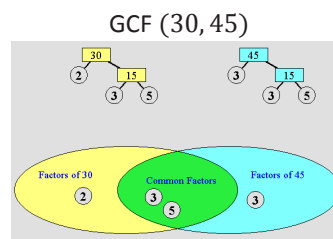
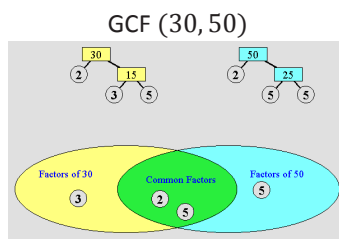
- b. Starting at 6:00 a.m., a bus stops at my street corner every 15 minutes. Also starting at 6:00 a.m., a taxi cab comes by every 12 minutes. What is the next time both a bus and a taxi are at the corner at the same time?

- c. Two gears in a machine are aligned by a mark drawn from the center of one gear to the center of the other. If the first gear has 24 teeth, and the second gear has 40 teeth, how many revolutions of the first gear are needed until the marks line up again?

- d. Is the LCM of a pair of numbers ever equal to one of the numbers? Explain with an example.
- e. Is the LCM of a pair of numbers ever less than both numbers? Explain with an example.

Station 3: Using Prime Factors to Determine GCF

Choose one of these problems that has not yet been solved. Solve it together on your student page. Then, use your marker to copy your work neatly on the chart paper and to cross out your choice so that the next group solves a different problem.



Next, choose one of these problems that has not yet been solved:

- a. Would you rather find all the factors of a number or find all the prime factors of a number? Why?
- b. Find the GCF of your original pair of numbers.
- c. Is the product of your LCM and GCF less than, greater than, or equal to the product of your numbers?
- d. Glenn's favorite number is very special because it reminds him of the day his daughter, Sarah, was born. The factors of this number do not repeat, and all the prime numbers are less than 12. What is Glenn's number? When was Sarah born?

Station 4: Applying Factors to the Distributive Property

Choose one of these problems that has not yet been solved. Solve it together on your student page. Then, use your marker to copy your work neatly on the chart paper and to cross out your choice so that the next group solves a different problem.

Find the GCF from the two numbers, and rewrite the sum using the distributive property.

1. $12 + 18 =$
2. $42 + 14 =$
3. $36 + 27 =$
4. $16 + 72 =$
5. $44 + 33 =$

Next, add another example to one of these two statements applying factors to the distributive property.

Choose any numbers for n , a , and b .

$$n(a) + n(b) = n(a + b)$$

$$n(a) - n(b) = n(a - b)$$

Problem Set

Complete the remaining stations from class.

Lesson 19: The Euclidean Algorithm as an Application of the Long Division Algorithm

Classwork

Opening Exercise

Euclid's algorithm is used to find the greatest common factor (GCF) of two whole numbers.

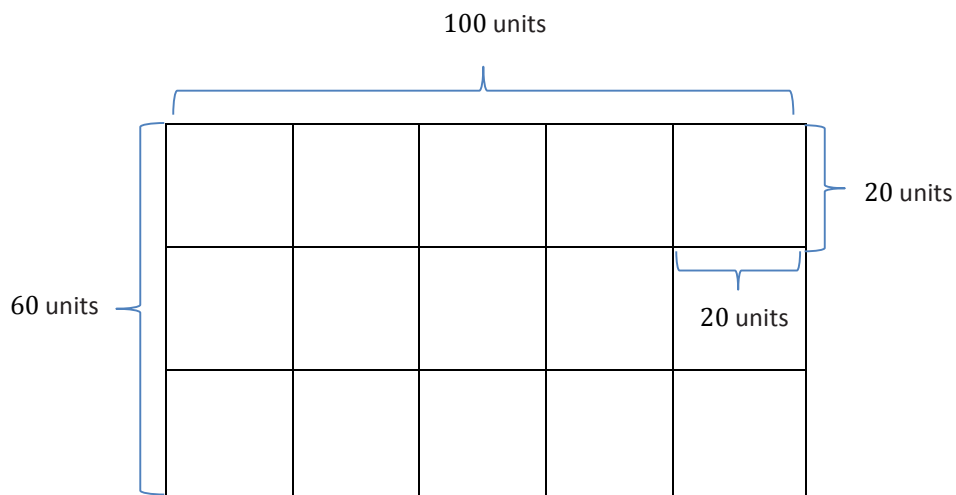
1. Divide the larger of the two numbers by the smaller one.
2. If there is a remainder, divide it into the divisor.
3. Continue dividing the last divisor by the last remainder until the remainder is zero.
4. The final divisor is the GCF of the original pair of numbers.

$$383 \div 4 =$$

$$432 \div 12 =$$

$$403 \div 13 =$$

Example 1: Euclid's Algorithm Conceptualized



Example 2: Lesson 18 Classwork Revisited

- a. Let's apply Euclid's algorithm to some of the problems from our last lesson.
 - i. What is the GCF of 30 and 50?

 - ii. Using Euclid's algorithm, we follow the steps that are listed in the Opening Exercise.

- b. Apply Euclid's algorithm to find the GCF (30, 45).

Example 3: Larger Numbers

GCF (96, 144)

GCF (660, 840)

Example 4: Area Problems

The greatest common factor has many uses. Among them, the GCF lets us find out the maximum size of squares that cover a rectangle. When we solve problems like this, we cannot have any gaps or any overlapping squares. Of course, the maximum size squares will be the minimum number of squares needed.

A rectangular computer table measures 30 inches by 50 inches. We need to cover it with square tiles. What is the side length of the largest square tile we can use to completely cover the table without overlap or gaps?



- a. If we use squares that are 10 by 10, how many do we need?

- b. If this were a giant chunk of cheese in a factory, would it change the thinking or the calculations we just did?

- c. How many 10 inch \times 10 inch squares of cheese could be cut from the giant 30 inch \times 50 inch slab?

Problem Set

1. Use Euclid's algorithm to find the greatest common factor of the following pairs of numbers:
 - a. GCF (12, 78)
 - b. GCF (18, 176)
2. Juanita and Samuel are planning a pizza party. They order a rectangular sheet pizza that measures 21 inches by 36 inches. They tell the pizza maker not to cut it because they want to cut it themselves.
 - a. All pieces of pizza must be square with none left over. What is the side length of the largest square pieces into which Juanita and Samuel can cut the pizza?
 - b. How many pieces of this size can be cut?
3. Shelly and Mickelle are making a quilt. They have a piece of fabric that measures 48 inches by 168 inches.
 - a. All pieces of fabric must be square with none left over. What is the side length of the largest square pieces into which Shelly and Mickelle can cut the fabric?
 - b. How many pieces of this size can Shelly and Mickelle cut?