Crafting the Dictator's Military: Loyalty, Efficiency, and the Guardianship Dilemma

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Abstract

Although some dictators construct coup-proofed and personally loyal militaries, others favor professional militaries that more efficiently repress outsider threats. Existing research analyzes the purportedly ubiquitous "loyalty-efficiency" tradeoff that dictators face and the "guardianship dilemma" that strong outsider threats create. This paper shows these two tradeoffs are intimately related by studying the *orientation* and *strength* of outsider threats. In the formal model, a dictator chooses between a personalist and professional military. The military can repress to defend the dictator, stage a coup, or transition to outsider rule. Non-revolutionary threats do not generate a loyalty-efficiency tradeoff. Personalist militaries' lower reservation value under outsider rule yields considerably stronger incentives than professional militaries to repress non-revolutionary threats—and, consequently, higher higher equilibrium repressive efficiency. The dictator's strict preference for the personalist military also eliminates the guardianship dilemma. However, revolutionary threats trigger both tradeoffs. A strong, revolutionary threat encourages choosing a professional military, raising coup likelihood.

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Among the tools that enable dictators to survive in power—such as patronage concessions, legislatures, and parties—perhaps the most fundamental is that dictators require a coercive apparatus to defeat threats posed by outsider opposition groups such as pro-democracy protesters, rebel groups, and foreign invaders. Considerable research analyzes two core tradeoffs that dictators face when constructing their militaries. First, there exists a ***loyalty-efficiency tradeoff***. Specifically, dictators trade off between coup loyalty—the propensity of the military to not attempt a coup—and repressive efficiency, the probability that the military defeats an outsider threat (Finer 1975, 93-5; 1997, 17-9; Quinlivan 1999; Roessler 2016). On the one hand, personalist militaries in which family members and unqualified co-ethnics stack the officer corps should be highly loyal. On the other hand, professional militaries distinguished by wide recruitment, meritocratic promotion, and disciplined hierarchical command chains should exhibit higher repressive efficiency. For example, as of 2010 (the eve of the Arab Spring protests), the relatively professional organization of Egypt's military contrasted with Syria's personalist military stacked with Alawites, co-ethnics of the al-Asad dynasty.

Second, the strength of outsider threats affects the type of military a dictator chooses—with consequences for the likelihood of a coup attempt. The more immediate threat of insider overthrow via a coup causes many dictators to "coup-proof" their military despite adverse consequences for repressive efficiency and prospects for outsider overthrow. However, this calculus changes when a dictator faces a strong outsider threat because it becomes more willing to sacrifice coup loyalty for increased repressive efficiency (Acemoglu, Vindigni and Ticchi 2010; Besley and Robinson 2010; Svolik 2013). This is known as the ***guardianship dilemma*** because the stronger guards needed to defeat a severe threat are better able to overthrow the dictator via a coup. However, others challenge the guardianship logic. Although strong outsider threats cause dictators to construct more coercively efficient militaries, coup probability does not necessarily increase. McMahon and Slantchev (2015) show formally why, in the face of a strong outsider threat, even a large and well-equipped military will remain loyal: the strong outsider threat lowers the value of holding office.

This paper provides a unified theory of military agency problems. It shows that these distinct debates—loyalty-efficiency tradeoff and guardianship dilemma—are intimately related by examining the *orientation* and *strength* of the outsider threat. I analyze a formal model in which a dictator faces an exogenous outsider threat that can overthrow the regime. The dictator first chooses how to organize its coercive apparatus by delegating authority to either a personalist or a professional military. The dictator faces a dual agency problem. To survive, it needs the military to defend the regime by exercising repression. However, the military can alternatively decide to either negotiate a transition with the outsider, or attempt a coup. Compared to a personalist military, a professional military is more likely to be *able* to successfully repress the opposition. However, because professional militaries recruit from broad segments of society, and merit rather than personal fealty to the incumbent dictator determines promotion decisions, professional militaries fare better than personalist militaries if the outsider takes power. Furthermore, professional militaries are more likely to have an opportunity to successfully stage a coup.

The main findings from the model challenge the two core arguments about (1) the existence of a loyalty-efficiency tradeoff and (2) whether or not strong outsider threats create a guardianship dilemma. First, existing loyalty-efficiency arguments are flawed because they overlook a central aspect of the repression calculus: the military's strategic decision to exercise repression when given orders.[1] For example, largely professional militaries in Tunisia and Egypt were ultimately unwilling to repress protesters in early 2011 amid Arab Spring protests.[2] Many democratic transitions in Latin America in the 1980s occurred when professionally oriented militaries negotiated deals with broad societal groups (e.g., Uruguay) or with moderate rebel groups (e.g., El Salvador). In all these cases, the military expected a relatively favorable fate following a transition to outsider rule—which affected its decision to not continue fighting.

More generally, I open up a key implicit assumption undergirding the loyalty-efficiency tradeoff by allowing the outsider threat to vary in its revolutionary orientation. Formalizing intuition from the aforementioned cases, I show that *non-revolutionary* threats do not generate a loyalty-efficiency tradeoff for the dictator. Facing a non-revolutionary threat—e.g., pro-democracy protests in Cairo in 2011 or moderate nationalist rebel groups in El Salvador in the 1980s—stacking the military with sycophants can reduce the probability of outsider overthrow (i.e., higher repression efficiency) relative to a more professional military. Whereas professional militaries have a relatively high-valued outside option to rule by a non-revolutionary actor, personalist militaries do not because of their patrimonial ties to the incumbent. This induces personalist militaries to defend the regime by exercising repression with greater likelihood, yielding a lower probability of outsider takeover despite lower endowed coercive ability. When encountering non-revolutionary threats,

[1] The next section discusses other formal contributions that address strategic repression choices (Myerson 2008; Dragu and Przeworski Forthcoming; Slantchev and Matush 2017; Tyson 2018).

[2] By contrast, personalist militaries in Bahrain, Syria, and (at least in part) Libya reacted with harsh crackdowns (Bellin 2012).

the dictator not only optimally chooses a personalist military, it *does not* face a tradeoff between military types: the personalist military exhibits higher repressive efficiency and lower coup propensity.

Instead, dictators only face a loyalty-efficiency tradeoff when encountering strong *revolutionary* threats, such as communist guerrillas in Malaysia in the 1960s and anti-monarchical rebellions in the Middle East in the 1950s and 1960s. These threats pose an existential crisis regardless of whether the military is personalist or professional because the outsider seeks to upend the existing social structure and elites. This creates strong incentives for either type of military to defend the regime if it can effectively exert repression. A strong threat causes the professional military to exhibit higher repressive efficiency because it can defeat the threat with considerably higher probability than the personalist military. Therefore, when facing a revolutionary threat, the dictator may prefer the professional military—despite its higher coup propensity—because it exhibits higher repressive efficiency.

The second result applies the revised loyalty-efficiency logic to untangle the guardianship dilemma debate. Revolutionary threats—which create a loyalty-efficiency tradeoff—are necessary and sufficient for a guardianship dilemma. The dictator's willingness to sacrifice coup loyalty for higher repressive efficiency as a revolutionary outsider threat grows in strength creates a non-monotonic relationship between threat size and equilibrium coup probability. The coup probability exhibits a discrete increase at an intermediate threat level in which the dictator switches from a personalist to a professional military—recovering the traditional guardianship dilemma logic. However, at all other threat levels, the equilibrium probability of a coup decreases in outsider threat strength because (1) optimal military choice is unchanged and (2) stronger outsider threats increase the difficulty of installing a military dictatorship, similar to McMahon and Slantchev's (2015) critique of the guardianship logic. The overall logic for this non-monotonic relationship contrasts with existing arguments for or against the guardianship dilemma.[3]

By contrast, non-revolutionary threats eliminate the guardianship dilemma for the same reason that non-revolutionary threats do not generate a loyalty-efficiency tradeoff. Because the personalist military is more repressively efficient regardless of threat strength, increasing the severity of a non-revolutionary threat does not cause the dictator to switch to the less loyal professional military. Therefore, equilibrium coup likelihood strictly decreases in the size of the threat.

[3]Acemoglu, Vindigni and Ticchi's (2010) and Svolik's (2013) models also generate a non-monotonic relationship, but rest on an opposing underlying logic (see the next section).

Despite adopting a core assumption from McMahon and Slantchev (2015), my critique of the traditional guardianship logic differs and is perhaps more fundamental. Imposing their assumption that stronger outsider threats diminish the value of a coup attempt is not sufficient to undermine the guardianship logic because it does not eliminate the dictator's incentive to switch from a personalist to a professional military when facing a strong outsider threat, as the case with a revolutionary threat demonstrates. Instead, a sufficient condition to eliminate the guardianship dilemma is for the dictator to not face a loyalty-efficiency tradeoff—which occurs when facing a non-revolutionary threat—because then the dictator does not choose a professional military even when facing a strong threat. This insight rests on modeling the military's endogenous repression choice and the orientation of the outsider threat, which McMahon and Slantchev (2015) do not analyze. Figure 1 summarizes the main findings.

Table 1: Summary of Main Findings

Non-revolutionary threat	Revolutionary threat
(Professional military has high value to outsider rule)	(Professional military has low value to outsider rule)
⇓	⇓
Personalist military more repressively efficient regardless of threat size	Personalist more efficient if weak threat Professional more efficient if strong threat
⇓	⇓
No loyalty-efficiency tradeoff	Loyalty-efficiency tradeoff
⇓	⇓
No guardianship dilemma	Guardianship dilemma

1 Related Theories of Coups and Repression

1.1 Loyalty-Efficiency Tradeoff

The present contribution departs from existing studies of the loyalty-efficiency tradeoff by establishing the *conditions* under which a loyalty-efficiency tradeoff exists, rather than assuming dictators necessarily trade off between loyalty and efficiency. Finer's (1975, 93-5; 1997, 17-9) wide-ranging survey of historical forms of military organization discusses the loyalty-efficiency tradeoff in early modern Europe by contrasting efficient foreign mercenary troops with more loyal paid domestic troops. Focusing on contemporary polities, Powell (2014, 2) argues that leaders "find themselves mired in a paradox in which a weak military can leave them vulnerable to invasion or civil war, while a strong military could expedite their exit through a coup d'etat." Similarly, Greitens (2016, 4) proclaims: "Because coup-proofing calls for fragmented and socially exclusive organizations, while protecting against popular unrest demands unitary and exclusive ones, autocrats cannot simultaneously maximize their defenses against both threats." This tradeoff provides incentives for rulers to "coup-proof" the military despite considerable evidence that protecting against disloyalty diminishes military efficiency (Quinlivan 1999; Pilster and Böhmelt 2011; Talmadge 2015). Roessler (2016) characterizes a similar tradeoff whereby fear of a coup may cause a ruler to exclude rival ethnic groups from power. This hinders the state's counterinsurgency capacity by disrupting the government's intelligence network in the excluded group's regional base.

Several contributions from the formal theoretic literature examine how dictators choose between competent and incompetent agents. Zakharov (2016) characterizes a dynamic loyalty-efficiency tradeoff between high-quality advisers that generate a high fixed payoff for the dictator, and low-quality advisers that endogenously demonstrate higher loyalty to the incumbent dictator because they have a lower outside option to betraying the incumbent than high-quality advisers. This resembles the present idea that professional militaries have a higher reservation value to negotiating a transition with society. However, in the present model, the dictator's utility from its military depends on whether the military *chooses* to exert repressive effort, contrary to Zakharov's (2016) assumption that dictators accrue a *fixed* rent from particular military types. Therefore, whereas rulers always face a loyalty-efficiency tradeoff in his model, here, a personalist military may exhibit greater repressive efficiency than a professional despite a weaker coercive endowment. This discrepancy is crucial for explaining the conditions under which a dictator faces a loyalty-efficiency tradeoff and, consequently, a guardianship dilemma. My model also departs from Egorov and Sonin's (2011) analysis in which rulers always face a loyalty-efficiency tradeoff because of different informational endowments. In their model, the types of agents do not differ in their coercive ability to defend the regime.

1.2 Guardianship Dilemma

The present contribution departs from existing debates about the guardianship dilemma by showing that its existence depends fundamentally on whether or not the dictator faces a loyalty-efficiency tradeoff. Linking these two tradeoffs generates new insights into the theoretical relationship between outsider threat strength and coup propensity. In Acemoglu, Vindigni and Ticchi's (2010) formal model of persistent civil wars, a strong outsider threat that can cause long-lasting civil war may encourage the government to build a larger military to end the civil war, but at the risk of a coup attempt. Svolik (2013) studies a moral hazard model with a similar tradeoff, and also shows that larger outsider threats can induce the dictator to build a bigger military despite heightening coup risk. Acemoglu, Ticchi and Vindigni (2010) and Besley and Robinson (2010) present related formal analyses of this tradeoff, and Huntington (1957) and Feaver (1999) provide non-formal discussions.

The present model is not the first to generate a non-monotonic relationship between outsider threat strength and equilibrium coup probability, but the logic differs by evaluating the standard guardianship logic in combination with allowing the external threat to endogenously affect the value of holding office. Acemoglu, Vindigni and Ticchi (2010) show that strong threats induce rulers to choose large militaries, and assume that governments can commit to continually pay large militaries but not small or intermediate-sized militaries. Svolik (2013) shows that the contracting problem between a government and its military dissipates as the military becomes large—the government's equilibrium response when facing a large threat—because the military can control policy without actually intervening (what he calls a "military tutelage" regime). Both these models assume that more severe outsider threats increase the military's bargaining leverage relative to the government, and that the size of the external threat does not affect the military's consumption. By contrast, here, greater external threats in expectation *lower* the value of a coup attempt, as in McMahon and Slantchev (2015). However, despite this feature, the overall relationship is non-monotonic in the present model because large threats may induce the dictator to switch to the professional military—recovering the guardianship dilemma mechanism that McMahon and Slantchev (2015) critique.

1.3 Endogenous Repression Compliance

Existing formal analyses of the guardianship dilemma do not analyze how the orientation of the outsider threat affects the military's payoff if the outsider threat takes over. Most existing models assume that the outsider threat does not directly affect the military's utility. McMahon and Slantchev (2015) advance beyond this simplifying premise, but instead assume that the military consumes 0 if the outsider rules. By contrast, other research examines the agency problem involved with inducing security agents to repress on behalf of the regime. Qualitative research on specific instances of military decisions regarding whether to repress social protesters provides informative discussions (e.g., McLauchlin 2010; Bellin 2012), but does not explicate general mechanisms that affect militaries' strategic choices. Modeling repressive effort as a *strategic* choice by the military improves upon the implicit assumption in much research that militaries with greater coercive ability *conditional* on choosing to fight are necessarily more efficient in *equilibrium*. Recent formal theory research also addresses this oversight by examining militaries' choices over exercising repression (Myerson 2008; Slantchev and Matush 2017; Tyson 2018) and whether security agents use resources they receive from the regime to help the dictator survive (Dragu and Przeworski Forthcoming).

The present contribution departs from existing formal theoretic analyses of endogenous repression compliance by studying this consideration alongside how dictators choose among different types of militaries, and how militaries' coup choice additionally affects their likelihood of exercising repression. This yields new insights for the two central tradeoffs examined in existing research regarding how dictators construct their militaries: the loyalty-efficiency tradeoff and the guardianship dilemma. Furthermore, the central focus on how militaries expect to fare following societal transition relates to broader considerations in the political regimes literature. Geddes (1999) argues that military regimes often acquiesce to democratization because they will survive as an intact organization. Debs (2016) proposes a different mechanism, also based on expected fate: military dictators are more willing than other types of dictators to democratize because they are less likely to face punishment for their comparative advantage in coercion under a democratic than an authoritarian regime. More broadly, Albertus and Menaldo (2018) argue that dictators more willingly democratize after enacting a constitution that affords elite protection against political participation by the masses.

2 Setup of Baseline Model

Appendix Table A.1 summarizes the formal notation.

2.1 Players and Moves

Two strategic players make sequential choices. First, a dictator facing an exogenous outsider threat decides whether to create a personalist or a professional military. Second, the military decides whether to repress the outsider threat to defend the incumbent dictator, to stage a coup and repress the outsider to install a military dictatorship, or to negotiate a regime transition that hands power to the outsider. In between these moves, Nature determines the military's repressive effectiveness, its cost to repressing, and its coup opportunity. The coercive endowments for the outsider threat, military, and dictator are respectively denoted as $\theta_T \in \left(0, \overline{\theta}_T\right)$, for $\overline{\theta}_T > 0$; $\theta_M \in \left\{\underline{\theta}_M, \overline{\theta}_M\right\}$; and $\theta_D \in \left(0, \overline{\theta}_D\right)$, for $\overline{\theta}_D > 0$. The key difference between military types is that a personalist military has a lower coercive endowment, $\underline{\theta}_M$, than a professional military, $\overline{\theta}_M > \underline{\theta}_M$.

Rulers throughout history have organized their militaries in various manners (Huntington 1957; Finer 1975, 1997, 2002). The present distinction between personalist and professional militaries captures in a parsimonious manner some important differences among empirical military types, while also abstracting away from many nuances that could be intriguing to analyze in future work. The difference in coercive endowments reflects a greater extent of coup-proofing measures, such as parallel military forces (e.g., presidential guards), inherent in personalist militaries, which undermines their fighting capacity (Quinlivan 1999; Pilster and Böhmelt 2011; Talmadge 2015). The dichotomy can also capture the breadth of individuals and groups from which the ruler recruits for the military. For simplicity, we can imagine that multiple identity groups populate society, and the dictator can decide whether to recruit either (1) only from its group (personalist military) or (2) broadly across groups (professional military). Consequently, generals in personalist militaries likely fear worse fates if a member of a different identity group seizes power, and personalist militaries tend to be less effective at fighting. The government needs people to fight, and recruiting solely from one group can create manpower deficits (Quinlivan 1999). Furthermore, ethnically biased recruiting can undermine intelligence networks in areas populated by excluded ethnic groups, which hinders counterinsurgency (Roessler 2016).

This motivation for the difference between personalist and professional militaries relates loosely to core ideas from Bueno de Mesquita et al.'s (2003) selectorate theory. They assume that regimes are composed of winning coalition members, and that incumbent rulers face a challenger that can offer anyone in the "selectorate" (the group of people that can participate in politics) a place in the winning coalition if the challenger takes power. The smaller is the ratio of winning coalition size to selectorate size, the more cheaply the incumbent ruler can buy off members of the current winning coalition. A large selectorate lowers the probability of any current winning coalition member gaining inclusion in the challenger's winning coalition. Therefore, holding fixed the size of the selectorate, the choice in the present model over military type relates to choosing the size of the winning coalition, which can be either small (personalist military) or large (professional military)—while additionally assuming that winning coalition size carries implications for the ability to defeat outsider threats.[4]

[4]Zakharov (2016) provides an alternative setup with endogenous reservation values to study the loyalty-

2.2 Dictator's Payoff

The dictator's only goal is political survival: it consumes 1 if it survives in power, and 0 otherwise—i.e., if either insider overthrow (coup) or outsider takeover occurs. To avoid parameters not needed to generate the core tradeoffs, the dictator does not pay costs of repression or military-building.

2.3 Military's Payoff to Defending the Regime

If the military exercises repression to defend the regime, then it pays a cost μ drawn from a smooth density function $F(\cdot)$ with full support over $\left[0, \overline{\mu}\right]$. The associated probability density function is $f(\cdot)$.[5] The military's consumption under the status quo regime equals $\omega_D \in (0,1)$ for either type of military. The benefit of defending the regime also depends on whether Nature chooses the military to be effective or ineffective at repression. If the military is effective at repression, then repression defeats the outsider with probability 1, whereas if the military is ineffective at repression, then the repression defeats the outsider with probability 0. Both of these Nature moves occur in between the dictator's and the military's moves, implying that the military knows the draw whereas the dictator only knows the underlying distribution. Overall, the military's expected utility to defending the regime is:

$$\begin{cases} \underbrace{\omega_D}_{\text{Payoff to s.q. regime}} - \underbrace{\mu}_{\text{Cost of repressing}} & \text{w/ Pr} = p\big(\theta_M, \theta_T\big) \\ -\mu & \text{w/ Pr} = 1 - p\big(\theta_M, \theta_T\big) \end{cases}$$

Removing the military's uncertainty about repression success considerably simplifies the exposition, and Section 5.2 alters the setup such that the military only knows the probability distribution for repression success but not the realized Nature draw. Section 5.3 relaxes the assumption that both military types consume the same amount under the incumbent regime.

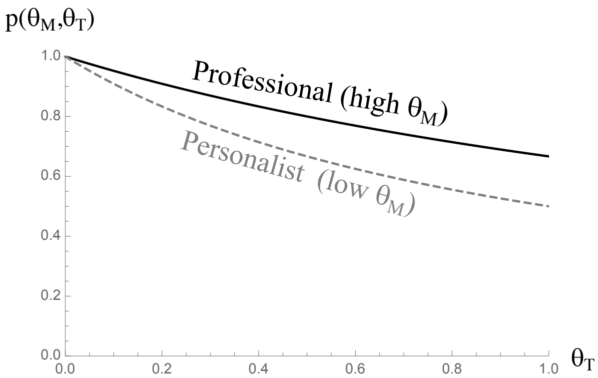
Figure 1 depicts key assumptions about the probability that repression is effective, and also summarizes the main takeaways in tabular form. Either type of military likely can defeat a weak threat. Although the effectiveness probability decreases in the strength of the outsider threat for both military types, this decline

efficiency tradeoff.

[5] Several proofs require the additional assumption $f'(\cdot) \leq 0$ which, for example, the uniform distribution satisfies. I define $\overline{\mu} > 0$ below.

is sharper for a personalist military, i.e., the negative effect of θ_T on $p(\cdot)$ decreases in θ_M. This assumed substitutability captures the sensible idea that the higher capacity of professional militaries enables retaining moderately high fighting capacity even against a strong threat, whereas personalist militaries may simply lack the ability to defeat strong threats.[6]

Figure 1: Assumptions about Repression Effectiveness



Probability of effective repression

	Low θ_T	High θ_T
Professional military	High	Medium
Personalist military	High	Low

Notes: Figure 1 uses the parameter values $\underline{\theta}_M = 1$ and $\overline{\theta}_M = 2$, and assumes $p(\theta_M, \theta_T) = \frac{\theta_M}{\theta_M + \theta_T}$.

2.4 Military's Payoff to Staging a Coup

Installing a military dictatorship—the goal of a coup—requires defeating the dictator *and* defeating the outsider threat, in which case the military consumes 1, which exceeds its consumption under the status quo regime. Implicitly, $\omega_D < 1$ implies that the dictator faces some limitations under the status quo regime to committing to make the military as well off as under military rule. Under military dictatorship, generals can invest in preferred military technology and also use the military as they feel appropriate, although Section 5.3 considers the possibility that the military prefers the existing regime to military rule.

As with defending the incumbent, the military cannot install a new dictatorship unless the military suc-

[6]Formally, $p(\cdot)$ satisfies $\frac{\partial p}{\partial \theta_M} > 0$, $\frac{\partial p}{\partial \theta_T} < 0$, and $\frac{\partial^2 p}{\partial \theta_M \partial \theta_T} > 0$; and the boundary conditions $p(\theta_M, 0) = 1$ for all $\theta_M > 0$ and $p(\theta_M, \theta_T) \in (0,1)$ for all $\theta_T > 0$. The ratio-form contest function $p(\theta_M, \theta_T) = \frac{\theta_M}{\theta_M + \theta_T}$ satisfies these assumptions, including the complementarity assumption for any $\theta_M > \theta_T > 0$, which rests on the reasonable presumption that the government's military has a greater coercive endowment than the outsider threat. The linear contest function $p(\theta_M, \theta_T) = 1 - \theta_T \cdot (1 - \theta_M)$ satisfies the cross-partial assumption for all $(\theta_M, \theta_T) \in (0,1)^2$.

cessfully represses the outsider threat. Therefore, the military's utility to a coup attempt depends in part on the same Nature move that governs its repression cost, μ. This assumption follows a key innovation in McMahon and Slantchev's (2015) setup: the outsider threat does not magically disappear after the military displaces the incumbent. The need to defeat the outsider affects the military's utility to holding office, which many prior analyses of the guardianship dilemma did not incorporate.

But the military also needs a coup opportunity to displace the dictator. Coup opportunities arise with probability $q(\theta_M, \theta_D)$, in which case a coup attempt topples the dictator with probability 1. With complementary probability, the military lacks a coup opportunity and wins a coup attempt with probability 0. The inherent secrecy and stealth involved with executing a coup imply that such opportunity may not always be available, even for an aggrieved military. As with the other Nature moves, the military knows the realization for whether or not it has a coup opportunity, whereas the dictator only knows the underlying probability distribution. Section 5.3 considers an alternative setup in which even a successful coup attempt may yield a transition to outsider rule.

Overall, the military's utility to staging a coup is:

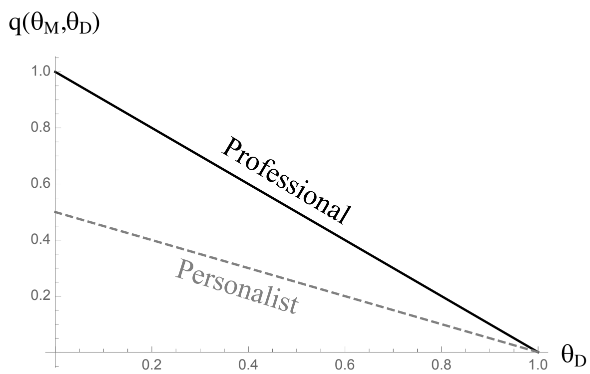
$$\begin{cases} \underbrace{1}_{\text{Payoff to mil. dict.}} - \underbrace{\mu}_{\text{Cost of repressing}} & \text{w/ Pr} = q\big(\theta_M, \theta_D\big) \cdot p\big(\theta_M, \theta_T\big) \\ -\mu & \text{w/ Pr} = 1 - q\big(\theta_M, \theta_D\big) \cdot p\big(\theta_M, \theta_T\big) \end{cases}$$

Figure 2 depicts key assumptions about the probability of a coup opportunity. Either type of military faces severe impediments to launching a coup if θ_D is high, which reflects strong institutions and high societal support for the regime that impedes launching a coup. Luttwak (2016) describes how rulers seek to rally the masses and other segments of the military to their defense during a coup attempt, which can thwart an attempted takeover. Alternatively, the dictator may enjoy military support from a foreign sponsor if a coup attempt occurs.[7] Although, for both military types, the probability of a coup opportunity increases as the dictator's coup-proofing ability decreases, this increase is sharper for professional militaries—which generally

[7]High θ_D may also reflect coup-proofing institutions, such as a presidential guard, that a ruler inherits. However, it is clearer for the present purposes to conceive of these aspects of a dictator's strength as embedded in its personalist/professional military choice and, hence, θ_M.

have more frequent opportunities to stage a coup. This reflects a standard assumption in the guardianship dilemma literature (Acemoglu, Vindigni and Ticchi 2010; Besley and Robinson 2010; Svolik 2013; McMahon and Slantchev 2015), and is sensible when considering that the lower θ_M inherent in personalist militaries often results from structuring such militaries to prevent communication among different branches and to counterbalancing the conventional military with a presidential guard (Quinlivan 1999). Therefore, whereas weak institutions and low societal support (low θ_D) should completely incapacitate the dictator's ability to prevent a coup by a professional military, it should still possess some ability to prevent a coup by a personalist military with built-in coup-proofing measures.[8]

Figure 2: Assumptions about Coup Opportunity



Probability of coup opportunity

	Low θ_D	High θ_D
Professional military	High	Low
Personalist military	Medium	Low

Notes: Figure 2 uses the parameter values $\underline{\theta}_M = 1$ and $\overline{\theta}_M = 2$, and assumes $q(\theta_M, \theta_D) = \big(\theta_M / \overline{\theta}_M\big) \cdot (1 - \theta_D)$.

2.5 Military's Payoff to Negotiated Transition

The military's utility to negotiating a transition to outsider rule depends on its type, θ_M, and on the orientation of the outsider threat, $r \in \big(\underline{r}, \overline{r}\big)$, for $\underline{r} < \overline{r}$. Higher r corresponds to a more revolutionary threat. Overall, the military's consumption following a transition is $\omega_T\big(\theta_M, r\big) \in \big(0, \omega_D\big)$. Assuming $\omega_T < \omega_D$

[8]Formally, $q(\cdot)$ satisfies $\frac{\partial q}{\partial \theta_M} > 0$, $\frac{\partial q}{\partial \theta_D} < 0$, and $\frac{\partial^2 q}{\partial \theta_M \partial \theta_D} < 0$. I also assume the cross-partial—i.e., the extent to which decreasing θ_D complements an increase in θ_M to raise q—is large in magnitude, which Appendix Assumption A.1 formalizes. Finally, I impose boundary conditions such that if the dictator has the lowest coup-proofing ability, then it cannot prevent a coup attempt by the professional military. Furthermore, at the highest coup-proofing ability for the dictator, neither a personalist nor professional military can stage a coup. Formally, $q\big(\underline{\theta}_M, 0\big) < q\big(\overline{\theta}_M, 0\big) = 1$, $q\big(\overline{\theta}_M, \overline{\theta}_D\big) = q\big(\overline{\theta}_M, \overline{\theta}_D\big) = 0$, and $q(\theta_M, \theta_D) \in (0, 1)$ for all other θ_M and θ_D.

focuses the analysis on the non-trivial case in which any military receives certain perks under the incumbent regime that it would lose following a transition.

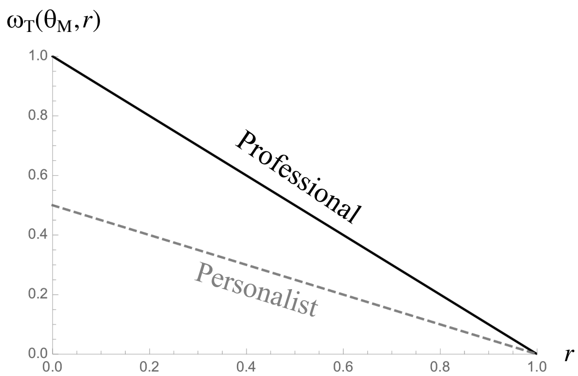
Departing from existing guardianship dilemma models, I model variance in the military's expected fate under outsider rule. This draws from important ideas about militaries' democratization incentives (e.g., Geddes 1999), but offers a novel argument for variance in militaries' willingness to negotiate transitions. A revolutionary threat corresponds to an outsider that seeks to effect a broad change in the composition of the elite class, and perhaps the entire social structure. Many revolutionary movement involve arbitrary arrests and executions, as with the formation of the Soviet Union, or general chaos and destruction even in cases that lack a distinctive revolutionary ideology, as with Genghis Khan's takeover of most of Asia and Europe. These aims and actions characterize many 20th century communist movements. For example, the Chinese Communist party implemented a massive land reform during and after its struggle to capture power in 1949. This was necessary to "destroy the gentry-landlord class (and thus eliminate a potential counterrevolutionary threat), establish Communist political power within the villages, and thus promote the building of a centralized state with firm administrative control over the countryside" (Meisner 1999, 92). Generalizing beyond the China case, Levitsky and Way (2013, 7) discuss communists' goals of destroying traditional ruling and religious institutions, political parties and the old army: "In most revolutions, preexisting armies either dissolved with the fall of the dictator (Cuba and Nicaragua) or were destroyed by civil war (China, Mexico, and Russia)." Conversely, in cases with an established communist regime, the regime would perceive any *non-communist* threat as revolutionary, such as a possible invasion of North Korea by South Korea and the United States. Other examples of revolutionary threats include movements in the Middle East in the 1950s and 1960s to replace monarchies and their historical power bases with pan-Arabist regimes (e.g., Egypt, Iraq, Yemen),[9] or ancient cases in which conquerors such as Genghis Khan in Asia in the 13th century and armies of the Prophet Muhammad and his disciples in the Middle East in the 7th century sought to conquer and to dismantle existing social structures.

Figure 3 depicts key assumptions about the military's payoff following a negotiated transition. Both military types expect dire fates under a highly revolutionary threat (high r) because both expect executions, disbandment, and other punishments. For the communism and anti-monarchy examples, the outsider sought

[9] Batatu (1978) describes the traditional landed elite in Iraq and the rise of revolutionary actors such as Ba'thists and communists.

to upend the existing social structure regardless of whether the military was organized professionally or personally.[10]

Figure 3: Assumptions about Military's Payoff Under Outsider Rule



Payoff to outsider rule

	Low r	High r
Professional military	High	Low
Personalist military	Medium	Low

Notes: Figure 3 uses the parameter values $\underline{\theta}_M = 1$, $\overline{\theta}_M = 2$, and $\omega_D = 0.5$, and assumes $\omega_T(\theta_M, r) = \left(\theta_M / \overline{\theta}_M\right) \cdot (1 - r) \cdot \omega_D$.

Although the payoff under outsider rule increases as r decreases for both military types, this increase is sharper for a professional military—which generally fares better under outsider rule than a personalist military because it is recruited from broader strata of society. Facing non-revolutionary outsiders (low r), professional and personalist militaries face sharp discrepancies in their outside options. A professional military expects minimal restructuring because of its continued ability to serve a new regime, whereas a personalist military composed largely of soldiers tied to the previous regime expects greater purging and restructuring. For example, consider pro-democracy protests that emerged across Arab countries in early 2011. Whereas the more professional Egyptian army eventually acquiesced to regime transition, the ethnically stacked and personally organized army in Syria feared its fate if non-Alawites took power. The military's willingness to fight outsiders triggered a civil war still ongoing as of early 2019.[11] Other examples of non-revolutionary threats include moderate nationalist and pro-democracy movements in Latin America in the 1980s (includ-

[10]Although revolutionary movements often arise in countries with personalist regimes (Goodwin and Skocpol 1989), other cases feature more professionally organized militaries, such as the British and, later, Malaysian armies that squared off against communists in Malaysia in the 1940s through 1960s.

[11]Although members of al-Asad's regime in Syria may consider the Sunni opposition as "revolutionary," in cases such as this, professional militaries would likely consider the protesters' and rebels' espoused democratization goals as non-revolutionary—implying that disparities in repression incentives arise from personalist-professional differences rather than from the orientation of the threat.

ing cases with rebel groups, such as El Salvador),[12] and the U.S. invasion of Iraq in 2003 where the goal was to replace a specific personalist regime rather than to more broadly reconstruct society.[13]

3 Analysis of Military's Decision

3.1 Repression, Coup, or Transition?

Table 2: Military's Optimal Choice

	Coup opportunity Pr=$q(\theta_M, \theta_D)$	**Not** Pr=$1 - q(\theta_M, \theta_D)$
Repression is effective Pr=$p(\theta_M, \theta_T)$	Coup	Repress if μ is low Transition if μ is medium
Ineffective Pr=$1 - p(\theta_M, \theta_T)$	Transition	Transition

Solving backwards, Table 2 summarizes the military's optimal choices under different Nature draws. If repression is ineffective, then the military optimally negotiates a transition to outsider rule because it cannot defeat the outsider to either defend the regime or install a military dictatorship. If instead the military has both a coup opportunity and repression is effective, then the military strictly prefers a coup to create a military dictatorship over defending the regime because $\omega_D < 1$. Furthermore, assuming the maximum repression cost is sufficiently low, the military also always prefers coups to transitions in this case.[14] Finally, if repression is effective but the military lacks a coup opportunity, then its optimal choice depends on repression costs. If the cost satisfies $\mu < \hat{\mu}(\theta_M) \equiv \omega_D - \omega_T(\theta_M, r)$, then the military will defend the regime, whereas the military optimally negotiates a transition in response to higher μ. Importantly, the critical cost threshold $\hat{\mu}(\theta_M)$ depends on θ_M because this affects the military's payoff under outsider rule.

[12] Geddes's (1999) examples of military dictatorships willing to acquiesce to democratization draw mainly from Latin America, which tend to feature professional militaries facing non-revolutionary threats.

[13] Formally, $\omega_T(\cdot)$ satisfies $\frac{\partial \omega_T}{\partial \theta_M} > 0$, $\frac{\partial \omega_T}{\partial r} < 0$, $\frac{\partial^2 \omega_T}{\partial \theta_M \partial r} < 0$. I also assume the cross-partial—i.e., the extent to which decreasing r complements an increase in θ_M to raise ω_T—is large in magnitude, which Appendix Assumption A.1 formalizes. Finally, I impose boundary conditions. At $\overline{r}$—the most revolutionary threat—both types of militaries consume 0. By contrast, at $\underline{r}$—the least revolutionary threat—the professional military consumes the same as under the status quo authoritarian regime. Formally, $\omega_T(\theta_M, \overline{r}) = 0$ for $\theta_M \in \{\underline{\theta}_M, \overline{\theta}_M\}$, and $\omega_T(\overline{\theta}_M, \underline{r}) = \omega_D$.

[14] Throughout, the analysis assumes the upper bound $\overline{\mu} = 1 - \omega_D$.

The assumed distributions for the Nature variables enables writing the probability of each outcome conditional on the dictator's military choice.

Lemma 1 (Outcome probabilities conditional on military type)**.** *Given the military choice θ_M, the equilibrium probability of each outcome is:*

$$\textit{Pr(repress)} = \underbrace{\Big[1 - q\big(\theta_M, \theta_D\big)\Big]}_{\textit{No coup opportunity}} \cdot \underbrace{p\big(\theta_M, \theta_T\big)}_{\textit{Effective repression}} \cdot \underbrace{F\Big(\omega_D - \omega_T\big(\theta_M, r\big)\Big)}_{\textit{Low repression costs}}$$

$$\textit{Pr(coup)} = \underbrace{q\big(\theta_M, \theta_D\big) \cdot p\big(\theta_M, \theta_T\big)}_{\textit{Effective repression and coup opportunity}}$$

$$\textit{Pr(transition)} = \underbrace{1 - p\big(\theta_M, \theta_T\big)}_{\textit{Ineffective repression}} + \Big[1 - q\big(\theta_M, \theta_D\big)\Big] \cdot p\big(\theta_M, \theta_T\big) \cdot \underbrace{\Big[1 - F\Big(\omega_D - \omega_T\big(\theta_M, r\big)\Big)\Big]}_{\textit{High repression costs}}$$

Lemma 1 yields two immediate implications. First, a professional military attempts a coup with higher probability than a personalist military. Second, conditional on effective repression without a coup opportunity, the personalist military defends the regime with higher probability.

Lemma 2 (Professional military and probability of a coup)**.**

$$\underbrace{p\big(\overline{\theta}_M, \theta_T\big) \cdot q\big(\overline{\theta}_M, r\big)}_{\textit{Professional}} > \underbrace{p\big(\underline{\theta}_M, \theta_T\big) \cdot q\big(\underline{\theta}_M, r\big)}_{\textit{Personalist}}$$

Lemma 3 (Personalist military and conditional probability of defending regime)**.**

$$\underbrace{F\Big(\omega_D - \omega_T\big(\overline{\theta}_M, r\big)\Big)}_{\textit{Professional}} < \underbrace{F\Big(\omega_D - \omega_T\big(\underline{\theta}_M, r\big)\Big)}_{\textit{Personalist}}$$

3.2 Equilibrium Loyalty and Efficiency of Professional and Personalist Militaries

These results enable characterizing the relative advantages of each military type for the dictator, which Table 3 summarizes. Recovering conventional wisdom about the loyalty-efficiency tradeoff, personalist militaries exhibit higher coup ***loyalty*** (Lemma 2). The two individually necessary and jointly sufficient conditions for the military to stage a coup are to have a coup opportunity and for repression to be effective, both of which advantage professional militaries. Notably, the coup loyalty result follows solely from differential *opportunities* to stage a coup rather than differences in the military's *preferences* for the incumbent. In other words, conventional ideas such as officers favoring co-ethnic rule are not necessary to generate the loyalty side of the loyalty-efficiency tradeoff. Section 5.3 discusses this consideration in more detail and demonstrates alternative loyalty assumptions that generate a similar finding.

With regard to repressive ***efficiency***, professional and personalist militaries exhibit mixed considerations. On the one hand, a professional military's higher probability of effective repression creates a repressive efficiency advantage.[15] However, the professional military's higher reservation value to outsider rule creates a countervailing implication for repressive efficiency. Conditional on effective repression, a professional military is less likely to defend the regime (Lemma 3). This countervailing efficiency mechanism—largely overlooked in existing studies positing a loyalty-efficiency tradeoff—creates the possibility that a personalist military can exhibit higher repressive efficiency despite its weaker coercive endowment.

Table 3: Relative Advantages of Each Military Type for Dictator

Mechanism	***Probability term***	***Professional***	***Personalist***
Loyalty	Pr(coup)		✓
Efficiency #1	Pr(effective repression)	✓	
Efficiency #2	Pr(defend regime | effective rep.)		✓

3.3 Non-Revolutionary Threats and Repressive Advantages of Personalist Militaries

Does the dictator trade off between coup loyalty and repressive efficiency? Repressive efficiency equals the probability of no outsider overthrow conditional on no coup:

$$E^*\big(\theta_M, \theta_T, r\big) \equiv \underbrace{p\big(\theta_M, \theta_T\big)}_{\text{Pr(effective repression)}} \cdot \underbrace{F\Big(\omega_D - \omega_T\big(\theta_M, r\big)\Big)}_{\text{Pr(defend regime | effective rep.)}} \tag{1}$$

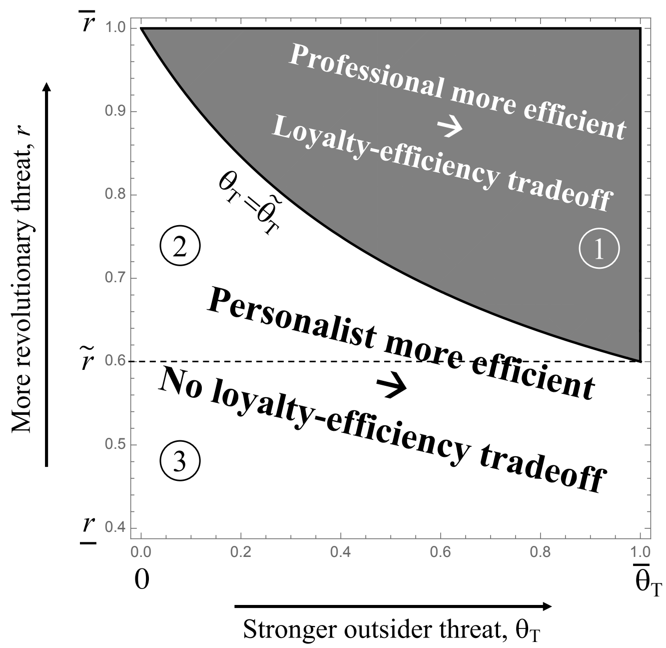
Figure 4 presents a region plot as a function of outsider threat strength, θ_T (horizontal axis), and the orientation of the outsider threat, r (vertical axis). The professional military is more efficient than the personalist military, $E^*\big(\underline{\theta}_M\big) < E^*\big(\overline{\theta}_M\big)$, in the gray region ①, whereas the opposite holds in the white regions ② and ③.

Region ① recovers the conventional wisdom that professional militaries exhibit greater repressive efficiency than personalist militaries, which follows from two factors. First, the *large*-magnitude threat implies that the professional military is considerably more likely to be able to repress effectively. Its higher coercive

[15] This effect arises from assuming $p\big(\theta_M, \theta_T\big)$ strictly increases in θ_M.

endowment $\overline{\theta}_M$ more effectively counteracts the negative effect of θ_T on the probability $p(\theta_M, \theta_T)$ of effective repression (see Figure 1). This implies that the magnitude of the first efficiency mechanism in Table 3 is large. Second, regarding the endogenous choice to exercise repression, the *revolutionary* orientation of the threat implies that a professional military fares only slightly better under outsider rule than a personalist military because both suffer low $\omega_T(\theta_M, r)$ under revolutionary rule (see Figure 3). This implies that the magnitude of the second efficiency mechanism in Table 3 is small.

Figure 4: Repressive Efficiency



Notes: Figure 4 uses the same parameter values and functional form assumptions as Figures 1 through 3, and $\omega_D = 0.8$ and $\mu \sim U(0, 1 - \omega_D)$.

However, region ② shows that even if the threat is revolutionary, at low values of θ_T, the *personalist* military is more repressively efficient. Facing a weak threat, the gap between $p(\overline{\theta}_M, \theta_T)$ and $p(\underline{\theta}_M, \theta_T)$ is small because either type of military is likely to effectively repress a weak threat. Region ③ shows that the dictator also does not face a loyalty-efficiency tradeoff if the threat is non-revolutionary—regardless of its severity. When facing a non-revolutionary threat, the professional military fares considerably better under outsider rule than the personalist military—i.e., $\omega_T(\overline{\theta}_M, r)$ is considerably larger than $\omega_T(\underline{\theta}_M, r)$—which creates a large gap in the two militaries' probability of exercising repression conditional on repression being effective.

In both these cases, the second efficiency mechanism highlighted in Table 3 that favors a personalist military dominates the first efficiency mechanism that favors a professional military, causing the personalist military to exhibit greater repressive efficiency. Coupled with the personalist military's higher coup loyalty (see Table 3), this implies that the personalist military is both more loyal and more efficient—and therefore the dictator does not face a loyalty-efficiency tradeoff—unless the threat is strong and revolutionary.

Lemma 4 (Repressive efficiency)**.** *There exist unique thresholds* $\tilde{r} \in \left(\underline{r}, \overline{r}\right)$ *and* $\tilde{\theta}_T \in \left(0, \overline{\theta}_T\right)$ *with the following properties:*

Part a. Non-revolutionary threat. *If* $r < \tilde{r}$*, then the personalist military exhibits higher repressive efficiency for all* $\theta_T \in \left(0, \overline{\theta}_T\right)$*:* $E^*\left(\underline{\theta}_M, \theta_T, r\right) > E^*\left(\overline{\theta}_M, \theta_T, r\right)$*. This is region* ③ *of Figure 4.*

Part b. Revolutionary threat. *If* $r > \tilde{r}$*, then:*

- *If* $\theta_T < \tilde{\theta}_T$*, then the personalist military exhibits higher repressive efficiency:* $E^*\left(\underline{\theta}_M, \theta_T, r\right) > E^*\left(\overline{\theta}_M, \theta_T, r\right)$*. This is region* ② *of Figure 4.*
- *If* $\theta_T > \tilde{\theta}_T$*, then the professional military exhibits higher repressive efficiency:* $E^*\left(\underline{\theta}_M, \theta_T, r\right) < E^*\left(\overline{\theta}_M, \theta_T, r\right)$*. This is region* ① *of Figure 4.*

4 Analysis of Dictator's Decision

4.1 Optimal Military Choice

When choosing its military, the dictator takes into account both coup propensity and repressive efficiency. It maximizes its probability of survival, which equals the probability that the military defends the regime:

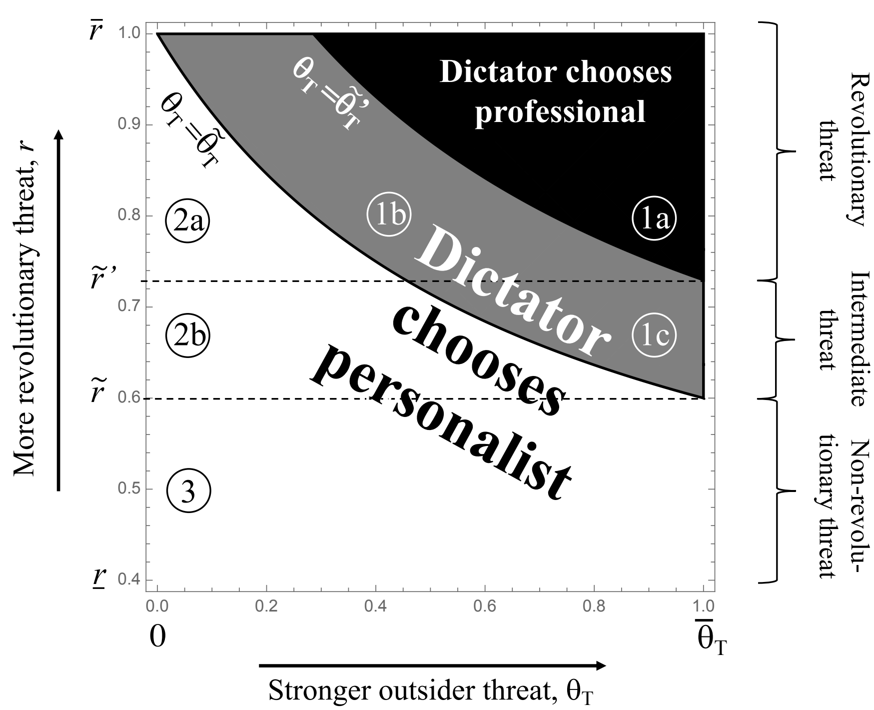
$$S^*\left(\theta_M, \theta_T, r, \theta_D\right) \equiv \underbrace{\Big[1 - q\big(\theta_M, \theta_D\big)\Big]}_{\text{No coup opportunity}} \cdot \underbrace{p\big(\theta_M, \theta_T\big) \cdot F\Big(\omega_D - \omega_T\big(\theta_M, r\big)\Big)}_{\text{Repressive efficiency}} \tag{2}$$

There are two cases to consider. First, if the professional military's coup likelihood is sufficiently high, then the dictator will choose the personalist military regardless of repressive efficiency considerations. This occurs when institutions are very weak and the dictator cannot harness the (possible) repressive advantages of a professional military.

Proposition 1 (Optimal military choice under ineffective coup-proofing)**.** *There exists a unique threshold* $\tilde{\theta}_D \in \left(0, \overline{\theta}_D\right)$ *such that if* $\theta_D < \tilde{\theta}_D$*, then* D *chooses the personalist military:* $S^*\left(\underline{\theta}_M, \theta_T, r, \theta_D\right) > S^*\left(\overline{\theta}_M, \theta_T, r, \theta_D\right)$*.*

The second and more strategically interesting case arises for higher θ_D. Then, the dictator's choice over military type depends on the above considerations about repressive efficiency. Because the professional military is more likely to attempt a coup, the dictator clearly will choose the personalist military under all parameter values in Lemma 4 in which the personalist military exhibits higher repressive efficiency—if the outsider threat is non-revolutionary and/or weak in magnitude. These are regions (2) and (3) in Figures 4 and 5. However, even for parameter values in which the professional military is more repressively efficient, the loyalty-efficiency tradeoff implies that the dictator does not necessarily choose the professional military. Although the dictator follows a similar threshold strategy as characterized in Lemma 4, it optimally chooses the professional military for a smaller range of parameter values than those for which the professional military exhibits higher repressive efficiency. Figure 5 shows this by distinguishing region (1a) in black, in which the dictator chooses a professional military, from the gray regions (1b) and (1c). Collectively, these three areas compose region (1) in Figure 4. Because the critical r threshold differs for repressive efficiency and for the dictator's optimal military choice, I refer to $r \in (\tilde{r}, \tilde{r}')$ as the "intermediate threat" range.

Figure 5: Optimal Military Choice and Consequences



Notes: Figure 5 uses the same parameter values and functional form assumptions as the previous figures.

Proposition 2 (Optimal military choice under effective coup-proofing). *Assume $\theta_D > \tilde{\theta}_D$, for $\tilde{\theta}_D$ defined in Proposition 1. Given the thresholds defined in Lemma 4, there exist unique thresholds $\tilde{r}' \in (\tilde{r}, \overline{r})$ and $\tilde{\theta}_T' \in (\tilde{\theta}_T, \overline{\theta}_T)$ with the following properties:*

Part a. Non-revolutionary (and intermediate) threat. *If $r < \tilde{r}'$, then G chooses a personalist military: $S^*(\underline{\theta}_M, \theta_T, r, \theta_D) > S^*(\overline{\theta}_M, \theta_T, r, \theta_D)$. This is regions (1c), (2b), and (3) in Figure 5.*

Part b. Revolutionary threat. *If $r > \tilde{r}'$, then:*

- *If $\theta_T < \tilde{\theta}_T'$, then D chooses a personalist military: $S^*(\underline{\theta}_M, \theta_T, r, \theta_D) > S^*(\overline{\theta}_M, \theta_T, r, \theta_D)$, for $S^*(\cdot)$ defined in Equation 2. This is regions (1b) and (2a) in Figure 5.*
- *If $\theta_T > \tilde{\theta}_T'$, then D chooses a professional military: $S^*(\underline{\theta}_M, \theta_T, r, \theta_D) < S^*(\overline{\theta}_M, \theta_T, r, \theta_D)$. This is region (1a) in Figure 5.*

Propositions 1 and 2, combined with the actions stated in Table 2, characterize the unique subgame perfect Nash equilibrium.

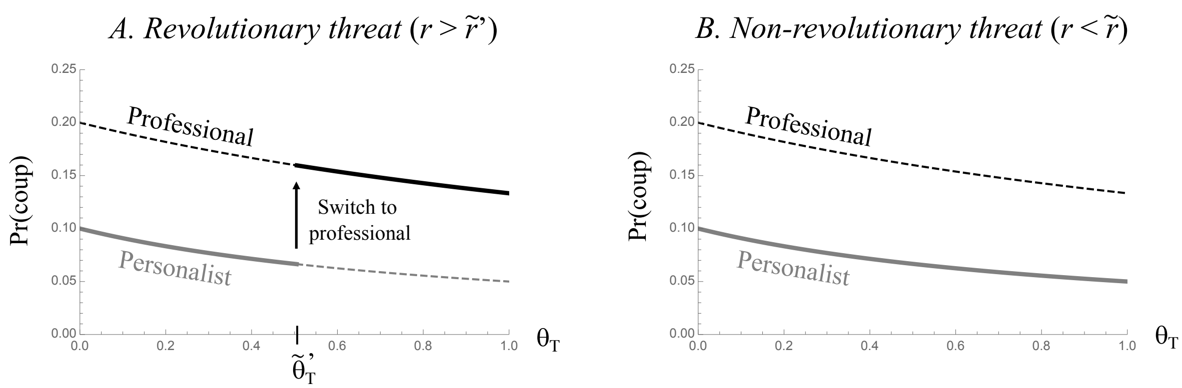
4.2 Consequences of the Loyalty-Efficiency Tradeoff for the Guardianship Dilemma

How does the strength of an outsider threat affect the equilibrium probability of a coup attempt? This is the central question for understanding the widely debated "guardianship dilemma" logic. This section demonstrates the close relationship among revolutionary threats, the loyalty-efficiency tradeoff, and the guardianship dilemma. One important implication is that existing arguments only characterize select parts of the overall guardianship logic.

Figure 6 depicts the relationship between θ_T and equilibrium coup probability, distinguishing between revolutionary (Panel A) and non-revolutionary (Panel B) threats. An increase in θ_T generates both a direct and an indirect effect. The direct effect is that higher θ_T decreases the probability with which the military will retain office conditional on displacing the dictator (i.e., lower probability of effective repression). Contrary to the guardianship logic, this mechanism yields a *negative* relationship between outsider threat strength and equilibrium coup probability. This logic is independent of military type or the orientation of the outsider threat, as shown by the downward slope of all four lines in Figure 6. This resembles the main finding from McMahon and Slantchev (2015): stronger outsider threats diminish equilibrium coup likelihood by decreasing the value of holding office.

However, the indirect effect of increasing θ_T recovers the traditional guardianship dilemma argument, contrary to McMahon and Slantchev's (2015) critique. If the outsider threat is revolutionary ($r > \tilde{r}'$), then the dictator faces a loyalty-efficiency tradeoff if the threat is strong, as regions (1a) and (1b) in Figure 5 show. As the threat increases in magnitude from region (1b) to (1a)—which occurs at $\theta_T = \tilde{\theta}'_T$—the dictator switches from a personalist to a professional military. This yields a discrete *increase* in the equilibrium coup probability, as Panel A of Figure 6 shows, because professional militaries exhibit higher coup propensity than personalist militaries (see Table 3). Therefore, a revolutionary threat generates both a loyalty-efficiency tradeoff and a guardianship dilemma.

Figure 6: Equilibrium Probability of a Coup Attempt



Notes: Solid lines correspond with parameter values in which the dictator optimally chooses the specified type of military, and dashed lines correspond with off-the-equilibrium path outcomes. Therefore, the equilibrium coup probability equals the piecewise function created by the solid lines. Both panels use the same parameter values and functional form assumptions as previous figures. In Panel A, $r = 0.3$. In Panel B, $r = 0.9$.

By contrast, the relationship between threat strength and equilibrium coup probability differs if the threat is non-revolutionary ($r < \tilde{r}$). Lemma 4 shows that the dictator does not face a loyalty-efficiency tradeoff. Consequently, the dictator never switches to the professional military, and therefore there are no θ_T values at which the equilibrium coup probability exhibits a discrete increase—hence eliminating the guardianship dilemma. Panel B of Figure 6 depicts this result.

Although a loyalty-efficiency tradeoff is necessary for a guardianship dilemma, the intermediate threat range—regions (1c) and (2b) in Figure 5—shows that it is not sufficient. The higher coup propensity of a professional military yields intermediate values $r \in \left(\tilde{r}, \tilde{r}'\right)$. Although the professional military is more efficient than the personalist military for high enough θ_T, the dictator prefers the personalist military even at $\theta_T = \overline{\theta}_T$ because the difference in repressive efficiency is not large enough to compensate for the difference in coup likelihood. Because the dictator never switches to the professional military, there are no θ_T values at which the equilibrium coup probability exhibits a discrete increase—the same result as the non-revolutionary threat case discussed in the previous paragraph.

Proposition 3 (Threat strength and equilibrium coup probability)**.** *Given the thresholds stated in Propositions 1 and 2:*

> ***Part a. Revolutionary threat.*** *If $\theta_D > \tilde{\theta}_D$ and $r > \tilde{r}'$, then equilibrium coup probability strictly decreases in θ_T for $\theta_T \in \left(0, \tilde{\theta}'_T\right) \cup \left(\tilde{\theta}'_T, \overline{\theta}_T\right)$, and exhibits a discrete increase at $\theta_T = \tilde{\theta}'_T$.*
>
> ***Part b. Non-revolutionary/intermediate threat.*** *If $\theta_D < \tilde{\theta}_D$ or $r < \tilde{r}'$, then equilibrium coup probability strictly decreases in θ_T for all $\theta_T \in \left(0, \overline{\theta}_T\right)$.*

Proposition 4 (Threat orientation, loyalty-efficiency tradeoff, and guardianship dilemma)**.** *Given the thresholds stated in Lemma 4 and Propositions 1 and 2, if $\theta_D > \tilde{\theta}_D$, then:*

> ***Part a. Revolutionary threat.*** *If $r > \tilde{r}'$, then the dictator faces both a loyalty-efficiency tradeoff and a guardianship dilemma.*
>
> ***Part b. Non-revolutionary threat.*** *If $r < \tilde{r}$, then the dictator faces neither a loyalty-efficiency tradeoff nor a guardianship dilemma.*
>
> ***Part c. Intermediate range.*** *If $r \in \left(\tilde{r}, \tilde{r}'\right)$, then the dictator faces a loyalty-efficiency tradeoff but not a guardianship dilemma.*

5 Additional Results and Extensions

5.1 Effects of Dictator Strength

Although the analysis focuses primarily on how characteristics of the external threat affect the dictator's optimal military choice, the dictator's endowed strength θ_D—which encompasses broader political institutions and popular support—also affects its choice. A dictator with high θ_D faces low coup vulnerability. Existing arguments posit that dictators should favor more broadly based professional militaries when facing a low coup threat. For example, Greitens (2016, 18) argues that dictators resolve their dual coup and outsider rebellion threats by "configuring their internal security apparatus to address the *dominant perceived threat* at the time they come to power. Prioritizing the threat of a coup leads to higher fragmentation and exclusivity, whereas focusing on the threat of popular uprising leads to a more unitary and socially inclusive apparatus."

The model produces two findings about the effects of increasing θ_D. The first supports Greitens' argument and the second does not. First, higher θ_D causes the dictator to weight repressive efficiency more heavily than the coup threat in its objective function because the probability of a coup attempt decreases. As $\theta_D \to \overline{\theta}_D$, the probability of a coup attempt goes to 0 and the dictator's survival objective function in Equation 2 becomes equivalent to equilibrium repressive efficiency (Equation 1). Graphically, as $\theta_D \to \overline{\theta}_D$, the black region in which the dictator prefers the professional military in Figure 5 converges to the gray region in Figure 4 in which the professional military exhibits higher repressive efficiency.

However, the second finding is that lowering the coup threat does not necessarily cause the dictator to choose a professional military. The revised loyalty-efficiency logic explained by the model (see Figure 4) implies that when facing a non-revolutionary threat, a personalist military exhibits higher repressive efficiency regardless of the strength of the threat. Therefore, if $r < \tilde{r}$, then low coup threat does not cause the dictator to switch to a "more unitary and socially inclusive apparatus" (Greitens 2016, 18). The existing argument is true only if the threat has a decidedly revolutionary orientation, $r > \tilde{r}$, which generates a loyalty-efficiency tradeoff for the dictator. In this case, higher θ_D increases the range of parameter values in which the dictator chooses the professional military. Appendix Proposition A.1 formalizes this logic.

5.2 Probabilistic Repression Success

To simplify the exposition, the baseline model assumed that the military knew whether it was effective at repression (wins with probability 1) or ineffective (wins with probability 0) when making its choice. If instead the military faces the same source of uncertainty as the dictator at the military choice stage, then one aspect of the military's calculus changes. Specifically, if the military defends the regime, then it succeeds with probability $p(\theta_M, \theta_T)$ and the military consumes $\omega_D - \mu$, and with complementary probability repression fails and the military consumes $-\mu$. To highlight the main difference that arises with this alteration, I focus only on repressive efficiency here (i.e., examining the military's choice between repression and negotiated transition), which now equals:

$$E_p^*(\theta_M, \theta_T, r) \equiv \underbrace{p(\theta_M, \theta_T)}_{\text{Pr(repression succeeds)}} \cdot \underbrace{F\Big(p(\theta_M, \theta_T) \cdot \omega_D - \omega_T(\theta_M, r)\Big)}_{\text{Pr(defend regime)}}, \tag{3}$$

where the subscript p in E_p^* expresses probabilistic repression success. Equation 3 differs from Equation 1 in one way: the possibility that repression can fail affects the military's probability of defending the regime. This term now equals $F\Big(p(\theta_M, \theta_T) \cdot \omega_D - \omega_T\big(\theta_M, r\big)\Big)$, as opposed to $F\Big(\omega_D - \omega_T\big(\theta_M, r\big)\Big)$ in the baseline model.

The key difference is the possibility that $F\Big(p(\overline{\theta}_M, \theta_T) \cdot \omega_D - \omega_T\big(\overline{\theta}_M, r\big)\Big) > F\Big(p(\underline{\theta}_M, \theta_T) \cdot \omega_D - \omega_T\big(\underline{\theta}_M, r\big)\Big)$, whereas in the baseline model the personalist military was more likely to defend the regime (conditional on effective repression) because of its lower reservation value to outsider rule. However, with the extension, the professional military's higher probability of winning increases its incentives to defend the regime relative to the personalist military. However, despite this difference, Appendix Section A.2.2 shows that the overall logic is similar to that in Lemma 4. The only additional required assumptions are that the cross-partials are large in magnitude, which ensures that the direct effects that drive Lemma 4 are larger in magnitude than the indirect effects created by assuming the military is uncertain if repression will succeed.

5.3 Unpacking Loyalty

The baseline model shows a setup in which the personalist military is less likely than the professional military to stage a coup—despite omitting conventional loyalty mechanisms. Personalist militaries are less likely to stage a coup because of lesser *opportunity*: more prevalent coup-proofing institutions inherent in a personalist military diminish its likelihood of having an opportunity to successfully stage a coup. This section alters the setup in two different ways to show how alternative conceptualizations of loyalty can yield a similar survival objective function for the dictator as Equation 2.

5.3.1 Inherent Loyalty

The first alteration assumes that the personalist military enjoys higher expected consumption under the incumbent regime to capture the idea of *inherent loyalty*. The only differences from the baseline model are that coup attempts *always* succeed with probability 1 (i.e., the military has a coup opportunity with probability 1), and that Nature determines the value of ω_D in between the dictator's and the military's moves. Nature draws ω_D from a Bernoulli distribution in which $\omega_D = \underline{\omega}_D \in (0, 1)$ with probability $q(\theta_M, \theta_D)$, and $\omega_D = \overline{\omega}_D > 1$ with probability $1 - q(\theta_M, \theta_D)$. The same assumptions as the baseline model apply to $q(\cdot)$, most important, it strictly increases in θ_M. The difficulty of governing and the deleterious effect it exerts on maintaining a hierarchical command chain plausibly cause militaries to prefer civilian rule in some circumstances (Finer 2002), which corresponds with $\omega_D > 1$. This possibility contrasts with the assumption in the baseline model that the military necessarily prefers military dictatorship.

The military's optimal choices and the associated probabilities are unchanged from the baseline model (see Table 2).[16] If $\omega_D = \underline{\omega}_D$, then the military (weakly) prefers a coup over repression because it consumes more in a military dictatorship than in the incumbent regime. This is strategically identical to the military having a coup opportunity in the baseline model. By contrast, if $\omega_D = \overline{\omega}_D$, then the military (weakly) prefers repression to a coup—even though a coup attempt succeeds with probability 1—because it consumes more under the incumbent regime than in a military dictatorship. Consequently, the dictator's survival objective function in Equation 2 is unaltered for this alternative setup and yields identical implications.

The personalist military exhibits higher expected valuation for the incumbent dictatorship than the professional military, which implies lower coup propensity (because $\omega_D = \overline{\omega}_D$ is necessary for choosing to defend the regime). This result captures the idea of inherent loyalty because the personalist military's stronger preferences for the regime increases the dictator's survival probability. Existing research suggests many possible sources of higher inherent loyalty for narrowly constructed personalist militaries. One possibility is that officers gain some type of "warm glow" from co-ethnic governance.[17] Another is that members of different ethnic groups exhibit similar preferences over public goods, and higher expected ω_D expresses in reduced form that the personalist military consumes more because the dictator provides more-preferred public goods (Alesina et al. 1999). Other possibilities relate to the dictator's ability to commit to deliver spoils to the military. The descent-based characteristics of ethnic groups make it easier to commit to reward co-ethnics because it is difficult to hide or to change one's ethnicity (Caselli and Coleman 2013). Alternatively, co-ethnics may be better able to solve the coordination problems inherent in compelling the dictator to pay its subordinates after they have defended him in battle (Myerson 2008), resulting in higher consumption under the incumbent regime.

[16] Appendix Table A.2 changes the appropriate labels from Table 2 for the present extension.

[17] See, for example, Quinlivan (1999, 135) whose section "The Exploitation of Special Loyalties" begins by stating: "The building block of political action in Saudi Arabia, Iraq, and Syria is the 'community of trust' that is willing to act together."

5.3.2 Strategic Loyalty

The second alteration yields a strategic endogenous loyalty mechanism: the personalist military's lower reservation value to outsider rule lowers the likelihood of staging a coup. As in the previous extension, assume that the military can always topple the incumbent regime with probability 1, however, as in the baseline model, $\omega_D < 1$ is the same for both types of military. The key change to generate the strategic loyalty mechanism is that with probability $q \in (0,1)$, a coup establishes a military dictatorship; and with probability $1-q$, a coup yields societal rule.[18] Therefore, coups do not necessarily enable military rule. For example, militaries often stage a coup and then hold elections within several years (especially since the Cold War ended), which fits with the present conceptualization of a negotiated transition because the generals did not create a consolidated military dictatorship. In some cases, the military may indeed have planned to hand power over to civilians from the beginning, whereas in other cases the military may have gambled that it could hold on—but instead ended up negotiating a transition because of concerted domestic or international pressure. In between the dictator's and military's moves, Nature draws q from a smooth distribution $G(\cdot)$ with full support over $[0,1]$. Therefore, like the other Nature moves in the game, the dictator knows only the distribution of q whereas the military knows its exact value. The bounds of the support for $G(\cdot)$ are strategically equivalent to lacking a coup opportunity in the baseline model ($q = 0$) because the military cannot establish a military dictatorship, and to having a coup opportunity in the baseline model ($q = 1$) because a coup attempt for sure establishes a military dictatorship.

This altered setup does not alter the first necessary condition for the military to defend the regime from the baseline model: the military needs to prefer repression to negotiating a transition. As in the baseline model, this decision hinges on the draw for repression costs, μ. However, the second necessary condition—the military prefers repression to a coup—is met if and only if:

$$\underbrace{\omega_D - \mu}_{\text{Military's utility to defending regime}} > \underbrace{q + (1-q) \cdot \omega_T\big(\theta_M, r\big) - \mu}_{\text{Military's utility to coup}} \tag{4}$$

Solving Equation 4 for q and imposing the assumed probability distribution implies that the dictator's sur-

[18] These probabilities are not a function of θ_M or other parameters.

vival objective function is:

$$S^*_{sl}\left(\theta_M,\theta_T,r,\theta_D\right)\equiv \underbrace{G\left(\frac{\omega_D-\omega_T\left(\theta_M,r\right)}{1-\omega_T\left(\theta_M,r\right)}\right)}_{\text{Prefers repression to coup}}\cdot \underbrace{p\left(\theta_M,\theta_T\right)\cdot F\left(\omega_D-\omega_T\left(\theta_M,r\right)\right)}_{\text{Repressive efficiency}}, \tag{5}$$

where the subscript sl in S^*_{sl} expresses strategic loyalty. Although the efficiency component of the survival function is identical to the baseline model, the coup component is different. Instead of the military preferring a coup to repression if the military has a coup opportunity, and preferring repression to a coup otherwise, the military's preference between defending the regime and staging a coup depends not only on the draw of q but also on its reservation value to outsider rule, $\omega_T\left(\theta_M,r\right)$. The personalist military's lower reservation value deters coups because of the possibility that the coup will create an opening for the outsider to take over. This captures the idea of *strategic loyalty* because the personalist military's fear of the outsider drives its decision to support the dictator—despite, compared to the professional military, enjoying the same consumption amount under the incumbent regime and having the same opportunity to overthrow the dictator.[19] Appendix Proposition A.2 formalizes this logic.

6 Discussion and Empirical Implications

This paper presents a model in which a dictator facing an outsider threat chooses between a personalist and a professional military, and the military can choose to defend the regime by exercising repression, stage a coup, or negotiate a transition to outsider rule. The main results challenge two important premises regarding the existence of a loyalty-efficiency tradeoff and its consequences for the guardianship dilemma.

The model analysis also highlights several challenges to empirically studying the relationships theoretically examined here. This does not imply that the inferential challenges are insurmountable, but does highlight important selection effects that empirical work needs to carefully consider. First, studying the relationship between how the military is organized and its likelihood of repressing protesters or exerting concerted effort to defeat an armed group (as opposed to mutinying) requires taking into account the magnitude and

[19] Unlike in the baseline game, the coup term in the dictator's objective function, $G\left(\frac{\omega_D-\omega_T(\theta_M,r)}{1-\omega_T(\theta_M,r)}\right)$, does not equal the equilibrium probability of a coup attempt. Appendix Section A.2.3 discusses this point in more depth.

orientation of the outsider threat. Facing a strong threat, a personalist military may fail to defend the regime because its repression is ineffective—despite its low reservation value to societal rule. By contrast, facing a weak (but non-revolutionary) threat, a professional military may fail to defend the regime because of its high reservation value to societal rule—despite a high likelihood of defeating the threat. Sources such as Finer (1975, 93-5; 1997, 17-9) offer invaluable accounts of various military forms throughout history, but future work should complement this analysis by also examining how attributes of outsider threats affect militaries' actions.

Second, there are considerable empirical difficulties to uncovering the relationship between the strength of outsider threats and the likelihood of coup attempts—an idea central to the guardianship dilemma—because of the crucial intervening variable of military type. Consider, for example, a regression specification from Powell's (2012) widely cited article on coup determinants. He finds that instability (including guerrilla activity and riots) positively correlates with coup attempts and success (1030)—consistent with the guardianship logic that stronger outsider threats breed coups. However, his regression models include covariates for intervening channels that the present model posits as strategic reactions to outsider threats, including military expenditures, military personnel, and military regime. Although this reasonably guards against one problem—since excluding these control variables would likely induce omitted variable bias—future empirical work could attempt to explicitly model some of the strategic channels posited here to avoid post-treatment and other forms of bias.

Third, an important selection effect also confounds studying the relationship between military type and coup likelihood. Although the model implies that professional militaries should be more likely than personalist militaries to stage coups, empirically, personalist militaries often attempt coups. Related, Huntington (1957) argues that a professional ethos decreases a military's disposition to intervene politically. However, analyzing the effect of θ_D in the model shows that dictators only choose professional militaries if θ_D is relatively high (i.e., good institutions)—which decreases the likelihood of coups. Therefore, any empirical relationship between professionalism and a lack of coups may reflect selection effects, and, all else equal, personalist militaries' greater reliance on the incumbent dictator may decrease their coup likelihood. Overall, scrutinizing the logical relationships implied by the model and their empirical implications will hopefully spur productive future research on the central questions of how dictators craft their militaries how this choice affects regime survival.

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A Supplementary Information for Formal Model

Table A.1: Summary of Parameters and Choice Variables

Aspect of game	Variables/description
Coercive endowments	• Dictator: θ_D, with maximum value $\overline{\theta}_D$ • Outsider threat: θ_T, with maximum value $\overline{\theta}_T$ • Military: $\underline{\theta}_M$ for personalist and $\overline{\theta}_M$ for professional
Military's utility to defending the regime	• ω_D: Military's consumption under incumbent dictator • $p(\theta_M, \theta_T)$: Probability repression is effective • μ: Military's cost of repression with maximum value $\overline{\mu}$ • $F(\cdot)$: Distribution function for repression cost, with pdf $f(\cdot)$
Military's utility to coup	• $q(\theta_M, \theta_D)$: Probability the military has a coup opportunity
Military's utility to negotiated transition	• r: Orientation of outsider threat (higher is more revolutionary) • $\omega_T(\theta_M, r)$: Military's consumption under outsider rule

A.1 Proofs for Baseline Model

Lemmas 1 through 3 follow trivially from the assumptions. I use the following to prove Lemma 4.

Lemma A.1. *For E^* defined in Equation 1:*

Part a.
$$\frac{d^2 E^*}{d\theta_M dr} > 0$$

Part b.
$$\frac{d^2 E^*}{d\theta_M d\theta_T} > 0$$

Proof. The first derivative is:

$$\frac{dE^*}{d\theta_M} = \overbrace{\underbrace{\frac{\partial p}{\partial \theta_M} \cdot F\big(\omega_D - \omega_T(\theta_M, r)\big)}_{(+)\uparrow \text{Pr(effective repression)}}}^{①} - \overbrace{\underbrace{p(\theta_M, \theta_T) \cdot f\big(\omega_D - \omega_T(\theta_M, r)\big) \cdot \frac{\partial \omega_T}{\partial \theta_M}}_{(-)\downarrow \text{Pr(defend regime | effective rep.)}}}^{②} >< 0$$

Part a.

$$\frac{d^2 E^*}{d\theta_M dr} = \underbrace{\frac{\partial p}{\partial \theta_M} \cdot f\big(\omega_D - \omega_T(\theta_M, r)\big) \cdot \left(-\frac{\partial \omega_T}{\partial r}\right)}_{(+)\uparrow \text{ magnitude of ① by } \uparrow \text{Pr(defend regime | effective rep.)}}$$

$$\underbrace{+p(\theta_M,\theta_T)\cdot f\big(\omega_D-\omega_T(\theta_M,r)\big)\cdot\left(-\frac{\partial^2\omega_T}{\partial\theta_M\partial r}\right)}_{\text{(+)}\downarrow\text{ magnitude of \textcircled{2} by}\downarrow\text{ effect of }\theta_M\text{ on }\omega_T}$$

$$\underbrace{+p(\theta_M,\theta_T)\cdot\Big[-f'\big(\omega_D-\omega_T(\theta_M,r)\big)\Big]\cdot\left(-\frac{\partial\omega_T}{\partial r}\right)\cdot\frac{\partial\omega_T}{\partial\theta_M}}_{\text{(+)}\downarrow\text{ magnitude of \textcircled{2} by}\uparrow\text{ Pr(defend regime | effective rep.)}}>0$$

Part b.

$$\frac{d^2E^*}{d\theta_M d\theta_T} = \underbrace{\frac{\partial^2 p}{\partial\theta_M\partial\theta_T}\cdot F\big(\omega_D-\omega_T(\theta_M,r)\big)}_{\text{(+)}\uparrow\text{ magnitude of \textcircled{1} by}\uparrow\text{ effect of }\theta_M\text{ on Pr(effective rep.)}}$$

$$+\underbrace{\left(-\frac{\partial p}{\partial\theta_T}\right)\cdot f\big(\omega_D-\omega_T(\theta_M,r)\big)\cdot\frac{\partial\omega_T}{\partial\theta_M}}_{\text{(+)}\downarrow\text{ magnitude of \textcircled{2} by}\downarrow\text{ Pr(effective rep.)}}>0$$

■

The first derivative shows the two countervailing effects of an increase in θ_M on equilibrium repressive efficiency. Mechanism ① is positive because higher θ_M raises the probability that the military can repress effectively. Mechanism ② is negative because higher θ_M decreases the probability that the military defends the regime conditional on effective repression, which follows from $\frac{\partial\omega_T}{\partial\theta_M}>0$. The effects encompassed in the second derivatives are:

- **Part a.** An increase in r increases the magnitude of $\frac{dE^*}{d\theta_M}$ if that term is positive, and decreases its magnitude if it negative, through three effects:
  - Increases the magnitude of mechanism ① by increasing the probability that the military defends the regime conditional on effective repression because $\frac{\partial\omega_T}{\partial r}<0$.
  - Decreases the magnitude of mechanism ② by decreasing the magnitude of the positive effect of θ_M on ω_T because $\frac{\partial^2\omega_T}{\partial\theta_M\partial r}<0$.
  - Decreases the magnitude of mechanism ② by increasing the probability that the military defends the regime conditional on effective repression because $\frac{\partial\omega_T}{\partial r}<0$.
- **Part b.** An increase in θ_T increases the magnitude of $\frac{dE^*}{d\theta_M}$ if that term is positive, and decreases its magnitude if it negative, through two effects:
  - Increases the magnitude of mechanism ① by increasing the magnitude of the positive effect of θ_M on the probability the military can repress effectively because $\frac{\partial^2 p}{\partial\theta_M\partial\theta_T}>0$.
  - Decreases the magnitude of mechanism ② by decreasing the probability the military can repress effectively because $\frac{\partial p}{\partial\theta_T}<0$.

Proof of Lemma 4, part a. Part b of Lemma A.1 implies that if $E^*\left(\underline{\theta}_M, \overline{\theta}_T, r\right) > E^*\left(\overline{\theta}_M, \overline{\theta}_T, r\right)$, then this inequality holds for all $\theta_T \in \left(0, \overline{\theta}_T\right)$. Showing that the conditions for the intermediate value theorem hold establishes the existence of at least one $\tilde{r} \in \left(\underline{r}, \overline{r}\right)$ such that $E^*\left(\underline{\theta}_M, \overline{\theta}_T, \tilde{r}\right) = E^*\left(\overline{\theta}_M, \overline{\theta}_T, \tilde{r}\right)$:

- $E^*\left(\underline{\theta}_M, \underline{\theta}_T, \underline{r}\right) = p\left(\underline{\theta}_M, \overline{\theta}_T\right) \cdot \underbrace{F\Big(\omega_D - \omega_T\left(\underline{\theta}_M, r\right)\Big)}_{>0} < p\left(\overline{\theta}_M, \overline{\theta}_T\right) \cdot \underbrace{F(0)}_{=0} = E^*\left(\overline{\theta}_M, \overline{\theta}_T, \underline{r}\right)$

  follows from assuming $\omega_T\left(\overline{\theta}_M, \underline{r}\right) = \omega_D$.
- $E^*\left(\underline{\theta}_M, \overline{\theta}_T, \overline{r}\right) = p\left(\underline{\theta}_M, \overline{\theta}_T\right) \cdot F\left(\omega_D\right) < p\left(\overline{\theta}_M, \overline{\theta}_T\right) \cdot F\left(\omega_D\right) = E^*\left(\overline{\theta}_M, \overline{\theta}_T, \overline{r}\right)$ follows from assuming $\omega_T\left(\theta_M, \overline{r}\right) = 0$ for $\theta_M \in \left\{\underline{\theta}_M, \overline{\theta}_M\right\}$.
- Continuity trivially holds.

The unique threshold claim for $\tilde{r}$ follows from $\frac{d^2 E^*}{d\theta_M dr} > 0$ (part a of Lemma A.1).

Part b. Showing that the conditions for the intermediate value theorem hold establishes that if $r < \tilde{r}$, then there exists at least one $\tilde{\theta}_T \in \left(0, \overline{\theta}_T\right)$ such that $E^*\left(\underline{\theta}_M, \tilde{\theta}_T, r\right) = E^*\left(\overline{\theta}_M, \tilde{\theta}_T, r\right)$:

- $E\left(\underline{\theta}_M, 0, r\right) = F\Big(\omega_D - \omega_T\left(\underline{\theta}_M, r\right)\Big) > F\Big(\omega_D - \omega_T\left(\overline{\theta}_M, r\right)\Big) = E\left(\overline{\theta}_M, 0, r\right)$ follows from assuming $p(\theta_M, 0) = 1$ for $\theta_M \in \left\{\underline{\theta}_M, \overline{\theta}_M\right\}$.
- $E\left(\underline{\theta}_M, \overline{\theta}_T, r\right) < E\left(\overline{\theta}_M, \overline{\theta}_T, r\right)$ follows from assuming $r < \tilde{r}$ (see the proof for part a).
- Continuity trivially holds.

The unique threshold claim for $\tilde{\theta}_T$ follows from $\frac{d^2 E^*}{d\theta_M d\theta_T} > 0$ (part b of Lemma A.1). ■

The following assumption characterizes the lower bounds for the magnitude of two second derivatives mentioned in the text.

Assumption A.1. *The proof for Lemma A.2 defines the following thresholds.*

Part a.

$$-\frac{\partial^2 \omega_T}{\partial \theta_M \partial r} > \underline{\partial^2 \omega_T}$$

Part b.

$$-\frac{\partial^2 q}{\partial \theta_M \partial \theta_D} > \underline{\partial^2 q}$$

I use the following technical lemma to prove the propositions.

Lemma A.2. *For S^* defined in Equation 2:*

Part a. $$\frac{d^2 S^*}{d\theta_M dr} > 0$$

Part b. $$\frac{d^2 S^*}{d\theta_M d\theta_D} > 0$$

Part c. $$\frac{d^2 S^*}{d\theta_M d\theta_T} > 0$$

Proof. The first derivative is:

$$\frac{dS^*}{d\theta_M} = \big[1 - q(\theta_M, \theta_D)\big] \cdot \underbrace{\frac{dE^*}{d\theta_M}}_{\text{(+/–) Lemma A.1}} \overbrace{\underbrace{-\frac{\partial q}{\partial \theta_M} \cdot p(\theta_M, \theta_T) \cdot F\Big(\omega_D - \omega_T(\theta_M, r)\Big)}_{\text{(–) }\uparrow\text{ Pr(coup opportunity)}}}^{③} >< 0$$

Part a.

$$\frac{d^2 S^*}{d\theta_M dr} = \big[1 - q(\theta_M, \theta_D)\big] \cdot \underbrace{\frac{d^2 E^*}{d\theta_M dr}}_{\text{(+) Lemma A.1}} \underbrace{-\frac{\partial q}{\partial \theta_M} \cdot p(\theta_M, \theta_T) \cdot f\Big(\omega_D - \omega_T(\theta_M, r)\Big) \cdot \Big(-\frac{\partial \omega_T}{\partial r}\Big)}_{\text{(–) }\downarrow\text{ magnitude of ③ by }\uparrow\text{ Pr(defend regime | effective rep.)}} >< 0$$

Eliding the terms in parentheses, substituting in terms for $\frac{d^2 E^*}{d\theta_M dr}$ from the Lemma A.1 proof shows that the overall term is strictly positive if and only if:

$$-\frac{\partial^2 \omega_T}{\partial \theta_M \partial r} > \underline{\partial^2 \omega_T} \equiv$$

$$\left\{(1-q) \cdot \left[\frac{\partial q}{\partial \theta_M} \cdot p \cdot f(\cdot) - \frac{\partial p}{\partial \theta_M} \cdot f(\cdot) + p \cdot \Big[-f'(\cdot)\Big] \cdot \frac{\partial \omega_T}{\partial \theta_M}\right]\right\} \cdot \Big(-\frac{\partial \omega_T}{\partial r}\Big) \cdot \frac{1}{(1-q) \cdot p \cdot f(\cdot)}, \quad \text{(A.1)}$$

which part a of Assumption A.1 assumes is true.

Part b.

$$\frac{d^2 S^*}{d\theta_M d\theta_D} = \underbrace{\Big(-\frac{\partial q}{\partial \theta_D}\Big) \cdot \frac{dE^*}{d\theta_M}}_{\text{(+/–) }\uparrow\text{ magn. of }\frac{dE^*}{d\theta_M}\text{ by }\downarrow\text{ Pr(coup opp.)}} + \underbrace{\Big(-\frac{\partial^2 q}{\partial \theta_M \partial \theta_D}\Big) \cdot p(\theta_M, \theta_T) \cdot F\big(\omega_D - \omega_T(\theta_M, r)\big)}_{\text{(+) }\downarrow\text{ magnitude of ③ by }\downarrow\text{ effect of }\theta_M\text{ on }q} >< 0$$

Eliding the terms in parentheses, substituting in terms for $\frac{dE^*}{d\theta_M}$ from the Lemma A.1 proof shows that the overall term is strictly positive if and only if:

$$-\frac{\partial^2 q}{\partial\theta_M \partial\theta_D} > \underline{\partial^2 q} \equiv \left(-\frac{\partial q}{\partial\theta_D}\right)\cdot\left[p\cdot f(\cdot)\cdot\frac{\partial\omega_T}{\partial\theta_M} - \frac{\partial p}{\partial\theta_M}\cdot F(\cdot)\right]\cdot\frac{1}{p\cdot F(\cdot)}, \tag{A.2}$$

which part b of Assumption A.1 assumes is true.

Part c.

$$\frac{d^2 S^*}{d\theta_M d\theta_T} = \big[1 - q(\theta_M, \theta_D)\big]\cdot \underbrace{\frac{d^2 E^*}{d\theta_M d\theta_T}}_{\text{(+) Lemma A.1}} + \underbrace{\frac{\partial q}{\partial\theta_M}\cdot\left(-\frac{\partial p}{\partial\theta_T}\right)\cdot F\big(\omega_D - \omega_T(\theta_M, r)\big)}_{\text{(+)} \downarrow \text{ magnitude of } (3) \text{ by} \downarrow \text{Pr(effective rep.)}} > 0$$

■

Remark A.1 simplifies the complementarity thresholds from Assumption A.1 using the functional form assumptions from Figures 4 and 6.

Remark A.1 (Illustration of complementarity thresholds)**.** *Assume the following functional forms:*

- $p(\theta_M, \theta_T) = 1 - \theta_T\cdot(1-\theta_M)$
- $\omega_T(\theta_M, r) = \big(\theta_M/\overline{\theta}_M\big)\cdot(1-r)\cdot\omega_D$
- $\mu \sim U(0, 1-\omega_D)$
- $q(\theta_M, \theta_D) = \big(\theta_M/\overline{\theta}_M\big)\cdot(1-\theta_D)$

Part a. *If $\theta_D > \frac{1}{2}$, then Part a of Assumption A.1 holds for all $\theta_T \in \big(0, \overline{\theta}_T\big)$ and $\theta_M \in \big\{\underline{\theta}_M, \overline{\theta}_M\big\}$.*

Part b. *If $r > \frac{1}{2}$, then Part b of Assumption A.1 holds for all $\theta_T \in \big(0, \overline{\theta}_T\big)$ and $\theta_M \in \big\{\underline{\theta}_M, \overline{\theta}_M\big\}$.*

Proof. The following preliminary result shows that the right-hand side of Equations A.1 and A.2 reach their upper bound at $\theta_T = 0$:

$$\frac{d}{d\theta_T}\left[-\frac{\partial p}{\partial\theta_M}\cdot\frac{1}{p(\theta_M, \theta_T)}\right] = -\left[\frac{\partial^2 p}{\partial\theta_M\partial\theta_T}\cdot\frac{1}{p} + \frac{\partial p}{\partial\theta_M}\cdot\frac{-\frac{\partial p}{\partial\theta_T}}{p^2}\right] < 0$$

Therefore, if the inequalities hold at $\theta_T = 0$, then they hold for all $\theta_T \in \big(0, \overline{\theta}_T\big)$.

Part a. Substituting the functional form assumptions and $\theta_T = 0$ into Equation A.1 yields:

$$\frac{\omega_D}{\overline{\theta}_M} > \frac{1-\theta_D}{\overline{\theta}_M}\cdot\frac{1}{1 - \frac{\theta_M}{\overline{\theta}_M}\cdot(1-\theta_D)}\cdot\frac{\theta_M}{\overline{\theta}_M}\cdot\omega_D,$$

which simplifies to:

$$\theta_D > 1 - \frac{1}{2} \cdot \frac{\overline{\theta}_M}{\theta_M}$$

Because the right-hand side achieves its upper bound at $\theta_M = \overline{\theta}_M$, substituting in $\theta_M = \overline{\theta}_M$ yields the claim.

Part b. Substituting the functional form assumptions and $\theta_T = 0$ into Equation A.2 yields:

$$\frac{1}{\overline{\theta}_M} > \frac{1}{\omega_D - \frac{\theta_M}{\overline{\theta}_M} \cdot (1-r) \cdot \omega_D} \cdot \frac{1}{\overline{\theta}_M} \cdot (1-r) \cdot \omega_D \cdot \frac{\theta_M}{\overline{\theta}_M},$$

which simplifies to:

$$r > 1 - \frac{1}{2} \cdot \frac{\overline{\theta}_M}{\theta_M}$$

Because the right-hand side achieves its upper bound at $\theta_M = \overline{\theta}_M$, substituting in $\theta_M = \overline{\theta}_M$ yields the claim. ■

Proof of Proposition 1. Parts a and c of Lemma A.2 imply that if $S^*\left(\underline{\theta}_M, \overline{\theta}_T, \overline{r}, \theta_D\right) > S^*\left(\overline{\theta}_M, \overline{\theta}_T, \overline{r}, \theta_D\right)$, then this inequality holds for all $\theta_T \in \left(0, \overline{\theta}_T\right)$ and $r \in \left(\underline{r}, \overline{r}\right)$. Showing that the conditions for the intermediate value theorem hold establishes the existence of at least one $\tilde{\theta}_D \in \left(0, \overline{\theta}_D\right)$ such that $S^*\left(\underline{\theta}_M, \overline{\theta}_T, \overline{r}, \tilde{\theta}_D\right) = S^*\left(\overline{\theta}_M, \overline{\theta}_T, \overline{r}, \tilde{\theta}_D\right)$:

- If $\theta_D = 0$, then $q\left(\underline{\theta}_M, 0\right) < q\left(\overline{\theta}_M, 0\right) = 1$, which implies $S^*\left(\underline{\theta}_M, \overline{\theta}_T, \overline{r}, 0\right) > S^*\left(\overline{\theta}_M, \overline{\theta}_T, \overline{r}, 0\right) = 0$.
- If $\theta_D = \overline{\theta}_D$, then $q\left(\underline{\theta}_M, \overline{\theta}_D\right) = q\left(\overline{\theta}_M, \overline{\theta}_D\right) = 0$. This implies that $S^*\left(\underline{\theta}_M, \overline{\theta}_T, \overline{r}, \overline{\theta}_D\right) = E^*\left(\underline{\theta}_M, \overline{\theta}_T, \overline{r}\right)$ and $S^*\left(\overline{\theta}_M, \overline{\theta}_T, \overline{r}, \overline{\theta}_D\right) = E^*\left(\overline{\theta}_M, \overline{\theta}_T, \overline{r}\right)$. The proof for part b of Lemma 4 shows that $E^*\left(\underline{\theta}_M, \overline{\theta}_T, \overline{r}\right) < E^*\left(\overline{\theta}_M, \overline{\theta}_T, \overline{r}\right)$.
- Continuity is trivially satisfied.

The unique threshold claim for $\tilde{\theta}_D$ follows from $\frac{d^2 S^*}{d\theta_M d\theta_D} > 0$ (part b of Lemma A.2). ■

Proof of Proposition 2, part a. Part c of Lemma A.2 implies that if $S^*\left(\underline{\theta}_M, \overline{\theta}_T, r, \theta_D\right) > S^*\left(\overline{\theta}_M, \overline{\theta}_T, r, \theta_D\right)$, then this inequality holds for all $\theta_T \in \left(0, \overline{\theta}_T\right)$. Showing that the conditions for the intermediate value theorem hold establishes that if $\theta_D > \tilde{\theta}_D$, then there exists at least one $\tilde{r}' \in \left(\tilde{r}, \overline{r}\right)$ such that $S^*\left(\underline{\theta}_M, \overline{\theta}_T, \tilde{r}', \theta_D\right) = S^*\left(\overline{\theta}_M, \overline{\theta}_T, \tilde{r}', \theta_D\right)$:

- $S^*\left(\underline{\theta}_M, \overline{\theta}_T, \tilde{r}, \theta_D\right) > S^*\left(\overline{\theta}_M, \overline{\theta}_T, \tilde{r}, \theta_D\right)$ simplifies to $q\left(\overline{\theta}_M, \theta_D\right) > q\left(\underline{\theta}_M, \theta_D\right)$, a true statement, because the two types of military exhibit the same repressive efficiency at these parameter values (see the definition of $\tilde{r}$ in the proof for part b of Lemma 4).
- $S^*\left(\underline{\theta}_M, \overline{\theta}_T, \overline{r}, \theta_D\right) < S^*\left(\overline{\theta}_M, \overline{\theta}_T, \overline{r}, \theta_D\right)$ follows from assuming $\theta_D > \tilde{\theta}_D$ (see the proof for Proposition 1).

- Continuity trivially holds.

The unique threshold claim for $\tilde{r}'$ follows from $\frac{d^2 S^*}{d\theta_M dr} > 0$ (part a of Lemma A.2).

Part b. Showing that the conditions for the intermediate value theorem hold establishes the existence of at least one $\tilde{\theta}'_T \in (\tilde{\theta}_T, \overline{\theta}_T)$ such that if $\theta_D > \tilde{\theta}_D$ and $r > \tilde{r}'$, then $S^*(\underline{\theta}_M, \tilde{\theta}'_T, r, \theta_D) = S^*(\overline{\theta}_M, \tilde{\theta}'_T, r, \theta_D)$:

- $S^*(\underline{\theta}_M, \tilde{\theta}_T, r, \theta_D) > S^*(\overline{\theta}_M, \tilde{\theta}_T, r, \theta_D)$ simplifies to $q(\overline{\theta}_M, \theta_D) > q(\underline{\theta}_M, \theta_D)$, a true statement, because the two types of military exhibit the same repressive efficiency at these parameter values (see the definition of $\tilde{\theta}_T$ in the proof for part b of Lemma 4).
- $S^*(\underline{\theta}_M, \overline{\theta}_T, r, \theta_D) < S^*(\overline{\theta}_M, \overline{\theta}_T, r, \theta_D)$ follows from assuming $\theta_D > \tilde{\theta}_D$ and $r > \tilde{r}'$ (see the proof for part a).
- Continuity trivially holds.

The unique threshold claim for $\tilde{\theta}'_T$ follows from $\frac{d^2 S^*}{d\theta_M d\theta_T} > 0$ (part c of Lemma A.2). ■

Proof of Proposition 4. The equilibrium probability of a coup is:

$$Pr(coup) = \begin{cases} q(\underline{\theta}_M, \theta_D) \cdot p(\underline{\theta}_M, \theta_T) & \text{if } \theta_T < \tilde{\theta}'_T \\ q(\overline{\theta}_M, \theta_D) \cdot p(\overline{\theta}_M, \theta_T) & \text{if } \theta_T > \tilde{\theta}'_T \end{cases}$$

Assuming $\frac{\partial p}{\partial \theta_T} < 0$ implies that this function strictly decreases at all $\theta_T \in (0, \tilde{\theta}'_T) \cup (\tilde{\theta}'_T, \overline{\theta}_T)$. Lemma 2 implies that the function exhibits a discrete increase at $\theta_T = \tilde{\theta}'_T$. ■

A.2 Proofs for Additional Results and Extensions

A.2.1 Effects of Dictator Strength

Define the difference between equilibrium repressive efficiency and the dictator's equilibrium survival probability (for a given choice of θ_M):

$$\Delta \equiv E^*(\theta_M, \theta_T, r) - S^*(\theta_M, \theta_T, r, \theta_D) \tag{A.3}$$

Visually, Figure 5 shows that for any value r, Δ equals the distance between $\tilde{\theta}'$ and $\tilde{\theta}$, i.e., the gray region.

Proposition A.1 (Effects of Dictator Strength)**.**

Part a. *If the professional military exhibits higher repressive efficiency than the personalist military ($r > \tilde{r}$ and $\theta_T > \tilde{\theta}_T$; see Lemma 4), then for Δ defined in Equation A.3:*

- *An increase in θ_D increases the dictator's likelihood of choosing the professional military: $\frac{d\Delta}{d\theta_D} < 0$.*
- *As the dictator becomes perfectly able to prevent coups, the dictator chooses the professional military: $\lim\limits_{\theta_D \to \overline{\theta}_D} \Delta = 0$*

Part b. *If the personalist military exhibits higher repressive efficiency than the professional military ($r < \tilde{r}$ or $\theta_T < \tilde{\theta}_T$), then an increase in θ_D does not affect the dictator's optimal military choice.*

Proof of part a. Equation A.3 simplifies to $q\big(\theta_M, \theta_D\big) \cdot p\big(\theta_M, \theta_T\big) \cdot F\Big(\omega_D - \omega_T\big(\theta_M, r\big)\Big)$. By assumption, $\frac{\partial q}{\partial \theta_D} < 0$ and $\lim\limits_{\theta_D \to \overline{\theta}_D} q\big(\theta_M, \theta_D\big) = 0$, which establishes the claim.

Part b. Follows because the dictator chooses the personalist military if $E^*\big(\underline{\theta}_M, \theta_T, r\big) > E^*\big(\overline{\theta}_M, \theta_T, r\big)$, and θ_D does not affect this inequality. ∎

A.2.2 Probabilistic Repression Success

Under the extension in which the military knows the probability with which repression succeeds but not the outcome of the Nature draw, a formal result identical in structure to Lemma 4 holds under the following assumptions about two of the cross-partials (for both, the magnitude of complementarities for the direct effects to dominate the indirect effects). Lemma A.2 shows that these assumptions generate an identical statement as Lemma A.1.

Assumption A.2. *The proof for Lemma A.3 defines the following thresholds.*

Part a.

$$-\frac{\partial^2 \omega_T}{\partial \theta_M \partial r} > \overline{\partial^2 \omega_T}$$

Part b.

$$-\frac{\partial^2 p}{\partial \theta_M \partial \theta_T} > \overline{\partial^2 p}$$

Lemma A.3. *For E_p^* defined in Equation 3:*

Part a.

$$\frac{d^2 E_p^*}{d\theta_M dr} > 0$$

Part b.

$$\frac{d^2 E_p^*}{d\theta_M d\theta_T} > 0$$

Proof. The structure of the proof is identical to that for Lemma A.1. The terms in blue are the additional terms that arise from probabilistic repression success. The first derivative is:

$$\frac{dE_p^*}{d\theta_M} = \overbrace{\underbrace{\frac{\partial p}{\partial \theta_M} \cdot F\big(p(\theta_M, \theta_T) \cdot \omega_D - \omega_T(\theta_M, r)\big)}_{\text{(+)} \uparrow \text{Pr(repression succeeds)}}}^{\textcircled{1}}$$

$$\overbrace{\underbrace{-p(\theta_M, \theta_T) \cdot f\big(p(\theta_M, \theta_T) \cdot \omega_D - \omega_T(\theta_M, r)\big) \cdot \left[\frac{\partial \omega_T}{\partial \theta_M} - \frac{\partial p}{\partial \theta_M} \cdot \omega_D\right]}_{\text{(–)} \downarrow \text{Pr(defend regime) if term in brackets} > 0}}^{\textcircled{2}} >< 0$$

Part a.

$$\frac{d^2 E_p^*}{d\theta_M dr} = \underbrace{\frac{\partial p}{\partial \theta_M} \cdot f\big(p(\theta_M, \theta_T) \cdot \omega_D - \omega_T(\theta_M, r)\big) \cdot \left(-\frac{\partial \omega_T}{\partial r}\right)}_{\text{(+)} \uparrow \text{ magnitude of } \textcircled{1} \text{ by } \uparrow \text{Pr(defend regime)}}$$

$$\underbrace{+p(\theta_M, \theta_T) \cdot f\big(p(\theta_M, \theta_T) \cdot \omega_D - \omega_T(\theta_M, r)\big) \cdot \left(-\frac{\partial^2 \omega_T}{\partial \theta_M \partial r}\right)}_{\text{(+)} \downarrow \text{ magnitude of } \textcircled{2} \text{ by } \downarrow \text{ effect of } \theta_M \text{ on } \omega_T}$$

$$\underbrace{+p(\theta_M, \theta_T) \cdot \Big[-f'\big(p(\theta_M, \theta_T) \cdot \omega_D - \omega_T(\theta_M, r)\big)\Big] \cdot \left(-\frac{\partial \omega_T}{\partial r}\right) \cdot \left[\frac{\partial \omega_T}{\partial \theta_M} - \frac{\partial p}{\partial \theta_M} \cdot \omega_D\right]}_{\text{(+)} \downarrow \text{ magnitude of } \textcircled{2} \text{ by } \uparrow \text{Pr(defend regime) if term in brackets} > 0} >< 0$$

Eliding the terms in brackets, the overall term is strictly positive if and only if:

$$-\frac{\partial^2 \omega_T}{\partial \theta_M \partial r} > \overline{\partial^2 \omega_T} \equiv \left\{\frac{\partial p}{\partial \theta_M} \cdot f(\cdot) \cdot \left(-\frac{\partial \omega_T}{\partial r}\right) + p \cdot \Big[-f'(\cdot)\Big] \cdot \left(-\frac{\partial \omega_T}{\partial r}\right) \cdot \left[\frac{\partial \omega_T}{\partial \theta_M} - \frac{\partial p}{\partial \theta_M} \cdot \omega_D\right]\right\} \cdot \frac{1}{p \cdot f(\cdot)},$$

which part a of Assumption A.2 assumes is true.

Part b.

$$\frac{d^2 E_p^*}{d\theta_M d\theta_T} = \underbrace{\frac{\partial^2 p}{\partial \theta_M \partial \theta_T} \cdot F\big(p(\theta_M, \theta_T) \cdot \omega_D - \omega_T(\theta_M, r)\big)}_{\text{(+)} \uparrow \text{ magnitude of } \textcircled{1} \text{ by } \uparrow \text{ effect of } \theta_M \text{ on Pr(repression succeeds)}}$$

$$\underbrace{+\left(-\frac{\partial p}{\partial \theta_T}\right) \cdot f\big(p(\theta_M, \theta_T) \cdot \omega_D - \omega_T(\theta_M, r)\big) \cdot \left[\frac{\partial \omega_T}{\partial \theta_M} - \frac{\partial p}{\partial \theta_M} \cdot \omega_D\right]}_{\text{(+)} \downarrow \text{ magnitude of } \textcircled{2} \text{ by } \downarrow \text{Pr(repression succeeds) if term in brackets} > 0}$$

$$\underbrace{-\frac{\partial p}{\partial \theta_M} \cdot f\big(p(\theta_M, \theta_T) \cdot \omega_D - \omega_T(\theta_M, r)\big) \cdot \left(-\frac{\partial p}{\partial \theta_T}\right) \cdot \omega_D}_{\text{(–)} \downarrow \text{ magnitude of } \textcircled{1} \text{ b/c diminish Pr(defend)}}$$

$$+\underbrace{p(\theta_M,\theta_T)\cdot f\big(p(\theta_M,\theta_T)\cdot\omega_D-\omega_T(\theta_M,r)\big)\cdot\frac{\partial^2 p}{\partial\theta_M\partial\theta_T}\cdot\omega_D}_{(+)\downarrow\text{ magnitude of (2) because }\uparrow\text{ effect of }\theta_M\text{ on Pr(defend)}}$$

$$-\underbrace{p(\theta_M,\theta_T)\cdot\Big[-f'\big(p(\theta_M,\theta_T)\cdot\omega_D-\omega_T(\theta_M,r)\big)\Big]\cdot\Big(-\frac{\partial p}{\partial\theta_T}\Big)\cdot\omega_D\cdot\Big[\frac{\partial\omega_T}{\partial\theta_M}-\frac{\partial p}{\partial\theta_M}\cdot\omega_D\Big]}_{(-)\uparrow\text{ magnitude of (2) by }\uparrow\text{ Pr(defend regime) if term in brackets}>0}><0$$

Eliding the terms in parentheses, the overall term is strictly positive if and only if:

$$\frac{\partial^2 p}{\partial\theta_M\partial\theta_T}>\overline{\partial^2 p}\equiv$$

$$\bigg\{\Big(-\frac{\partial p}{\partial\theta_T}\Big)\cdot\Big[\frac{\partial\omega_T}{\partial\theta_M}-\frac{\partial p}{\partial\theta_M}\cdot\omega_D\Big]\cdot\Big[f(\cdot)-p\cdot[-f'(\cdot)]\cdot\omega_D\Big]-\frac{\partial p}{\partial\theta_M}\cdot f(\cdot)\cdot\Big(-\frac{\partial p}{\partial\theta_T}\Big)\cdot\omega_D\Big]\bigg\}\cdot\frac{1}{F(\cdot)+p\cdot f(\cdot)\cdot\omega_D},$$

which part b of Assumption A.2 assumes is true.

■

How do the expressions in the proof for Lemma A.2 differ from those in the proof for Lemma A.1? To explain the differences, the following copy and pastes the text that follows the proof of Lemma A.1 in black, and the blue text comments on the differences in Lemma A.2. To avoid repetition, I do not comment on the change that arises within every $F(\cdot)$, $f(\cdot)$, and $f'(\cdot)$ term because ω_D is multiplied by $p\big(\theta_M,\theta_T\big)$.

The first derivative shows the two countervailing effects of an increase in θ_M on equilibrium repressive efficiency. Mechanism (1) is positive because higher θ_M raises the probability that the military can repress effectively. This term is unchanged, although is now phrased as the probability that the military succeeds at repression. Mechanism (2) is negative because higher θ_M decreases the probability that the military defends the regime conditional on effective repression, which follows from $\frac{\partial\omega_T}{\partial\theta_M}>0$. This mechanism no longer requires the qualifying statement about effective repression. More important, this mechanism is not necessarily negative because of an additional effect of θ_M on the military's probability of defending the regime: θ_M increases the probability that repression succeeds, which increases the military's incentive to defend the regime. Mechanism (2) is negative if and only if:

$$\frac{\partial\omega_T}{\partial\theta_M}>\frac{\partial p}{\partial\theta_M}\cdot\omega_D. \tag{A.4}$$

If Equation A.4 does not hold, then the professional military's higher probability of winning dominates the personalist military's lower value to outsider rule to yield a higher probability of defending the incumbent for professional militaries.

The effects encompassed in the second derivatives are:

- **Part a.** An increase in r increases the magnitude of $\frac{dE^*}{d\theta_M}$ if that term is positive, and decreases its magnitude if it negative, through three effects:
  - Increases the magnitude of mechanism (1) by increasing the probability that the military defends the regime conditional on effective repression because $\frac{\partial\omega_T}{\partial r}<0$. The sign of this term is

unchanged, although does not require the phrase about effective repression.

- Decreases the magnitude of mechanism (2) by decreasing the magnitude of the positive effect of θ_M on ω_T because $\frac{\partial^2 \omega_T}{\partial \theta_M \partial r} < 0$. Unchanged.
- Decreases the magnitude of mechanism (2) by increasing the probability that the military defends the regime conditional on effective repression because $\frac{\partial \omega_T}{\partial r} < 0$. This mechanism no longer requires the qualifying statement about effective repression. More important, this effect is positive if Equation A.4 holds, and negative otherwise. In the latter case, $-\frac{\partial^2 \omega_T}{\partial \theta_M \partial r}$ must be large enough in magnitude for the overall derivative in part a to be positive. (Alternatively, the statement in part a is true without imposing an assumption about the magnitude of $-\frac{\partial^2 \omega_T}{\partial \theta_M \partial r}$ if $f'(\cdot) = 0$ which, for example, the uniform distribution satisfies.)

- **Part b.** An increase in θ_T increases the magnitude of $\frac{dE^*}{d\theta_M}$ if that term is positive, and decreases its magnitude if it negative, through two effects:
  - Increases the magnitude of mechanism (1) by increasing the magnitude of the positive effect of θ_M on the probability the military can repress effectively because $\frac{\partial^2 p}{\partial \theta_M \partial \theta_T} > 0$. This term is unchanged, although is now phrased as the probability that the military succeeds at repression.
  - Decreases the magnitude of mechanism (2) by decreasing the probability the military can repress effectively because $\frac{\partial p}{\partial \theta_T} < 0$. This mechanism no longer requires the qualifying statement about effective repression. More important, this effect is positive if Equation A.4 holds and negative otherwise. In the latter case, $\frac{\partial^2 p}{\partial \theta_M \partial \theta_T}$ must be large enough in magnitude for the overall derivative in part b to be positive. The intuition for the countervailing effect is as follows. If Equation A.4 does not hold, then mechanism (2) is positive. In this case, a decrease in the probability that repression succeeds caused by higher θ_T diminishes the magnitude of a positive effect on repressive efficiency, hence the negative sign.
  - Three additional expressions (lines 3 through 5 in the proof for part b) arise from assuming probabilistic repression success.

A.2.3 Unpacking Loyalty

Inherent loyalty. Table A.2 is identical to Table 2 except it changes the description of the columns to correspond with the inherent loyalty extension.

Table A.2: Military's Optimal Choice with Inherent Loyalty

	Low value s.q. dictator Pr=$q(\theta_M, \theta_D)$	**High value s.q. dictator** Pr=$1 - q(\theta_M, \theta_D)$
Repression is effective Pr=$p(\theta_M, \theta_T)$	Coup	Repress if μ is low Transition if μ is medium
Ineffective Pr=$1 - p(\theta_M, \theta_T)$	Transition	Transition

Strategic loyalty. Proposition A.2 formalizes the discussion from the text regarding the conditions under which the military prefers defending the regime over attempting a coup.

Proposition A.2 (Strategic loyalty mechanism)**.** *The coup component of the dictator's survival objective function,* $G\left(\frac{\omega_D-\omega_T(\theta_M,r)}{1-\omega_T(\theta_M,r)}\right)$ *in Equation 5, strictly decreases in* θ_M *through the effect of* θ_M *on* ω_T*.*

Proof. Expressing $G\left(\frac{\omega_D-\omega_T(\theta_M,r)}{1-\omega_T(\theta_M,r)}\right)$ as G:

$$\frac{dG}{d\theta_M} = \frac{\partial G}{\partial \theta_M} + \frac{\partial G}{\partial \omega_T} \cdot \frac{d\omega_T}{d\theta_M}$$

$$\frac{\partial G}{\partial \theta_M} = 0$$

$$\frac{\partial G}{\partial \omega_T} = -\frac{1-\omega_D}{(1-\omega_T)^2}$$

$$\frac{d\omega_T}{d\theta_M} > 0 \text{ by assumption}$$

This implies that:

$$\frac{\partial G}{\partial \omega_T} \cdot \frac{d\omega_T}{d\theta_M} < 0$$

■

Unlike in the baseline setup, the coup term in the dictator's objective function for the strategic loyalty setup, $G\left(\frac{\omega_D-\omega_T(\theta_M,r)}{1-\omega_T(\theta_M,r)}\right)$, does not equal the equilibrium probability of a coup attempt. For low enough q, the military prefers a negotiated transition to a coup for any draw of μ. This creates the possibility that the military prefers negotiated transition to a coup for parameter values in which the military strictly prefers a coup to repression, which is not possible in the baseline model. This consideration is irrelevant for the dictator's objective function—conditional on the military choosing not to repress, its coup/transition choice does not affect the dictator's consumption—but does affect the equilibrium probability of a coup attempt. Instead, this probability equals:

$$\underbrace{\int_0^{\omega_D-\omega_T} \overbrace{\int_{\frac{\omega_D-\omega_T}{1-\omega_T}}^{1} dG(q)}^{\text{Prefers coup to repression}} \cdot dF(\mu)}_{\text{Prefers repression to transition}} + \underbrace{\int_{\omega_D-\omega_T}^{\overline{\mu}} \overbrace{\int_{\frac{\mu}{1-\omega_T}}^{1} dG(q)}^{\text{Prefers coup to transition}} \cdot dF(\mu)}_{\text{Prefers transition to repression}}$$

The outer integral for each term expresses the probability that the military prefers repression to transitioning or vice versa, and the inner integral expresses the probability that the military prefers a coup to the most-preferred alternative. The range of the outer integrals is the same as in the baseline model: the military prefers repression to transition if $\mu \in \big(0, \omega_D-\omega_T\big)$, and prefers transition to repression if $\mu \in \big(\omega_D-\omega_T, \overline{\mu}\big)$. The range of the inner integral in the first term expresses that the military prefers a coup over repression if $q \in \big(\frac{\omega_D-\omega_T}{1-\omega_T}, 1\big)$, which follows from the discussion in the text. The range of the inner integral in the second term expresses that the military prefers a coup over transition if $q \in \big(\frac{\mu}{1-\omega_T}, 1\big)$, which follows from solving

$q + (1-q) \cdot \omega_T - \mu > \omega_T$ for μ. The entire expression simplifies to:

$$\left[1 - G\left(\frac{\omega_D - \omega_T}{1-\omega_T}\right)\right] \cdot F\big(\omega_D - \omega_T\big) + \int_{\omega_D - \omega_T}^{\overline{\mu}} \left[1 - G\left(\frac{\mu}{1-\omega_T}\right)\right] \cdot dF(\mu)$$