Math 1496 - Calc I

Def^n $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Do we really need to go to this def^n every time? No - we will create some rules to help us.

(1) Constant Rule

$$\frac{d}{dx}(c) = 0$$

Proof Let $f(x) = c$

so $$f'(x) = \lim_{h \to 0} \frac{c - c}{h} = 0 \quad \boxed{4} \checkmark$$

so $\frac{d}{dx} 7 = 0 \quad \frac{d}{dx} -3 = 0$ etc.

(2) <u>Power Rule</u>

We seen

$$\frac{d}{dx}x = 1, \quad \frac{d}{dx}x^2 = 2x \quad \frac{d}{dx}x^{1/2} = \frac{1}{2}x^{-1/2}$$

$$\frac{d}{dx}\frac{1}{x} = \frac{d}{dx}x^{-1} = -1x^{-2}$$

So there is a rule here

$$\frac{d}{dx}x^n = nx^{n-1}$$

<u>Proof</u> Let $f(x) = x^n$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$$ ← use binomial thm

$$= \lim_{h \to 0} \frac{\cancel{x^n} + nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2 + \cdots h^n - \cancel{x^n}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}\left(nx^{n-1} + \underbrace{\frac{n(n-1)}{2}x^{n-2}h + \cdots h^{n-1}}_{\text{these terms} \to 0}\right)}{\cancel{h}}$$

$$= nx^{n-1} \quad \blacksquare$$

(3) Constant Multiple

$$\frac{d}{dx} cf(x) = c f'(x)$$

Proof

$$\frac{d}{dx} c f(x) = \lim_{h\to 0} \frac{c f(x+h) - c f(x)}{h}$$

$$= c \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \leftarrow \text{def}^n \text{ of deriv.}$$

$$= c f'(x) \quad \square$$

(4) Sum/Difference

$$\frac{d}{dx} f(x) \pm g(x) = f'(x) \pm g'(x)$$

Proof- Sum

$$\lim_{h\to 0} \frac{f(x+h) + g(x+h) - (f(x) + g(x))}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} + \frac{g(x+h) - g(x)}{h}$$

$$= f'(x) + g'(x) \quad \square$$

Ex if $f(x) = 3x+2$

$$f'(x) = \frac{d}{dx}(3x+2) = 3\frac{d}{dx}x + \frac{d}{dx}2 = 3$$

Ex if $f(x) = -x^2+2x$

$$f' = -\frac{d}{dx}x^2 + 2\frac{d}{dx}(x) = -2x+2$$ much quicker

Ex Find the eq^n of the tangent of

$f(x) = x^{1/3}$ at $x = 8$

1st The point is $f(8) = 8^{1/3} = 2$ or $(8,4)$

we need $f'(x) = \lim_{h \to 0} \frac{(x+h)^{1/3} - x^{1/3}}{h}$ ← good luck

or $f'(8) = \lim_{x \to 8} \frac{(x^{1/3} - 2)}{x-8}$ still hard

but $f'(x) = \frac{1}{3}x^{-2/3}$ $f'(8) = \frac{1}{3(8)^{2/3}} = \frac{1}{3 \cdot 4} = \frac{1}{12}$

tangent $y - 2 = \frac{1}{12}(x-8)$

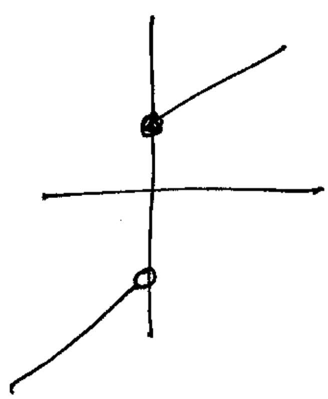
A word of caution. Consider

$$f(x) = \begin{cases} x-1 & x<0 \\ x+1 & x \geq 0 \end{cases}$$

One might say

$$f'(x) = \begin{cases} 1 & x<0 \\ 1 & x \geq 0 \end{cases}$$ saying that f is differentiable

but $f(x)$ looks like



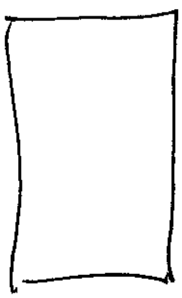
and f isn't even cont's at $x=0$ & using the def^n

$$\lim_{x \to 0^-} \frac{x-1-1}{x} = \lim_{x \to 0^-} \frac{x-2}{x} \text{ DNE}$$

$$\lim_{x \to 0^+} \frac{x+1-1}{x} = \lim_{x \to 0^+} \frac{x}{x} = 1$$

An Application

Suppose we are at the top of a building 100 ft high and we drop a ball. Suppose we know its position at any time and its given by



$$s(t) = 100 - 16t^2 \text{ ft}$$

Find the velocity at any time

$$\lim_{h \to 0} \frac{s(t+h) - s(t)}{h} = s'(t)$$

so $s' = \frac{d}{dt}\, 100 - 16t^2 = -32t$

so at $t = 1$ $\quad s'(1) = -32$ ft/sec

$t = 2$ $\quad s'(2) = -32(2) = -64$ ft/sec.