

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise A, Question 1

#### Question:

Find the mid-point of the line joining these pairs of points:

(a)  $(4, 2)$  ,  $(6, 8)$

(b)  $(0, 6)$  ,  $(12, 2)$

(c)  $(2, 2)$  ,  $(-4, 6)$

(d)  $(-6, 4)$  ,  $(6, -4)$

(e)  $(-5, 3)$  ,  $(7, 5)$

(f)  $(7, -4)$  ,  $(-3, 6)$

(g)  $(-5, -5)$  ,  $(-11, 8)$

(h)  $(6a, 4b)$  ,  $(2a, -4b)$

(i)  $(2p, -q)$  ,  $(4p, 5q)$

(j)  $(-2s, -7t)$  ,  $(5s, t)$

(k)  $(-4u, 0)$  ,  $(3u, -2v)$

(l)  $(a + b, 2a - b)$  ,  $(3a - b, -b)$

(m)  $(4\sqrt{2}, 1)$  ,  $(2\sqrt{2}, 7)$

(n)  $(-\sqrt{3}, 3\sqrt{5})$  ,  $(5\sqrt{3}, 2\sqrt{5})$

(o)  $(\sqrt{2} - \sqrt{3}, 3\sqrt{2} + 4\sqrt{3})$  ,  $(3\sqrt{2} + \sqrt{3}, -\sqrt{2} + 2\sqrt{3})$

#### Solution:

(a)  $(x_1, y_1) = (4, 2)$  ,  $(x_2, y_2) = (6, 8)$

$$\text{So } \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{4 + 6}{2}, \frac{2 + 8}{2} \right) = \left( \frac{10}{2}, \frac{10}{2} \right) = (5, 5)$$

(b)  $(x_1, y_1) = (0, 6)$  ,  $(x_2, y_2) = (12, 2)$

$$\text{So } \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{0 + 12}{2}, \frac{6 + 2}{2} \right) = \left( \frac{12}{2}, \frac{8}{2} \right) = (6, 4)$$

(c)  $(x_1, y_1) = (2, 2)$  ,  $(x_2, y_2) = (-4, 6)$

$$\text{So } \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{2 + (-4)}{2}, \frac{2 + 6}{2} \right) = \left( \frac{-2}{2}, \frac{8}{2} \right) = (-1, 4)$$

$$(d) (x_1, y_1) = (-6, 4), (x_2, y_2) = (6, -4)$$

$$\text{So } \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{-6 + 6}{2}, \frac{4 + (-4)}{2} \right) = \left( \frac{0}{2}, \frac{0}{2} \right) = \left( 0, 0 \right)$$

$$(e) (x_1, y_1) = (-5, 3), (x_2, y_2) = (7, 5)$$

$$\text{So } \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{-5 + 7}{2}, \frac{3 + 5}{2} \right) = \left( \frac{2}{2}, \frac{8}{2} \right) = \left( 1, 4 \right)$$

$$(f) (x_1, y_1) = (7, -4), (x_2, y_2) = (-3, 6)$$

$$\text{So } \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{7 + (-3)}{2}, \frac{-4 + 6}{2} \right) = \left( \frac{4}{2}, \frac{2}{2} \right) = \left( 2, 1 \right)$$

$$(g) (x_1, y_1) = (-5, -5), (x_2, y_2) = (-11, 8)$$

$$\text{So } \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{-5 + (-11)}{2}, \frac{-5 + 8}{2} \right) = \left( \frac{-16}{2}, \frac{3}{2} \right) = \left( -8, \frac{3}{2} \right)$$

$$(h) (x_1, y_1) = (6a, 4b), (x_2, y_2) = (2a, -4b)$$

$$\text{So } \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{6a + 2a}{2}, \frac{4b + (-4b)}{2} \right) = \left( \frac{8a}{2}, \frac{0}{2} \right) = \left( 4a, 0 \right)$$

$$(i) (x_1, y_1) = (2p, -q), (x_2, y_2) = (4p, 5q)$$

$$\text{So } \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{2p + 4p}{2}, \frac{-q + 5q}{2} \right) = \left( \frac{6p}{2}, \frac{4q}{2} \right) = \left( 3p, 2q \right)$$

$$(j) (x_1, y_1) = (-2s, -7t), (x_2, y_2) = (5s, t)$$

$$\text{So } \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{-2s + 5s}{2}, \frac{-7t + t}{2} \right) = \left( \frac{3s}{2}, \frac{-6t}{2} \right) = \left( \frac{3s}{2}, -3t \right)$$

$$(k) (x_1, y_1) = (-4u, 0), (x_2, y_2) = (3u, -2v)$$

$$\text{So } \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{-4u + 3u}{2}, \frac{0 + (-2v)}{2} \right) = \left( \frac{-u}{2}, -v \right)$$

$$(l) (x_1, y_1) = (a + b, 2a - b), (x_2, y_2) = (3a - b, -b)$$

$$\text{So } \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{a + b + 3a - b}{2}, \frac{2a - b + (-b)}{2} \right) = \left( \frac{4a}{2}, \frac{2a - 2b}{2} \right) = \left( 2a, a - b \right)$$

$$(m) (x_1, y_1) = (4\sqrt{2}, 1), (x_2, y_2) = (2\sqrt{2}, 7)$$

$$\text{So } \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{4\sqrt{2} + 2\sqrt{2}}{2}, \frac{1 + 7}{2} \right) = \left( \frac{6\sqrt{2}}{2}, \frac{8}{2} \right) = \left( 3\sqrt{2}, 4 \right)$$

$$(n) (x_1, y_1) = (-\sqrt{3}, 3\sqrt{5}), (x_2, y_2) = (5\sqrt{3}, 2\sqrt{5})$$

$$\text{So } \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{-\sqrt{3} + 5\sqrt{3}}{2}, \frac{3\sqrt{5} + 2\sqrt{5}}{2} \right) = \left( \frac{4\sqrt{3}}{2}, \frac{5\sqrt{5}}{2} \right) = \left( 2\sqrt{3}, \frac{5\sqrt{5}}{2} \right)$$

$$(o) (x_1, y_1) = (\sqrt{2} - \sqrt{3}, 3\sqrt{2} + 4\sqrt{3}), (x_2, y_2) = (3\sqrt{2} + \sqrt{3}, -\sqrt{2} + 2\sqrt{3})$$

$$\text{So } \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{\sqrt{2} - \sqrt{3} + 3\sqrt{2} + \sqrt{3}}{2}, \frac{3\sqrt{2} + 4\sqrt{3} + (-\sqrt{2} + 2\sqrt{3})}{2} \right)$$

$$= \left( \frac{\sqrt{2} - \sqrt{3} + 3\sqrt{2} + \sqrt{3}}{2}, \frac{3\sqrt{2} + 4\sqrt{3} - \sqrt{2} + 2\sqrt{3}}{2} \right)$$

$$= \left( \frac{4\sqrt{2}}{2}, \frac{2\sqrt{2} + 6\sqrt{3}}{2} \right)$$

$$= (2\sqrt{2}, \sqrt{2} + 3\sqrt{3})$$

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# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise A, Question 2

#### Question:

The line  $PQ$  is a diameter of a circle, where  $P$  and  $Q$  are  $(-4, 6)$  and  $(7, 8)$  respectively. Find the coordinates of the centre of the circle.

#### Solution:

$$(x_1, y_1) = (-4, 6), (x_2, y_2) = (7, 8)$$
$$\text{So } \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{-4 + 7}{2}, \frac{6 + 8}{2} \right) = \left( \frac{3}{2}, \frac{14}{2} \right) = \left( \frac{3}{2}, 7 \right)$$

$$\text{The centre is } \left( \frac{3}{2}, 7 \right).$$

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# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise A, Question 3

#### Question:

The line  $RS$  is a diameter of a circle, where  $R$  and  $S$  are  $\left(\frac{4a}{5}, -\frac{3b}{4}\right)$  and  $\left(\frac{2a}{5}, \frac{5b}{4}\right)$  respectively. Find the coordinates of the centre of the circle.

#### Solution:

$$\begin{aligned}\left(x_1, y_1\right) &= \left(\frac{4a}{5}, -\frac{3b}{4}\right), \quad \left(x_2, y_2\right) = \left(\frac{2a}{5}, \frac{5b}{4}\right) \\ \text{So } \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) &= \left(\frac{\frac{4a}{5} + \frac{2a}{5}}{2}, \frac{-\frac{3b}{4} + \frac{5b}{4}}{2}\right) = \left(\frac{\frac{6a}{5}}{2}, \frac{\frac{2b}{4}}{2}\right) = \left(\frac{3a}{5}, \frac{b}{4}\right)\end{aligned}$$

The centre is  $\left(\frac{3a}{5}, \frac{b}{4}\right)$ .

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise A, Question 4

#### Question:

The line  $AB$  is a diameter of a circle, where  $A$  and  $B$  are  $(-3, -4)$  and  $(6, 10)$  respectively. Show that the centre of the circle lies on the line  $y = 2x$ .

#### Solution:

$$(x_1, y_1) = (-3, -4), (x_2, y_2) = (6, 10)$$
$$\text{So } \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{-3 + 6}{2}, \frac{-4 + 10}{2} \right) = \left( \frac{3}{2}, \frac{6}{2} \right) = \left( \frac{3}{2}, 3 \right)$$

Substitute  $x = \frac{3}{2}$  into  $y = 2x$ :

$$y = 2 \left( \frac{3}{2} \right) = 3 \quad \checkmark$$

So the centre is on the line  $y = 2x$ .

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## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise A, Question 5

#### Question:

The line  $JK$  is a diameter of a circle, where  $J$  and  $K$  are  $\left(\frac{3}{4}, \frac{4}{3}\right)$  and  $\left(-\frac{1}{2}, 2\right)$  respectively. Show that the centre of the circle lies on the line  $y = 8x + \frac{2}{3}$ .

#### Solution:

$$\begin{aligned} \left(x_1, y_1\right) &= \left(\frac{3}{4}, \frac{4}{3}\right), \quad \left(x_2, y_2\right) = \left(-\frac{1}{2}, 2\right) \\ \text{So } \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) &= \left(\frac{\frac{3}{4} + \left(-\frac{1}{2}\right)}{2}, \frac{\frac{4}{3} + 2}{2}\right) = \left(\frac{\frac{1}{4}}{2}, \frac{\frac{10}{3}}{2}\right) = \left(\frac{1}{8}, \frac{5}{3}\right) \end{aligned}$$

Substitute  $x = \frac{1}{8}$  into  $y = 8x + \frac{2}{3}$ :

$$y = 8 \left(\frac{1}{8}\right) + \frac{2}{3} = 1 + \frac{2}{3} = \frac{5}{3} \checkmark$$

So the centre is on the line  $y = 8x + \frac{2}{3}$ .

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## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise A, Question 6

#### Question:

The line  $AB$  is a diameter of a circle, where  $A$  and  $B$  are  $(0, -2)$  and  $(6, -5)$  respectively. Show that the centre of the circle lies on the line  $x - 2y - 10 = 0$ .

#### Solution:

$$(x_1, y_1) = (0, -2), (x_2, y_2) = (6, -5)$$
$$\text{So } \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{0 + 6}{2}, \frac{-2 + (-5)}{2} \right) = \left( \frac{6}{2}, \frac{-7}{2} \right) = \left( 3, \frac{-7}{2} \right)$$

Substitute  $x = 3$  and  $y = \frac{-7}{2}$  into  $x - 2y - 10 = 0$ :

$$\left( 3 \right) - 2 \left( \frac{-7}{2} \right) - 10 = 3 + 7 - 10 = 0 \quad \checkmark$$

So the centre is on the line  $x - 2y - 10 = 0$ .



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## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise A, Question 7

#### Question:

The line  $FG$  is a diameter of the circle centre  $(6, 1)$ . Given  $F$  is  $(2, -3)$ , find the coordinates of  $G$ .

#### Solution:

$$(x_1, y_1) = (a, b), (x_2, y_2) = (2, -3)$$

The centre is  $(6, 1)$  so

$$\left( \frac{a+2}{2}, \frac{b+(-3)}{2} \right) = (6, 1)$$

$$\frac{a+2}{2} = 6$$

$$a+2 = 12$$

$$a = 10$$

$$\frac{b+(-3)}{2} = 1$$

$$\frac{b-3}{2} = 1$$

$$b-3 = 2$$

$$b = 5$$

The coordinates of  $G$  are  $(10, 5)$ .

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## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise A, Question 8

#### Question:

The line  $CD$  is a diameter of the circle centre  $(-2a, 5a)$ . Given  $D$  has coordinates  $(3a, -7a)$ , find the coordinates of  $C$ .

#### Solution:

$$(x_1, y_1) = (p, q), (x_2, y_2) = (3a, -7a)$$

The centre is  $(-2a, 5a)$  so

$$\left( \frac{p+3a}{2}, \frac{q+(-7a)}{2} \right) = \left( -2a, 5a \right)$$

$$\frac{p+3a}{2} = -2a$$

$$p+3a = -4a$$

$$p = -7a$$

$$\frac{q+(-7a)}{2} = 5a$$

$$\frac{q-7a}{2} = 5a$$

$$q-7a = 10a$$

$$q = 17a$$

The coordinates of  $C$  are  $(-7a, 17a)$ .

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## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise A, Question 9

#### Question:

The points  $M(3, p)$  and  $N(q, 4)$  lie on the circle centre  $(5, 6)$ . The line  $MN$  is a diameter of the circle. Find the value of  $p$  and  $q$ .

#### Solution:

$(x_1, y_1) = (3, p)$ ,  $(x_2, y_2) = (q, 4)$  so

$$\left( \frac{3+q}{2}, \frac{p+4}{2} \right) = (5, 6)$$

$$\frac{3+q}{2} = 5$$

$$3+q = 10$$

$$q = 7$$

$$\frac{p+4}{2} = 6$$

$$p+4 = 12$$

$$p = 8$$

So  $p = 8, q = 7$

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## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise A, Question 10

#### Question:

The points  $V(-4, 2a)$  and  $W(3b, -4)$  lie on the circle centre  $(b, 2a)$ . The line  $VW$  is a diameter of the circle. Find the value of  $a$  and  $b$ .

#### Solution:

$(x_1, y_1) = (-4, 2a)$ ,  $(x_2, y_2) = (3b, -4)$  so

$$\left( \frac{-4 + 3b}{2}, \frac{2a - 4}{2} \right) = (b, 2a)$$

$$\frac{-4 + 3b}{2} = b$$

$$-4 + 3b = 2b$$

$$-4 = -b$$

$$b = 4$$

$$\frac{2a - 4}{2} = 2a$$

$$2a - 4 = 4a$$

$$-4 = 2a$$

$$a = -2$$

So  $a = -2$ ,  $b = 4$

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## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

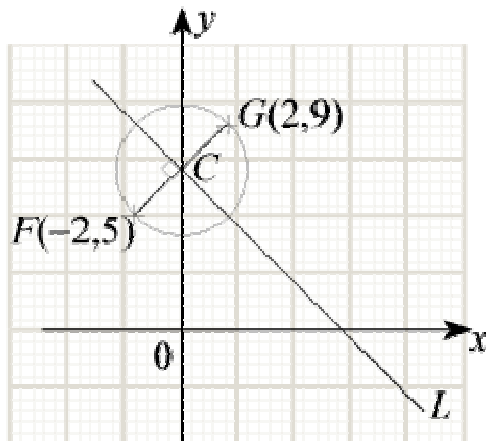
#### Exercise B, Question 1

#### Question:

The line  $FG$  is a diameter of the circle centre  $C$ , where  $F$  and  $G$  are  $(-2, 5)$  and  $(2, 9)$  respectively. The line  $l$  passes through  $C$  and is perpendicular to  $FG$ . Find the equation of  $l$ .

#### Solution:

(1)



(2) The gradient of  $FG$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 5}{2 - (-2)} = \frac{4}{4} = 1$$

(3) The gradient of a line perpendicular to  $FG$  is  $\frac{-1}{(1)} = -1$ .

(4)  $C$  is the mid-point of  $FG$ , so the coordinates of  $C$  are

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{-2 + 2}{2}, \frac{5 + 9}{2} \right) = \left( \frac{0}{2}, \frac{14}{2} \right) = (0, 7)$$

(5) The equation of  $l$  is

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 7 &= -1(x - 0) \\ y - 7 &= -x \\ y &= -x + 7 \end{aligned}$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

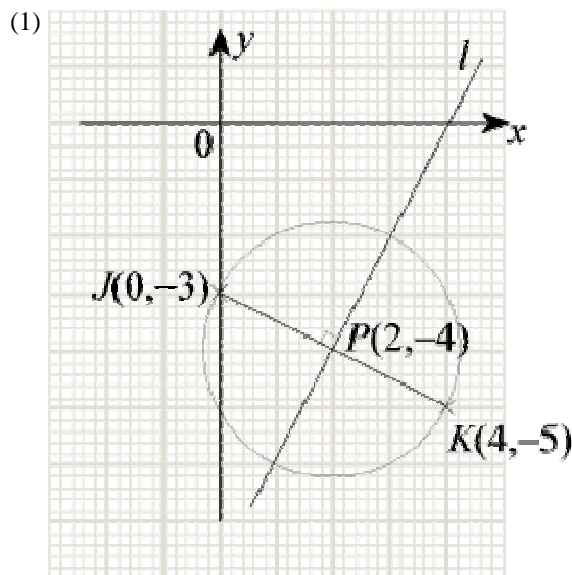
### Coordinate geometry in the (x,y) plane

#### Exercise B, Question 2

#### Question:

The line  $JK$  is a diameter of the circle centre  $P$ , where  $J$  and  $K$  are  $(0, -3)$  and  $(4, -5)$  respectively. The line  $l$  passes through  $P$  and is perpendicular to  $JK$ . Find the equation of  $l$ . Write your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers.

#### Solution:



(2) The gradient of  $JK$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - (-3)}{4 - 0} = \frac{-5 + 3}{4} = \frac{-2}{4} = \frac{-1}{2}$$

(3) The gradient of a line perpendicular to  $JK$  is  $\frac{-1}{\left(\frac{-1}{2}\right)} = 2$

(4)  $P$  is the mid-point of  $JK$ , so the coordinates of  $P$  are

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{0 + 4}{2}, \frac{-3 + (-5)}{2} \right) = \left( \frac{4}{2}, \frac{-8}{2} \right) = (2, -4)$$

(5) The equation of  $l$  is

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-4) &= 2(x - 2) \\ y + 4 &= 2x - 4 \\ 0 &= 2x - y - 4 - 4 \\ 2x - y - 8 &= 0 \end{aligned}$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise B, Question 3

#### Question:

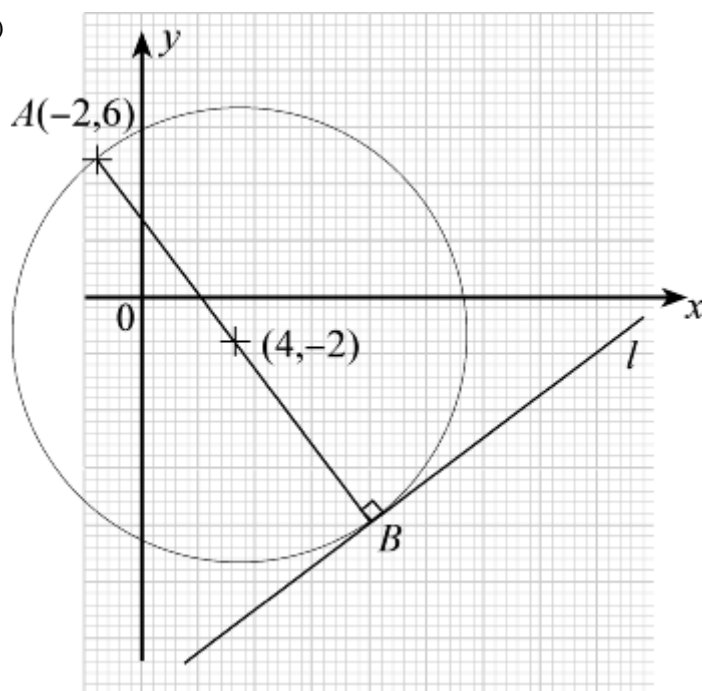
The line  $AB$  is a diameter of the circle centre  $(4, -2)$ . The line  $l$  passes through  $B$  and is perpendicular to  $AB$ . Given that  $A$  is  $(-2, 6)$ ,

(a) find the coordinates of  $B$ .

(b) Hence, find the equation of  $l$ .

#### Solution:

(1)



(2) Let the coordinates of  $B$  be  $(a, b)$ .

$(4, -2)$  is the mid-point of  $AB$  so

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = (4, -2)$$

$$\text{i.e. } \left( \frac{-2 + a}{2}, \frac{6 + b}{2} \right) = (4, -2)$$

So

$$\frac{-2 + a}{2} = 4$$

$$-2 + a = 8$$

$$a = 10$$

and

$$\frac{6 + b}{2} = -2$$

$$6 + b = -4$$

$$b = -10$$

(a) The coordinates of  $B$  are  $(10, -10)$ .

(3) Using  $(-2, 6)$  and  $(4, -2)$ , the gradient of  $AB$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 6}{4 - (-2)} = \frac{-8}{6} = \frac{-4}{3}$$

(4) The gradient of a line perpendicular to  $AB$  is  $\frac{-1}{\left(\frac{-4}{3}\right)} = \frac{3}{4}$

(5) The equation of  $l$  is

$$y - y_1 = m(x - x_1)$$

$$y - \left(-10\right) = \frac{3}{4} \left(x - 10\right)$$

$$y + 10 = \frac{3x}{4} - \frac{30}{4}$$

$$y = \frac{3x}{4} - \frac{30}{4} - 10$$

$$y = \frac{3x}{4} - \frac{70}{4}$$

$$y = \frac{3x}{4} - \frac{35}{2}$$

(b) The equation of  $l$  is  $y = \frac{3}{4}x - \frac{35}{2}$ .



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## Edexcel Modular Mathematics for AS and A-Level

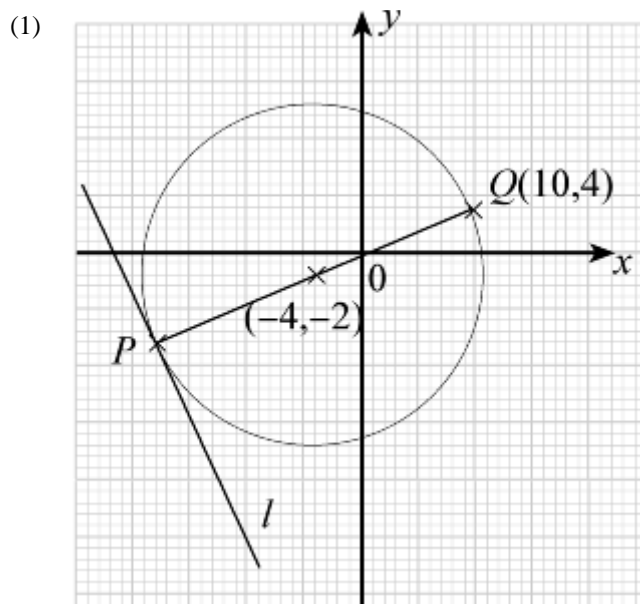
### Coordinate geometry in the (x,y) plane

#### Exercise B, Question 4

#### Question:

The line  $PQ$  is a diameter of the circle centre  $(-4, -2)$ . The line  $l$  passes through  $P$  and is perpendicular to  $PQ$ . Given that  $Q$  is  $(10, 4)$ , find the equation of  $l$ .

#### Solution:



(2) Let the coordinates of  $P$  be  $(a, b)$ .  
 $(-4, -2)$  is the mid-point of  $PQ$  so

$$\left( \frac{10+a}{2}, \frac{4+b}{2} \right) = (-4, -2)$$

$$\frac{10+a}{2} = -4$$

$$10+a = -8$$

$$a = -18$$

$$\frac{4+b}{2} = -2$$

$$4+b = -4$$

$$b = -8$$

The coordinates of  $P$  are  $(-18, -8)$ .

(3) The gradient of  $PQ$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-2)}{10 - (-4)} = \frac{6}{14} = \frac{3}{7}$$

(4) The gradient of a line perpendicular to  $PQ$  is  $\frac{-1}{\left(\frac{3}{7}\right)} = \frac{-7}{3}$ .

(5) The equation of  $l$  is

$$y - y_1 = m ( x - x_1 )$$

$$y - \begin{pmatrix} -8 \end{pmatrix} = \frac{-7}{3} \left[ x - \begin{pmatrix} -18 \end{pmatrix} \right]$$

$$y + 8 = \frac{-7}{3} \begin{pmatrix} x + 18 \end{pmatrix}$$

$$y + 8 = \frac{-7}{3} x - 42$$

$$y = \frac{-7}{3} x - 50$$

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# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise B, Question 5

#### Question:

The line  $RS$  is a chord of the circle centre  $(5, -2)$ , where  $R$  and  $S$  are  $(2, 3)$  and  $(10, 1)$  respectively. The line  $l$  is perpendicular to  $RS$  and bisects it. Show that  $l$  passes through the centre of the circle.

#### Solution:

(1) The gradient of  $RS$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 3}{10 - 2} = \frac{-2}{8} = \frac{-1}{4}$$

(2) The gradient of a line perpendicular to  $RS$  is  $\frac{-1}{\left(\frac{-1}{4}\right)} = 4$ .

(3) The mid-point of  $RS$  is

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{2 + 10}{2}, \frac{3 + 1}{2} \right) = \left( \frac{12}{2}, \frac{4}{2} \right) = (6, 2)$$

(4) The equation of  $l$  is

$$y - y_1 = m(x - x_1)$$

$$y - 2 = 4(x - 6)$$

$$y - 2 = 4x - 24$$

$$y = 4x - 22$$

(5) Substitute  $x = 5$  into  $y = 4x - 22$ :

$$y = 4(5) - 22 = 20 - 22 = -2 \checkmark$$

So  $l$  passes through the centre of the circle.

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise B, Question 6

#### Question:

The line  $MN$  is a chord of the circle centre  $\left(1, -\frac{1}{2}\right)$ , where  $M$  and  $N$  are  $(-5, -5)$  and  $(7, 4)$

respectively. The line  $l$  is perpendicular to  $MN$  and bisects it. Find the equation of  $l$ . Write your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers.

#### Solution:

(1) The gradient of  $MN$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-5)}{7 - (-5)} = \frac{4 + 5}{7 + 5} = \frac{9}{12} = \frac{3}{4}$$

(2) The gradient of a line perpendicular to  $MN$  is  $\frac{-1}{\left(\frac{3}{4}\right)} = \frac{-4}{3}$ .

(3) The coordinates of the mid-point of  $MN$  are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-5 + 7}{2}, \frac{-5 + 4}{2}\right) = \left(\frac{2}{2}, \frac{-1}{2}\right) = \left(1, \frac{-1}{2}\right)$$

(4) The equation of  $l$  is

$$y - y_1 = m(x - x_1)$$

$$y - \left(\frac{-1}{2}\right) = \frac{-4}{3} \left(x - 1\right)$$

$$y + \frac{1}{2} = \frac{-4}{3} \left(x - 1\right)$$

$$y + \frac{1}{2} = \frac{-4}{3}x + \frac{4}{3}$$

$$(\times 6)$$

$$6y + 3 = -8x + 8$$

$$8x + 6y + 3 = 8$$

$$8x + 6y - 5 = 0$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise B, Question 7

#### Question:

The lines  $AB$  and  $CD$  are chords of a circle. The line  $y = 2x + 8$  is the perpendicular bisector of  $AB$ . The line  $y = -2x - 4$  is the perpendicular bisector of  $CD$ . Find the coordinates of the centre of the circle.

#### Solution:

$$y = 2x + 8$$

$$y = -2x - 4$$

$$\underline{2y = 4}$$

$$y = 2$$

Substitute  $y = 2$  into  $y = 2x + 8$ :

$$2 = 2x + 8$$

$$-6 = 2x$$

$$x = -3$$

#### Check.

Substitute  $x = -3$  and  $y = 2$  into  $y = -2x - 4$ :

$$(2) = -2(-3) - 4$$

$$2 = 6 - 4$$

$$2 = 2 \quad \checkmark$$

The coordinates of the centre of the circle are  $(-3, 2)$ .

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise B, Question 8

#### Question:

The lines  $EF$  and  $GH$  are chords of a circle. The line  $y = 3x - 24$  is the perpendicular bisector of  $EF$ . Given  $G$  and  $F$  are  $(-2, 4)$  and  $(4, 10)$  respectively, find the coordinates of the centre of the circle.

#### Solution:

(1) The gradient of  $GF$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 4}{4 - (-2)} = \frac{6}{6} = 1$$

(2) The gradient of a line perpendicular to  $GF$  is  $-\frac{1}{(1)} = -1$ .

(3) The mid-point of  $GF$  is

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{-2 + 4}{2}, \frac{4 + 10}{2} \right) = \left( \frac{2}{2}, \frac{14}{2} \right) = (1, 7)$$

(4) The equation of the perpendicular bisector is

$$y - y_1 = m(x - x_1)$$

$$y - 7 = -1(x - 1)$$

$$y - 7 = -x + 1$$

$$y = -x + 8$$

(5) Solving  $y = -x + 8$  and  $y = 3x - 24$  simultaneously:

$$-x + 8 = 3x - 24$$

$$-4x = -32$$

$$x = \frac{-32}{-4}$$

$$x = 8$$

Substitute  $x = 8$  into  $y = -x + 8$ :

$$y = -(8) + 8$$

$$y = -8 + 8$$

$$y = 0$$

So the centre of the circle is  $(8, 0)$ .

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise B, Question 9

#### Question:

The points  $P(3, 16)$ ,  $Q(11, 12)$  and  $R(-7, 6)$  lie on the circumference of a circle.

(a) Find the equation of the perpendicular bisector of

(i)  $PQ$

(ii)  $PR$ .

(b) Hence, find the coordinates of the centre of the circle.

#### Solution:

(a) (i) The gradient  $PQ$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{12 - 16}{11 - 3} = \frac{-4}{8} = \frac{-1}{2}$$

The gradient of a line perpendicular to  $PQ$  is  $\frac{-1}{\left(\frac{-1}{2}\right)} = 2$ .

The mid-point of  $PQ$  is

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{3 + 11}{2}, \frac{16 + 12}{2} \right) = (7, 14)$$

The equation of the perpendicular bisector of  $PQ$  is

$$y - y_1 = m(x - x_1)$$

$$y - 14 = 2(x - 7)$$

$$y - 14 = 2x - 14$$

$$y = 2x$$

(ii) The gradient of  $PR$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 16}{-7 - 3} = \frac{-10}{-10} = 1$$

The gradient of a line perpendicular to  $PR$  is  $-\frac{1}{(1)} = -1$ .

The mid-point of  $PR$  is

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{3 + (-7)}{2}, \frac{16 + 6}{2} \right) = \left( \frac{3 - 7}{2}, \frac{22}{2} \right) = (-2, 11)$$

The equation of the perpendicular bisector of  $PR$  is

$$y - y_1 = m(x - x_1)$$

$$y - 11 = -1[x - (-2)]$$

$$y - 11 = -1(x + 2)$$

$$y - 11 = -x - 2$$

$$y = -x + 9$$

(b) Solving  $y = 2x$  and  $y = -x + 9$  simultaneously:

$$2x = -x + 9$$

$$3x = 9$$

$$x = 3$$

Substitute  $x = 3$  in  $y = 2x$ :

$$y = 2(3)$$

$$y = 6$$

**Check.**

Substitute  $x = 3$  and  $y = 6$  into  $y = -x + 9$ :

$$(6) = -(3) + 9$$

$$6 = -3 + 9$$

$$6 = 6 \quad \checkmark$$

The coordinates of the centre are  $(3, 6)$ .

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# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise B, Question 10

#### Question:

The points  $A(-3, 19)$ ,  $B(9, 11)$  and  $C(-15, 1)$  lie on the circumference of a circle. Find the coordinates of the centre of the circle.

#### Solution:

(1) The gradient of  $AB$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{11 - 19}{9 - (-3)} = \frac{-8}{12} = \frac{-2}{3}$$

The gradient of a line perpendicular to  $AB$  is  $\frac{-1}{\left(\frac{-2}{3}\right)} = \frac{3}{2}$ .

The mid-point of  $AB$  is

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{-3 + 9}{2}, \frac{19 + 11}{2} \right) = \left( \frac{6}{2}, \frac{30}{2} \right) = (3, 15)$$

The equation of the perpendicular bisector of  $AB$  is

$$y - y_1 = m(x - x_1)$$

$$y - 15 = \frac{3}{2} \left( x - 3 \right)$$

$$y - 15 = \frac{3}{2}x - \frac{9}{2}$$

$$y = \frac{3}{2}x + \frac{21}{2}$$

(2) The gradient of  $BC$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 11}{-15 - 9} = \frac{-10}{-24} = \frac{5}{12}$$

The gradient of a line perpendicular to  $BC$  is  $\frac{-1}{\left(\frac{5}{12}\right)} = \frac{-12}{5}$

The mid-point of  $BC$  is

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{9 + (-15)}{2}, \frac{11 + 1}{2} \right) = \left( \frac{9 - 15}{2}, \frac{11 + 1}{2} \right) = \left( \frac{-6}{2}, \frac{12}{2} \right) = (-3, 6)$$

The equation of the perpendicular bisector of  $BC$  is

$$y - y_1 = m(x - x_1)$$

$$y - 6 = \frac{-12}{5} \left[ x - \left( -3 \right) \right]$$

$$y - 6 = \frac{-12}{5} (x + 3)$$

$$y - 6 = \frac{-12}{5}x - \frac{36}{5}$$

$$y = \frac{-12}{5}x - \frac{6}{5}$$

(3) Solving  $y = \frac{-12}{5}x - \frac{6}{5}$  and  $y = \frac{3}{2}x + \frac{21}{2}$  simultaneously:

$$\frac{3}{2}x + \frac{21}{2} = \frac{-12}{5}x - \frac{6}{5}$$

$$\frac{3}{2}x + \frac{12}{5}x = \frac{-6}{5} - \frac{21}{2}$$

$$\frac{39}{10}x = -\frac{117}{10}$$

$$39x = -117$$

$$x = -3$$

Substitute  $x = -3$  into  $y = \frac{3}{2}x + \frac{21}{2}$ :

$$y = \frac{3}{2} \left( -3 \right) + \frac{21}{2}$$

$$y = \frac{-9}{2} + \frac{21}{2}$$

$$y = \frac{12}{2}$$

$$y = 6$$

**Check.**

Substitute  $x = -3$  and  $y = 6$  into  $y = \frac{-12}{5}x - \frac{6}{5}$ :

$$\left( \begin{array}{c} 6 \\ 6 \end{array} \right) = \frac{-12}{5} \left( \begin{array}{c} -3 \\ -3 \end{array} \right) - \frac{6}{5}$$

$$6 = \frac{36}{5} - \frac{6}{5}$$

$$6 = \frac{30}{5}$$

$$6 = 6 \quad \checkmark$$

The centre of the circle is  $(-3, 6)$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise C, Question 1

#### Question:

Find the distance between these pairs of points:

(a)  $(0, 1)$  ,  $(6, 9)$

(b)  $(4, -6)$  ,  $(9, 6)$

(c)  $(3, 1)$  ,  $(-1, 4)$

(d)  $(3, 5)$  ,  $(4, 7)$

(e)  $(2, 9)$  ,  $(4, 3)$

(f)  $(0, -4)$  ,  $(5, 5)$

(g)  $(-2, -7)$  ,  $(5, 1)$

(h)  $(-4a, 0)$  ,  $(3a, -2a)$

(i)  $(-b, 4b)$  ,  $(-4b, -2b)$

(j)  $(2c, c)$  ,  $(6c, 4c)$

(k)  $(-4d, d)$  ,  $(2d, -4d)$

(l)  $(-e, -e)$  ,  $(-3e, -5e)$

(m)  $(3\sqrt{2}, 6\sqrt{2})$  ,  $(2\sqrt{2}, 4\sqrt{2})$

(n)  $(-\sqrt{3}, 2\sqrt{3})$  ,  $(3\sqrt{3}, 5\sqrt{3})$

(o)  $(2\sqrt{3} - \sqrt{2}, \sqrt{5} + \sqrt{3})$  ,  $(4\sqrt{3} - \sqrt{2}, 3\sqrt{5} + \sqrt{3})$

#### Solution:

(a)  $(x_1, y_1) = (0, 1)$  ,  $(x_2, y_2) = (6, 9)$

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(6 - 0)^2 + (9 - 1)^2} \\ &= \sqrt{6^2 + 8^2} \\ &= \sqrt{36 + 64} \\ &= \sqrt{100} \\ &= 10 \end{aligned}$$

(b)  $(x_1, y_1) = (4, -6)$  ,  $(x_2, y_2) = (9, 6)$

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(9 - 4)^2 + [6 - (-6)]^2} \\ &= \sqrt{5^2 + 12^2} \\ &= \sqrt{25 + 144} \\ &= \sqrt{169} \\ &= 13 \end{aligned}$$

$$(c) (x_1, y_1) = (3, 1), (x_2, y_2) = (-1, 4)$$

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-1 - 3)^2 + (4 - 1)^2} \\ &= \sqrt{(-4)^2 + 3^2} \\ &= \sqrt{16 + 9} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

$$(d) (x_1, y_1) = (3, 5), (x_2, y_2) = (4, 7)$$

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(4 - 3)^2 + (7 - 5)^2} \\ &= \sqrt{1^2 + 2^2} \\ &= \sqrt{1 + 4} \\ &= \sqrt{5} \end{aligned}$$

$$(e) (x_1, y_1) = (2, 9), (x_2, y_2) = (4, 3)$$

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(4 - 2)^2 + (3 - 9)^2} \\ &= \sqrt{2^2 + (-6)^2} \\ &= \sqrt{4 + 36} \\ &= \sqrt{40} \\ &= \sqrt{4 \times 10} \\ &= \sqrt{4} \times \sqrt{10} \\ &= 2\sqrt{10} \end{aligned}$$

$$(f) (x_1, y_1) = (0, -4), (x_2, y_2) = (5, 5)$$

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(5 - 0)^2 + [5 - (-4)]^2} \\ &= \sqrt{5^2 + 9^2} \\ &= \sqrt{25 + 81} \\ &= \sqrt{106} \end{aligned}$$

$$(g) (x_1, y_1) = (-2, -7), (x_2, y_2) = (5, 1)$$

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[5 - (-2)]^2 + [1 - (-7)]^2} \\ &= \sqrt{(5 + 2)^2 + (1 + 7)^2} \\ &= \sqrt{7^2 + 8^2} \\ &= \sqrt{49 + 64} \\ &= \sqrt{113} \end{aligned}$$

$$(h) (x_1, y_1) = (-4a, 0), (x_2, y_2) = (3a, -2a)$$

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[3a - (-4a)]^2 + (-2a - 0)^2} \\ &= \sqrt{(3a + 4a)^2 + (-2a)^2} \\ &= \sqrt{(7a)^2 + (-2a)^2} \\ &= \sqrt{49a^2 + 4a^2} \\ &= \sqrt{53a^2} \\ &= \sqrt{53} \sqrt{a^2} \\ &= a\sqrt{53} \end{aligned}$$

$$(i) (x_1, y_1) = (-b, 4b), (x_2, y_2) = (-4b, -2b)$$

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[-4b - (-b)]^2 + (-2b - 4b)^2} \\ &= \sqrt{(-4b + b)^2 + (-6b)^2} \\ &= \sqrt{(-3b)^2 + (-6b)^2} \\ &= \sqrt{9b^2 + 36b^2} \\ &= \sqrt{45b^2} \end{aligned}$$

$$\begin{aligned}
 &= \sqrt{9 \times 5 \times b^2} \\
 &= \sqrt{9} \sqrt{5} \sqrt{b^2} \\
 &= 3b \sqrt{5}
 \end{aligned}$$

$$(j) (x_1, y_1) = (2c, c), (x_2, y_2) = (6c, 4c)$$

$$\begin{aligned}
 &\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(6c - 2c)^2 + (4c - c)^2} \\
 &= \sqrt{(4c)^2 + (3c)^2} \\
 &= \sqrt{16c^2 + 9c^2} \\
 &= \sqrt{25c^2} \\
 &= \sqrt{25} \sqrt{c^2} \\
 &= 5c
 \end{aligned}$$

$$(k) (x_1, y_1) = (-4d, d), (x_2, y_2) = (2d, -4d)$$

$$\begin{aligned}
 &\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{[2d - (-4d)]^2 + (-4d - d)^2} \\
 &= \sqrt{(2d + 4d)^2 + (-5d)^2} \\
 &= \sqrt{(6d)^2 + (-5d)^2} \\
 &= \sqrt{36d^2 + 25d^2} \\
 &= \sqrt{61d^2} \\
 &= \sqrt{61} \sqrt{d^2} \\
 &= d\sqrt{61}
 \end{aligned}$$

$$(l) (x_1, y_1) = (-e, -e), (x_2, y_2) = (-3e, -5e)$$

$$\begin{aligned}
 &\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{[-3e - (-e)]^2 + [-5e - (-e)]^2} \\
 &= \sqrt{(-3e + e)^2 + (-5e + e)^2} \\
 &= \sqrt{(-2e)^2 + (-4e)^2} \\
 &= \sqrt{4e^2 + 16e^2} \\
 &= \sqrt{20e^2} \\
 &= \sqrt{4 \times 5 \times e^2} \\
 &= \sqrt{4} \times \sqrt{5} \times \sqrt{e^2} \\
 &= 2\sqrt{5}e
 \end{aligned}$$

$$(m) (x_1, y_1) = (3\sqrt{2}, 6\sqrt{2}), (x_2, y_2) = (2\sqrt{2}, 4\sqrt{2})$$

$$\begin{aligned}
 &\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(2\sqrt{2} - 3\sqrt{2})^2 + (4\sqrt{2} - 6\sqrt{2})^2} \\
 &= \sqrt{(-\sqrt{2})^2 + (-2\sqrt{2})^2} \\
 &= \sqrt{2 + 8} \\
 &= \sqrt{10}
 \end{aligned}$$

$$(n) (x_1, y_1) = (-\sqrt{3}, 2\sqrt{3}), (x_2, y_2) = (3\sqrt{3}, 5\sqrt{3})$$

$$\begin{aligned}
 &\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{[3\sqrt{3} - (-\sqrt{3})]^2 + (5\sqrt{3} - 2\sqrt{3})^2} \\
 &= \sqrt{(3\sqrt{3} + \sqrt{3})^2 + (3\sqrt{3})^2} \\
 &= \sqrt{(4\sqrt{3})^2 + (3\sqrt{3})^2} \\
 &= \sqrt{48 + 27} \\
 &= \sqrt{75} \\
 &= \sqrt{25 \times 3} \\
 &= \sqrt{25} \times \sqrt{3} \\
 &= 5\sqrt{3}
 \end{aligned}$$

$$(o) (x_1, y_1) = (2\sqrt{3} - \sqrt{2}, \sqrt{5} + \sqrt{3}), (x_2, y_2) = (4\sqrt{3} - \sqrt{2}, 3\sqrt{5} + \sqrt{3})$$

$$\begin{aligned}
 &\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{[4\sqrt{3} - \sqrt{2} - (2\sqrt{3} - \sqrt{2})]^2 + [3\sqrt{5} + \sqrt{3} - (\sqrt{5} + \sqrt{3})]^2} \\
 &= \sqrt{(4\sqrt{3} - \sqrt{2} - 2\sqrt{3} + \sqrt{2})^2 + (3\sqrt{5} + \sqrt{3} - \sqrt{5} - \sqrt{3})^2} \\
 &= \sqrt{(2\sqrt{3})^2 + (2\sqrt{5})^2}
 \end{aligned}$$

$$\begin{aligned} &= \sqrt{12 + 20} \\ &= \sqrt{32} \\ &= \sqrt{16 \times 2} \\ &= \sqrt{16} \times \sqrt{2} \\ &= 4 \sqrt{2} \end{aligned}$$

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# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x,y) plane  
Exercise C, Question 2

**Question:**

The point  $(4, -3)$  lies on the circle centre  $(-2, 5)$ . Find the radius of the circle.

**Solution:**

$$\begin{aligned}(x_1, y_1) &= (4, -3), (x_2, y_2) = (-2, 5) \\ \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-2 - 4)^2 + [5 - (-3)]^2} \\ &= \sqrt{(-6)^2 + 8^2} \\ &= \sqrt{36 + 64} \\ &= \sqrt{100} \\ &= 10\end{aligned}$$

Radius of circle = 10.

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# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise C, Question 3

#### Question:

The point  $(14, 9)$  is the centre of the circle radius 25. Show that  $(-10, 2)$  lies on the circle.

#### Solution:

$$\begin{aligned}(x_1, y_1) &= (-10, 2), (x_2, y_2) = (14, 9) \\ \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[14 - (-10)]^2 + (9 - 2)^2} \\ &= \sqrt{24^2 + 7^2} \\ &= \sqrt{576 + 49} \\ &= \sqrt{625} \\ &= 25\end{aligned}$$

So  $(-10, 2)$  is on the circle.



# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise C, Question 4

#### Question:

The line  $MN$  is a diameter of a circle, where  $M$  and  $N$  are  $(6, -4)$  and  $(0, -2)$  respectively. Find the radius of the circle.

#### Solution:

$$\begin{aligned}
 (x_1, y_1) &= (6, -4), (x_2, y_2) = (0, -2) \\
 &\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(0 - 6)^2 + [-2 - (-4)]^2} \\
 &= \sqrt{(-6)^2 + (-2 + 4)^2} \\
 &= \sqrt{(-6)^2 + (2)^2} \\
 &= \sqrt{36 + 4} \\
 &= \sqrt{40} \\
 &= \sqrt{4 \times 10} \\
 &= \sqrt{4} \times \sqrt{10} \\
 &= 2\sqrt{10}
 \end{aligned}$$

The diameter has length  $2\sqrt{10}$ .

So the radius has length  $\frac{2\sqrt{10}}{2} = \sqrt{10}$ .

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise C, Question 5

#### Question:

The line  $QR$  is a diameter of the circle centre  $C$ , where  $Q$  and  $R$  have coordinates  $(11, 12)$  and  $(-5, 0)$  respectively. The point  $P$  is  $(13, 6)$ .

- (a) Find the coordinates of  $C$ .
- (b) Show that  $P$  lies on the circle.

#### Solution:

(a) The mid-point of  $QR$  is

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{11 + (-5)}{2}, \frac{12 + 0}{2} \right) = \left( \frac{11 - 5}{2}, \frac{12}{2} \right) = \left( \frac{6}{2}, \frac{12}{2} \right) = (3, 6)$$

(b) The radius of the circle is

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(11 - 3)^2 + (12 - 6)^2} \\ &= \sqrt{8^2 + 6^2} \\ &= \sqrt{64 + 36} \\ &= \sqrt{100} \\ &= 10 \end{aligned}$$

The distance between  $C$  and  $P$  is

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(13 - 3)^2 + (6 - 6)^2} \\ &= \sqrt{10^2 + 0^2} \\ &= \sqrt{10^2} \\ &= 10 \end{aligned}$$

So  $P$  is on the circle.

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise C, Question 6

#### Question:

The points  $(-3, 19)$ ,  $(-15, 1)$  and  $(9, 1)$  are vertices of a triangle. Show that a circle centre  $(-3, 6)$  can be drawn through the vertices of the triangle.

#### Solution:

$$(1) (x_1, y_1) = (-3, 6), (x_2, y_2) = (-3, 19)$$

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[(-3) - (-3)]^2 + (19 - 6)^2} \\ &= \sqrt{(-3 + 3)^2 + (13)^2} \\ &= \sqrt{0^2 + 13^2} \\ &= \sqrt{13^2} \\ &= 13 \end{aligned}$$

$$(2) (x_1, y_1) = (-3, 6), (x_2, y_2) = (-15, 1)$$

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[-15 - (-3)]^2 + (1 - 6)^2} \\ &= \sqrt{(-12)^2 + (-5)^2} \\ &= \sqrt{144 + 25} \\ &= \sqrt{169} \\ &= 13 \end{aligned}$$

$$(3) (x_1, y_1) = (-3, 6), (x_2, y_2) = (9, 1)$$

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[9 - (-3)]^2 + (1 - 6)^2} \\ &= \sqrt{(12)^2 + (-5)^2} \\ &= \sqrt{144 + 25} \\ &= \sqrt{169} \\ &= 13 \end{aligned}$$

The distance of each vertex of the triangle to  $(-3, 6)$  is 13. So a circle centre  $(-3, 6)$  and radius 13 can be drawn through the vertices of the triangle.

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise C, Question 7

#### Question:

The line  $ST$  is a diameter of the circle  $c_1$ , where  $S$  and  $T$  are  $(5, 3)$  and  $(-3, 7)$  respectively.

The line  $UV$  is a diameter of the circle  $c_2$  centre  $(4, 4)$ . The point  $U$  is  $(1, 8)$ .

(a) Find the radius of (i)  $c_1$  (ii)  $c_2$ .

(b) Find the distance between the centres of  $c_1$  and  $c_2$ .

#### Solution:

(a) (i) The centre of  $c_1$  is

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{5 + (-3)}{2}, \frac{3 + 7}{2} \right) = \left( \frac{2}{2}, \frac{10}{2} \right) = (1, 5)$$

The radius of  $c_1$  is

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(5 - 1)^2 + (3 - 5)^2} \\ &= \sqrt{4^2 + (-2)^2} \\ &= \sqrt{16 + 4} \\ &= \sqrt{20} \\ &= \sqrt{4 \times 5} \\ &= \sqrt{4} \times \sqrt{5} \\ &= 2\sqrt{5} \end{aligned}$$

(ii) The radius of  $c_2$  is

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(4 - 1)^2 + (4 - 8)^2} \\ &= \sqrt{3^2 + (-4)^2} \\ &= \sqrt{9 + 16} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

(b) The distance between the centres is

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(1 - 4)^2 + (5 - 4)^2} \\ &= \sqrt{(-3)^2 + (1)^2} \\ &= \sqrt{9 + 1} \\ &= \sqrt{10} \end{aligned}$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise C, Question 8

#### Question:

The points  $U(-2, 8)$ ,  $V(7, 7)$  and  $W(-3, -1)$  lie on a circle.

- (a) Show that  $\triangle UVW$  has a right angle.  
 (b) Find the coordinates of the centre of the circle.

#### Solution:

(a) (1) The distance  $UV$  is

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{|7 - (-2)|^2 + (7 - 8)^2} \\ &= \sqrt{(7 + 2)^2 + (-1)^2} \\ &= \sqrt{9^2 + (-1)^2} \\ &= \sqrt{81 + 1} \\ &= \sqrt{82} \end{aligned}$$

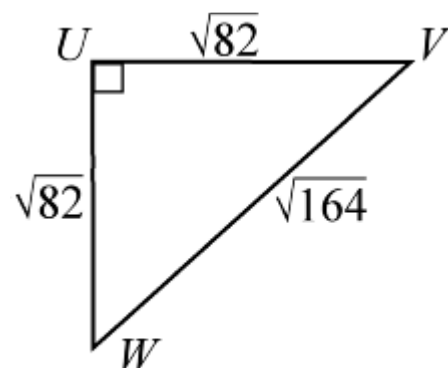
(2) The distance  $VW$  is

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-3 - 7)^2 + (-1 - 7)^2} \\ &= \sqrt{(-10)^2 + (-8)^2} \\ &= \sqrt{100 + 64} \\ &= \sqrt{164} \end{aligned}$$

(3) The distance  $UW$  is

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{|-3 - (-2)|^2 + (-1 - 8)^2} \\ &= \sqrt{(-3 + 2)^2 + (-9)^2} \\ &= \sqrt{(-1)^2 + (-9)^2} \\ &= \sqrt{1 + 81} \\ &= \sqrt{82} \end{aligned}$$

$$\text{Now } (\sqrt{82})^2 + (\sqrt{82})^2 = (\sqrt{164})^2$$



$$\text{i.e. } UV^2 + UW^2 = VW^2$$

So, by Pythagoras' theorem,  $\triangle UVW$  has a right angle at  $U$ .

- (b) The angle in a semicircle is a right angle. So  $VW$  is a diameter of the circle.  
 The mid-point of  $VW$  is

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{7 + (-3)}{2}, \frac{7 + (-1)}{2} \right) = \left( \frac{7-3}{2}, \frac{7-1}{2} \right) = (2, 3)$$

The centre of the circle is  $(2, 3)$ .

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# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise C, Question 9

#### Question:

The points  $A(2, 6)$ ,  $B(5, 7)$  and  $C(8, -2)$  lie on a circle.

(a) Show that  $\triangle ABC$  has a right angle.

(b) Find the area of the triangle.

#### Solution:

(a) (1) The distance  $AB$  is

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(5 - 2)^2 + (7 - 6)^2} \\ &= \sqrt{3^2 + 1^2} \\ &= \sqrt{9 + 1} \\ &= \sqrt{10} \end{aligned}$$

(2) The distance  $BC$  is

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(8 - 5)^2 + (-2 - 7)^2} \\ &= \sqrt{3^2 + (-9)^2} \\ &= \sqrt{9 + 81} \\ &= \sqrt{90} \end{aligned}$$

(3) The distance  $AC$  is

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(8 - 2)^2 + (-2 - 6)^2} \\ &= \sqrt{6^2 + (-8)^2} \\ &= \sqrt{36 + 64} \\ &= \sqrt{100} \end{aligned}$$

$$\text{Now } (\sqrt{10})^2 + (\sqrt{90})^2 = (\sqrt{100})^2$$

$$\text{i.e. } AB^2 + BC^2 = AC^2$$

So, by Pythagoras' theorem, there is a right angle at  $B$ .

(b) The area of the triangle is

$$\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times AB \times BC = \frac{1}{2} \sqrt{10} \sqrt{90} = \frac{1}{2} \sqrt{900} = \frac{1}{2} \times 30 = 15$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise C, Question 10

#### Question:

The points  $A(-1, 9)$ ,  $B(6, 10)$ ,  $C(7, 3)$  and  $D(0, 2)$  lie on a circle.

(a) Show that  $ABCD$  is a square.

(b) Find the area of  $ABCD$ .

(c) Find the centre of the circle.

#### Solution:

(a) (1) The length of  $AB$  is

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[6 - (-1)]^2 + (10 - 9)^2} \\ &= \sqrt{7^2 + 1^2} \\ &= \sqrt{49 + 1} \\ &= \sqrt{50} \end{aligned}$$

(2) The length of  $BC$  is

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(7 - 6)^2 + (3 - 10)^2} \\ &= \sqrt{1^2 + (-7)^2} \\ &= \sqrt{1 + 49} \\ &= \sqrt{50} \end{aligned}$$

(3) The length of  $CD$  is

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(0 - 7)^2 + (2 - 3)^2} \\ &= \sqrt{(-7)^2 + (-1)^2} \\ &= \sqrt{49 + 1} \\ &= \sqrt{50} \end{aligned}$$

(4) The length of  $DA$  is

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-1 - 0)^2 + (9 - 2)^2} \\ &= \sqrt{(-1)^2 + 7^2} \\ &= \sqrt{1 + 49} \\ &= \sqrt{50} \end{aligned}$$

The sides of the quadrilateral are equal.

(5) The gradient of  $AB$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 9}{6 - (-1)} = \frac{1}{7}$$

The gradient of  $BC$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 10}{7 - 6} = \frac{-7}{1} = -7$$

The product of the gradients =  $-1$   $\left( \frac{1}{7} \times -7 = -1 \right)$ .

So the line  $AB$  is perpendicular to  $BC$ .

So the quadrilateral  $ABCD$  is a square.

(b) The area =  $\sqrt{50} \times \sqrt{50} = 50$



(c) The mid-point of  $AC$  is

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{-1 + 7}{2}, \frac{9 + 3}{2} \right) = \left( \frac{6}{2}, \frac{12}{2} \right) = (3, 6)$$

So the centre of the circle is  $(3, 6)$ .

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# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise D, Question 1

#### Question:

Write down the equation of these circles:

- (a) Centre  $(3, 2)$ , radius 4
- (b) Centre  $(-4, 5)$ , radius 6
- (c) Centre  $(5, -6)$ , radius  $2\sqrt{3}$
- (d) Centre  $(2a, 7a)$ , radius  $5a$
- (e) Centre  $(-2\sqrt{2}, -3\sqrt{2})$ , radius 1

#### Solution:

- (a)  $(x_1, y_1) = (3, 2)$ ,  $r = 4$   
 So  $(x - 3)^2 + (y - 2)^2 = 4^2$   
 or  $(x - 3)^2 + (y - 2)^2 = 16$
- (b)  $(x_1, y_1) = (-4, 5)$ ,  $r = 6$   
 So  $[x - (-4)]^2 + (y - 5)^2 = 6^2$   
 or  $(x + 4)^2 + (y - 5)^2 = 36$
- (c)  $(x_1, y_1) = (5, -6)$ ,  $r = 2\sqrt{3}$   
 So  $(x - 5)^2 + [y - (-6)]^2 = (2\sqrt{3})^2$   
 $(x - 5)^2 + (y + 6)^2 = 2^2(\sqrt{3})^2$   
 $(x - 5)^2 + (y + 6)^2 = 4 \times 3$   
 $(x - 5)^2 + (y + 6)^2 = 12$
- (d)  $(x_1, y_1) = (2a, 7a)$ ,  $r = 5a$   
 So  $(x - 2a)^2 + (y - 7a)^2 = (5a)^2$   
 or  $(x - 2a)^2 + (y - 7a)^2 = 25a^2$
- (e)  $(x_1, y_1) = (-2\sqrt{2}, -3\sqrt{2})$ ,  $r = 1$   
 So  $[x - (-2\sqrt{2})]^2 + [y - (-3\sqrt{2})]^2 = 1^2$   
 or  $(x + 2\sqrt{2})^2 + (y + 3\sqrt{2})^2 = 1$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise D, Question 2

#### Question:

Write down the coordinates of the centre and the radius of these circles:

(a)  $(x + 5)^2 + (y - 4)^2 = 9^2$

(b)  $(x - 7)^2 + (y - 1)^2 = 16$

(c)  $(x + 4)^2 + y^2 = 25$

(d)  $(x + 4a)^2 + (y + a)^2 = 144a^2$

(e)  $(x - 3\sqrt{5})^2 + (y + \sqrt{5})^2 = 27$

#### Solution:

(a)  $(x + 5)^2 + (y - 4)^2 = 9^2$

or  $[x - (-5)]^2 + (y - 4)^2 = 9^2$

The centre of the circle is  $(-5, 4)$  and the radius is 9.

(b)  $(x - 7)^2 + (y - 1)^2 = 16$

or  $(x - 7)^2 + (y - 1)^2 = 4^2$

The centre of the circle is  $(7, 1)$  and the radius is 4.

(c)  $(x + 4)^2 + y^2 = 25$

or  $[x - (-4)]^2 + (y - 0)^2 = 5^2$

The centre of the circle is  $(-4, 0)$  and the radius is 5.

(d)  $(x + 4a)^2 + (y + a)^2 = 144a^2$

or  $[x - (-4a)]^2 + [y - (-a)]^2 = (12a)^2$

The centre of the circle is  $(-4a, -a)$  and the radius is  $12a$ .

(e)  $(x - 3\sqrt{5})^2 + (y + \sqrt{5})^2 = 27$

or  $(x - 3\sqrt{5})^2 + [y - (-\sqrt{5})]^2 = (\sqrt{27})^2$

Now  $\sqrt{27} = \sqrt{9 \times 3} = \sqrt{9} \times \sqrt{3} = 3\sqrt{3}$

The centre of the circle is  $(3\sqrt{5}, -\sqrt{5})$  and the radius is  $3\sqrt{3}$ .

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise D, Question 3

#### Question:

Find the centre and radius of these circles by first writing in the form  $(x - a)^2 + (y - b)^2 = r^2$

(a)  $x^2 + y^2 + 4x + 9y + 3 = 0$

(b)  $x^2 + y^2 + 5x - 3y - 8 = 0$

(c)  $2x^2 + 2y^2 + 8x + 15y - 1 = 0$

(d)  $2x^2 + 2y^2 - 8x + 8y + 3 = 0$

#### Solution:

(a)  $x^2 + y^2 + 4x + 9y + 3 = 0$

$$x^2 + 4x + y^2 + 9y = -3$$

$$(x + 2)^2 - 4 + \left(y + \frac{9}{2}\right)^2 - \frac{81}{4} = -3$$

$$(x + 2)^2 + \left(y + \frac{9}{2}\right)^2 = \frac{85}{4}$$

So the centre is  $(-2, -4.5)$  and the radius is 4.61 (2 d.p.)

(b)  $x^2 + y^2 + 5x - 3y - 8 = 0$

$$x^2 + 5x + y^2 - 3y = 8$$

$$\left(x + \frac{5}{2}\right)^2 - \frac{25}{4} + \left(y - \frac{3}{2}\right)^2 - \frac{9}{4} = 8$$

$$\left(x + \frac{5}{2}\right)^2 + \left(y - \frac{3}{2}\right)^2 = 16.5$$

So the centre is  $(-2.5, 1.5)$  and the radius is 4.06 (2 d.p.)

(c)  $2x^2 + 2y^2 + 8x + 15y - 1 = 0$

$$x^2 + y^2 + 4x + \frac{15}{2}y - \frac{1}{2} = 0$$

$$x^2 + 4x + y^2 + \frac{15}{2}y = \frac{1}{2}$$

$$(x + 2)^2 - 4 + \left(y + \frac{15}{4}\right)^2 - \frac{225}{16} = \frac{1}{2}$$

$$(x + 2)^2 + \left(y + \frac{15}{4}\right)^2 = 18\frac{9}{16}$$

So the centre is  $(-2, -3.75)$  and the radius is 4.31 (2 d.p.)

(d)  $2x^2 + 2y^2 - 8x + 8y + 3 = 0$

$$x^2 + y^2 - 4x + 4y + \frac{3}{2} = 0$$

$$x^2 - 4x + y^2 + 4y = -\frac{3}{2}$$

$$(x - 2)^2 - 4 + (y + 2)^2 - 4 = -\frac{3}{2}$$

$$(x - 2)^2 + (y + 2)^2 = \frac{13}{2}$$

So the centre is (2, -2) and the radius is 2.55 (2 d.p.)

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# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise D, Question 4

#### Question:

In each case, show that the circle passes through the given point:

(a)  $(x - 2)^2 + (y - 5)^2 = 13$ ,  $(4, 8)$

(b)  $(x + 7)^2 + (y - 2)^2 = 65$ ,  $(0, -2)$

(c)  $x^2 + y^2 = 25^2$ ,  $(7, -24)$

(d)  $(x - 2a)^2 + (y + 5a)^2 = 20a^2$ ,  $(6a, -3a)$

(e)  $(x - 3\sqrt{5})^2 + (y - \sqrt{5})^2 = (2\sqrt{10})^2$ ,  $(\sqrt{5}, -\sqrt{5})$

#### Solution:

(a) Substitute  $x = 4$ ,  $y = 8$  into  $(x - 2)^2 + (y - 5)^2 = 13$

$$(x - 2)^2 + (y - 5)^2 = (4 - 2)^2 + (8 - 5)^2 = 2^2 + 3^2 = 4 + 9 = 13 \quad \checkmark$$

So the circle passes through  $(4, 8)$ .

(b) Substitute  $x = 0$ ,  $y = -2$  into  $(x + 7)^2 + (y - 2)^2 = 65$

$$(x + 7)^2 + (y - 2)^2 = (0 + 7)^2 + (-2 - 2)^2 = 7^2 + (-4)^2 = 49 + 16 = 65 \quad \checkmark$$

So the circle passes through  $(0, -2)$ .

(c) Substitute  $x = 7$  and  $y = -24$  into  $x^2 + y^2 = 25^2$

$$x^2 + y^2 = 7^2 + (-24)^2 = 49 + 576 = 625 = 25^2 \quad \checkmark$$

So the circle passes through  $(7, -24)$ .

(d) Substitute  $x = 6a$ ,  $y = -3a$  into  $(x - 2a)^2 + (y + 5a)^2 = 20a^2$

$$(x - 2a)^2 + (y + 5a)^2 = (6a - 2a)^2 + (-3a + 5a)^2 = (4a)^2 + (2a)^2 = 16a^2 + 4a^2 = 20a^2 \quad \checkmark$$

So the circle passes through  $(6a, -3a)$ .

(e) Substitute  $x = \sqrt{5}$ ,  $y = -\sqrt{5}$  into  $(x - 3\sqrt{5})^2 + (y - \sqrt{5})^2 = (2\sqrt{10})^2$

$$(x - 3\sqrt{5})^2 + (y - \sqrt{5})^2 = (\sqrt{5} - 3\sqrt{5})^2 + (-\sqrt{5} - \sqrt{5})^2 = (-2\sqrt{5})^2 + (-2\sqrt{5})^2$$

$$= 4 \times 5 + 4 \times 5 = 20 + 20 = 40 = (2\sqrt{10})^2$$

$$\text{Now } \sqrt{40} = \sqrt{4 \times 10} = \sqrt{4} \times \sqrt{10} = 2\sqrt{10} \quad \checkmark$$

So the circle passes through  $(\sqrt{5}, -\sqrt{5})$ .

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise D, Question 5

#### Question:

The point  $(4, -2)$  lies on the circle centre  $(8, 1)$ . Find the equation of the circle.

#### Solution:

The radius of the circle is

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(8 - 4)^2 + [1 - (-2)]^2} \\ &= \sqrt{4^2 + 3^2} \\ &= \sqrt{16 + 9} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

The centre of the circle is  $(8, 1)$  and the radius is 5.

$$\text{So } (x - 8)^2 + (y - 1)^2 = 5^2$$

$$\text{or } (x - 8)^2 + (y - 1)^2 = 25$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise D, Question 6

#### Question:

The line  $PQ$  is the diameter of the circle, where  $P$  and  $Q$  are  $(5, 6)$  and  $(-2, 2)$  respectively. Find the equation of the circle.

#### Solution:

(1) The centre of the circle is

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{5 + (-2)}{2}, \frac{6 + 2}{2} \right) = \left( \frac{3}{2}, \frac{8}{2} \right) = \left( \frac{3}{2}, 4 \right)$$

(2) The radius of the circle is

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{\left(5 - \frac{3}{2}\right)^2 + (6 - 4)^2} \\ &= \sqrt{\left(\frac{7}{2}\right)^2 + (2)^2} \\ &= \sqrt{\frac{49}{4} + 4} \\ &= \sqrt{\frac{49}{4} + \frac{16}{4}} \\ &= \sqrt{\frac{65}{4}} \end{aligned}$$

So the equation of the circle is

$$\left(x - \frac{3}{2}\right)^2 + (y - 4)^2 = \left(\sqrt{\frac{65}{4}}\right)^2$$

$$\text{or } \left(x - \frac{3}{2}\right)^2 + (y - 4)^2 = \frac{65}{4}$$



# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise D, Question 7

#### Question:

The point  $(1, -3)$  lies on the circle  $(x - 3)^2 + (y + 4)^2 = r^2$ . Find the value of  $r$ .

#### Solution:

Substitute  $x = 1$ ,  $y = -3$  into  $(x - 3)^2 + (y + 4)^2 = r^2$

$$(1 - 3)^2 + (-3 + 4)^2 = r^2$$

$$(-2)^2 + (1)^2 = r^2$$

$$4 + 1 = r^2$$

$$5 = r^2$$

$$\text{So } r = \sqrt{5}$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise D, Question 8

#### Question:

The line  $y = 2x + 13$  touches the circle  $x^2 + (y - 3)^2 = 20$  at  $(-4, 5)$ . Show that the radius at  $(-4, 5)$  is perpendicular to the line.

#### Solution:

- (1) The centre of the circle  $x^2 + (y - 3)^2 = 20$  is  $(0, 3)$ .  
(2) The gradient of the line joining  $(0, 3)$  and  $(-4, 5)$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 3}{-4 - 0} = \frac{2}{-4} = -\frac{1}{2}$$

- (3) The gradient of  $y = 2x + 13$  is 2.  
(4) The product of the gradients is

$$-\frac{1}{2} \times 2 = -1$$

So the radius is perpendicular to the line.

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise D, Question 9

#### Question:

The line  $x + 3y - 11 = 0$  touches the circle  $(x + 1)^2 + (y + 6)^2 = 90$  at  $(2, 3)$ .

- (a) Find the radius of the circle.
- (b) Show that the radius at  $(2, 3)$  is perpendicular to the line.

#### Solution:

(a) The radius of the circle  $(x + 1)^2 + (y + 6)^2 = 90$  is  $\sqrt{90}$ .  
 $\sqrt{90} = \sqrt{9 \times 10} = \sqrt{9} \times \sqrt{10} = 3\sqrt{10}$

(b) (1) The centre of the circle  $(x + 1)^2 + (y + 6)^2 = 90$  is  $(-1, -6)$ .  
 (2) The gradient of the line joining  $(-1, -6)$  and  $(2, 3)$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - (-6)}{2 - (-1)} = \frac{3 + 6}{2 + 1} = \frac{9}{3} = 3$$

(3) Rearrange  $x + 3y - 11 = 0$  into the form  $y = mx + c$

$$x + 3y - 11 = 0$$

$$3y - 11 = -x$$

$$3y = -x + 11$$

$$y = -\frac{1}{3}x + \frac{11}{3}$$

So the gradient of  $x + 3y - 11 = 0$  is  $-\frac{1}{3}$ .

(4) The product of the gradients is

$$3 \times -\frac{1}{3} = -1$$

So the radius is perpendicular to the line.

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise D, Question 10

#### Question:

The point  $P(1, -2)$  lies on the circle centre  $(4, 6)$ .

- (a) Find the equation of the circle.  
 (b) Find the equation of the tangent to the circle at  $P$ .

#### Solution:

(a) (1) The radius of the circle is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(4 - 1)^2 + [6 - (-2)]^2} = \sqrt{3^2 + 8^2} = \sqrt{9 + 64} = \sqrt{73}$$

(2) The equation of the circle is

$$(x - 4)^2 + (y - 6)^2 = (\sqrt{73})^2$$

$$\text{or } (x - 4)^2 + (y - 6)^2 = 73$$

(b) (1) The gradient of the line joining  $(1, -2)$  and  $(4, 6)$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - (-2)}{4 - 1} = \frac{6 + 2}{3} = \frac{8}{3}$$

(2) The gradient of the tangent is  $\frac{-1}{\left(\frac{8}{3}\right)} = -\frac{3}{8}$ .

(3) The equation of the tangent to the circle at  $(1, -2)$  is

$$y - y_1 = m(x - x_1)$$

$$y - \left(-2\right) = -\frac{3}{8} \left(x - 1\right)$$

$$y + 2 = -\frac{3}{8} \left(x - 1\right)$$

$$8y + 16 = -3(x - 1)$$

$$8y + 16 = -3x + 3$$

$$3x + 8y + 16 = 3$$

$$3x + 8y + 13 = 0$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise E, Question 1

#### Question:

Find where the circle  $(x - 1)^2 + (y - 3)^2 = 45$  meets the  $x$ -axis.

#### Solution:

Substitute  $y = 0$  into  $(x - 1)^2 + (y - 3)^2 = 45$

$$(x - 1)^2 + (-3)^2 = 45$$

$$(x - 1)^2 + 9 = 45$$

$$(x - 1)^2 = 36$$

$$x - 1 = \pm \sqrt{36}$$

$$x - 1 = \pm 6$$

$$\text{So } x - 1 = 6 \Rightarrow x = 7$$

$$\text{and } x - 1 = -6 \Rightarrow x = -5$$

The circle meets the  $x$ -axis at  $(7, 0)$  and  $(-5, 0)$ .

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise E, Question 2

#### Question:

Find where the circle  $(x - 2)^2 + (y + 3)^2 = 29$  meets the y-axis.

#### Solution:

Substitute  $x = 0$  into  $(x - 2)^2 + (y + 3)^2 = 29$

$$(-2)^2 + (y + 3)^2 = 29$$

$$4 + (y + 3)^2 = 29$$

$$(y + 3)^2 = 25$$

$$y + 3 = \pm \sqrt{25}$$

$$y + 3 = \pm 5$$

$$\text{So } y + 3 = 5 \Rightarrow y = 2$$

$$\text{and } y + 3 = -5 \Rightarrow y = -8$$

The circle meets the y-axis at  $(0, 2)$  and  $(0, -8)$ .

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise E, Question 3

#### Question:

The circle  $(x - 3)^2 + (y + 3)^2 = 34$  meets the  $x$ -axis at  $(a, 0)$  and the  $y$ -axis at  $(0, b)$ . Find the possible values of  $a$  and  $b$ .

#### Solution:

(1) Substitute  $x = a, y = 0$  into  $(x - 3)^2 + (y + 3)^2 = 34$

$$(a - 3)^2 + (3)^2 = 34$$

$$(a - 3)^2 + 9 = 34$$

$$(a - 3)^2 = 25$$

$$a - 3 = \pm \sqrt{25}$$

$$a - 3 = \pm 5$$

$$\text{So } a - 3 = 5 \Rightarrow a = 8$$

$$\text{and } a - 3 = -5 \Rightarrow a = -2$$

The circle meets the  $x$ -axis at  $(8, 0)$  and  $(-2, 0)$ .

(2) Substitute  $x = 0, y = b$  into  $(x - 3)^2 + (y + 3)^2 = 34$

$$(-3)^2 + (b + 3)^2 = 34$$

$$9 + (b + 3)^2 = 34$$

$$(b + 3)^2 = 25$$

$$b + 3 = \pm \sqrt{25}$$

$$b + 3 = \pm 5$$

$$\text{So } b + 3 = 5 \Rightarrow b = 2$$

$$\text{and } b + 3 = -5 \Rightarrow b = -8$$

The circle meets the  $y$ -axis at  $(0, 2)$  and  $(0, -8)$ .

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise E, Question 4

#### Question:

The line  $y = x + 4$  meets the circle  $(x - 3)^2 + (y - 5)^2 = 34$  at  $A$  and  $B$ . Find the coordinates of  $A$  and  $B$ .

#### Solution:

Substitute  $y = x + 4$  into  $(x - 3)^2 + (y - 5)^2 = 34$

$$(x - 3)^2 + [(x + 4) - 5]^2 = 34$$

$$(x - 3)^2 + (x + 4 - 5)^2 = 34$$

$$(x - 3)^2 + (x - 1)^2 = 34$$

$$x^2 - 6x + 9 + x^2 - 2x + 1 = 34$$

$$2x^2 - 8x + 10 = 34$$

$$2x^2 - 8x - 24 = 0$$

$$x^2 - 4x - 12 = 0$$

$$(x - 6)(x + 2) = 0$$

So  $x = 6$  and  $x = -2$

Substitute  $x = 6$  into  $y = x + 4$

$$y = 6 + 4$$

$$y = 10$$

Substitute  $x = -2$  into  $y = x + 4$

$$y = -2 + 4$$

$$y = 2$$

The coordinates of  $A$  and  $B$  are  $(6, 10)$  and  $(-2, 2)$ .



# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise E, Question 5

#### Question:

Find where the line  $x + y + 5 = 0$  meets the circle  $(x + 3)^2 + (y + 5)^2 = 65$ .

#### Solution:

Rearranging  $x + y + 5 = 0$

$$y + 5 = -x$$

$$y = -x - 5$$

Substitute  $y = -x - 5$  into  $(x + 3)^2 + (y + 5)^2 = 65$

$$(x + 3)^2 + [(-x - 5) + 5]^2 = 65$$

$$(x + 3)^2 + (-x - 5 + 5)^2 = 65$$

$$(x + 3)^2 + (-x)^2 = 65$$

$$x^2 + 6x + 9 + x^2 = 65$$

$$2x^2 + 6x + 9 = 65$$

$$2x^2 + 6x - 56 = 0$$

$$x^2 + 3x - 28 = 0$$

$$(x + 7)(x - 4) = 0$$

So  $x = -7$  and  $x = 4$

Substitute  $x = -7$  into  $y = -x - 5$

$$y = -(-7) - 5$$

$$y = 7 - 5$$

$$y = 2$$

Substitute  $x = 4$  into  $y = -x - 5$

$$y = -(4) - 5$$

$$y = -4 - 5$$

$$y = -9$$

So the line meets the circle at  $(-7, 2)$  and  $(4, -9)$ .

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise E, Question 6

#### Question:

Show that the line  $y = x - 10$  does not meet the circle  $(x - 2)^2 + y^2 = 25$ .

#### Solution:

Substitute  $y = x - 10$  into  $(x - 2)^2 + y^2 = 25$

$$(x - 2)^2 + (x - 10)^2 = 25$$

$$x^2 - 4x + 4 + x^2 - 20x + 100 = 25$$

$$2x^2 - 24x + 104 = 25$$

$$2x^2 - 24x + 79 = 0$$

$$\text{Now } b^2 - 4ac = (-24)^2 - 4(2)(79) = 576 - 632 = -56$$

As  $b^2 - 4ac < 0$  then  $2x^2 - 24x + 79 = 0$  has no real roots.

So the line does not meet the circle.

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise E, Question 7

#### Question:

Show that the line  $x + y = 11$  is a tangent to the circle  $x^2 + (y - 3)^2 = 32$ .

#### Solution:

Rearranging  $x + y = 11$

$$y = 11 - x$$

Substitute  $y = 11 - x$  into  $x^2 + (y - 3)^2 = 32$

$$x^2 + [(11 - x) - 3]^2 = 32$$

$$x^2 + (11 - x - 3)^2 = 32$$

$$x^2 + (8 - x)^2 = 32$$

$$x^2 + 64 - 16x + x^2 = 32$$

$$2x^2 - 16x + 64 = 32$$

$$2x^2 - 16x + 32 = 0$$

$$x^2 - 8x + 16 = 0$$

$$(x - 4)(x - 4) = 0$$

The line meets the circle at  $x = 4$  (only).

So the line is a tangent.

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise E, Question 8

#### Question:

Show that the line  $3x - 4y + 25 = 0$  is a tangent to the circle  $x^2 + y^2 = 25$ .

#### Solution:

Rearrange  $3x - 4y + 25 = 0$

$$3x + 25 = 4y$$

$$4y = 3x + 25$$

$$y = \frac{3}{4}x + \frac{25}{4}$$

Substitute  $y = \frac{3}{4}x + \frac{25}{4}$  into  $x^2 + y^2 = 25$

$$x^2 + \left( \frac{3}{4}x + \frac{25}{4} \right)^2 = 25$$

$$x^2 + \frac{9}{16}x^2 + \frac{150}{16}x + \frac{625}{16} = 25$$

$$\frac{25}{16}x^2 + \frac{150}{16}x + \frac{225}{16} = 0$$

$$25x^2 + 150x + 225 = 0$$

$$x^2 + 6x + 9 = 0$$

$$(x + 3)(x + 3) = 0$$

The line meets the circle at  $x = -3$  (only).

So the line is a tangent.

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise E, Question 9

#### Question:

The line  $y = 2x - 2$  meets the circle  $(x - 2)^2 + (y - 2)^2 = 20$  at  $A$  and  $B$ .

- (a) Find the coordinates of  $A$  and  $B$ .
- (b) Show that  $AB$  is a diameter of the circle.

#### Solution:

(a) Substitute  $y = 2x - 2$  into  $(x - 2)^2 + (y - 2)^2 = 20$

$$(x - 2)^2 + [(2x - 2) - 2]^2 = 20$$

$$(x - 2)^2 + (2x - 4)^2 = 20$$

$$x^2 - 4x + 4 + 4x^2 - 16x + 16 = 20$$

$$5x^2 - 20x + 20 = 20$$

$$5x^2 - 20x = 0$$

$$5x(x - 4) = 0$$

So  $x = 0$  and  $x = 4$

Substitute  $x = 0$  into  $y = 2x - 2$

$$y = 2(0) - 2$$

$$y = 0 - 2$$

$$y = -2$$

Substitute  $x = 4$  into  $y = 2x - 2$

$$y = 2(4) - 2$$

$$y = 8 - 2$$

$$y = 6$$

So the coordinates of  $A$  and  $B$  are  $(0, -2)$  and  $(4, 6)$ .

(b) (1) The length of  $AB$  is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(4 - 0)^2 + [6 - (-2)]^2}$$

$$= \sqrt{4^2 + (6 + 2)^2}$$

$$= \sqrt{4^2 + 8^2}$$

$$= \sqrt{16 + 64}$$

$$= \sqrt{80}$$

$$= \sqrt{4 \times 20}$$

$$= \sqrt{4} \times \sqrt{20}$$

$$= 2\sqrt{20}$$

The radius of the circle  $(x - 2)^2 + (y - 2)^2 = 20$  is  $\sqrt{20}$ .

So the length of the chord  $AB$  is twice the length of the radius.

$AB$  is a diameter of the circle.

(2) Substitute  $x = 2$ ,  $y = 2$  into  $y = 2x - 2$

$$2 = 2(2) - 2 = 4 - 2 = 2 \quad \checkmark$$

So the line  $y = 2x - 2$  joining  $A$  and  $B$  passes through the centre  $(2, 2)$  of the circle.

So  $AB$  is a diameter of the circle.

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise E, Question 10

#### Question:

The line  $x + y = a$  meets the circle  $(x - p)^2 + (y - 6)^2 = 20$  at  $(3, 10)$ , where  $a$  and  $p$  are constants.

(a) Work out the value of  $a$ .

(b) Work out the two possible values of  $p$ .

#### Solution:

(a) Substitute  $x = 3, y = 10$  into  $x + y = a$

$$(3) + (10) = a$$

So  $a = 13$

(b) Substitute  $x = 3, y = 10$  into  $(x - p)^2 + (y - 6)^2 = 20$

$$(3 - p)^2 + (10 - 6)^2 = 20$$

$$(3 - p)^2 + 4^2 = 20$$

$$(3 - p)^2 + 16 = 20$$

$$(3 - p)^2 = 4$$

$$(3 - p) = \pm\sqrt{4}$$

$$3 - p = \pm 2$$

$$\text{So } 3 - p = 2 \Rightarrow p = 1$$

$$\text{and } 3 - p = -2 \Rightarrow p = 5$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise F, Question 1

#### Question:

The line  $y = 2x - 8$  meets the coordinate axes at  $A$  and  $B$ . The line  $AB$  is a diameter of the circle. Find the equation of the circle.

#### Solution:

Substitute  $x = 0$  into  $y = 2x - 8$

$$y = 2(0) - 8$$

$$y = -8$$

Substitute  $y = 0$  into  $y = 2x - 8$

$$0 = 2x - 8$$

$$2x = 8$$

$$x = 4$$

The line meets the coordinate axes at  $(0, -8)$  and  $(4, 0)$

The coordinates of the centre of the circle is

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{0 + 4}{2}, \frac{-8 + 0}{2} \right) = \left( \frac{4}{2}, -\frac{8}{2} \right) = (2, -4)$$

The length of the diameter is

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(4 - 0)^2 + [0 - (-8)]^2} \\ &= \sqrt{4^2 + 8^2} \\ &= \sqrt{16 + 64} \\ &= \sqrt{80} \\ &= \sqrt{16 \times 5} \\ &= \sqrt{16} \times \sqrt{5} \\ &= 4\sqrt{5} \end{aligned}$$

So the length of the radius is  $\frac{4\sqrt{5}}{2} = 2\sqrt{5}$ .

The centre of the circle is  $(2, -4)$  and the radius is  $2\sqrt{5}$ .

So the equation is

$$(x - x_1)^2 + (y - y_1)^2 = r^2$$

$$(x - 2)^2 + [y - (-4)]^2 = (2\sqrt{5})^2$$

$$(x - 2)^2 + (y + 4)^2 = 20$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise F, Question 2

#### Question:

The circle centre  $(8, 10)$  meets the  $x$ -axis at  $(4, 0)$  and  $(a, 0)$ .

(a) Find the radius of the circle.

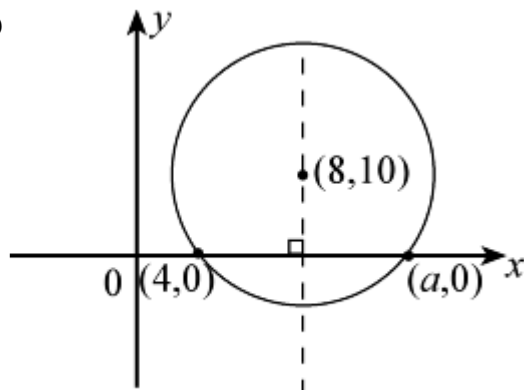
(b) Find the value of  $a$ .

#### Solution:

(a) The radius is

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(8 - 4)^2 + (10 - 0)^2} \\ &= \sqrt{4^2 + 10^2} \\ &= \sqrt{16 + 100} \\ &= \sqrt{116} \\ &= 2\sqrt{29} \end{aligned}$$

(b)



The centre is on the perpendicular bisector of  $(4, 0)$  and  $(a, 0)$ . So

$$\frac{4+a}{2} = 8$$

$$4 + a = 16$$

$$a = 12$$



# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise F, Question 3

#### Question:

The circle  $(x - 5)^2 + y^2 = 36$  meets the  $x$ -axis at  $P$  and  $Q$ . Find the coordinates of  $P$  and  $Q$ .

#### Solution:

Substitute  $y = 0$  into  $(x - 5)^2 + y^2 = 36$

$$(x - 5)^2 = 36$$

$$x - 5 = \sqrt{36}$$

$$x - 5 = \pm 6$$

$$\text{So } x - 5 = 6 \Rightarrow x = 11$$

$$\text{and } x - 5 = -6 \Rightarrow x = -1$$

The coordinates of  $P$  and  $Q$  are  $(-1, 0)$  and  $(11, 0)$ .

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# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise F, Question 4

#### Question:

The circle  $(x + 4)^2 + (y - 7)^2 = 121$  meets the y-axis at  $(0, m)$  and  $(0, n)$ . Find the value of  $m$  and  $n$ .

#### Solution:

Substitute  $x = 0$  into  $(x + 4)^2 + (y - 7)^2 = 121$

$$4^2 + (y - 7)^2 = 121$$

$$16 + (y - 7)^2 = 121$$

$$(y - 7)^2 = 105$$

$$y - 7 = \pm \sqrt{105}$$

$$\text{So } y = 7 \pm \sqrt{105}$$

The values of  $m$  and  $n$  are  $7 + \sqrt{105}$  and  $7 - \sqrt{105}$ .

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

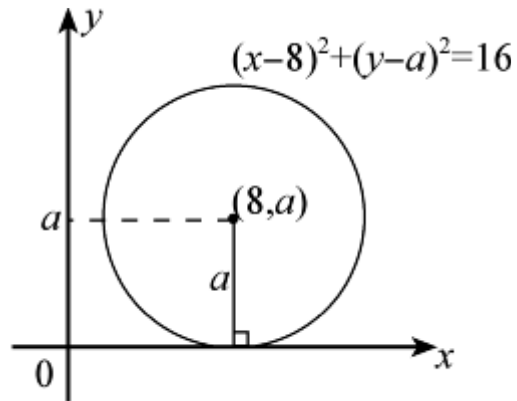
### Coordinate geometry in the (x,y) plane

#### Exercise F, Question 5

#### Question:

The line  $y = 0$  is a tangent to the circle  $(x - 8)^2 + (y - a)^2 = 16$ . Find the value of  $a$ .

#### Solution:



The radius of the circle is  $\sqrt{16} = 4$ .

So  $a = 4$

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# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise F, Question 6

#### Question:

The point  $A(-3, -7)$  lies on the circle centre  $(5, 1)$ .  
Find the equation of the tangent to the circle at  $A$ .

#### Solution:

The gradient of the line joining  $(-3, -7)$  and  $(5, 1)$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-7)}{5 - (-3)} = \frac{1+7}{5+3} = \frac{8}{8} = 1$$

So the gradient of the tangent is  $-\frac{1}{(1)} = -1$ .

The equation of the tangent is

$$y - y_1 = m(x - x_1)$$

$$y - (-7) = -1[x - (-3)]$$

$$y + 7 = -1(x + 3)$$

$$y + 7 = -x - 3$$

$$y = -x - 10 \text{ or } x + y + 10 = 0$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise F, Question 7

#### Question:

The circle  $(x + 3)^2 + (y + 8)^2 = 100$  meets the positive coordinate axes at  $A(a, 0)$  and  $B(0, b)$ .

(a) Find the value of  $a$  and  $b$ .

(b) Find the equation of the line  $AB$ .

#### Solution:

(a) Substitute  $y = 0$  into  $(x + 3)^2 + (y + 8)^2 = 100$

$$(x + 3)^2 + 8^2 = 100$$

$$(x + 3)^2 + 64 = 100$$

$$(x + 3)^2 = 36$$

$$x + 3 = \pm \sqrt{36}$$

$$x + 3 = \pm 6$$

$$\text{So } x + 3 = 6 \Rightarrow x = 3$$

$$\text{and } x + 3 = -6 \Rightarrow x = -9$$

As  $a > 0$ ,  $a = 3$ .

Substitute  $x = 0$  into  $(x + 3)^2 + (y + 8)^2 = 100$

$$3^2 + (y + 8)^2 = 100$$

$$9 + (y + 8)^2 = 100$$

$$(y + 8)^2 = 91$$

$$y + 8 = \pm \sqrt{91}$$

$$\text{So } y + 8 = \sqrt{91} \Rightarrow y = \sqrt{91} - 8$$

$$\text{and } y + 8 = -\sqrt{91} \Rightarrow y = -\sqrt{91} - 8$$

As  $b > 0$ ,  $b = \sqrt{91} - 8$ .

(b) The equation of the line joining  $(3, 0)$  and  $(0, \sqrt{91} - 8)$  is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 0}{(\sqrt{91} - 8) - 0} = \frac{x - 3}{0 - 3}$$

$$\frac{y}{\sqrt{91} - 8} = \frac{x - 3}{-3}$$

$$y = \left( \sqrt{91} - 8 \right) \times \left( \frac{x - 3}{-3} \right)$$

$$y = \left( \frac{\sqrt{91} - 8}{-3} \right) (x - 3)$$

$$y = \left( \frac{8 - \sqrt{91}}{3} \right) (x - 3)$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise F, Question 8

#### Question:

The circle  $(x + 2)^2 + (y - 5)^2 = 169$  meets the positive coordinate axes at  $C(c, 0)$  and  $D(0, d)$ .

(a) Find the value of  $c$  and  $d$ .

(b) Find the area of  $\triangle OCD$ , where  $O$  is the origin.

#### Solution:

(a) Substitute  $y = 0$  into  $(x + 2)^2 + (y - 5)^2 = 169$

$$(x + 2)^2 + (-5)^2 = 169$$

$$(x + 2)^2 + 25 = 169$$

$$(x + 2)^2 = 144$$

$$x + 2 = \pm \sqrt{144}$$

$$x + 2 = \pm 12$$

$$\text{So } x + 2 = 12 \Rightarrow x = 10$$

$$\text{and } x + 2 = -12 \Rightarrow x = -14$$

As  $c > 0$ ,  $c = 10$ .

Substitute  $x = 0$  into  $(x + 2)^2 + (y - 5)^2 = 169$

$$2^2 + (y - 5)^2 = 169$$

$$4 + (y - 5)^2 = 169$$

$$(y - 5)^2 = 165$$

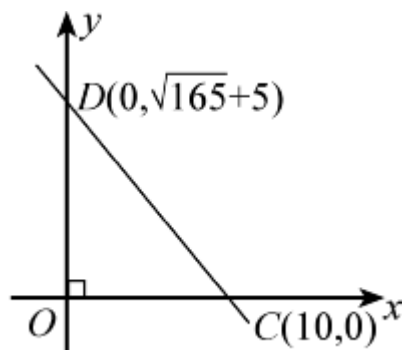
$$y - 5 = \pm \sqrt{165}$$

$$\text{So } y - 5 = \sqrt{165} \Rightarrow y = \sqrt{165} + 5$$

$$\text{and } y - 5 = -\sqrt{165} \Rightarrow y = -\sqrt{165} + 5$$

As  $d > 0$ ,  $d = \sqrt{165} + 5$ .

(b)



The area of  $\triangle OCD$  is

$$\frac{1}{2} \times 10 \times \left( \sqrt{165} + 5 \right) = 5 \left( \sqrt{165} + 5 \right)$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise F, Question 9

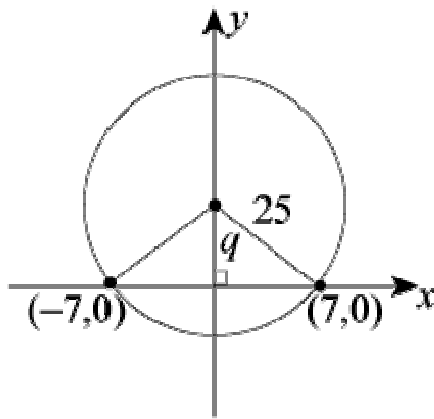
#### Question:

The circle, centre  $(p, q)$  radius 25, meets the  $x$ -axis at  $(-7, 0)$  and  $(7, 0)$ , where  $q > 0$ .

- (a) Find the value of  $p$  and  $q$ .
- (b) Find the coordinates of the points where the circle meets the  $y$ -axis.

#### Solution:

- (a) By symmetry  $p = 0$ .



Using Pythagoras' theorem

$$q^2 + 7^2 = 25^2$$

$$q^2 + 49 = 625$$

$$q^2 = 576$$

$$q = \pm \sqrt{576}$$

$$q = \pm 24$$

As  $q > 0$ ,  $q = 24$ .

- (b) The circle meets the  $y$ -axis at  $q \pm r$ ; i.e.  
 at  $24 + 25 = 49$   
 and  $24 - 25 = -1$   
 So the coordinates are  $(0, 49)$  and  $(0, -1)$ .

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise F, Question 10

#### Question:

Show that  $(0, 0)$  lies inside the circle  $(x - 5)^2 + (y + 2)^2 = 30$ .

#### Solution:

The distance between  $(0, 0)$  and  $(5, -2)$  is

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(5 - 0)^2 + (-2 - 0)^2} \\ &= \sqrt{5^2 + (-2)^2} \\ &= \sqrt{25 + 4} \\ &= \sqrt{29} \end{aligned}$$

The radius of the circle is  $\sqrt{30}$ .

As  $\sqrt{29} < \sqrt{30}$   $(0, 0)$  lies inside the circle.



# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise F, Question 11

#### Question:

The points  $A(-4, 0)$ ,  $B(4, 8)$  and  $C(6, 0)$  lie on a circle. The lines  $AB$  and  $BC$  are chords of the circle. Find the coordinates of the centre of the circle.

#### Solution:

(1) The gradient of  $AB$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 0}{4 - (-4)} = \frac{8}{4 + 4} = \frac{8}{8} = 1$$

(2) The gradient of a line perpendicular to  $AB$  is  $\frac{-1}{(1)} = -1$ .

(3) The mid-point of  $AB$  is

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{-4 + 4}{2}, \frac{0 + 8}{2} \right) = \left( \frac{0}{2}, \frac{8}{2} \right) = (0, 4)$$

(4) The equation of the perpendicular bisector of  $AB$  is

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 4 &= -1(x - 0) \\ y - 4 &= -x \\ y &= -x + 4 \end{aligned}$$

(5) The gradient of  $BC$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 8}{6 - 4} = \frac{-8}{2} = -4$$

(6) The gradient of a line perpendicular to  $BC$  is  $-\frac{1}{(-4)} = \frac{1}{4}$ .

(7) The mid-point of  $BC$  is

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{4 + 6}{2}, \frac{8 + 0}{2} \right) = \left( \frac{10}{2}, \frac{8}{2} \right) = (5, 4)$$

(8) The equation of the perpendicular bisector of  $BC$  is

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 4 &= \frac{1}{4}(x - 5) \\ y - 4 &= \frac{1}{4}x - \frac{5}{4} \\ y &= \frac{1}{4}x + \frac{11}{4} \end{aligned}$$

(9) Solving  $y = -x + 4$  and  $y = \frac{1}{4}x + \frac{11}{4}$  simultaneously

$$\frac{1}{4}x + \frac{11}{4} = -x + 4$$

$$\frac{5}{4}x + \frac{11}{4} = 4$$

$$\frac{5}{4}x = \frac{5}{4}$$

$$x = 1$$

Substitute  $x = 1$  into  $y = -x + 4$

$$y = -1 + 4$$

$$y = 3$$

So coordinates of the centre of the circle are  $(1, 3)$ .

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# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise F, Question 12

#### Question:

The points  $R(-4, 3)$ ,  $S(7, 4)$  and  $T(8, -7)$  lie on a circle.

(a) Show that  $\triangle RST$  has a right angle.

(b) Find the equation of the circle.

#### Solution:

(a) (1) The distance between  $R$  and  $S$  is

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[7 - (-4)]^2 + (4 - 3)^2} \\ &= \sqrt{(7 + 4)^2 + 1^2} \\ &= \sqrt{11^2 + 1^2} \\ &= \sqrt{121 + 1} \\ &= \sqrt{122} \end{aligned}$$

(2) The distance between  $S$  and  $T$  is

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(8 - 7)^2 + (-7 - 4)^2} \\ &= \sqrt{1^2 + (-11)^2} \\ &= \sqrt{1 + 121} \\ &= \sqrt{122} \end{aligned}$$

(3) The distance between  $R$  and  $T$  is

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[8 - (-4)]^2 + (-7 - 3)^2} \\ &= \sqrt{(8 + 4)^2 + (-10)^2} \\ &= \sqrt{12^2 + (-10)^2} \\ &= \sqrt{144 + 100} \\ &= \sqrt{244} \end{aligned}$$

By Pythagoras' theorem

$$(\sqrt{122})^2 + (\sqrt{122})^2 = (\sqrt{244})^2$$

So  $\triangle RST$  has a right angle (at  $S$ ).

(b) (1) The radius of the circle is

$$\frac{1}{2} \times \text{diameter} = \frac{1}{2} \sqrt{244} = \frac{1}{2} \sqrt{4 \times 61} = \frac{1}{2} \sqrt{4} \times \sqrt{61} = \frac{1}{2} \times 2 \sqrt{61} = \sqrt{61}$$

(2) The centre of the circle is the mid-point of  $RT$ :

$$\begin{aligned} & \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left( \frac{-4 + 8}{2}, \frac{3 + (-7)}{2} \right) = \left( \frac{4}{2}, -\frac{4}{2} \right) = \left( 2, -2 \right) \end{aligned}$$

So the equation of the circle is

$$(x - 2)^2 + (y + 2)^2 = (\sqrt{61})^2$$

$$\text{or } (x - 2)^2 + (y + 2)^2 = 61$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise F, Question 13

#### Question:

The points  $A(-7, 7)$ ,  $B(1, 9)$ ,  $C(3, 1)$  and  $D(-7, 1)$  lie on a circle. The lines  $AB$  and  $CD$  are chords of the circle.

(a) Find the equation of the perpendicular bisector of (i)  $AB$  (ii)  $CD$ .

(b) Find the coordinates of the centre of the circle.

#### Solution:

(a) (i) (1) The gradient of the line joining  $A$  and  $B$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 7}{1 - (-7)} = \frac{2}{1 + 7} = \frac{2}{8} = \frac{1}{4}$$

(2) The gradient of a line perpendicular to  $AB$  is  $-\frac{1}{m} = \frac{-1}{\left(\frac{1}{4}\right)} = -4$

(3) The mid-point of  $AB$  is

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{-7 + 1}{2}, \frac{7 + 9}{2} \right) = \left( \frac{-6}{2}, \frac{16}{2} \right) = (-3, 8)$$

(4) The equation of the perpendicular bisector of  $AB$  is

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 8 &= -4[x - (-3)] \\ y - 8 &= -4(x + 3) \\ y - 8 &= -4x - 12 \\ y &= -4x - 4 \end{aligned}$$

(ii) (1) The gradient of the line joining  $C$  and  $D$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 1}{-7 - 3} = \frac{0}{-10} = 0$$

So the line is horizontal.

(2) The mid-point of  $CD$  is  $\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{3 + (-7)}{2}, \frac{1 + 1}{2} \right) = \left( \frac{-4}{2}, \frac{2}{2} \right) = (-2, 1)$

(3) The equation of the perpendicular bisector of  $CD$  is  $x = -2$   
i.e. the vertical line through  $(-2, 1)$

(b) Solving  $y = -4x - 4$  and  $x = -2$  simultaneously,

substitute  $x = -2$  into  $y = -4x - 4$

$$y = -4(-2) - 4 = 8 - 4 = 4$$

So the centre of the circle is  $(-2, 4)$ .

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise F, Question 14

#### Question:

The centres of the circles  $(x - 8)^2 + (y - 8)^2 = 117$  and  $(x + 1)^2 + (y - 3)^2 = 106$  are  $P$  and  $Q$  respectively.

(a) Show that  $P$  lies on  $(x + 1)^2 + (y - 3)^2 = 106$ .

(b) Find the length of  $PQ$ .

#### Solution:

(a) The centre of  $(x - 8)^2 + (y - 8)^2 = 117$  is  $(8, 8)$ .

Substitute  $(8, 8)$  into  $(x + 1)^2 + (y - 3)^2 = 106$

$$(8 + 1)^2 + (8 - 3)^2 = 9^2 + 5^2 = 81 + 25 = 106 \quad \checkmark$$

So  $(8, 8)$  lies on the circle  $(x + 1)^2 + (y - 3)^2 = 106$ .

(b) As  $Q$  is the centre of the circle  $(x + 1)^2 + (y - 3)^2 = 106$  and  $P$  lies on this circle, the length  $PQ$  must equal the radius.

$$\text{So } PQ = \sqrt{106}$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise F, Question 15

#### Question:

The line  $y = -3x + 12$  meets the coordinate axes at  $A$  and  $B$ .

- Find the coordinates of  $A$  and  $B$ .
- Find the coordinates of the mid-point of  $AB$ .
- Find the equation of the circle that passes through  $A$ ,  $B$  and  $O$ , where  $O$  is the origin.

#### Solution:

(a)  $y = -3x + 12$

(1) Substitute  $x = 0$  into  $y = -3x + 12$

$$y = -3(0) + 12 = 12$$

So  $A$  is  $(0, 12)$ .

(2) Substitute  $y = 0$  into  $y = -3x + 12$

$$0 = -3x + 12$$

$$3x = 12$$

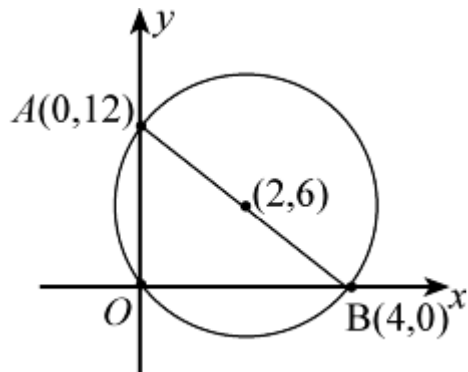
$$x = 4$$

So  $B$  is  $(4, 0)$ .

(b) The mid-point of  $AB$  is

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{0 + 4}{2}, \frac{12 + 0}{2} \right) = (2, 6)$$

(c)



$\angle AOB = 90^\circ$ , so  $AB$  is a diameter of the circle.

The centre of the circle is the mid-point of  $AB$ , i.e.  $(2, 6)$ .

The length of the diameter  $AB$  is

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(4 - 0)^2 + (0 - 12)^2} \\ &= \sqrt{4^2 + (-12)^2} \\ &= \sqrt{16 + 144} \\ &= \sqrt{160} \end{aligned}$$

So the radius of the circle is  $\frac{\sqrt{160}}{2}$ .

The equation of the circle is

$$(x - 2)^2 + (y - 6)^2 = \left( \frac{\sqrt{160}}{2} \right)^2$$

$$(x - 2)^2 + (y - 6)^2 = \frac{160}{4}$$

$$(x - 2)^2 + (y - 6)^2 = 40$$

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# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

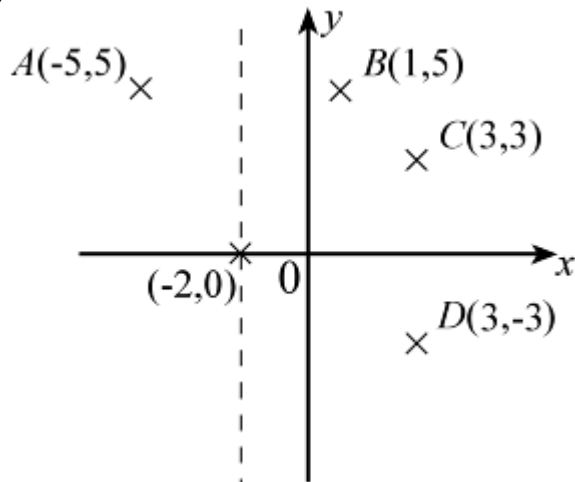
#### Exercise F, Question 16

#### Question:

The points  $A(-5, 5)$ ,  $B(1, 5)$ ,  $C(3, 3)$  and  $D(3, -3)$  lie on a circle. Find the equation of the circle.

#### Solution:

(1)



(2) The mid-point of  $AB$  is

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{-5 + 1}{2}, \frac{5 + 5}{2} \right) = \left( \frac{-4}{2}, \frac{10}{2} \right) = (-2, 5)$$

So the equation of the perpendicular bisector of  $AB$  is  $x = -2$ .

(3) The mid-point of  $CD$  is

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{3 + 3}{2}, \frac{3 + (-3)}{2} \right) = \left( \frac{6}{2}, \frac{3 - 3}{2} \right) = \left( 3, \frac{0}{2} \right) = (3, 0)$$

So the equation of the perpendicular bisector of  $CD$  is  $y = 0$ .

(4) The perpendicular bisectors intersect at  $(-2, 0)$ .

(5) The radius is the distance between  $(-2, 0)$  and  $(-5, 5)$

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[-5 - (-2)]^2 + (5 - 0)^2} \\ &= \sqrt{(-5 + 2)^2 + (5)^2} \\ &= \sqrt{(-3)^2 + (5)^2} \\ &= \sqrt{9 + 25} \\ &= \sqrt{34} \end{aligned}$$

(6) So the equation of the circle centre  $(-2, 0)$  and radius  $\sqrt{34}$  is

$$\begin{aligned} [x - (-2)]^2 + (y - 0)^2 &= (\sqrt{34})^2 \\ (x + 2)^2 + y^2 &= 34 \end{aligned}$$



# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

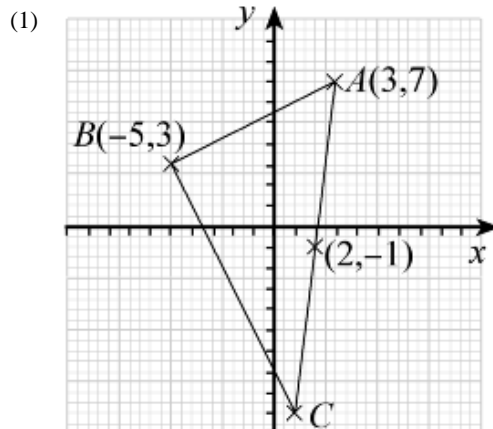
### Coordinate geometry in the (x,y) plane

#### Exercise F, Question 17

#### Question:

The line  $AB$  is a chord of a circle centre  $(2, -1)$ , where  $A$  and  $B$  are  $(3, 7)$  and  $(-5, 3)$  respectively.  $AC$  is a diameter of the circle. Find the area of  $\triangle ABC$ .

#### Solution:



(2) Let the coordinates of  $C$  be  $(p, q)$ .  
 $(2, -1)$  is the mid-point of  $(3, 7)$  and  $(p, q)$

$$\text{So } \frac{3+p}{2} = 2 \text{ and } \frac{7+q}{2} = -1$$

$$\frac{3+p}{2} = 2$$

$$3+p=4$$

$$p=1$$

$$\frac{7+q}{2} = -1$$

$$7+q = -2$$

$$q = -9$$

So the coordinates of  $C$  are  $(1, -9)$ .

(3) The length of  $AB$  is

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-5 - 3)^2 + (3 - 7)^2} \\ &= \sqrt{(-8)^2 + (-4)^2} \\ &= \sqrt{64 + 16} \\ &= \sqrt{80} \end{aligned}$$

The length of  $BC$  is

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-5 - 1)^2 + [3 - (-9)]^2} \\ &= \sqrt{(-6)^2 + (3 + 9)^2} \\ &= \sqrt{(-6)^2 + (12)^2} \\ &= \sqrt{36 + 144} \\ &= \sqrt{180} \end{aligned}$$

(4) The area of  $\triangle ABC$  is

$$\frac{1}{2} \sqrt{180} \sqrt{80} = \frac{1}{2} \sqrt{14400} = \frac{1}{2} \sqrt{144 \times 100} = \frac{1}{2} \sqrt{144} \times \sqrt{100} = \frac{1}{2} \times 12 \times 10 = 60$$

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise F, Question 18

#### Question:

The points  $A(-1, 0)$ ,  $B\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$  and  $C\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$  are the vertices of a triangle.

(a) Show that the circle  $x^2 + y^2 = 1$  passes through the vertices of the triangle.

(b) Show that  $\triangle ABC$  is equilateral.

#### Solution:

(a) (1) Substitute  $(-1, 0)$  into  $x^2 + y^2 = 1$

$$(-1)^2 + (0)^2 = 1 + 0 = 1 \quad \checkmark$$

So  $(-1, 0)$  is on the circle.

(2) Substitute  $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$  into  $x^2 + y^2 = 1$

$$\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = 1 \quad \checkmark$$

So  $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$  is on the circle.

(3) Substitute  $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$  into  $x^2 + y^2 = 1$

$$\left(\frac{1}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = 1 \quad \checkmark$$

So  $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$  is on the circle.

(b) (1) The distance between  $(-1, 0)$  and  $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$  is

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{\left[\frac{1}{2} - (-1)\right]^2 + \left(\frac{\sqrt{3}}{2} - 0\right)^2} \\ &= \sqrt{\left(\frac{1}{2} + 1\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} \\ &= \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} \\ &= \sqrt{\frac{9}{4} + \frac{3}{4}} \end{aligned}$$

$$= \sqrt{\frac{12}{4}}$$

$$= \sqrt{3}$$

(2) The distance between  $(-1, 0)$  and  $\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$  is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{\left[\frac{1}{2} - (-1)\right]^2 + \left(\frac{-\sqrt{3}}{2} - 0\right)^2}$$

$$= \sqrt{\left(\frac{1}{2} + 1\right)^2 + \left(\frac{-\sqrt{3}}{2}\right)^2}$$

$$= \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{-\sqrt{3}}{2}\right)^2}$$

$$= \sqrt{\frac{9}{4} + \frac{3}{4}}$$

$$= \sqrt{\frac{12}{4}}$$

$$= \sqrt{3}$$

(3) The distance between  $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$  and  $\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$  is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{\left(\frac{1}{2} - \frac{1}{2}\right)^2 + \left(\frac{-\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right)^2}$$

$$= \sqrt{0^2 + (-\sqrt{3})^2}$$

$$= \sqrt{0 + 3}$$

$$= \sqrt{3}$$

So  $AB$ ,  $BC$  and  $AC$  all equal  $\sqrt{3}$ .  
 $\triangle ABC$  is equilateral.

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise F, Question 19

#### Question:

The points  $P(2, 2)$ ,  $Q(2 + \sqrt{3}, 5)$  and  $R(2 - \sqrt{3}, 5)$  lie on the circle  $(x - 2)^2 + (y - 4)^2 = r^2$ .

(a) Find the value of  $r$ .

(b) Show that  $\triangle PQR$  is equilateral.

#### Solution:

(a) Substitute  $(2, 2)$  into  $(x - 2)^2 + (y - 4)^2 = r^2$

$$(2 - 2)^2 + (2 - 4)^2 = r^2$$

$$0^2 + (-2)^2 = r^2$$

$$r^2 = 4$$

$$r = 2$$

(b) (1) The distance between  $(2, 2)$  and  $(2 + \sqrt{3}, 5)$  is

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(2 + \sqrt{3} - 2)^2 + (5 - 2)^2} \\ &= \sqrt{(\sqrt{3})^2 + 3^2} \\ &= \sqrt{3 + 9} \\ &= \sqrt{12} \end{aligned}$$

(2) The distance between  $(2, 2)$  and  $(2 - \sqrt{3}, 5)$  is

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(2 - \sqrt{3} - 2)^2 + (5 - 2)^2} \\ &= \sqrt{(-\sqrt{3})^2 + (3)^2} \\ &= \sqrt{3 + 9} \\ &= \sqrt{12} \end{aligned}$$

(3) The distance between  $(2 + \sqrt{3}, 5)$  and  $(2 - \sqrt{3}, 5)$  is

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[(2 - \sqrt{3}) - (2 + \sqrt{3})]^2 + (5 - 5)^2} \\ &= \sqrt{(2 - \sqrt{3} - 2 - \sqrt{3})^2 + 0^2} \\ &= \sqrt{(-2\sqrt{3})^2} \\ &= \sqrt{(-2)^2 \times (\sqrt{3})^2} \\ &= \sqrt{4 \times 3} \\ &= \sqrt{12} \end{aligned}$$

So  $PQ$ ,  $QR$  and  $PR$  all equal  $\sqrt{12}$ .

$\triangle PQR$  is equilateral.

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

### Coordinate geometry in the (x,y) plane

#### Exercise F, Question 20

#### Question:

The points  $A(-3, -2)$ ,  $B(-6, 0)$  and  $C(p, q)$  lie on a circle centre  $\left(-\frac{5}{2}, 2\right)$ . The line  $BC$  is a diameter of the circle.

- (a) Find the value of  $p$  and  $q$ .
- (b) Find the gradient of (i)  $AB$  (ii)  $AC$ .
- (c) Show that  $AB$  is perpendicular to  $AC$ .

#### Solution:

(a) The mid-point of  $(-6, 0)$  and  $(p, q)$  is  $\left(-\frac{5}{2}, 2\right)$ .

$$\text{So } \left(\frac{-6+p}{2}, \frac{0+q}{2}\right) = \left(-\frac{5}{2}, 2\right)$$

$$\frac{-6+p}{2} = -\frac{5}{2}$$

$$-6+p = -5$$

$$p = -5 + 6$$

$$p = 1$$

$$\frac{0+q}{2} = 2$$

$$\frac{q}{2} = 2$$

$$q = 4$$

(b) (i) The gradient of the line joining  $(-3, -2)$  and  $(-6, 0)$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - (-2)}{-6 - (-3)} = \frac{2}{-6+3} = \frac{2}{-3} = -\frac{2}{3}$$

(ii) The gradient of the line joining  $(-3, -2)$  and  $(1, 4)$  is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-2)}{1 - (-3)} = \frac{4+2}{1+3} = \frac{6}{4} = \frac{3}{2}$$

(c) Two lines are perpendicular if  $m_1 \times m_2 = -1$ .

$$\text{Now } -\frac{2}{3} \times \frac{3}{2} = -1 \quad \checkmark$$

So  $AB$  is perpendicular to  $AC$ .